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ROYAL SOCIETY OPEN SCIENCE

Self-induced class stratification in competitive societies of agents: Nash stability in the presence of envy

Claudius Gros

Note: Reports are unedited and appear as submitted by the referee. The review history appears in chronological order.

Review History

RSOS-200411.R0 (Original submission)

Review form: Reviewer 1

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Do you have any ethical concerns with this paper?
No

Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Accept as is

Comments to the Author(s)
The manuscript "Self induced class stratification in competitive societies of agents: Nash stability in the presence of envy." by author Gros presents a model on the dynamical effects of envy in social dynamics and organisation that is interesting and to my knowledge new and therefore should be published.

It represents envy in a game theoretical setting of a so-called shopping trouble model in a delightful and creative way and observes a self-organised stratification of the toy model society as a function of sensible parameters. The observed ordering effects from this physics of complex adaptive system's perspective are interesting and the implicit claim of practical relevance via universality makes this a prime system for further discussion in the field of sociophysics. I like the model much. My personal caveat is that the model definition eq. 2.2 is very well suited for physicists --- but perhaps not so much as a minimalistic toy model for less mathematically inclined, and therefore will probably not turn viral in the sociology community right away. I do not have any ad hoc suggestion, though, other than encouraging the author to think about more realistic, yet at the same time simpler payoff functions or scenarios in a second paper.

This paper should be published as is (after correcting the mixed up variables alpha and epsilon in caption 2) and will definitely motivate interesting follow-up research.

Review form: Reviewer 2

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Do you have any ethical concerns with this paper?
No

Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Major revision is needed (please make suggestions in comments)

Comments to the Author(s)
Summary of the paper:

The author studies increasing levels of envy in a game-theoretic setting and shows via numerical simulations that this in itself causes a separation of society into a lower and an upper class. Further findings include that class stratification is Nash stable and strict when members of the same class receive the same rewards; upper-class members play only pure strategies, whereas lower-class members play the same mixed strategy; the fraction of upper-class members decreases progressively with larger levels of envy until just a single upper class agent is left. The performed numerical simulations are based on a reference model, the so-called "shopping trouble model." The author also discusses the impact of his findings for human societies.

Recommendation:

The findings of this paper, which by and large is well written (but see also the listed points of criticism and the minor suggestions for improvement below), are certainly interesting and novel and deserve publication. My main point of criticism is that the paper seems to be written by an expert for other experts only. It is not well accessible for a wider audience. Therefore, I recommend to accept this paper subject to a major revision.

Some points of criticism:

Many of the notions and concepts used come without explanation or definition, so for someone working in a different special area it is really hard to understand the results of this paper. For example, it should first be said what specific type of game-theoretic model is used and how it is formally defined, including utility functions and their domain and range.

The notion of envy is informally explained on page 2 but should also be formally defined.

The same applies to the notions of Nash state, Nash stability, and so on. For example, in the setting of hedonic games (or, more generally, coalition formation games), Nash stability means that no player prefers moving to another coalition. But since we here are in a non-cooperative game (I guess), I assume that Nash stability means something else here. However, I cannot understand what it really means if no formal definition is given.

Regarding the (shopping trouble) model explained on pages 3--4, in particular through the displayed formulas (2.1), (2.2), and (2.3), it is unclear whether the author introduces this model here of whether it has been already proposed in the literature and he merely keeps studying it. Also, it would be nice to compare this specific model with other (perhaps related, perhaps similar) models occurring in the literature and to explain what the main advantages of the present model are.

Minor suggestions for improvement:

page 1, abstract:
"into a lower- and an upper class": Delete the hyphen.

page 1, bottom:
In the last line, after "selecting the same option are" there is some empty space that should be filled.

page 2, line 19:
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It is unclear whether you consider apples, banana and kiwis as
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independently whether => independently of whether

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"jackpot": A jackpot typically contains everything, i.e., the
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by "second best option".

page 3, line 52:
Fix "support of a strategies".

page 4, line 6:
where have defined => where we have defined

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a functionality that is equivalently =>
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page 8, line 28:
in contrast to the lower- and upper class rewards =>
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page 9, line 52:
Delete the comma at the end of Eq. (3.5).

page 10, line 48:
This sentence reads better if you shift "numerically" to the end of the
sentence. In the next sentence, "Shown is" looks also a bit misplaced.

