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Dynamically Generated Inflationary ΛCDM

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Abstract: Our primary objective is to construct a plausible, unified model of inflation, dark energy and dark matter from a fundamental Lagrangian action first principle, wherein all fundamental ingredients are systematically dynamically generated starting from a very simple model of modified gravity interacting with a single scalar field employing the formalism of non-Riemannian spacetime volume-elements. The non-Riemannian volume element in the initial scalar field action leads to a hidden, nonlinear Noether symmetry which produces an energy-momentum tensor identified as the sum of a dynamically generated cosmological constant and dust-like dark matter. The non-Riemannian volume-element in the initial Einstein–Hilbert action upon passage to the physical Einstein-frame creates, dynamically, a second scalar field with a non-trivial inflationary potential and with an additional interaction with the dynamically generated dark matter. The resulting Einstein-frame action describes a fully dynamically generated inflationary model coupled to dark matter. Numerical results for observables such as the scalar power spectral index and the tensor-to-scalar ratio conform to the latest 2018 *PLANCK* data.

Keywords: inflation; dark energy; dark matter

1. Introduction

In the last decade or so, a groundbreaking concept emerged regarding the intrinsic necessity to modify (extend) gravity theories beyond the framework of the norm—Einstein's general relativity. The main motivation for these developments was to overcome the limitations of the latter coming from: (i) cosmology—for solving the problems of dark energy and dark matter and explaining the large scale structure of the Universe [1–3]; (ii) quantum field theory in curved spacetime—because of the non-renormalizabilty of ultraviolet divergences in higher loops [4–9]; (iii) modern string theory—because of the natural appearance of higher-order curvature invariants and scalar-tensor couplings in low-energy effective field theories [10–14].

Another parallel crucial development is the emergence of the theoretical framework based on the concept of "inflation," which is a necessary part of the standard model of cosmology, since it provides the solution to the fundamental puzzles of the old Big Bang theory, such as the horizon, the flatness and the monopole problems [15–22]. It can be achieved through various mechanisms; for instance, through the introduction of primordial scalar field(s) [23–76], or through correction terms into the modified gravitational action [77–122].

Additionally, inflation was proved crucial in providing a framework for the generation of primordial density perturbations [123,124]. Since these perturbations affect the cosmic background radiation (CMB), the inflationary effect on observations can be investigated through the prediction of the scalar spectral index of the curvature perturbations and its running, for the tensor spectral index and for the tensor-to-scalar ratio.

Various classes of modified gravity theories have been employed to construct viable inflationary models—$f(R)$-gravity; scalar-tensor gravity; Gauss–Bonnet gravity (see [125,126] for a detailed account)—and recently, models have been based on non-local gravity ([127] and references therein) or based on brane-world scenarios ([128] and references therein). The first successful cosmological model based on the extended $f(R) = R + R^2$-gravity produces the classical Starobinsky inflationary scalar field potential [16].

Dynamically generated models of inflation from modified/extended gravity such as the Starobinsky model [18,126,129,130] still remain viable and produce some of the best fits to existing observational data compared to other inflationary models [131].

Unification of inflation with dark energy and dark matter have been widely discussed [79,81,132–142]. It is indeed challenging to describe both phases of acceleration using a single scalar field minimally couple to gravity, without affecting the thermal history of the universe, which has been verified to a good degree of accuracy. In order to enable slow-roll behavior, the scalar field potential should exhibit shallow behavior early on, followed by a steep region for most of the universe's history, and shallow behavior once again for later times. Although a simple exponential potential does not comply with the above picture, here we present a simple modified gravity model naturally providing a dynamically generated scalar potential, whose inflationary dynamics are compatible with the recent observational data. On the other hand, the task of describing particle creation will be discussed in our future work.

Another specific but broad class of modified (extended) gravitational theories is based on the formalism of *non-Riemannian spacetime volume-elements*. It was originally proposed in [143–147], and a subsequent, concise geometric formulation was proposed in in [148–150]. This formalism was used as a basis for constructing a series of extended gravity-matter models describing unified dark energy and dark matter scenarios [151,152]; quintessential cosmological models with gravity-assisted and inflaton-assisted dynamical suppression (in the "early" universe) or dynamical generation (in the post-inflationary universe) of electroweak spontaneous symmetry breaking and charge confinement [153,154]; and a novel mechanism for the dynamical supersymmetric Brout–Englert–Higgs effect in supergravity [148].

In the present paper our principal aim is to construct a plausible unified model, i.e., describing (most of the) principal physical manifestations of a unification of inflation and dark energy interacting with dark matter, where the formalism of the non-Riemannian spacetime volume-elements will play a fundamental role. To this end, we will consider a simple modified gravity interacting with a single scalar field where the Einstein–Hilbert part and the scalar field part of the action are constructed within the formalism of the non-Riemannian volume-elements—alternatives to the canonical Riemannian one $\sqrt{-g}$. The non-Riemannian volume element in the initial scalar field action leads to a hidden, nonlinear Noether symmetry which produces an energy-momentum tensor identified as the sum of a dynamically generated cosmological constant and dynamically generated dust-like dark matter. The non-Riemannian volume-element in the initial Einstein–Hilbert action, upon passage to the physical Einstein-frame, creates, dynamically, a second scalar field with a non-trivial inflationary potential and with an additional interaction with the dynamically generated dark matter. The resulting Einstein-frame action describes a fully dynamically generated, unified model of inflation, dark energy and dark matter. Numerical results for observables such as the scalar power spectral index and the tensor-to-scalar ratio conform to the latest 2018 *PLANCK* data.

Let us briefly recall the essence of the non-Riemannian volume-form (volume-element) formalism. In integrals over differentiable manifolds (not necessarily a Riemannian one, so *no* metric is needed) volume-forms are given by non-singular maximal rank differential forms ω:

$$\int_{\mathcal{M}} \omega(\ldots) = \int_{\mathcal{M}} dx^D\, \Omega(\ldots)\;, \tag{1}$$

where

$$\omega = \frac{1}{D!}\omega_{\mu_1\ldots\mu_D} dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_D} \quad , \quad \omega_{\mu_1\ldots\mu_D} = -\varepsilon_{\mu_1\ldots\mu_D}\Omega\;. \tag{2}$$

Our conventions for the alternating symbols $\varepsilon^{\mu_1,\ldots,\mu_D}$ and $\varepsilon_{\mu_1,\ldots,\mu_D}$ are: $\varepsilon^{01\ldots D-1}=1$ and $\varepsilon_{01\ldots D-1}=-1$).

The volume element Ω transforms as scalar density under general coordinate reparametizations.

In Riemannian D-dimensional spacetime manifolds, a standard, generally-covariant volume-form is defined through the "D-bein" (frame-bundle) canonical one-forms, $e^A = e^A_\mu dx^\mu$ ($A=0,\ldots,D-1$):

$$\omega = e^0\wedge\ldots\wedge e^{D-1} = \det\|e^A_\mu\|\, dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_D}, \tag{3}$$

yields:

$$\Omega = \det\|e^A_\mu\| = \sqrt{-\det\|g_{\mu\nu}\|}\;. \tag{4}$$

To construct modified gravitational theories as alternatives to ordinary standard theories in Einstein's general relativity, instead of $\sqrt{-g}$ we can employ one or more alternative *non-Riemannian* volume element(s) as in (1) given by non-singular *exact* D-forms $\omega = dA$ where:

$$A = \frac{1}{(D-1)!}A_{\mu_1\ldots\mu_{D-1}} dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_{-1}} \tag{5}$$

so that the *non-Riemannian* volume element reads:

$$\Omega \equiv \Phi(A) = \frac{1}{(D-1)!}\varepsilon^{\mu_1\ldots\mu_D}\,\partial_{\mu_1}A_{\mu_2\ldots\mu_D}\;. \tag{6}$$

Thus, a non-Riemannian volume element is defined in terms of the (scalar density of the) dual field-strength of an auxiliary rank $D-1$ tensor gauge field $A_{\mu_1\ldots\mu_{D-1}}$.

