



CFS Working Paper Series

No. 684

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Fixed and Variable Longevity Annuities in
Defined Contribution Plans: Optimal
Retirement Portfolios Taking Social
Security into Account

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January 8, 2023

Abstract

This paper investigates retirees' optimal purchases of fixed and variable longevity income annuities using their defined contribution (DC) plan assets and given their expected Social Security benefits. As an alternative, we also evaluate using plan assets to boost Social Security benefits through delayed claiming. We determine that including deferred income annuities in DC accounts is welfare enhancing for all sex/education groups examined. We also show that providing access to well-designed variable deferred annuities with some equity exposure further enhances retiree wellbeing, compared to having access only to fixed annuities. Nevertheless, for the least educated, delaying claiming Social Security is preferred, whereas the most educated benefit more from using accumulated DC plan assets to purchase deferred annuities.

Keywords: lifecycle saving; retirement plan; annuity; longevity; delayed claiming; household finance

JEL codes: G11, G22, D14, D91

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Acknowledgments: The authors are grateful for comments from John Ameriks, Mark Iwry, Jake Lund, and Richard Shea, and for participants at Goethe University's ICIR Conference on "Private and Social Insurance Implications of Demographic Change." We also acknowledge research support from the TIAA Institute as well as funding provided by the German Investment and Asset Management Association (BVI) and the Pension Research Council/Boettner Center at The Wharton School of the University of Pennsylvania. Opinions and any errors are solely those of the authors and not those of any individuals cited or any institutions with which the authors are affiliated. ©2023 Horneff, Maurer, and Mitchell

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The US Social Security retirement system pays retirees a lifetime annuity with fixed real benefits that depend (progressively) on retirees' earning histories and claiming ages. For this reason, if a retiree receives a substantial portion of her income through a Social Security annuity, it stands to reason that her remaining financial portfolio should include substantial exposure to risky equities, through a target date fund or with annuities whose payments are linked at least in part to the performance of an equity portfolio. Additionally, Social Security replacement rates are higher for lifetime low-earners, and lower for lifetime high-earners. As a result, low lifetime earners receiving a higher replacement rate could decide to devote a greater proportion of their remaining financial wealth to risky equities, through a target date fund or direct equity investment. Conversely, higher lifetime-earning retirees receiving a relatively low Social Security replacement rate may wish to purchase larger private annuities in old age from their tax-qualified retirement accounts, to provide a predictable income stream sufficient to cover necessities. Moreover, the recent 2019 SECURE Act encouraged the inclusion of annuities in defined contribution (DC) plans and Individual Retirement Plans (IRAs) worth \$21.3 trillion (ICI 2021), providing plan sponsors "safe harbor" rules for their inclusion.

This paper focuses on how these two instruments – annuities with lifelong benefits purchased using DC plan assets, and Social Security annuities – should be considered jointly to optimize household welfare. Understanding how these interact is of key importance in order to generate efficient retirement portfolios. Additionally, there is likely to be substantial heterogeneity in the demand for longevity annuities across the retiree population, depending on their assets inside and outside tax-qualified retirement plans, their mortality assumptions, their accrued Social Security benefits, and longevity.

Federal regulation requires compliance with various conditions for the purchase of an annuity to qualify for favorable tax treatment (known as a qualifying life annuity contract, or QLAC). The purchase price of the QLAC cannot exceed 25% of the assets in the DC plan and, at the same time, cannot exceed \$130,000; moreover, the lifetime income payments must begin no later than age 85. Except for a refundable premium option, the QLAC may not include death

benefits. Of particular interest in this paper are the regulations regarding the nature of lifetime payments. The Internal Revenue Service explicitly stated (Federal Register Vol. 79, No. 12/, July 2, 2014) that QLACs must generally make predictable (nominal or real) fixed lifetime payments. Linking payments to a stock market index or to a portfolio of mutual funds is expressly not permitted, even if there is a minimum guaranteed income under such contracts, also known as variable or investment-linked annuities.¹

We analyze several research questions, as follows: What should be the retiree's optimal portfolio fraction of DC plan assets allocated to longevity annuities, bonds, and risky stocks? When should DC plan assets be used to permit retirees to (a) defer claiming Social Security benefits and cover spending needs in the meanwhile, or (b) buy a deferred annuity while simultaneously claiming Social Security benefits? To what extent should a longevity annuity in a DC plan offer fixed versus variable payouts, where the latter are linked to the return of an underlying portfolio of risky stocks and bonds (e.g., in a participating annuity or investment-linked annuity)? What welfare gains are feasible, given these annuities? How does the selected deferral period of the QLAC alter outcomes: do people prefer short deferrals (as in the case of immediate annuities), or will retirees favor the maximum possible regulatory deferral period until age 85? Is something in between even better?