Decision letter (RSOS-200411.R0)

We hope you are keeping well at this difficult and unusual time. We continue to value your support of the journal in these challenging circumstances. If Royal Society Open Science can assist you at all, please don't hesitate to let us know at the email address below.

Dear Dr Gros

On behalf of the Editors, I am pleased to inform you that your Manuscript RSOS-200411 entitled "Self induced class stratification in competitive societies of agents: Nash stability in the presence of envy" has been accepted for publication in Royal Society Open Science subject to minor revision in accordance with the referee suggestions. Please find the referees' comments at the end of this email.

The reviewers and handling editors have recommended publication, but also suggest some minor revisions to your manuscript. Therefore, I invite you to respond to the comments and revise your manuscript.

• Ethics statement
If your study uses humans or animals please include details of the ethical approval received, including the name of the committee that granted approval. For human studies please also detail whether informed consent was obtained. For field studies on animals please include details of all permissions, licences and/or approvals granted to carry out the fieldwork.

• Data accessibility
It is a condition of publication that all supporting data are made available either as supplementary information or preferably in a suitable permanent repository. The data accessibility section should state where the article's supporting data can be accessed. This section

should also include details, where possible of where to access other relevant research materials such as statistical tools, protocols, software etc can be accessed. If the data has been deposited in an external repository this section should list the database, accession number and link to the DOI for all data from the article that has been made publicly available. Data sets that have been deposited in an external repository and have a DOI should also be appropriately cited in the manuscript and included in the reference list.

If you wish to submit your supporting data or code to Dryad (http://datadryad.org/), or modify your current submission to dryad, please use the following link: http://datadryad.org/submit?journalID=RSOS&manu=RSOS-200411

• Competing interests
Please declare any financial or non-financial competing interests, or state that you have no competing interests.

• Authors' contributions
All submissions, other than those with a single author, must include an Authors' Contributions section which individually lists the specific contribution of each author. The list of Authors should meet all of the following criteria; 1) substantial contributions to conception and design, or acquisition of data, or analysis and interpretation of data; 2) drafting the article or revising it critically for important intellectual content; and 3) final approval of the version to be published.

All contributors who do not meet all of these criteria should be included in the acknowledgements.

We suggest the following format:
AB carried out the molecular lab work, participated in data analysis, carried out sequence alignments, participated in the design of the study and drafted the manuscript; CD carried out the statistical analyses; EF collected field data; GH conceived of the study, designed the study, coordinated the study and helped draft the manuscript. All authors gave final approval for publication.

• Acknowledgements
Please acknowledge anyone who contributed to the study but did not meet the authorship criteria.

• Funding statement
Please list the source of funding for each author.

Please ensure you have prepared your revision in accordance with the guidance at https://royalsociety.org/journals/authors/author-guidelines/ -- please note that we cannot publish your manuscript without the end statements. We have included a screenshot example of the end statements for reference. If you feel that a given heading is not relevant to your paper, please nevertheless include the heading and explicitly state that it is not relevant to your work.

Because the schedule for publication is very tight, it is a condition of publication that you submit the revised version of your manuscript before 28-May-2020. Please note that the revision deadline will expire at 00.00am on this date. If you do not think you will be able to meet this date please let me know immediately.

To revise your manuscript, log into https://mc.manuscriptcentral.com/rsos and enter your Author Centre, where you will find your manuscript title listed under "Manuscripts with Decisions". Under "Actions," click on "Create a Revision." You will be unable to make your revisions on the originally submitted version of the manuscript. Instead, revise your manuscript and upload a new version through your Author Centre.