The modified gravity Lagrangain actions based on the non-Riemannian volume-elements' formalism are of the following generic form (here and in what follows we will use units with $16\pi G_{\text{Newton}}=1$):

$$S = \int d^4x\,\Phi_1(B)\big(R+\mathcal{L}_1\big) + \int d^4x\,\Phi_0(A)\mathcal{L}_0 + \int d^4x\,\sqrt{-g}\,\mathcal{L}_2\;, \tag{7}$$

where $\Phi_0(A)$ and $\Phi_1(B)$ are of the form (6) (for $D=4$), R is the scalar curvature, and the Lagrangian densities $\mathcal{L}_{0,1,2}$ contain the matter fields (and possibly higher curvature terms; e.g., R^2).

A basic property of the class of actions (7) is that the equations of motion with respect to auxiliary gauge fields, defining the non-Riemannian volume-elements $\Phi_0(A)$ and $\Phi_1(B)$ as in (6), produce dynamically generated *free integration constants* M_1, M_0:

$$\begin{aligned} \partial_\mu\big(R+\mathcal{L}_1\big) = 0 \;\to\; R+\mathcal{L}_1 = -M_1 \\ \partial_\mu\mathcal{L}_0 = 0 \;\to\; \mathcal{L}_0 = -2M_0\;, \end{aligned} \tag{8}$$

(cf. Equations (15) and (28) below) whose appearance will play an instrumental role in the sequel.

Further, let us stress the following important characteristic feature of the modified gravity-matter actions (7). When considering the gravity part in the first order (Palatini) framework (i.e., $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$ with a priori independent metric $g_{\mu\nu}$ and affine connection $\Gamma^\lambda_{\mu\nu}$), then the auxiliary rank 3 tensor gauge fields defining the non-Riemannian volume-elements in (7) are almost *pure-gauge* degrees of freedom;

i.e., they *do not* introduce any additional propagating gravitational degrees of freedom when passing to the physical Einstein-frame except for few discrete degrees of freedom with conserved canonical momenta appearing as arbitrary integration constants. This has been explicitly shown within the canonical Hamiltonian treatment [149,153].

On the other hand, when we treat (7) in the second order (metric) formalism (the affine connection $\Gamma^{\lambda}_{\mu\nu}$ is the canonical Levi–Civitta connection in terms of $g_{\mu\nu}$), while passing to the physical Einstein-frame via conformal transformation (cf. Equation (30) below):

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = \chi_1 \, g_{\mu\nu}, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \,, \tag{9}$$

the first non-Riemannian volume element $\Phi_1(A)$ in (7) is not any more (almost) "pure gauge," but creates a new dynamical canonical scalar field u via $\chi_1 = \exp \frac{u}{\sqrt{3}}$, which will play the role both of an inflaton field at early times, as well as driving late-time de Sitter expansion (see Section 3 below).

In Section 2 we briefly review our construction in [152] of a simple gravity-scalar-field model—a specific member of the class of modified gravitational models (7) of the form (10) below, which yields an explicit dynamical generation of independent (non-interacting among each other) dark energy and dark matter components in an unified description as a manifestation of a single material entity ("darkon" scalar field)— the simplest realization of a ΛCDM model.

In Section 3 we extend the previous construction to dynamically generate, apart from dark matter, early-time inflation and late-time de Sitter expansion as well—via dynamical creation of an additional canonical scalar field u ("inflaton") out of a non-Riemannian volume-element with the following properties: (i) u acquires, dynamically, a non-trivial inflationary type scalar field potential driving inflation at early times of the universe's evolution; (ii) At late times the same evolving u flows towards a stable critical point of the pertinent dynamical system describing the cosmological evolution, driving a late-time de Sitter expansion in a dark energy dominated epoch; (iii) In this case the field u induces a specific interaction between the dark energy and dark matter.

In Section 4 we study the cosmological implications of the latter dynamically generated inflationary model with interacting dark energy and dark matter. In Section 5 several plots of the numerical solutions for the evolution of the dynamical inflationary field and for the behavior of the relevant inflationary slow-roll parameters and the corresponding observables are presented. Section 6 contains our conclusions and outlook.

2. A Simple Model of Unification of Dark Energy and Dark Matter

In [152] we started with the following non-conventional gravity-scalar-field action—a simple particular case of the class (7)—containing one metric-independent non-Riemannian volume-element alongside with the standard Riemannian one:

$$S = \int d^4x \sqrt{-g}\, R(g) + \int d^4x \big(\sqrt{-g} + \Phi_0(A)\big) L(\varphi, X) \,, \tag{10}$$

with the following notations:

- The first term in (10) is the standard Einstein–Hilbert action with $R(g)$ denoting the scalar curvature with respect to metric $g_{\mu\nu}$ in the second order (metric) formalism;
- $\Phi_0(A)$ is particular representative of a $D = 4$ non-Riemannian volume-element density (6):

$$\Phi_0(A) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu A_{\nu\kappa\lambda} \,. \tag{11}$$

- $L(\varphi, X)$ is general-coordinate invariant Lagrangian of a single scalar field $\varphi(x)$:

$$L(\varphi, X) = X - V(\varphi), \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \,. \tag{12}$$

Varying (10) with respect to $g^{\mu\nu}$, φ and $A_{\mu\nu\lambda}$ yield the following equations of motion, respectively:

$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = \frac{1}{2}T_{\mu\nu} \quad , \quad T_{\mu\nu} = g_{\mu\nu}L(\varphi, X) + \Big(1 + \frac{\Phi_0(A)}{\sqrt{-g}}\Big)\partial_\mu\varphi\,\partial_\nu\varphi\,; \tag{13}$$

$$-\frac{\partial V}{\partial\varphi} + \big(\Phi_0(A) + \sqrt{-g}\big)^{-1}\partial_\mu\Big[\big(\Phi_0(A) + \sqrt{-g}\big)g^{\mu\nu}\partial_\nu\varphi\Big] = 0\,; \tag{14}$$

$$\partial_\mu L(\varphi, X) = 0 \quad \longrightarrow \quad L(\varphi, X) \equiv X - V(\varphi) = -2M_0 = \text{const}\,, \tag{15}$$

where M_0 is arbitrary integration constant (the factor 2 is for later convenience).

As stressed in [152], the scalar field dynamics are determined entirely by the first-order differential equation—the dynamical constraint Equation (15). The usual second order differential Equation (14) for φ is in fact a consequence of (15) together with the energy-momentum conservation:

$$\nabla^\mu T_{\mu\nu} = 0. \tag{16}$$

Additionally, as exhibited in [152], the specific form of the scalar field potential $V(\varphi)$ does not affect the dynamics of the system (10); see the remark below following (18). The same phenomenon occurs in the extension of (10) to the model (24) in Sections 3 and 4 below.

The canonical Hamiltonian analysis in [152] of the action (10) reveals that the auxiliary gauge field $A_{\mu\nu\lambda}$ is in fact an almost pure-gauge; i.e., it is a non-propagating field-theoretic degree of freedom with the integration constant $(-2M_0)$ identified with the conserved Dirac-constrained canonical momentum conjugated to the "pure gauge," "magnetic" component of $A_{\mu\nu\lambda}$. For a general canonical Hamiltonian treatment of Lagrangian action with one or more non-Riemannian volume-elements, we refer to [155].