There are three literatures to which our work is related: economic studies on life cycle financial decision making, the decision to purchase annuities in retirement, and delayed claiming of Social Security benefits. Excellent reviews of the first area include Gomes (2020) and Gomes et al. (2021) who discuss dynamic consumption and portfolio choice models in discrete time. For the second area, we build on previous studies about the optimal demand for annuities (e.g., Huang et al. 2017; Horneff et al. 2010, 2020; Inkmann et al. 2011; Milevsky 2005) by exploring different deferral ages for the lifelong annuity. A third literature examines the pros and cons of delaying Social Security claiming (e.g., Hubener et al. 2016; Shoven and Slavov 2014). We bring these three threads together by integrating the decision to delay claiming and annuitization. Closest to our work is Munnell et al. (2022) who discussed the possibility of using DC plan assets at retirement to finance delayed claiming or buy fixed annuities. Compared to the latter paper, our contributions are to embed the decisions in a full life cycle model, which starts at age 25, runs to 100, and it also incorporates optimal saving and investing across bonds and risky stocks, consumption, and

¹ Interestingly, however, participating life annuities, where payments are linked to the overall investment experience of a life insurance company, are consistent with the regulatory requirements of a QLAC; see Maurer et al. (2016).

withdrawal patterns for assets inside as well as outside the DC plan. Our model also includes heterogeneity in lifetime earnings, assets, and mortality across education groups, and importantly, we incorporate important institutional aspects including the progressive and complex US income tax code and Social Security benefits formula. And finally, we investigate optimal annuitization ratios for both fixed as well as variable annuities and alternative deferral ages.

In what follows, we outline the methodological foundations of our life cycle model which we use to answer these questions. Subsequently we illustrate how we realistically calibrate the model parameters, and we use a matching procedure to select preference parameters so that the model results match the empirically observable assets invested by US workers in tax-qualified defined contribution retirement plans as closely as possible. Next, we use our model to analyze the demand for and welfare consequences of four alternative settings: claiming Social Security at age 66 or 67 without access to deferred income annuities (DIAs); and claiming Social Security at age 66 with access to fixed or variable DIAs. We extend prior research by comparing the value of people purchasing private annuities, versus delaying Social Security benefits using assets from their DC accounts to finance consumption. We document that using retirement account assets to purchase at least some fixed deferred income annuities is welfare enhancing for all sex/education groups examined, and allowing payout annuities to have a small exposure to equity can further enhance welfare. Nevertheless, for the least educated, delaying claiming Social Security benefits is preferred, whereas the most educated benefit more from using accumulated DC plan assets to purchase deferred annuities.

I. Life Cycle Model: Methodology

Our discrete time dynamic portfolio and consumption model posits a utility-maximizing worker who decides how much to consume optimally and how much to invest in risky stocks, bonds, and annuities over her lifetime. We model utility as depending on consumption and bequests, while constraints include a realistic characterization of income profiles, taxes, and the opportunity to invest in risky stocks and riskless bonds both in a DC tax-qualified retirement plan (up to a limit) as well as in non-tax-qualified accounts. At retirement (assumed here to be age 66), the individual determines how much of her retirement account she wishes to convert to a deferred income annuity (DIA), with the remainder held in stocks and bonds. We also take into account the Required Minimum Distribution

rules relevant to the US DC setting, as well as a realistic formulation of Social Security benefits and mortality heterogeneity across educational categories.²

A. Preferences

The individual's decision period starts at $t = 1$ (age of 25) and ends at $T = 76$ (age 100); accordingly, each period corresponds to a year. The individual's subjective probability of survival from time t until $t + 1$ is denoted by p_t^s . Preferences at time t are specified by a time-separable *Epstein-Zin* utility function defined over current consumption, C_t . The parameter ρ represents the coefficient of relative risk aversion, ψ the elasticity of intertemporal substitution (*EIS*) and β is the time preference rate on future utility. The term Q_{t+1} denotes the level of bequest at time $t+1$. The strength of the individual bequest motive is controlled by variable b . Then the recursive definition of the corresponding value function is given by:

$$J_t = \left\{ (1 - \beta)C_t^{1-1/\psi} + \beta \left(\sum_i \Pi_{ij,t} E_t \left(p_t^s J_{t+1}^{1-\rho} + (1 - p_t^s) b \left(\frac{Q_{t+1}}{b} \right)^{1-\rho} \right) \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \quad (1)$$

where the $\Pi_{ij,t} = Prob(l_{t+1} = i | l_t = j)$ is a time-dependent transition matrix representing the probability to move from current (t) income level j to income level i one year later ($t + 1$).

B. Annuity: Pricing and Payouts

At age 66 (K), the individual determines how much (up to 25%) of her DC plan assets ($DIA_K \leq \min(0.25L_K, \$130,000)$) she will switch to a deferred annuity paying lifetime income benefits starting at age τ (age 67, 80, or 85). The idea of using small amounts of accumulated assets to purchase deferred annuities was originally proposed by Milevsky (2005), who favored these over a more costly single premium immediate annuity. We consider two alternative products.