When submitting your revised manuscript, you will be able to respond to the comments made by the referees and upload a file "Response to Referees" in "Section 6 - File Upload". You can use this to document any changes you make to the original manuscript. In order to expedite the processing of the revised manuscript, please be as specific as possible in your response to the referees. We strongly recommend uploading two versions of your revised manuscript:

1) Identifying all the changes that have been made (for instance, in coloured highlight, in bold text, or tracked changes);
2) A 'clean' version of the new manuscript that incorporates the changes made, but does not highlight them.

When uploading your revised files please make sure that you have:

1) A text file of the manuscript (tex, txt, rtf, docx or doc), references, tables (including captions) and figure captions. Do not upload a PDF as your "Main Document";
2) A separate electronic file of each figure (EPS or print-quality PDF preferred (either format should be produced directly from original creation package), or original software format);
3) Included a 100 word media summary of your paper when requested at submission. Please ensure you have entered correct contact details (email, institution and telephone) in your user account;
4) Included the raw data to support the claims made in your paper. You can either include your data as electronic supplementary material or upload to a repository and include the relevant doi within your manuscript. Make sure it is clear in your data accessibility statement how the data can be accessed;
5) All supplementary materials accompanying an accepted article will be treated as in their final form. Note that the Royal Society will neither edit nor typeset supplementary material and it will be hosted as provided. Please ensure that the supplementary material includes the paper details where possible (authors, article title, journal name).

Supplementary files will be published alongside the paper on the journal website and posted on the online figshare repository (https://rs.figshare.com/). The heading and legend provided for each supplementary file during the submission process will be used to create the figshare page, so please ensure these are accurate and informative so that your files can be found in searches. Files on figshare will be made available approximately one week before the accompanying article so that the supplementary material can be attributed a unique DOI.

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Once again, thank you for submitting your manuscript to Royal Society Open Science and I look forward to receiving your revision. If you have any questions at all, please do not hesitate to get in touch.

Kind regards,
Anita Kristiansen
Editorial Coordinator

Royal Society Open Science
openscience@royalsociety.org

on behalf of Professor Tim Rogers (Associate Editor) and Mark Chaplain (Subject Editor)
openscience@royalsociety.org

Associate Editor Comments to Author (Professor Tim Rogers):

Comments to the Author:

Please implement the changes recommended by both referees. In particular, care should be taken to ensure the manuscript is self contained and all necessary terms are precisely defined, or suitable references given.

Reviewer comments to Author:
Reviewer: 1

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Reviewer: 2

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page 1, bottom:
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or indivisible goods, like apples, banana and kiwis":
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What does "f.i." mean? It is not very common.

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accumulation of unwon rewards. So I don't know what you mean
by "second best option".

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Fix "support of a strategies".

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where have defined => where we have defined

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This sentence reads better if you shift "numerically" to the end of the
sentence. In the next sentence, "Shown is" looks also a bit misplaced.

Author's Response to Decision Letter for (RSOS-200411.R0)

See Appendix A.

Decision letter (RSOS-200411.R1)

We hope you are keeping well at this difficult and unusual time. We continue to value your support of the journal in these challenging circumstances. If Royal Society Open Science can assist you at all, please don't hesitate to let us know at the email address below.

Dear Dr Gros,

It is a pleasure to accept your manuscript entitled "Self induced class stratification in competitive societies of agents: Nash stability in the presence of envy" in its current form for publication in Royal Society Open Science.

You can expect to receive a proof of your article in the near future. Please contact the editorial office (openscience_proofs@royalsociety.org) and the production office (openscience@royalsociety.org) to let us know if you are likely to be away from e-mail contact -- if you are going to be away, please nominate a co-author (if available) to manage the proofing process, and ensure they are copied into your email to the journal.