A crucial property of the model (10) is the existence of a hidden nonlinear Noether symmetry revealed in [152]. Indeed, both Equations (14) and (15) can be equivalently rewritten in the following current-conservation law form:

$$\nabla_\mu J^\mu = 0 \quad , \quad J^\mu \equiv \Big(1 + \frac{\Phi_0(A)}{\sqrt{-g}}\Big)\sqrt{2X}g^{\mu\nu}\partial_\nu\varphi\,. \tag{17}$$

The covariantly conserved current J^μ (17) is the Noether current corresponding to the invariance (modulo total derivative) of the action (10) with respect to the following hidden nonlinear symmetry transformations:

$$\delta_\epsilon\varphi = \epsilon\sqrt{X} \quad , \quad \delta_\epsilon g_{\mu\nu} = 0 \quad , \quad \delta_\epsilon\mathcal{A}^\mu = -\epsilon\frac{1}{2\sqrt{X}}g^{\mu\nu}\partial_\nu\varphi\big(\Phi_0(A) + \sqrt{-g}\big)\,, \tag{18}$$

with $\mathcal{A}^\mu = \big(\mathcal{A}^0 \equiv \frac{1}{3!}\varepsilon^{mkl}A_{mkl}\ ,\mathcal{A}^i \equiv -\frac{1}{2}\varepsilon^{ikl}A_{0kl}\big)$— "dual" components of the auxiliary gauge field $A_{\mu\nu\lambda}$ (11).

Remark 1. *We notice that the existence of the hidden nonlinear symmetry (18) of the action (10) does not depend on the specific form of the scalar field potential $V(\varphi)$.*

The next important step is to rewrite $T_{\mu\nu}$ (13) and J^μ (17) in the relativistic hydrodynamical form (again taking into account (15)):

$$T_{\mu\nu} = \rho_0 u_\mu u_\nu - 2M_0 g_{\mu\nu} \quad , \quad J^\mu = \rho_0 u^\mu\,. \tag{19}$$

Here the integration constant M_0 appears as a dynamically generated cosmological constant and:

$$\rho_0 \equiv \Big(1 + \frac{\Phi_0(A)}{\sqrt{-g}}\Big)\,2X, \quad u_\mu \equiv -\frac{\partial_\mu\varphi}{\sqrt{2X}} \quad \big(\text{note } u^\mu u_\mu = -1\,\big)\,. \tag{20}$$

We now find that the covariant conservation laws for the energy-momentum tensor (19) $\nabla^\mu T_{\mu\nu} = 0$ and the J-current (17) acquire the form:

$$\nabla^\mu \left(\rho_0 u_\mu u_\nu\right) = 0 \quad , \quad \nabla^\mu \left(\rho_0 u_\mu\right) = 0 \,. \tag{21}$$

Equations (21) imply in turn the geodesic equation for the "fluid" 4-velocity u_μ:

$$u_\mu \nabla^\mu u_\nu = 0 \,. \tag{22}$$

Therefore, comparing (19) with the standard expression for a perfect fluid stress-energy tensor $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$, we see that $T_{\mu\nu}$ (19) consists of two additive parts which have the following interpretations according to the standard Λ-CDM model [156–163] (using notations $p = p_{\rm DM} + p_{\rm DE}$ and $\rho = \rho_{\rm DM} + \rho_{\rm DE}$):

- A dynamically generated dark energy part given by the second cosmological constant term in $T_{\mu\nu}$ (19) due to (15), where $p_{\rm DE} = -2M\,,\, \rho_{\rm DE} = 2M$.
- A dynamically generated dark matter part given by the first term in (19), where $p_{\rm DM} = 0$, $\rho_{\rm DM} = \rho_0$ with ρ_0 as in (20), which in fact according to (21) and (22) describes a *dust fluid* with fluid density ρ_0 flowing along geodesics. Thus, we will refer to the φ scalar field by the alias "darkon."

The conservation laws (21) due to the hidden nonlinear Noether symmetry (18) imply that in the model (10) there is *no interaction* between dark energy and dark matter— they are separately conserved.

3. Inflation and Unified Dark Energy and Dark Matter

Now we will extend the simple model (10) of unified dark energy and dark matter by introducing another metric-independent non-Riemannian volume-element:

$$\Phi_1(B) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \tag{23}$$

inside the gravity (Einstein–Hilbert) part of the action (using again units with $16\pi G_{\rm Newton} = 1$):

$$S = \int d^4x \left\{ \Phi_1(B) \Big[R(g) - 2\Lambda_0 \frac{\Phi_1(B)}{\sqrt{-g}} \Big] + \Big(\sqrt{-g} + \Phi_0(A) \Big) \Big[-\frac{1}{2} g^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi - V(\varphi) \Big] \right\} . \tag{24}$$

Here Λ_0 is a dimensionful parameter to be identified later on as energy scale of the inflationary universe's epoch.

The specific form of the action (24) may be justified by the requirement of global Weyl-scale invariance under the transformations:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \quad , \quad A_{\mu\nu\kappa} \to \lambda^2 A_{\mu\nu\kappa} \quad , \quad B_{\mu\nu\kappa} \to \lambda B_{\mu\nu\kappa} \quad , \quad \varphi \to \lambda^{-\frac{1}{2}} \varphi \,, \tag{25}$$

and provided we choose $V(\varphi) = \varphi^4$. Concerning global Weyl-scale invariance, let us note that it played an important role already since the original papers on the non-canonical volume-form formalism [146]. In particular, models with spontaneously broken dilatation symmetry have been constructed along these lines, which are free of the fifth force problem [147].

The equations of motion of the action (24) with respect to φ and $A_{\mu\nu\lambda}$ are the same as in (14) and (15); therefore, once again, (24) is invariant under the hidden, nonlinear Noether symmetry (18) with the associated Noether conserved current (17), which we rewrite here for later convenience, taking into account (15):

$$\nabla_\mu J^\mu = 0 \quad , \quad J^\mu = (1 + \chi_0) \sqrt{2\big(V(\varphi) - 2M_0\big)}\, g^{\mu\nu} \partial_\nu \varphi \quad , \quad \chi_0 \equiv \frac{\Phi_0(A)}{\sqrt{-g}} \,. \tag{26}$$

On the other hand, the equations of motion with respect to $g^{\mu\nu}$ and $B_{\mu\nu\lambda}$ now read:

$$R_{\mu\nu}(g) + \frac{1}{\chi_1}\big(g_{\mu\nu}\Box\chi_1 - \nabla_\mu\nabla_\nu\chi_1\big) - \Lambda_0\chi_1 g_{\mu\nu} = \frac{1}{2\chi_1}T_{\mu\nu}\,, \tag{27}$$

$$R(g) - 4\Lambda_0\chi_1 = -M_1 \quad , \quad \chi_1 \equiv \frac{\Phi_1(B)}{\sqrt{-g}}\,, \tag{28}$$

where $T_{\mu\nu}$ is the same energy-momentum tensor as in (13) or (19), which taking into account (15) and using short-hand notation χ_0 in (26)) reads $T_{\mu\nu} = -2M_0 g_{\mu\nu} + \big(1+\chi_0\big)\partial_\mu\varphi\partial_\nu\varphi$, and M_1 is another free integration constant similar to M_0 in (15). Taking a trace of (27) together with (28) implies a dynamical equation for χ_1 (χ_0 and χ_1 as defined in (26) and (28), respectively):

$$\Box\chi_1 - \frac{1}{3}M_1\chi_1 - \frac{1}{6}T = 0 \quad , \quad T \equiv g^{\mu\nu}T_{\mu\nu} = -8M_0 - 2(1+\chi_0)\big(V(\varphi) - 2M_0\big)\,, \tag{29}$$

