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Anomalous hydrodynamics kicks neutron stars

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ABSTRACT

Observations show that, at the beginning of their existence, neutron stars are accelerated briskly to velocities of up to a thousand kilometers per second. We argue that this remarkable effect can be explained as a manifestation of quantum anomalies on astrophysical scales. To theoretically describe the early stage in the life of neutron stars we use hydrodynamics as a systematic effective-field-theory framework. Within this framework, anomalies of the Standard Model of particle physics as underlying microscopic theory imply the presence of a particular set of transport terms, whose form is completely fixed by theoretical consistency. The resulting chiral transport effects in proto-neutron stars enhance neutrino emission along the internal magnetic field, and the recoil can explain the order of magnitude of the observed kick velocities.

1. Introduction

Proto-neutron stars are observed to receive kicks, i.e. a large change of momentum along a particular axis early in their evolution [1]. We spell out a new mechanism to explain these kicks, based on the modern formulation of hydrodynamics in presence of quantum anomalies. It allows us to give an analytic and systematic explanation for asymmetric emission of matter out of the neutron star, and the resulting recoil can explain the kick. On a qualitative level, the kicks have been linked to asymmetric neutrino emission already in [2–4]. However, to link the study of anomalies as microscopic effect to a macroscopic phenomenon like the neutron star kicks, one has to face the fact that the asymmetry may not survive thermal washout [5,6]. Working with hydrodynamics as effective-field-theory description from the outset avoids this issue, and allows us to work within one consistent framework throughout. The crucial ingredients are anomalies, i.e. the breaking of classical conservation laws by quantum effects. The presence of such anomalies in a microscopic theory is a robust feature that persists in effective-field-theory descriptions [7]. Consequently, anomalies were found to have striking implications in the hydrodynamic regime [8–12], where one might otherwise expect them to simply be washed out. The effects of the resulting anomalous transport phenomena have been studied extensively on microscopic length scales, e.g. in heavy-ion-collisions [13–16]. Previous attempts to link astrophysical observations and specifically neutron star kicks to anomalous transport turned out to be problematic [17–19]. In this work we will specifically study the early stage of the neutron star evolution, when neutrinos have a short mean free path and can not be isolated from the electrons. In contrast to [17–19], we will therefore focus on the lepton number current, which combines both and is actually conserved at the classical level. In that environment there are sizable anomalous transport effects in the presence of moderately large magnetic fields and, as we will see, they also affect the neutrinos. When the neutrinos are transported to the edge of the proto-neutron star and leave it, they carry away a large enough momentum component asymmetrically. The resulting mechanism is illustrated in Fig. 1. To estimate the kick we will build on existing numerical simulations for the full evolution of the neutron star and just compute the corrections due to anomalous transport effects, which turns out to be justified as we will argue in the main part.

The rest of this note is organized as follows. We start by introducing the hydrodynamic framework to fix notation. We then discuss the specific currents which receive large contributions from

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anomalous transport effects in a typical neutron star, and show how they can propel it to the observed velocities. We then close with a brief discussion.

2. Anomalous hydrodynamics

In recent years, hydrodynamics has been reinterpreted and developed systematically in effective field theory language [20,21]. The effective degrees of freedom are the classically conserved currents, and their hydrodynamic description is valid on length scales much larger than the mean free paths. One striking result of this program is that anomalies of the underlying microscopic quantum field theory cause macroscopic transport effects [8–10]. Any system which is described microscopically by a relativistic quantum field theory receives the following contributions to a current corresponding to a global symmetry (at first order in the hydrodynamic expansion in gradients) [10]

$$J_a^\mu = n_a u^\mu + \sigma_a{}^b V_b^\mu + \sigma_a^V \omega^\mu + \sigma_{ab}^B B^{b\,\mu} + \mathcal{O}(\partial^2)\,, \qquad (1)$$