Fixed annuity: In case of a fixed longevity annuity purchased at age K for a nonrefundable premium of DIA_K , the life insurer begins paying fixed lifelong benefits (FPA) from age τ as follows:

² Throughout this paper, we work in real terms (e.g., for labor income and asset returns). This is justified as the Social Security bend points, the brackets for income taxation, and the maximum contribution limits to retirement plans are updated annually for inflation.

$$FPA_t = \frac{DIA_K}{\ddot{a}_\tau} \quad (t \geq \tau), \quad (2)$$

where $\ddot{a}_{K,\tau} = p_{K,\tau}^a \sum_{s=0}^{121-\tau} (\prod_{i=\tau}^{\tau+s} p_i^a) R_f^{-(s+(\tau-K))}$ is the annuity factor. Here, R_f is the interest rate and p_i^a are the yearly survival probabilities the insurance company uses to price the annuity. These probabilities are derived from actuarial mortality tables and generally differ from subjective survival probabilities used in the utility function. Moreover, $p_{K,\tau}^a$ is the cumulative probability of surviving from age K until the end of the deferral period τ .

Variable Annuity: A variable payout annuity is a financial contract between a retiree and a life insurance company, whereby in exchange for paying an initial nonrefundable premium, the annuitant begins to receive lifelong payments when the deferral period is over, equal to the value of a pre-specified number of units in a specific asset portfolio (stocks, bonds, or some combination) represented by a mutual fund, index fund, or exchange-traded fund. Since payments depend on the value of the annuity fund units, they will be stochastic when the underlying portfolios hold risky assets. Hence variable longevity annuities offer both an investment element, in terms of a mutual fund-style subaccount, and an insurance element, in terms of pooling longevity risks across the retiree group. Following Maurer et al. (2013), the payouts from the variable annuity (*VPA*) purchased at age K which start at age τ are as follows:

$$VPA_{t+1} = \frac{VPA_t}{AIR} R_{t+1}^P \quad t \geq \tau. \quad (3)$$

Here, $R_{t+1}^P = R_f + \alpha_t(R_{t+1} - R_f)$ denotes the annual gross return of a portfolio invested in risky stocks with a share of α_t and $(1 - \alpha_t)$ in bonds. The first payment after the end of the deferral period is given by $VPA_\tau = \frac{DIA}{N_{K,\tau}} * \prod_{i=K}^{\tau} R_i^P$. Here $N_{K,\tau} = p_{K,\tau}^a \sum_{s=0}^{121-\tau} (\prod_{i=\tau}^{\tau+s} p_i^a) AIR^{-(s)}$ is the annuity factor, and $\prod_{i=K}^{\tau} R_i^P$ is the cumulative performance of the underlying asset portfolio within the deferral period. The payouts (after the deferral period) from the variable annuity are given by an updating rule that relates the annuity payouts VPA_{t+1} in the next year to the previous payout VPA_t and the realized investment return R_{t+1}^P on the underlying portfolio relative to the assumed interest rate (*AIR*) set by the insurance company.³ The annuity payment rises when $R_{t+1}^P > AIR$, it falls when $R_{t+1}^P < AIR$, and is constant when $R_{t+1}^P = AIR$.

C. Cash on Hand

³ See Horneff et al. (2010) for a detailed discussion of the role of the AIR in the annuity context.

While working, the individual has the opportunity to invest a portion (A_t) of her uncertain pre-tax salary Y_t (up to an annual limit of \$18,000)⁴ in a tax-qualified retirement plan as well as in non-tax-qualified risky stocks S_t and risk free bonds B_t :

$$X_t = \begin{cases} C_t + S_t + B_t + A_t, & t < K \\ C_t + S_t + B_t, & t \geq K. \end{cases} \quad (4)$$

Here X_t is after-tax cash on hand, C_t denotes consumption, and $C_t, A_t, S_t, B_t \geq 0$ and $A_t \leq \$18,000$ to age 51; additional retirement plan ‘catch-up’ contributions are permitted over age 51 of up to \$6,000. One year later, her cash on hand is given by the value of her stocks having earned an uncertain (real) gross return R_t , bonds having earned a riskless return of R_f , labor income Y_{t+1} reduced by housing costs h_t modeled as a percentage of labor income (as in Love 2010), and withdrawals (W_t) from her DC plan, where withdrawals before age 59 1/2 result in a 10% penalty tax and are restricted to certain amounts.⁵

$$X_{t+1} = \begin{cases} S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} - Y_{t+1} d_w, & t < K \\ S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} - Y_{t+1} d_r, & K \leq t < \tau \\ S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} + PA_t - Y_{t+1} d_r, & t \geq \tau. \end{cases} \quad (5)$$

During her worklife, the individual pays taxes which reduce cash on hand available for consumption and investment. We posit that labor income is reduced by 11.65% (d_w), which is the sum of the Medicare (1.45%), city/state (4%), and Social Security (6.2%) taxes (up to a cap of 127,200 per year). In addition, the worker also must pay income taxes (IT_{t+1}^{tax}) according to US federal progressive tax system rules (for details, see IRS 2017 and Appendix A).

The individual may save in a tax-qualified DC plan only during the working period, while non-pension saving in bonds and stocks is allowed over the entire life cycle. We model the exogenously-determined labor income process as $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$ with a deterministic trend $f(t)$, a permanent income component $P_{t+1} = P_t \cdot N_{t+1}$, and a transitory shock U_{t+1} , using data from the Panel Study of Income Dynamics (PSID).⁶ In our life cycle model, we work with a discrete Markov-

⁴ The \$18,000 limit was the legal limit on U.S. tax-deferred contributions to DC plans in 2017; also, if the worker’s plan permitted by the plan, employees age 50+ could make additional catch-up contributions of \$6,000 per year; see IRS (2017).