The passage to the Einstein-frame is accomplished via the conformal transformation:

$$g_{\mu\nu} \longrightarrow \bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu}\,, \tag{30}$$

on Equations (27) and (29), and upon using the known formulae for conformal transformations of Ricci curvature tensor and covariant Dalambertian (see e.g., [164]; bars indicate magnitudes in the $\bar{g}_{\mu\nu}$-frame):

$$R_{\mu\nu}(g) = R_{\mu\nu}(\bar{g}) - 3\frac{\bar{g}_{\mu\nu}}{\chi_1}\bar{g}^{\kappa\lambda}\partial_\kappa\chi_1^{1/2}\partial_\lambda\chi_1^{1/2} + \chi_1^{-1/2}\big(\bar{\nabla}_\mu\bar{\nabla}_\nu\chi_1^{1/2} + \bar{g}_{\mu\nu}\bar{\Box}\chi_1^{1/2}\big)\,, \tag{31}$$

$$\Box\chi_1 = \chi_1\Big(\bar{\Box}\chi_1 - 2\bar{g}^{\mu\nu}\frac{\partial_\mu\chi_1^{1/2}\partial_\nu\chi_1}{\chi_1^{1/2}}\Big)\,. \tag{32}$$

In the process we introduce the field redefinition $\chi_1 \;\to\; u$:

$$\chi_1 = \exp\Big\{\frac{u}{\sqrt{3}}\Big\}\,, \tag{33}$$

so that u appears as a canonical scalar field in the Einstein-frame transformed Equations (15), (27) and (29):

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{1}{2}\bar{T}_{\mu\nu}\,,$$
$$\bar{T}_{\mu\nu} = \partial_\mu u\,\partial_\nu u + \bar{g}_{\mu\nu}\Big[-\frac{1}{2}\bar{g}^{\kappa\lambda}\partial_\kappa u\,\partial_\lambda u - U_{\text{eff}}(u)\Big] + e^{-u/\sqrt{3}}(1+\chi_0)\partial_\mu\varphi\,\partial_\nu\varphi\,, \tag{34}$$

$$\bar{\Box}u - \frac{\partial U_{\text{eff}}(u)}{\partial u} + \frac{1}{\sqrt{3}}e^{-2u/\sqrt{3}}(1+\chi_0)\big(V(\varphi) - 2M_0\big) = 0\,, \tag{35}$$

$$\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + e^{-u/\sqrt{3}}\big(V(\varphi) - 2M_0\big) = 0\,, \tag{36}$$

and most importantly, u acquires a non-trivial, dynamically degenerated potential:

$$U_{\text{eff}}(u) = 2\Lambda_0 - M_1 e^{-u/\sqrt{3}} + 2M_0 e^{-2u/\sqrt{3}} \tag{37}$$

due to the appearance of the free integration constants from the equations of motion of the original-frame, non-Riemannian spacetime volume-elements. The hidden, nonlinear Noether symmetry current conservation (17), equivalent to the φ-equation of motion, becomes in the Einstein-frame:

$$\bar{\nabla}_\mu\bar{J}^\mu = 0 \quad , \quad \bar{J}^\mu = (1+\chi_0)e^{-u/\sqrt{3}}\sqrt{V(\varphi) - 2M_0}\,\bar{g}^{\mu\nu}\partial_\nu\varphi\,. \tag{38}$$

Thus, the Einstein-frame Lagrangian action producing the Einstein-frame equations of motion (34)–(38) reads:

$$S_{EF} = \int d^4x \sqrt{-\bar{g}} \Big[\bar{R} - \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu u \, \partial_\nu u - U_{\text{eff}}(u) \Big]$$
$$+ \int d^4x \sqrt{-\bar{g}} (1 + \chi_0) e^{-u/\sqrt{3}} \Big[-\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - e^{-u/\sqrt{3}} \big(V(\varphi) - 2M_0 \big) \Big] \,, \tag{39}$$

with $U_{\text{eff}}(u)$ as in (37), in which χ_0 (from (26)) becomes a simple Lagrange multiplier.

The upper line in S_{EF} (39) represents an inflationary Lagrangian action with dynamically generated inflationary potential $U_{\text{eff}}(u)$ (37) obtained in [107] from a pure gravity initial action (without any matter fields) in terms of non-Riemannian volume-elements:

$$S_0 = \int d^4x \, \Big\{ \Phi_1(B) \Big[R(g) - 2\Lambda_0 \Phi_1(B)/\sqrt{-g} \Big] + \big(\Phi_0(A) \big)^2 / \sqrt{-g} \Big\} \tag{40}$$

which, as graphically depicted on Figure 1, is a generalization of the classic Starobinsky inflationary potential [16]. In fact, the latter is a special case of (37) for the particular values of the parameters: $\Lambda_0 = M_0 = \frac{1}{4} M_1$.

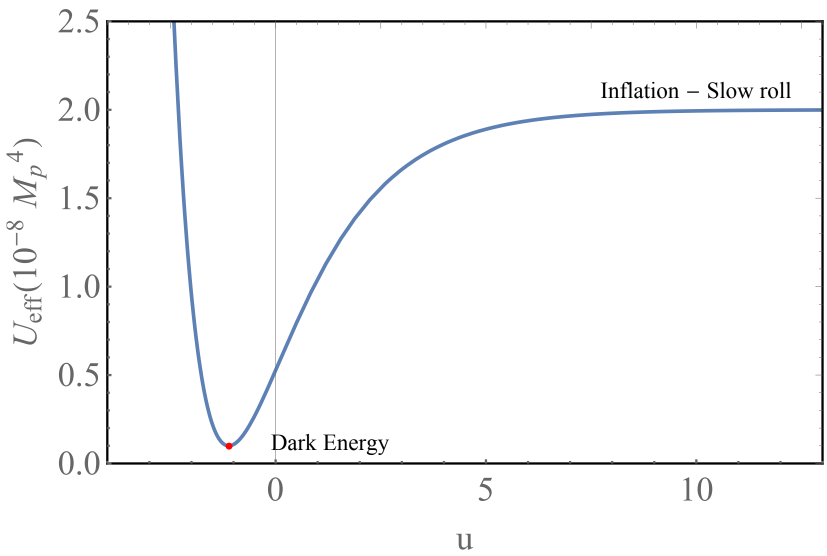

Figure 1. Shape of the effective potential $U_{\text{eff}}(u)$ in the Einstein-frame (37). The physical unit for u is $M_{Pl}/\sqrt{2}$.

$U_{\text{eff}}(u)$ (37) possesses two main features relevant for cosmological applications:

- (i) $U_{\text{eff}}(u)$ (37) has an almost flat region for large positive u: $U_{\text{eff}}(u) \simeq 2\Lambda_0$ for large u. This almost flat region corresponds to "early universe" inflationary evolution with energy scale $2\Lambda_0$, as will be evident from the autonomous dynamical system analysis of the cosmological dynamics in Section 4.
- (ii) $U_{\text{eff}}(u)$ (37) has a stable minimum for a small finite value $u = u_*$: $\frac{\partial U_{\text{eff}}}{\partial u} = 0$ for $u \equiv u_*$, where:

$$\exp(-\frac{u_*}{\sqrt{3}}) = \frac{M_1}{4M_0} \quad , \frac{\partial^2 U_{\text{eff}}}{\partial u^2}\bigg|_{u=u_*} = \frac{M_1^2}{12M_0} > 0 \,. \tag{41}$$

- (iii) As it will be explicitly exhibited in the dynamical system analysis in Section 4, the region of u around the stable minimum at $u = u_*$ (41) corresponds to the late-time de Sitter expansion of the universe with a slightly varied late-time Hubble parameter (dark energy dominated epoch), wherein the minimum value of the potential:

$$U_{\rm eff}(u_*) = 2\Lambda_0 - \frac{M_1^2}{8M_0} \equiv 2\Lambda_{\rm DE} \tag{42}$$

is the asymptotic value at $t \to \infty$ of the dynamical dark energy density [165,166].