where a, b label the currents in the theory, n_a is the net charge density, u^μ is the fluid velocity, $\sigma_a{}^b$ is the conductivity and $V_a^\mu = (E_a^\mu - T(\eta^{\mu\nu} + u^\mu u^\nu)\partial_\nu \frac{\mu_a}{T})$ with the field strength E_a. Furthermore, we have the temperature T, flat metric $\eta^{\mu\nu}$ and the chemical potential μ_a which is thermodynamically conjugate to n_a. The two remaining terms in equation (1) contain the chiral vortical coefficient σ_a^V, the vorticity $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu \partial_\rho u_\sigma$, the chiral magnetic coefficient σ_{ab}^B, and $B_b^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{b\,\mu\nu}$ with the field strength tensor F_b (for the electromagnetic $U(1)_{\rm em}$ this is the familiar magnetic field). The remarkable feature is that, using the standard hydrodynamic restriction of positivity of the local entropy production, the transport coefficients σ_a^V and σ_{ab}^B can be computed exactly from the anomalies of the underlying theory and thermodynamic quantities [10,22]. The explicit expressions depend on the chosen frame. In the fixed laboratory frame of [23], we have

$$\sigma_a^V = \frac{1}{2}C_{abc}\mu^b\mu^c - \beta_a T^2\,, \qquad \sigma_{ab}^B = C_{abc}\mu^c\,. \qquad (2)$$

The C_{abc} are the coefficients characterizing the anomalous conservation laws, $\langle \partial_\mu J_a^\mu \rangle = \frac{1}{8}C_{abc}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^b F_{\rho\sigma}^c$. In perturbative calculations, anomalies arise from triangle diagrams involving the three currents $j_{a/b/c}$. The diagrams are generally not anomalous when all currents are vector-like (V), but can be for diagrams with axial-vector (A) contributions of the form VVA or AAA. The T^2-term in (2) encodes the chiral transport effect at zero chemical potential, and is linked to mixed gauge-gravitational anomalies [24, 25]. When dynamical gauge fields contribute to the anomalies, the transport coefficients are not protected from renormalization and may receive additional contributions [26,27]. The explicit expressions for σ^V, σ^B in (2) will allow us to compute the magnitude and direction of chiral transport contributions in a proto-neutron star below.

3. Currents and anomalies

We now introduce the relevant currents and discuss their anomalies. For the neutron star kicks we will be interested in the leptonic currents, and to keep the discussion clear we make a number of simplifying assumptions: Since neutrino masses are very small compared to typical temperatures and chemical potentials in a neutron star, we will ignore them. The various lepton flavors are then conserved separately at the classical level. The relevant leptons for our purposes are electrons and electron neutrinos, since these are the flavors mostly produced in the relevant electroweak processes [28,29]. For our purposes lepton number therefore means electron number. The electron and neutrino currents alone are not conserved at the classical level due to the weak interactions, and in the highly interactive environment of a proto-neutron star they can not be separated. For the hydrodynamic description we therefore have to consider the classically conserved lepton number current combining both. At typical neutron star temperatures of $\mathcal{O}(10\text{ MeV})$, sphaleron processes are suppressed [30] and we will not take them into account. The electron mass is small compared to typical temperatures and chemical potentials as well, and it is tempting to just work with massless electrons. Despite being small, the electron mass was found to have drastic implications for the asymmetry between left-handed and right-handed electrons generated during the formation of a neutron star in [31]. We will assume here that the left-handed and right-handed lepton number currents, $J_{\ell L}$ and $J_{\ell R}$ are separately conserved within each local equilibration region to a good enough accuracy to be part of the hydrodynamic description. The holographic study in [32,33] has shown that the anomalous transport effects present for conserved currents persist when the conservation is slightly violated, and they were even enhanced in certain cases. We leave the question of whether or not a large electron asymmetry is generated during the formation of the neutron star open and consider both scenarios when we discuss anomalous transport in the next section.

To discuss the anomalies we will use the linear combinations $J_{\ell L} \pm J_{\ell R}$ and call them J_ℓ and $J_{\ell 5}$, respectively. The electron part in these currents is vector/axial vector like, while the neutrino part is purely left-handed for both. The charges under the respective symmetries $U(1)_{\ell/\ell 5}$ are 1 for all fields except for the right-handed electrons, which have -1 w.r.t. $U(1)_{\ell 5}$. Since neither of J_ℓ and $J_{\ell 5}$ are purely vector like, we get a rather large number of different anomalous triangle diagrams. To begin with, both symmetries have a $U(1)_a^3$ anomaly, yielding non-vanishing coefficients C_{aaa} with $a = \ell, \ell 5$. We also get non-vanishing $C_{\ell,\ell,\ell 5}$ and $C_{\ell,\ell 5,\ell 5}$. These will be relevant for the chiral effects due to the vorticity only. We also have mixed anomalies with the electromagnetic gauge field. Since the $U(1)_{\rm em}$ is vector like, we get these from VVA diagrams. In these diagrams the neutrinos do not contribute since they are not charged under $U(1)_{\rm em}$, and $J_{\ell/\ell 5}$ therefore actually behave vector/axial vector like. We get two non-vanishing anomaly contributions corresponding to $C_{{\rm em},\ell,\ell 5}$ and $C_{{\rm em},{\rm em},\ell 5}$. Computing the actual numerical values of all these coefficients is straightforward, and we do not need to list them here.