⁵ In the case of matching contributions, early withdrawals from tax-qualified retirement plans before age 60 usually require the employer’s approval, typically given only in exceptional circumstances (hardship) and within limits. To address this, our model permits early withdrawals prior to age 60 if the worker has less than \$20,000 cash on hand; early withdrawals of up to half of the retirement plan assets are allowed. Both a 10% penalty tax and income tax must be paid on such withdrawals.

⁶ The PSID is a project of the National Institute on Aging, fielded at the University of Michigan; for more information see [Panel Study of Income Dynamics \(PSID\) | National Institute on Aging \(nih.gov\)](https://www.nia.nih.gov/psid).

switching income process, $Y_{t+1} = I_{t+1}^l \cdot U_{t+1}^{MS}$, for three income profiles each, for men and women. The transitory shocks U_{t+1}^{MS} depend on age, sex, and income by educational categories.

During retirement, the individual saves in stocks and bonds, and she also consumes. The DIA pays lifelong benefits from age τ onwards. In retirement, she also receives lifelong Social Security benefits determined by her Primary Insurance Amount (PIA) which is a function of her average indexed lifetime earnings (the AIME).⁷ Her Social Security payments (Y_{t+1}) in retirement ($t \geq K$) are given by:

$$Y_{t+1} = PIA_K^l \cdot \varepsilon_{t+1}, \quad (6)$$

where ε_t is a lognormally-distributed transitory shock $\ln(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$ with a mean of one which reflects out-of-pocket medical and other expenditure shocks (Love 2010).⁸ During retirement, Social Security benefits are taxed (up to a limit)⁹ at the individual federal income tax rate as well as the city/state/Medicare tax rate.

D. The Tax Qualified DC Plan

Prior to retirement, the retiree's total value (L_{t+1}) of her DC assets at time $t + 1$ (for $t < K$) is determined by her previous period's value, minus any withdrawals ($W_t \leq L_t$), plus additional contributions (A_t), and returns from stocks and bonds:

$$L_{t+1} = \begin{cases} \omega_t^s(L_t - W_t + A_t + M_t)R_{t+1} + (1 - \omega_t^s)(L_t - W_t + A_t + M_t)R_f, & t < K \\ \omega_K^s(L_K - W_K - DIA_K)R_{K+1} + (1 - \omega_K^s)(L_K - W_K - DIA_K)R_f, & t = K \\ \omega_t^s(L_t - W_t)R_{t+1} + (1 - \omega_t^s)(L_t - W_t)R_f, & t \geq K. \end{cases} \quad (7)$$

Her retirement plan assets are invested in a Target Date Fund having a relative stock exposure (ω_t^s) that declines according to age, following the popular "125-Age rule" ($\omega_t^s = (125 - \text{Age})/100$).¹⁰

To be considered as a safe harbor DC plan and, therefore, avoid complex non-discrimination testing, we assume that the employers match 100% of employee contributions up to

⁷ The US Social Security benefit formula is a piece-wise linear function of the Average Indexed Monthly Earnings providing a replacement rate of 90% up to a first bend point, 32% between the first and a second bend point, and 15% above that.

⁸ The assumed transitory variances are $\sigma_\varepsilon^2 = 0.0784$ for high school and less than high school graduates, and $\sigma_\varepsilon^2 = 0.0767$ for college graduates (Love 2010).

⁹ For details on how we treat Social Security benefit taxation, see the Appendix A. Due to quite generous allowances, relatively few individuals pay income taxes on their Social Security benefits.

¹⁰ This approach satisfies the Qualified Default Investment Alternative (QDIA) rules as per US Department of Labor regulations (nd).

5% of yearly labor income.¹¹ Due to regulation, the matching rate was applied only to a maximum compensation of \$270,000 (in 2017), so the maximum employer contribution was \$13,500. The matching contribution is then given by:

$$M_t = \min(A_t, 0.05Y_t, \$13,500). \quad (8)$$

The amount used to buy the DIA reduces the value of her DC assets invested in stocks and bonds. Wealth dynamics of the DC account are given by the previous value L_t , withdrawals W_t , and investment returns on stocks and bonds.

The US Treasury stipulates that DC participants take required minimum withdrawals (RMDs) from their plans from age 70.5 onwards or else they must pay a substantial tax penalty (50%); these are defined as a specified age-dependent percentage (m_t) of plan assets.¹² Yet the value of the DIA is excluded when determining the retiree's RMD. Therefore, to avoid the excise penalty, plan payouts are set so that $mL_t \leq W_t < L_t$. Benefit payments from the deferred annuity are counted in taxable income. It should be noted that, at present, US regulation allows only fixed annuities to be excluded from the RMD calculation. Nevertheless, in our analysis below, we also examine outcomes if variable annuities were eligible for QLAC treatment, for purposes of comparison as a policy experiment.