The lower line in $S_{\rm EF}$ (39) represents the interaction between the dynamical inflaton field u and the "darkon" field φ; in other words, here we have unification of inflation, dark energy and dark-matter. This is reflected in the structure of the Einstein-frame energy-momentum tensor $\bar{T}_{\mu\nu}$ (34)—the first two terms being the stress-energy tensor of u and the last term being the "darkon" stress-energy tensor coupled to u.

4. Cosmological Implications

Let us now consider reduction of the Einstein-frame action (39) to the Friedmann–Lemaitre–Robertson–Walker (FLRW) framework with metric $ds^2 = -N^2 dt^2 + a(t)^2 d\vec{x}^2$, where $u = u(t)$ and $\varphi = \varphi(t)$:

$$S_{\rm FLRW} = \int d^4x \Big\{ -6\frac{a\,\dot{a}^2}{N} + Na^3 \Big[\frac{1}{2}\frac{\dot{u}^2}{N^2} + M_1 e^{-u/\sqrt{3}} - 2M_0 e^{-2u/\sqrt{3}} - 2\Lambda_0\Big] \\ + Na^3(1+\chi_0)e^{-u/\sqrt{3}}\Big[\frac{1}{2}\frac{\dot{\varphi}^2}{N^2} - e^{-u/\sqrt{3}}\big(V(\varphi) - 2M_0\big)\Big]\Big\}\,. \tag{43}$$

The equations of motion with respect to χ_0 and φ from (43) are equivalent to the FLRW reduction of the dynamical constraint (36) and the Noether current conservation (38), respectively:

$$\dot{\varphi}^2 = 2e^{-u/\sqrt{3}}\big(V(\varphi) - 2M_0\big) \quad , \quad \frac{d}{dt}\Big[a^3(1+\chi_0)e^{-u/\sqrt{3}}\sqrt{V(\varphi) - 2M_0}\;\dot{\varphi}\Big] = 0\,, \tag{44}$$

which imply the relation:

$$(1+\chi_0)e^{-u/\sqrt{3}}\big(V(\varphi) - 2M_0\big) = \frac{c_0}{a^3}e^{u/2\sqrt{3}}\,, \tag{45}$$

with c_0 a free integration constant. Taking into account (45), the FLRW reduction of the Einstein-frame energy-momentum tensor (34) becomes:

$$\bar{T}_{00} \equiv \rho \quad , \quad \bar{T}_{ij} \equiv a^2\delta_{ij}p \quad , \quad \bar{T}_{0i} = 0\,, \tag{46}$$

$$\rho = \frac{1}{2}\,\dot{u}^2 + U_{\rm eff}(u) + 2\frac{c_0}{a^3}e^{-u/2\sqrt{3}} \quad , \quad p = \frac{1}{2}\,\dot{u}^2 - U_{\rm eff}(u)\,. \tag{47}$$

Relations (47) explicitly show that the last term in ρ:

$$\rho_{\rm DM} \equiv 2\frac{c_0}{a^3}e^{-u/2\sqrt{3}} \tag{48}$$

represents the "dust" dark matter part of the total energy denisty—it is "dust" because of absence ot corresponding contribution for the pressure p in (47).

The equation of motion from (43) with respect to u is ($H = \dot{a}/a$ being the Hubble parameter):

$$\ddot{u} + 3H\,\dot{u} + \frac{\partial U_{\rm eff}}{\partial u} - \frac{1}{\sqrt{3}}\frac{c_0}{a^3}e^{-u/2\sqrt{3}} = 0, \tag{49}$$

and finally, the two Friedmann equations (varying (43) with respect to lapse N and a) read:

$$6H^2 = \frac{1}{2}\,\dot{u}^2 + U_{\rm eff}(u) + 2\frac{c_0}{a^3}e^{-u/2\sqrt{3}}\,, \tag{50}$$

$$\dot{H} = -\frac{1}{4}\left(\dot{u}^2 + 2\frac{c_0}{a^3}e^{-u/2\sqrt{3}}\right)\,. \tag{51}$$

Remark 2. *We observe that due to the hidden, nonlinear Noether symmetry current conservation (45), the FLRW dynamics given by (49)–(51) do not depend on the explicit form of the "darkon" part of the FLRW action (43)—the only trace of the "darkon" is embodied in the integration constant c_0.*

It is instructive to analyze the system FLRW Equations (49)–(51) as an autonomous dynamical system. To this end it is useful to rewrite the system (49)–(51) in terms of a set of dimensionless coordinates (following the approach in [167]):

$$x := \frac{\dot{u}}{\sqrt{12}H}, \quad y := \frac{\sqrt{U_{\rm eff}(u) - 2\Lambda_{\rm DE}}}{\sqrt{6}H}, \quad z := \frac{\sqrt{\Lambda_{\rm DE} + \rho_{\rm DM}}}{\sqrt{3}H}\,, \tag{52}$$

with $L_{\rm DE}$ as in (42) and $\rho_{\rm DM}$ as in (48). In these coordinates the system defines a closed orbit:

$$x^2 + y^2 + z^2 = 1\,, \tag{53}$$

which is equivalent to the first Friedmann Equation (50). Then, Equations (49) and (51) can be represented as a 3-dimensional autonomous dynamical system for the $(x, y, H$ variables (cf. (52)):

$$\begin{aligned} x' = \frac{3}{2}x\left[x^2 - 1 - y^2 - \frac{\Lambda_{\rm DE}}{3H^2}\right] + \frac{1}{2}\left(1 - x^2 - y^2 - \frac{\Lambda_{\rm DE}}{3H^2}\right) \\ -\frac{2y}{H}\sqrt{\frac{M_0}{3}}\left(\frac{M_1}{4M_0} - \sqrt{\frac{3}{M_0}}Hy\right)\,, \end{aligned} \tag{54}$$

$$y' = \frac{2x}{H}\sqrt{\frac{M_0}{3}}\left(\frac{M_1}{4M_0} - \sqrt{\frac{3}{M_0}}Hy\right) + \frac{3}{2}y\left[1 + x^2 - y^2 - \frac{\Lambda_{\rm DE}}{3H^2}\right]\,, \tag{55}$$

$$H' = -\frac{3}{2}H\left[1 + x^2 - y^2 - \frac{\Lambda_{\rm DE}}{3H^2}\right]\,, \tag{56}$$

where the primes indicate derivatives with respect to number of e-folds $\mathcal{N} = \log(a)$ (meaning $\frac{d}{d\mathcal{N}} = \frac{1}{H}\frac{d}{dt}$).