4. Chiral transport in proto-neutron stars

After the discussion of the general framework and the relevant currents above, we now focus on anomalous transport of leptons in the bulk of a neutron star. Electrons and neutrinos appear together in the classically conserved currents J_ℓ and $J_{\ell 5}$, and anomalous transport, if present, thus affects both. With the non-vanishing anomaly coefficients given in the previous section, we see that this indirectly communicates the presence of a magnetic field also to the neutrinos, despite the fact that they are not charged with respect to the electromagnetic U(1). The transparency properties of the crust will be discussed below.

To estimate the relative strength of the two anomalous transport effects in the neutron star we take a look at its vorticity. The star can be modeled as a rigidly rotating disk of radius $r_{\rm N}$, with vorticity $\omega = -2\Omega$ where Ω is the angular velocity. With $\Omega = 2\pi/\text{ms}$ as a ballpark figure [34], we then find $\omega \approx 10^{-17}$ MeV. The magnetic fields, on the other hand, can easily take values of $10^{12}\text{ G} \approx 0.1\text{ MeV}^2$ [35]. The dimensionful quantities entering the coefficients (2) are all of $\mathcal{O}(\text{MeV})$, and we thus expect the chiral effects due to the magnetic field to be dominant by many orders of magnitude.

For an order-of-magnitude estimate of the coefficient for the chiral effect due to vorticity we use [28,29]

$$\epsilon/n_\ell = \mu^\ell = 300\text{ MeV}\,, \qquad T = 10\text{ MeV}\,, \tag{3}$$

along with $\epsilon = 3P$. This yields $\sigma_\ell^V \approx (10^3 C_x - 10^2\beta)$ MeV^2, with an $\mathcal{O}(1)$ coefficient C_x parametrizing the contribution from U(1) anomalies and the second term representing the temperature-dependent contributions. The coefficient β, which includes the gravitational contributions, enters at essentially the same order of magnitude as the pure U(1) anomalies parametrized by C_x.

We now turn to the chiral effects due to the magnetic field, which we discuss in more detail. The coefficients we are interested in are $\sigma^B_{a,\mathrm{em}}$ with $a = \ell, \ell 5$, such that B is the magnetic field of $\mathrm{U(1)_{em}}$. The explicit form is $\sigma^B_{a,\mathrm{em}} = C_{a,\mathrm{em},c}\mu^c$. We see that at least one of the external fields in the triangle diagrams computing the C_{abc} is the electromagnetic gauge field. As explained above, in that case the only non-vanishing coefficients are $C_{\mathrm{em},\ell,\ell 5}$ and $C_{\mathrm{em,em},\ell 5}$. Assuming that the neutron star is neutral to a good approximation, we ignore the contribution due to the latter. With $C = C_{\mathrm{em},\ell,\ell 5} = 1/(2\pi^2)$ the explicit form of the coefficients becomes

$$\sigma^B_{\ell,\mathrm{em}} = C\mu^{\ell 5}, \qquad \sigma^B_{\ell 5,\mathrm{em}} = C\mu^\ell\,. \tag{4}$$

To estimate the resulting currents we use (3) for the values of μ^ℓ and n_ℓ, and for the corresponding values for $J_{\ell 5}$ we discuss the two cases corresponding to whether the electron mass can be ignored or not, as explained above. The first case is $\mu^{\ell 5} \approx 0$. Noting that $n_{\ell 5} = n_{e_\mathrm{L}} - n_{e_\mathrm{R}} + n_\nu$, this describes the case where electron chirality is preserved: the electroweak interactions may generate a large chiral asymmetry for electrons, but the combined number of left-handed electrons and neutrinos is conserved. The range of magnetic fields observed in neutron stars is rather wide, and spans several orders of magnitude [35]. For our quantitative estimate, we assume that inside the neutron star the effective magnetic field is approximately uniform. With the intermediate value $B = 0.1$ MeV^2 and (3) we find

$$\vec{J}_\ell \approx 0\,, \qquad \vec{J}_{\ell 5} = C\mu^\ell \vec{B} \approx \vec{e}_B \cdot 1\text{ MeV}^3\,. \tag{5}$$