II. Model calibration

To calibrate the model, we use financial market parameters of the risk-free interest rate at 1% and an equity risk premium of 4% with a return volatility of 18%. For the variable annuity, the assumed interest rate is set equal AIR is 2%, and we consider three allocations to equities: 20%, 50%, and according to the 125-age rule. Survival rates are taken from the US Population Life Table 2017 (Arias and Xu 2019) with heterogeneity across sex/educational categories (Krueger et al. 2015); and for annuity pricing, we use the US Annuity 2012 mortality table (unisex, trend function until 2017) provided by the Society of Actuaries (SOA 2012). Annuity survival rates are higher than those for the general population because they account for adverse selection by annuity purchasers. Social security old age benefits are based on the 35 best years of income and the bend points as of 2017 (US Social Security Administration, nd_a and _b).¹³ The age-dependent percentages (m_t) of Required Minimum

¹¹ See 401(k) HelpCenter.com (2017).

¹² The 2019 Secure Act raised the RMD age to 72, and there is an ongoing discussion about whether to raise the age further to age 75; see Tepper (2022).

¹³ Accordingly, the annual Primary Insurance Amount (or the unreduced Social Security benefit payment) equals 90 percent of (12 times) the first \$885 of average indexed monthly earnings, plus 32 percent of average indexed monthly

Distributions from DC plans are calculated as one divided by the retiree's remaining life expectancy using the Internal Revenue Service (IRS 2015). In line with US rules, federal income taxes are calculated based on the household's taxable income, six income tax brackets, and the corresponding marginal tax rates for each tax bracket (see Appendix A).

The labor income process during the worklife has both permanent and transitory components with uncorrelated and normally distributed shocks as $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$ and $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$. We estimate the deterministic component of the wage rate process w_t^i along with the variances of the permanent and transitory wage shocks N_t^i and U_t^i using the 1975–2017 waves of the PSID. These are computed separately by sex for three education levels: high school dropouts, high school graduates, and those with at least some college (<HS, HS, Coll+).¹⁴ Wage rates are converted into yearly income by assuming a 40-hour workweek and 52 weeks of employment per year. At age 66, the retiree receives a combined income stream from her DC plan and Social Security benefits, and from age 85 on, payments from the longevity annuity. Results for the six subgroups appear in Figure 1, where Panel A reports the expected income profiles for males, and Panel B for females, by education group. Using 200,000 simulation paths, we re-estimate each of labor income profiles using a Markov-switching model with three income levels, to generate the time-dependent transition matrices for permanent income as well as the age-dependent transitory shocks (see Appendix B).

Figure 1 here

We use dynamic stochastic programming to solve the individual's optimization problem. There are five state variables: wealth, the total value of the individual's DC assets, payments from the longevity income annuity, three income levels, and time.¹⁵ We also compute individual consumption and welfare gains under alternative scenarios using our modeling approach.

Values of the preference parameters for the six subgroups are selected so that the model generates DC wealth profiles consistent with empirical evidence. Specifically, we calibrate the model to PSID data 1999-2017 for five age groups (20-29, 30-39, 40-49, 50-59, and 60-69). To generate DC simulated balances, we first solve the lifecycle model where people claim at age 66 and lack access to longevity income annuities. Then we use the model to generate 200,000 lifecycle simulations weighted

earnings over \$885 and through \$5,336, plus 15 percent of average indexed monthly earnings over \$5,336 and through the cap \$10,600. All dollar values are reported in \$2017.

¹⁴ Additional details appear in Appendix C.

¹⁵ For discretization, we split the five dimensional state space by using a 40(X)×20(L)×15(PA)×3(l)×76(t) grid size. For each grid point we calculate the optimal policy and the value function.

for the six subgroups¹⁶ to generate the matching values. We find that a risk aversion coefficient of ρ of 7, ψ of 0.35, b of 1.1, and a time discount rate β of 0.95 are the parameters that closely match simulated model outcomes to empirical evidence on pension assets. Figure 2 displays simulated and empirical data for five age groups, and interestingly, our simulated outcomes are remarkably close to the empirically-observed account values.

Figure 2 here

III. Results

We analyze and compare DC wealth and withdrawal profiles, retirement income, and consumption behavior in four different settings: claim @66 w/o DIA; claim @67 w/o DIA; claim @66 w/ fixed DIA 85; and claim @66 w/variable DIA 85). In all four settings, the investor can consume and invest inside and outside her DC plan. In both of the first two settings, the retiree lacks access to a DIA, but the retirement claiming age is 66 in setting 1 versus 67 in setting 2. Since the retiree in the second setting receives no earned income or Social Security benefits during age 66, she must withdraw sufficient cash from her retirement plan to cover her consumption that year. The following year, her Social Security benefits are 8% higher and remain so from then on. In setting 3 (4), the retiree has access to a fixed (variable) DIA with payouts starting at age 85; her claiming age remains at 66. Figure 3 summarizes the different settings, assuming the variable DIA holds a flat 50% of its portfolio in equity and the remaining portion in bonds.