The dynamical system (54)–(56) possesses two critical points:

- (A) Stable critical point:

$$x_* = 0 \quad , \quad y_* = 0 \quad , \quad H_* = \sqrt{\frac{\Lambda_{\rm DE}}{3}}\,, \tag{57}$$

where all three eigenvalues of the stability matrix are negative or with negative real parts ($\lambda_1 = -3\,,\ \lambda_{2,3} = \frac{1}{2}\left[-3 + \sqrt{9 - \frac{M_1^2}{M_0\Lambda_{\rm DE}}}\right]$). The stable critical point (57) corresponds to the late-time asymptotics of the universe's evolution where according to the definitions (52) $u(t) \to u_*$—the stable minimum of the effective potential $U_{\rm eff}(u)$ (37), so that $U_{\rm eff}(u) \to 2\Lambda_{\rm DE}$, the dark matter energy density (48) $\rho_{\rm DM} \to 0$, and $\dot{H} \to 0$ according to (56); i.e., late-time accelerated expansion with $H_* = \sqrt{\frac{\Lambda_{\rm DE}}{3}}$.
- (B) Unstable critical point:

$$x_{**} = 0 \quad , \quad y_{**} = \sqrt{1 - \frac{\Lambda_{\rm DE}}{\Lambda_0}} = \frac{M_1}{4\sqrt{M_0\Lambda_0}} \quad , \quad H_{**} = \sqrt{\frac{\Lambda_0}{3}}\,, \tag{58}$$

where one of the three eigenvalues of the stability matrix is zero ($\lambda_1 = 0$, $\lambda_2 = -3$, $\lambda_3 = -3(1 - \Lambda_{\rm DE}/\Lambda_0) = -\frac{3M_1^2}{16M_0\Lambda_0}$). According to the definitions (52), in the vicinity of the unstable critical point (58), $u(t)$ is very large positive ($u \to \infty$), so that $U_{\rm eff}(u) \simeq 2\Lambda_0$, $\rho_{\rm DM}$ is vanishing $\rho_{\rm DM} \approx 0$ and we have there a slow-roll inflationary evolution with inflationary scale Λ_0 where the standard slow-roll parameters are very small:

$$\epsilon = -\frac{\dot{H}}{H^2} \approx \left(\frac{\frac{\partial U_{\rm eff}}{\partial u} - \frac{1}{2\sqrt{3}}\rho_{\rm DM}}{U_{\rm eff} + \rho_{\rm DM}}\right)^2 + \frac{3}{2}\frac{\rho_{\rm DM}}{U_{\rm eff} + \rho_{\rm DM}} \, , \tag{59}$$

$$\eta = -\frac{\dot{H}}{H^2} - \frac{\ddot{H}}{2H\,\dot{H}} \approx -2\frac{\frac{\partial^2 U_{\rm eff}}{\partial u^2} + \frac{1}{12}\rho_{\rm DM}}{U_{\rm eff} + \rho_{\rm DM}} + \mathrm{O}(\rho_{\rm DM}) \, . \tag{60}$$

5. Numerical Solutions

Going back to the system of Equations (49)–(51), we can use (50) to replace the term $\rho_{\rm DM} \equiv 2\frac{c_0}{a^3}e^{-u/2\sqrt{3}}$ in (49) and (51) so that we will obtain a closed system of two coupled nonlinear differential equations for $\big(u(t), H(t)\big)$ of second and first order, respectively:

$$\ddot{u} + 3H\,\dot{u} + \frac{\partial U_{\rm eff}}{\partial u} - \frac{1}{2\sqrt{3}}\Big[6H^2 - \frac{1}{2}\,\dot{u}^2 - U_{\rm eff}(u)\Big] = 0 \, , \tag{61}$$

$$\dot{H} = -\frac{1}{4}\Big[6H^2 + \frac{1}{2}\,\dot{u}^2 - U_{\rm eff}(u)\Big] \, , \tag{62}$$

where $U_{\rm eff}(u)$ is given by (37): $U_{\rm eff}(u) = 2\Lambda_0 - M_1 e^{-u/\sqrt{3}} + 2M_0 e^{-2u/\sqrt{3}}$.

Below we present several plots qualitatively illustrating the evolutionary behavior of the numerical solution of the system (61) and (62) with initial conditions conforming to the unstable critical point B (58): $\dot{u}\,(0) = 0$, $H(0) = \sqrt{\Lambda_0/3}$ and $u(0)$—some large initial value. As a numerical example, for the purpose of graphical illustration, we will take the following numerical values of the parameters (the physical units would be $10^{-9}M_{Pl}^4$):

$$\Lambda_0 = 50 \;\; , \;\; M_1 = 20 \;\; , \;\; M_0 = 0.501 \quad \longrightarrow \; \Lambda_{\rm DE} = 0.1 \tag{63}$$

according to (42) (in reality $\Lambda_{\rm DE}$ is much smaller than $1/500$ part of Λ_0: $\Lambda_0 \sim 10^{-8}M_{Pl}^4$ [168,169] and $\Lambda_{DE} \sim 10^{-122}M_{Pl}^4$; cf. [170]).

On Figure 2 below, the plot represents the overall evolution of $u(t)$, whereas on Figure 3 are the plots for the slow-roll parameters $\epsilon = -\frac{\dot{H}}{H^2}$ and $\eta = -\frac{\dot{H}}{H^2} - \frac{\ddot{H}}{2H\dot{H}}$ clearly indicating the end of inflation where their sharp grow-up starts.

Figure 4 represents the plot of the evolution of the Hubble parameter $H(t)$ with a clear indication of the two (quasi-)de Sitter epochs—during early-times inflation with much higher value of $H \simeq \sqrt{\frac{\Lambda_0}{3}}$, and in late-times with much smaller value of $H \simeq \sqrt{\frac{\Lambda_{\rm DE}}{3}}$.

The plots on Figure 5 depict the oscillations of $u(t)$ and $\dot{u}\,(t)$ occurring after the end of inflation.

Figure 6 contains the plots of the evolution of $w = p/\rho$—the parameter of the equation of state with a clear indication of a short time epoch of matter domination after end of inflation.

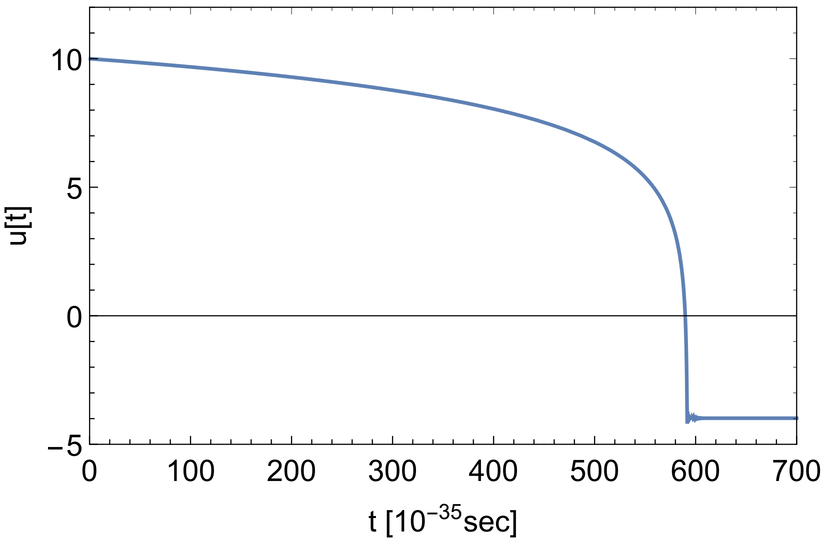

Figure 2. Numerical shape of the evolution of $u(t)$. The physical unit for u is $M_{Pl}/\sqrt{2}$.