The effect is illustrated in Fig. 1(b): $J_\ell \approx 0$ means that there are equal lepton currents moving parallel and antiparallel to the magnetic field. From the non-vanishing $J_{\ell 5}$ we conclude that left-handed and right-handed lepton currents are moving in opposite directions.

We now come to the second scenario, where the chiral asymmetry of the electrons is washed out during the formation of the neutron star [31]. In that case we do get a non-vanishing $\mu^{\ell 5}$ from the excess of left-handed particles due to the neutrinos. The number of left-handed and right-handed leptons in Fig. 1(b) then is not equal, resulting in a non-vanishing J_ℓ. The left-handed current $(J_\ell + J_{\ell 5})/2$, however, changes only by an $\mathcal{O}(1)$ factor compared to the previous scenario, and the same applies for its composition in terms of electrons and neutrinos. The crucial point for us is that only the neutrinos will be able to leave the neutron star, and the number of excess neutrinos moving along the magnetic field changes only by an $\mathcal{O}(1)$ factor. The order-of-magnitude estimate of the kick in the next section is therefore not affected. In either of these two scenarios, the resulting neutrino flux is linear in the uniform magnetic field.[1]

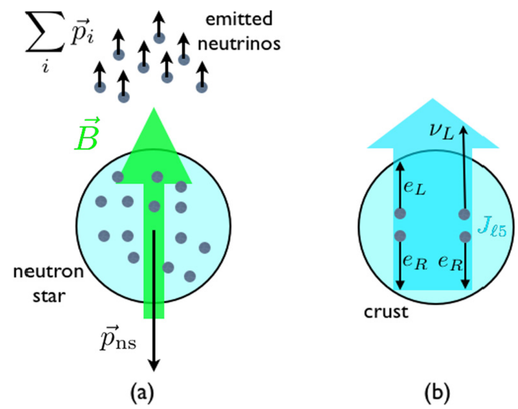

Fig. 1. Neutrinos are emitted from a proto-neutron star through chiral transport effects parallel to the magnetic field $\vec{B}$. (a) The general mechanism: each neutrino carries away the momentum $\vec{p}_i$, producing a recoil $\vec{p}_{\mathrm{ns}} = -\sum_i \vec{p}_i$ on the neutron star. (b) Illustration of the currents: $J_\ell \approx 0$ but sizable $J_{\ell 5}$ means that left-handed lepton number flows opposite to the right-handed one. Only the (left-handed) neutrinos can escape through the crust.

5. Kicks from chiral transport in proto-neutron stars

With the precise form of the transport coefficients and an estimate for the resulting currents, we can now estimate whether and how efficiently the resulting currents can accelerate the neutron star. In typical scenarios, the crust of a neutron star is transparent only to neutrinos, which are thus the only particles emitted, as illustrated in Fig. 1. This produces a recoil on the neutron star, which we estimate as follows.

To get the number of neutrinos leaving the neutron star, we compute the neutrino flux $\dot{N}_\nu = |\vec{J}|A$, with the area of the rotating disk $A = \pi r_\mathrm{N}^2$ and $r_\mathrm{N} = 10$ km. For the current we take the value for $J_{\ell 5}$ given in (5). As order-of-magnitude estimate we augment it by a factor of 1/2, to account for the fact that only the neutrinos can leave the neutron star. This is compatible with the number densities given in [28,29]. Converting to SI units, we find $\dot{N}_\nu \approx 10^{54}$/s, evaluated at the temperature and chemical potential stated in equation (3). Even for neutrinos the mean free path is only on the order of one meter, see e.g. [36], and the hydrodynamic approximation is valid. To compute the corresponding momentum current we could use the chiral transport coefficients for the energy–momentum tensor, as given in [23]. For the sake of simplicity, however, we just use the Fermi momentum as average momentum per neutrino, $\langle p_\nu\rangle \approx \mu^\ell$. This reproduces the same result. We note that the relevant scale for the anomalous transport is set by the chemical potential, which is more than an order of magnitude larger than the temperature. For the momentum of the neutron star after the kick we then get $\Delta P_{\mathrm{NS}} = \Delta t \dot{N}_\nu \langle p_\nu\rangle$, where $\Delta t \approx 10$ s is the time span we assume for the kick to last. With a neutron star mass of $m_{\mathrm{NS}} = 3 \cdot 10^{30}$ kg this yields