Figure 3 here

Expected Optimal Life Cycle Patterns in Retirement

Figure 4 provides a comparison of the expected optimal life cycle patterns for ages 60-100 for plan assets (Panel A) and withdrawals (Panel B); labor earnings, Social Security benefits, and DIA payments (Panel C); and consumption (Panel D), all based on our simulated data for the US population.¹⁷ In all panels, the solid black line shows outcomes when the retiree lacks access to DIA and claims benefits at age 66. The brown dotted line describes profiles relevant to the individual who also lacks access to the DIA but defers Social Security claiming by one year (to age 67). The green

¹⁶ To obtain national averages, we aggregate the 200,000 simulated subgroup life-cycle patterns using weights from the National Center on Education Statistics (2016). Specifically, the number of simulations by sex/education are: male <HS 13,000, HS 30,000, Coll+ 57,000; females <HS 11,000, HS 28,000, Coll+ 61,000.

¹⁷ Results for the complete life cycle from age 25 are available on request; we do not report these before age 60 here, as we are most interested in behavior during the retirement phase. Starting the simulation process at age 25 generates the heterogeneity in income, wealth, saving, investment, and Social Security accruals required to understand withdrawal, annuitization, and consumption patterns for different population subgroups.

dotted line portrays averages when the individual can purchase a fixed DIA using her DC plan assets up to the allowed limits. The green solid line shows results for the average individual permitted to purchase the variable 50/50 equity/bond DIA using her retirement plan assets.

Figure 4 here

During their work lives, individuals optimally save a portion of their pay in tax-qualified DC plans, building assets (including returns) worth an average of \$250,000 by age 65, in all four settings (Panel A). Large differences in retirement plan wealth are not apparent until age 66. In the “Claim @67, w/o DIA” setting, the retiree must withdraw enough to cover consumption in that year, generating a spike in withdrawals averaging \$23,000 (Panel B) due to having no Social Security benefits or earnings. The decrease in assets to finance consumption observed at age 66 for retirees having access to DIAs is attributable to their purchasing either a fixed or variable DIA: average withdrawals total approximately \$50,000, of which about \$14,000 is used to cover consumption costs, and \$36,000 for the DIA purchase. Thereafter, withdrawal amounts are similar under the four settings until age 85. The patterns diverge from age 85 onwards, where the retiree lacking access to DIAs continues withdrawing smoothly until the end of her life, whereas retirees with DIAs nearly exhaust their retirement accounts, relying instead on the fixed (variable) DIA benefits for life.¹⁸

In Panel C, we compare the optimal labor, Social Security, and DIA income profiles in the four settings. The black solid line traces out the drop in labor earnings at age 66, with Social Security benefits replacing about half of labor income on average. Deferring claiming by a year implies no labor income at age 67, and Social Security benefits are then higher by 8% for life due to the delay in claiming. When the retiree has access to a DIA, her income increases from age 85 onward. This is due to fixed DIA annual payouts of \$8,200 per annum, or for the variable DIA at age 85, the expected payout starts at \$13,000 and then rises to \$15,000 by age 100. The explanation for the rising annuity profile in the latter case is that the expected return on the portfolio backing the variable annuity (3%) holding 50% equity exceeds the insurer’s Assumed Interest Rate (2%).

Panel D reports expected consumption profiles in the same four settings. The lowest average consumption path is observed for the retiree lacking DIA access who claims Social Security at age 66. By comparison, the “Claim @67, w/o DIA” setting provides higher consumption between age 60-100 compared to the base case, since the 8% Social Security benefit boost more than compensates the retiree for having taken the large “bridge” withdrawal from her DC plan at 67. The consumption

¹⁸ This is similar to what Munnell et al. (2022) report, in line with their assumed rule of thumb drawdown plan when the retiree depletes all of her retirement assets by age 85 when the DIA begins paying out.

increase experienced in the second setting also rises as the retiree ages. A retiree with a fixed DIA has very slightly lower consumption before age 85, compared to the base case. Thereafter, consumption rises substantially when the DIA payout starts, and the gap grows with age. An even more pronounced change applies in the 50/50 stock/bond variable DIA case: the increase in expected consumption begins slightly earlier, yet the difference post age 85 is much higher. For instance, at the age of 90, expected consumption would total \$26,500 with no annuity; \$27,400 if the retiree delayed Social Security claiming; \$28,300 with a fixed DIA; and \$30,700 with a variable DIA.

A look at Panels B-D confirms that optimal consumption in retirement can exceed annual income. For example, the retiree who claims at 66 and has access to a fixed annuity would have Social Security income, DIA payments, and DC plan withdrawals at age 85 totaling \$34,100. Expenditures for taxes and consumption are lower, amounting to \$31,100. This confirms that the retiree behaving optimally is not always a hand-to-mouth consumer (as in Munnell et al. 2022).

Optimal Demand for DIA at Retirement

Let us now consider optimal investments at retirement in the two types of DIAs of most interest here. The first has a fixed payout from age 85 onward, which we call the “fixed DIA 85.” The second, labelled the “variable DIA 85,” is invested 50/50 in equities/bonds and provides a variable payout from age 85. The *DIA Ratio* indicates the optimal percentage of assets in the tax-qualified retirement plan that the retiree converts to a DIA at age 66. Panel A in Figure 5 compares the distribution of this ratio for the fixed versus the variable DIA; both distributions are based on 200,000 simulation paths for the US population discussed above (men/women for three education levels). The x-axis in both figures runs from 0% to the maximum value of 25%, where the latter results from the IRS tax qualification requirements for a longevity annuity to count under the RMD rules. Results show that about 84% of the population would be interested in a “fixed DIA 85;” by contrast, 88% of the population favors the “variable DIA 85.” Panel B shows that for a given DIA ratio, the demand for a variable DIA exceeds the demand for a fixed annuity.