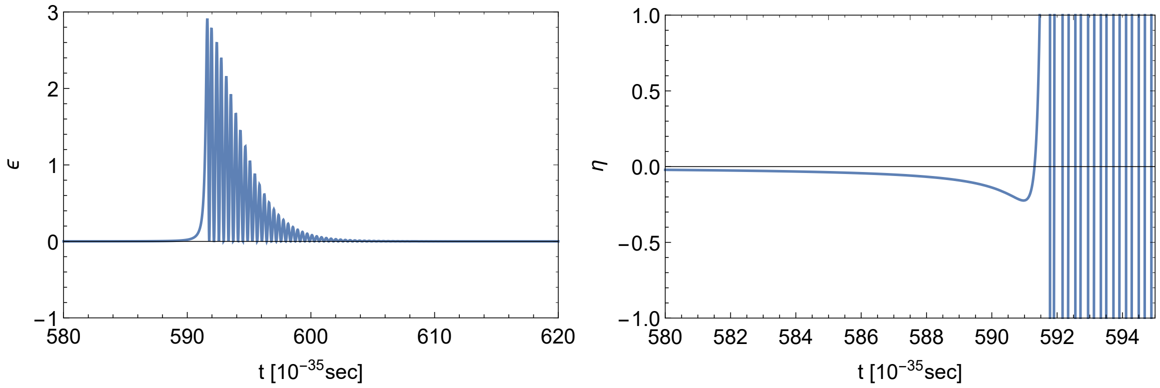

Figure 3. Slow-roll parameters ϵ and η before and around end of inflation. When $\epsilon = 1$ the inflation ends.

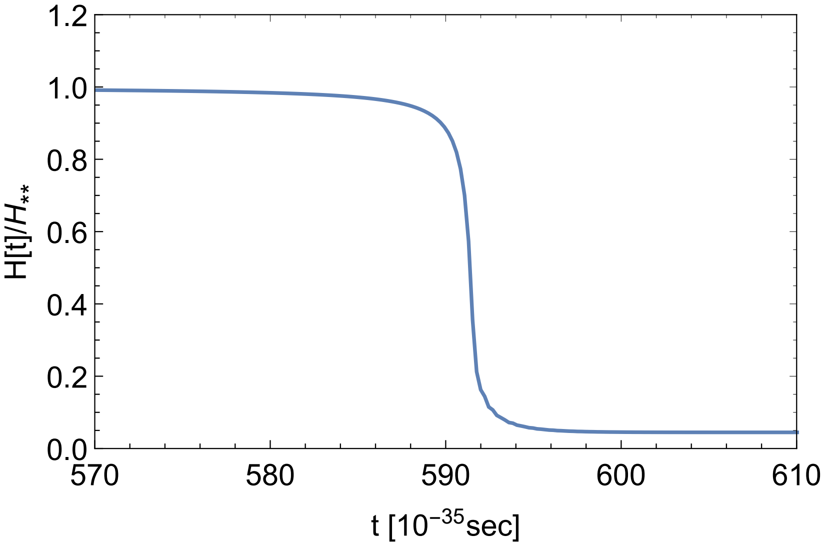

Figure 4. Numerical shape of the evolution of $H(t)$. Here $H_{**} \equiv \sqrt{\Lambda_0/3}$ as in (58).

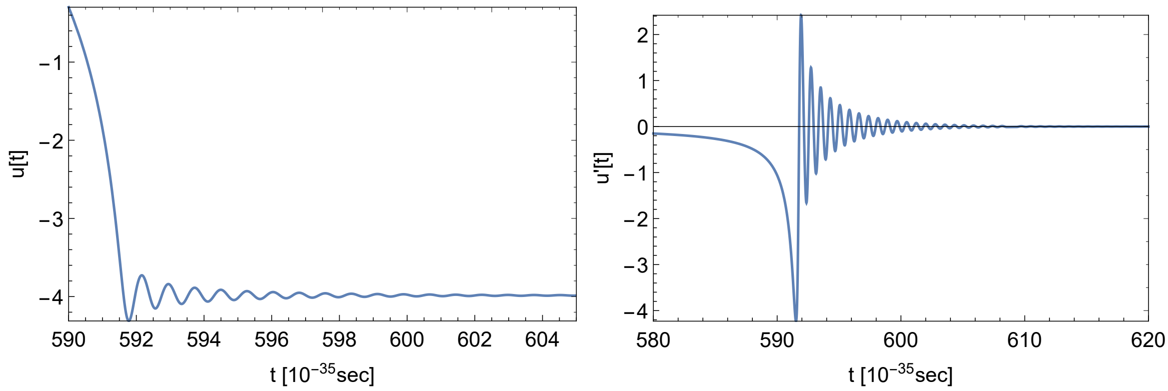

Figure 5. In the left panel—blown-up portion of the plot on Figure 2 around and after end of inflation depicting the oscillations of $u(t)$ after end of inflation. In the right panel—oscillations of $\dot{u}\ (t)$ after end of inflation.

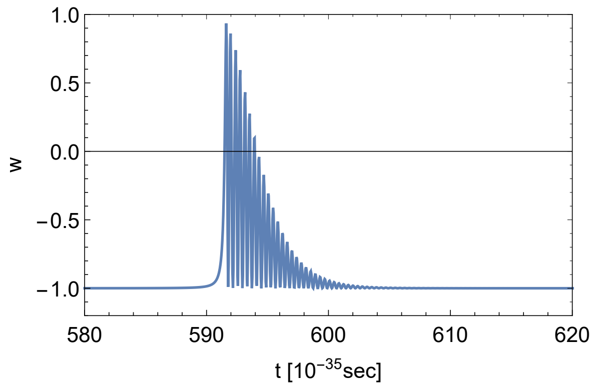

Figure 6. Evolution of w parameter of the equation of state with sharp growth above $w \approx -1$ for a short time interval after end of inflation—matter domination.

To obtain plausible values for the observables—the scalar power spectral index n_s and the tensor to scalar ratio r [53,105,171]—we need the functional dependence of the slow-roll parameters ϵ and η with respect to $\mathcal{N} = \log(a)$, the number of e-folds. More specifically, in $\mathcal{N}_f$ is the number of e-folds at the end of inflation defined as $\epsilon(\mathcal{N}_f) \approx 1$; then, we need the values $\epsilon(\mathcal{N}_i)$ and $\eta(\mathcal{N}_i)$ at $\mathcal{N}_i$—e-folds at the start of inflation, where it is assumed that $\mathcal{N}_f - \mathcal{N}_i \sim 60$. Then, according to [53,105]:

$$r \approx 16\epsilon(\mathcal{N}_i) \quad , \quad n_s \approx 1 - 6\epsilon(\mathcal{N}_i) + 2\eta(\mathcal{N}_i), \tag{64}$$

where the corresponding slow roll parameter reads:

$$\epsilon(\mathcal{N}_i) = -\frac{H'(\mathcal{N}_i)}{H(\mathcal{N}_i)} \quad , \quad \eta(\mathcal{N}_i) = -\frac{H'(\mathcal{N}_i)}{H(\mathcal{N}_i)} - \frac{H''(\mathcal{N}_i)}{2H(\mathcal{N}_i)H'(\mathcal{N}_i)}\ , \tag{65}$$

and where $H = H(\mathcal{N})$ is the functional dependence of Hubble parameter with respect to the number of e-folds. To this end we employ numerical simulation of the autonomous dynamical system Equations (54)–(56).