Due to rapid publication and an extremely tight schedule, if comments are not received, your paper may experience a delay in publication. Royal Society Open Science operates under a continuous publication model. Your article will be published straight into the next open issue and

this will be the final version of the paper. As such, it can be cited immediately by other researchers. As the issue version of your paper will be the only version to be published I would advise you to check your proofs thoroughly as changes cannot be made once the paper is published.

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Thank you for your fine contribution. On behalf of the Editors of Royal Society Open Science, we look forward to your continued contributions to the Journal.

Kind regards,
Andrew Dunn
Royal Society Open Science Editorial Office
Royal Society Open Science
openscience@royalsociety.org

on behalf of Professor Tim Rogers (Associate Editor) and Mark Chaplain (Subject Editor)
openscience@royalsociety.org

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Appendix A

Response to the comments of Reviewer 1

This paper should be published as is (after correcting the mixed up variables
alpha and epsilon in caption 2) and will definitely motivate interesting
follow-up research.

We thank the reviewer for her/his support and pointing out
that alpha and epsilon have been mixed in the caption
Fig. 2, which we corrected

Response to the comments of Reviewer 2

We thank the reviewer for their support and for
the valuable suggestions, which we implemented in
full.

Many of the notions and concepts used come without explanation or definition,
so for someone working in a different special area it is really hard to
understand the results of this paper. For example, it should first be said
what specific type of game-theoretic model is used and how it is formally
defined, including utility functions and their domain and range.
The notion of envy is informally explained on page 2 but should also be
formally defined.
The same applies to the notions of Nash state, Nash stability, and so on. For
example, in the setting of hedonic games (or, more generally, coalition
formation games), Nash stability means that no player prefers moving to another
coalition. But since we here are in a non-cooperative game (I guess), I assume
that Nash stability means something else here. However, I cannot understand
what it really means if no formal definition is given.

We introduced a new section, 4.5 Terminology, in which we present,
as we hope, an exhaustive list of formal definitions.

Regarding the (shopping trouble) model explained on pages 3--4, in particular
through the displayed formulas (2.1), (2.2), and (2.3), it is unclear whether
the author introduces this model here of whether it has been already proposed
in the literature and he merely keeps studying it. Also, it would be nice to
compare this specific model with other (perhaps related, perhaps similar)
models occurring in the literature and to explain what the main advantages of
the present model are.

True, this was never stated explicitly. In Sect. 2 we now point
out that the shopping trouble model is introduced in this paper, see
``The bare formulation of the here introduced shopping trouble game, ...''

We also introduced a new paragraph somewhat below Eq.(2.2), starting
with
``The utility function (2.2) of the shopping trouble model'',
where the relation of our model with status seeking games is
discussed.

Minor suggestions for improvement:
Thank you very much for these suggestions, which we implemented.

In the last line, after "selecting the same option are" there is some
empty space that should be filled.

This is actually a particularity of the Royal Society format,
which will be taken into account by the production editor.

"the problem of allocating multiple types of divisible
or indivisible goods, like apples, banana and kiwis":
It is unclear whether you consider apples, banana and kiwis as
divisible or indivisible goods.
True, reformulated.

What does "f.i." mean? It is not very common.

For instance. Has been written out in the revised version.

"jackpot": A jackpot typically contains everything, i.e., the
accumulation of unwon rewards. So I don't know what you mean
by "second best option".

This expression was, admittedly, too colloquial. The sentence
in question has bee rewritten.

Fix "support of a strategies".

Done.

where have defined => where we have defined

Done.

a functionality that is equivalently =>
a functionality that is equivalent

Actually, we believe that equivalently is fine here.

"we present to this extend": Do you mean "to this end" or "to this extent"?

The first, corrected.

in contrast to the lower- and upper class rewards =>
in contrast to the lower- and upper-class rewards

Done.

Delete the comma at the end of Eq. (3.5).

Done.

This sentence reads better if you shift "numerically" to the end of the
sentence. In the next sentence, "Shown is" looks also a bit misplaced.

Corrected / reformulated. Thanks.