Figure 5 here

Table 1 reports the average optimal DIA ratio for each of the sex/education groups considered, where we now vary the annuity’s deferral period. Naturally, this does not mean that a DIA which starts paying out at age 67 would provide a higher utility than a DIA starting at age 80. Rather, the comparisons posit that retirees in the two different settings have no choice between deferred DIAs with different payout start dates. We also examine the case of annuities that pay benefits from age 67, and separately, from ages 80 and 85, respectively. The optimal investment in a “fixed DIA 85” for a

Coll+ female (HS, <HS) is 12% (9%, 4%) in expectation. For a Coll+ male (HS, <HS), the optimal DIA Ratio is a bit higher, at 13% (9%, 6%). There are two reasons for these differences: first, the least educated have higher mortality rates; and second, the Social Security annuity is relatively higher for lower earners. For the age 67 annuity, our numbers for the optimal DIA ratio are in line with the 20% default ratio suggested by Munnell et al (2022), for both sexes and all ages examined. Moreover, results for the DIA payable at age 85 are similar to those reported by Horneff et al. (2020). Figure 5, Panel B shows that the optimal fraction of plan assets held in a fixed DIA would equal 10.7% versus 11.9% in a variable DIA, and only 5% would optimally elect DIA ratios of 20% or more. Accordingly, our optimal life cycle results are compatible with a substantially lower demand for DIAs with later deferral ages than the 20% default rate examined by Munnell et al. (2022) for similar products.

Moreover, for all educational groups, the demand for a variable DIA is 1-2% greater than for the fixed DIA. We also observe that for shorter deferral periods, the optimal DIA ratio increases regardless of sex, education, and type of DIA. The gap is also less for shorter deferral periods, comparing the most and the least educated. For example, the optimal demand for a fixed DIA payable at age 85 is three times higher for the Coll+ female, compared to the high school dropout. By contrast, the DIA ratio payable at age 67 is only 1.4 times higher for the most versus the least educated. Again, this is because the least educated face higher mortality rates, hence desire earlier payments, and they also enjoy relatively higher Social Security replacement rates.

Table 1 here

IV. Welfare Gains

Next, we compare the utility implications of the settings shown in Figure 3, where the values illustrate how much more money an individual in the “Claim @66, w/o DIA” reference setting would need in her retirement plan to achieve the same welfare as in the other settings. Specifically, Table 2 reports on three alternative outcomes. The left-most two columns identify how much additional money the retiree would require if she claimed Social Security at age 66 and could purchase either a fixed or a 50/50 stock/bond variable DIA commencing at age 85. The right-most column examines how much the retiree would need to be as well off by delaying claiming one year to age 67, compared to age 66, with no access to annuity in either case. Note that the leisure available in both regimes is identical since the individuals do not work longer, but they wait a year to apply for Social Security benefits.

Table 2 here

All values in Table 2 are positive, which means that retirees are always better off than they would be if they claimed at age 66 with no DIA. For example, the Coll+ female would require \$38,804 in additional wealth if she had to claim benefits at age 66 with no DIA, versus having a fixed annuity payable from age 85; \$41,305 if she lacked access to a variable DIA 85 invested 50/50 in equity/bonds; and \$20,594 if she lacked access to DIA but could claim a year later, at age 67.¹⁹ For Coll+ men, the welfare gains are, respectively, \$46,870, \$50,207, and \$26,809. Hence, for both educated subgroups, access to the variable DIA is more welfare-enhancing than claiming at 66 and having no DIA.

Table 2 here

The same is not true for all educational groups, however, illustrating the importance of population heterogeneity. For instance, a female high school dropout is financially better off if she claims later. Thus, she would need \$7,725 more assets if she could not claim at 67 instead of age 66. Conversely, if she lacks access to a fixed (variable) DIA, then she would require only \$3,410 (\$4,295) more to make her as well off as claiming at 66. A similar situation arises with the male high school dropouts. In other words, the least educated do relatively better by deferring claiming Social Security benefits for a year without any annuity, because they have a higher Social Security replacement rate (not capped) and a higher mortality risk; hence this group is less interested in deferred annuities of either type.

To evaluate how much additional money the retiree in the “Claim @67, w/o DIA” setting would need to have to attain the same welfare as in the “Claim @66, w/ fixed DIA” setting, we report results in Table 3 using this new reference case and three deferral ages (85, 80, and 67). Here all groups except the high school dropouts would favor a fixed DIA rather than claiming later. For instance, the Coll+ woman who could delay claiming but lacks access to the fixed DIA requires an additional \$17,367 in her DC plan to be as well off. The opposite is true for female high school dropouts: they are, on average, \$4,056 worse off if they cannot delay claiming but do have a DIA. A similar pattern applies to men. There are two reasons for this difference by educational levels. The first is that Social Security benefits are capped for high earners. Since college educated workers earn more than the Social Security earnings cap, when they delay claiming, they receive 8% more benefits but only to a maximum. Accordingly, the better educated benefit less at the margin from delayed claiming, compared to high school dropouts earning below the cap. The second reason has to do with different

¹⁹ This is similar to Munnell et al. (2022) who report that wealthier retirees would have higher welfare gains by using retirement assets to purchase a fixed DIA versus delaying claiming Social Security benefits.

survival probabilities by education: the least educated are likely to die earlier, implying that a DIA payable from age 85 is less attractive for the lower educated, versus a shorter deferral period.