From the inflationary scenario, we know that the observed value of the inflationary scale $\Lambda_0 \sim 10^{-8} M_{Pl}^4$ is far larger than the current value ($\sim 10^{-122} M_{Pl}^4$) of the cosmological constant $\Lambda_{\rm DE}$ (42). Thus, as in the numerical example above for the numerical solution of the system for $u(t), H(t)$ (61) and (62), we will take again the values for the parameters according to (63), meaning that we set the initial condition for the Hubble parameter to be according to (58) $H_{initial} = \sqrt{\frac{\Lambda_0}{3}} = \sqrt{\frac{50}{3}}$. With those numerical values, we find the observables (64) to be:

$$r \approx 0.003683 \quad , \quad n_s \approx 0.9638 \,, \tag{66}$$

which are well inside the last *PLANCK* observed constraints [131]:

$$0.95 < n_s < 0.97 \quad , \quad r < 0.064 \,. \tag{67}$$

In order to see the pattern of the general behavior depending on the initial conditions, we employ here Monte Carlo simulation with 10^4 samples for the initial conditions using a normal distribution: $\Lambda_0 = 50 \pm 10$, $M_1 = 20 \pm 10$, while the error bar is taken to be $1\,\sigma$.

Figure 7 shows how different values of initial conditions yield different numbers of e-folds until end of inflation (where $\epsilon = 1$), and accordingly, different values for the observables r and n_s, whereas Figure 8 depicts the corresponding relation between r and n_s. Nevertheless, all the values of the latter fall within the constraint (67).

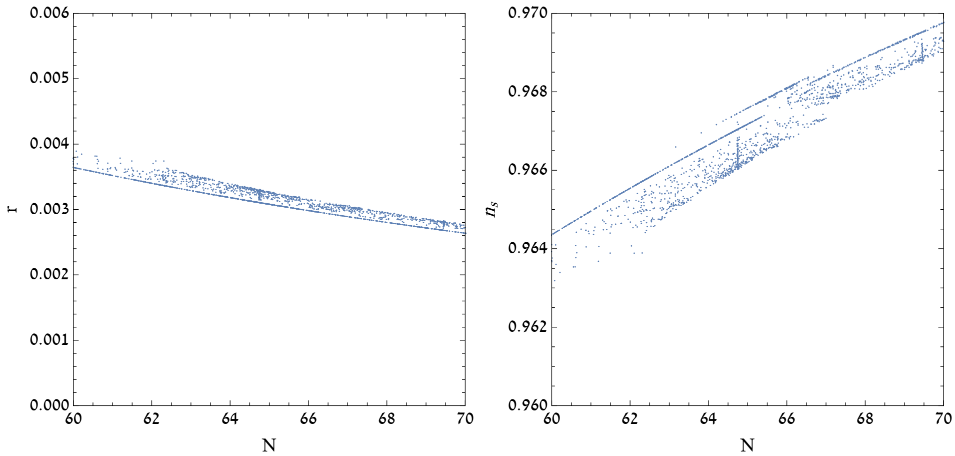

Figure 7. The scalar to tensor ratio r and the scalar spectral index n_s vs. the number of e-folds for different values of the initial conditions. The sampling of the latter is done with a normal distribution $\Lambda_0 = 50 \pm 10$, $M_1 = 20 \pm 10$.

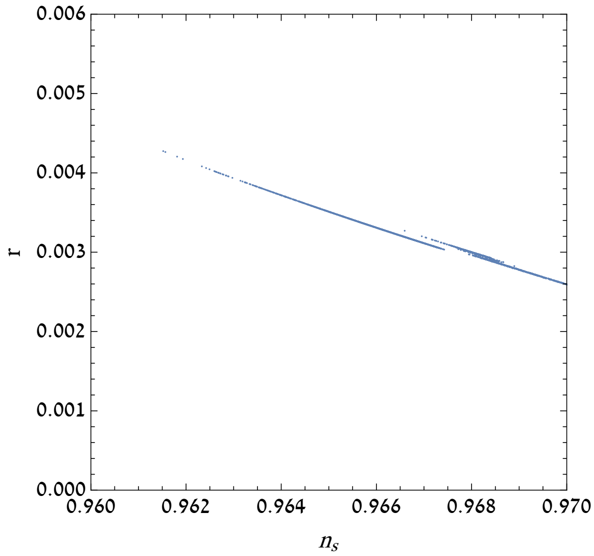

Figure 8. The relation between the scalar to tensor ratio r and the scalar spectral index n_s via sampled initial conditions with a normal distribution $\Lambda_0 = 50 \pm 10$, $M_1 = 20 \pm 10$. All of the sampled values fall well inside the Planck data constraint (67).

6. Conclusions and Outlook

In the present paper, starting from the basic first principle of Lagrangian field-theoretic actions combined with a non-canonical modification of gravity via employing non-Riemannian spacetime volume forms as alternatives to the standard Riemannian one given by $\sqrt{-g}$, we have constructed a unified model of dynamically generated inflation with dark energy and dark matter coupled among themselves. Upon passage to the physical Einstein frame, our model captures the main properties of the slow-roll inflationary epoch in early times, the short period of matter domination after the end of inflation and the late-time epoch of de Sitter expansion—all driven by a dynamically created scalar inflaton field. The numerical results for the observables (scalar power spectral index and tensor to scalar ratio) conform to the 2018 *PLANCK* constraints.

In the present model, dark matter in the form of a dust-like fluid is created already in the early universe's inflationary epoch without a significant impact on the inflationary dynamics. After the end of inflation, the dust-like dark matter, apart from a short period of matter domination, still does not exert a sufficient impact, which means that one has to further extend the present formulation in order to properly take the full dark matter contribution to the evolution into account.

One subject that has to be addressed is the "reheating of the universe," since of course we need temperature in the early universe to account for processes such as Bing Bang nucleosynthesis. There are many ways to achieve this, due to the oscillating nature of inflaton solutions near the minimum of the inflaton potential, which leads in general to particle creation. For example, one possible way to complement the modified gravity-scalar field model (24) in order to incorporate the effect of radiation after end of inflation is to include a coupling to the "topological" density of a electromagnetic field $\mathcal{A}_\mu$ with field strength $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ in the following way:

$$\widetilde{S} = \int d^4x \left\{ \Phi_1(B) \Big[R(g) - 2\Lambda_0 \frac{\Phi_1(B)}{\sqrt{-g}} \Big] - \frac{\sqrt{-g}}{\Phi_1(B)} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda} \right. \\ \left. + \Big(\sqrt{-g} + \Phi_0(A) \Big) \Big[-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \Big] \right\} . \tag{68}$$

Upon passage to the Einstein-frame via the conformal transformation (30), the action (68) becomes (cf. (39)):

$$\widetilde{S}_{\rm EF} = \int d^4x \Big[\sqrt{-\bar{g}} \Big(\bar{R} - \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu u\, \partial_\nu u - U_{\rm eff}(u) \Big) - e^{-u/\sqrt{3}} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda} \Big] \\ + \int d^4x \sqrt{-\bar{g}} (1 + \chi_0) e^{-u/\sqrt{3}} \Big[-\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \varphi\, \partial_\nu \varphi - e^{-u/\sqrt{3}} \big(V(\varphi) - 2M_0 \big) \Big] . \tag{69}$$

The coupling term $e^{-u/\sqrt{3}} \varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda}$ is suppressed in the inflationary stage where the derivative of u is small (because of the slow roll regime), whereas after end of inflation it may produce pairs of photons out of u due to the appreciable time-derivative of u resulting from the oscillations near the minimum of the effective potential. Of course, many other possible interaction terms can be introduced.

Finally, in the reheating stage, many particles can be produced; some of them could be no standard-model particles. If those are stable, they could provide additional "dark matter" apart from the "darkon" dust-like dark matter discussed here. Of course, if all created particles beyond those of the standard models turn out to be unstable, then we will be left with the "darkon" as the unique source of dark matter.

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