$$\Delta v \approx 10^3\text{ km/s}\,. \tag{6}$$

We thus find that the sudden momentum gains can indeed be explained by rapid neutrino emission due to the chiral transport effects, resulting in the simple picture shown in Fig. 1.

Before coming to the discussion of our results, we want to put the effect we obtained into perspective, by comparing the asym-

[1] For a non-uniformly distributed magnetic field, we expect various types of corrections. Within the hydrodynamic approximation there are higher order gradient corrections (e.g. gradients of the magnetic field). Within the hydrodynamic regime, various effects could also lead to a decreased value for the effective magnetic field or the effective area through which the currents flow, e.g. through confinement of field lines to flux tubes. A more severe systematic change would arise if large gradients of the electromagnetic vector potential would lead to the breakdown of the hydrodynamic approximation, in which case our estimates do not apply.

metric neutrino emission to the total number of neutrinos emitted by a typical neutron star. A ballpark figure for the latter is $\Delta N_\nu \approx 10^{58}$ [37]. Integrating our emission rate of $\dot{N}_\nu = 10^{54}$/s over 10 s, on the other hand, only gives about 10^{55}. So the fraction of neutrinos emitted asymmetrically due to the anomalous transport effects is very small. More detailed studies of time-dependent neutrino luminosities confirm this picture for the first order 10 s in the central region of the neutron star [38], which is the part relevant for our analysis. This suggests that for processes which do not rely specifically on the direction of emission, like cooling of the star, the anomalous transport effects produce only small corrections and ignoring them is a good approximation. It also shows that the additional emission is unlikely to significantly alter properties like chemical potentials. This a posteriori justifies our decoupling of the anomalous transport effects from the more complicated analysis of the full neutron star evolution.

6. Discussion

We have estimated anomalous transport effects in proto-neutron stars in a systematic hydrodynamic framework. A short mean free path here is a necessary ingredient for the hydrodynamic description to be valid. In the highly interactive early stage of a proto-neutron star, one has to describe electrons and neutrinos by the classically conserved lepton-number current containing both. This way the neutrinos are sensitive to anomalous transport effects through anomaly diagrams with photons, even though they are not charged themselves. The electrons are crucial in the bulk of the neutron star and are only filtered out at the crust, which leaves the neutrinos to escape and kick the neutron star. There are two independent effects, one causing neutrino emission along the axis of rotation, and the other one causing emission along the magnetic field of the proto-neutron star. The latter turns out to be dominant by many orders of magnitude, and the neutrino recoil can indeed accelerate a typical proto-neutron star to velocities of order 10^3 km/s, in agreement with observations. The scale of the anomalous transport effects is set by the chemical potential, which is an order of magnitude larger than the temperature.

The precise form of the transport terms also allows for phenomenological conclusions. On a qualitative level, we expect the kick to be aligned with the internal magnetic field of the neutron star. If the magnetic field is not aligned with the angular momentum and rotating quickly along with the star, the kick will be proportional to the net magnetic field along the axis of rotation. This can allow to distinguish our scenario e.g. from the mechanism of [39,40], which is insensitive to the neutron star spin. More quantitatively, we find a precise relation between the properties of the neutron star and the strength and direction of the kick. The chiral effect due to the vorticity in principle offers access to mixed gravitational anomalies, which result in a quadratic temperature dependence. For typical neutron stars the effect is out-shined by the chiral effects due to the magnetic field, but there may be situations where this is different [41].

Our estimates show that chiral transport can lead to sizable effects at least in certain phases of supernova explosions. Recent work investigating the effects of chiral transport on core collapse supernovae confirms this assertion [42,43]. For the future it would be interesting to incorporate anomalous transport effects directly into numerical models for the dynamical evolution of proto-neutron stars, possibly utilizing magnetohydrodynamics as suggested in [44,45].

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