Table 3 here

Table 3 also shows that fixed DIAs payable from age 80 onward are also most attractive to men and women of all education levels, versus a fixed DIA payable from age 67. For instance, female (male) college graduates have a 2.2-2.6 (2.6-3.0) times larger welfare gain from a deferral age of 80, compared to the DIA paying from age 67. This is true because the DIA for the younger deferral age is costly, while the deferred product costs much less and provides a much higher benefit to those who live long. Nevertheless, rapidly rising mortality risk beyond age 80 makes later deferral less appealing. For example, the probability that a male age 66 attains age 80 is 65%, but only 45% for reaching age 85.

Next, in Table 4 we extend the analysis of welfare results by examining three types of variable DIAs embodying stock: a 20% fixed fraction, a 50% fixed fraction, and a life cycle glide path with the equity share totaling $((125-\text{Age})/100)$. As in Table 3, the dollar values represent the additional assets the retiree would need if she held the respective DIA, versus claiming at age 67 with no annuity. We see positive values for both better educated groups across all deferral ages and all three variable annuity types. Once again, we note that the annuity deferral to age 80 is preferable to the two other deferral ages considered. By contrast, all results are negative for the high school drop outs, meaning that the least educated prefer delayed claiming to age 67, versus all of these variable annuities. This is because, for the least educated, delaying claiming Social Security and using their accumulated savings to bridge their consumption for a year is strongly preferred to an annuity offered by a life insurance company.

Table 4 here

It is also informative to compare Tables 3 and 4, to evaluate whether giving retirees access to some equities in their variable DIAs increases their welfare gains, versus only having access to a fixed DIA. Our analysis confirms that, for almost all of the education/sex groups, deferring ages, and stock fractions, the welfare gains from a variable annuity exceed those in the corresponding case in Table 3. For the variable DIA with either 50% equities or a life cycle glide path payable from age 80, educated women can expect an additional welfare gain of 13-15% (respectively, 24,074/20,989-1, and 23,667/20,989-1) compared to the fixed DIA; for men, the comparable gain is on the order of 22-27% (respectively, 26,492/21,729-1 and 26,728/20,989-1). Interestingly, even the smallest equity exposure

we study, of 20%, boosts welfare of the high school dropouts by more than 25% for females and males by 49%, compared to a fixed deferred annuity.²⁰

It is also worth noting that there are a few cases where the welfare gains with variable annuities are lower than for fixed annuities: one occurs for the highest-educated men and women having a low (20% equity exposure in the DIA) and a deferral age of 67. This interesting case arises because, here, the expected portfolio return is lower than the AIR assumed when pricing the annuity policy; this in turn produces a declining payout pattern which undermines the annuity's longevity protection feature. Such a clearly undesirable result can be rectified by assuming a lower AIR. For instance, if we reduce the AIR to 1%, the welfare gain from the variable DIA would be \$9,834 for Coll+ women and \$7,461 for Coll+ men, well above that found when using a fixed DIA (of \$7,926 and \$6,449, respectively). A similar problem arises for Coll+ women having an age-dependent DIA stock allocation and a deferral age of 67; this also can be rectified with a lower AIR. In this sense, our findings underscore the critical role of the AIR in deferred variable annuity design, as discussed in detail in Horneff et al. (2010).

III. Conclusions

This paper explored the welfare impact of providing access to longevity income annuities inside tax-qualified retirement accounts. We incorporate the Social Security benefit and tax structure, income taxes, and other institutionally relevant details including required minimum distributions. Our life cycle model recognizes key heterogeneity among the US population in terms of earnings and survival patterns. We extend prior research by comparing the value of purchasing private annuities, versus using funded retirement accounts for bridge financing which permits retirees to receive higher lifelong Social Security benefits by deferring claiming.

Our results show that using retirement account assets to purchase at least some fixed deferred income annuities is welfare enhancing for all sex/education groups examined. Nevertheless, better educated males and females benefit far more – 7 to 11 times more – compared to the least educated. We also find that a deferral age of 80 is strongly preferred to an immediate annuity, and also to the maximum deferral age of 85 allowed under IRS rules.

We also see that the better educated favor using retirement plan assets to purchase DIAs, versus delaying claiming Social Security benefits by a year and financing consumption from retirement plan

²⁰ We also note that the DIA with 20% stocks behaves relatively similarly to the payout structure of a participating annuity, where the latter will depend on the overall investment experience of a life insurance company investing in roughly 80% bonds with the remainder in risky assets like stocks. For more on participating annuities, see Maurer et al. (2016).

withdrawals. By contrast, lower educated retirees prefer the opposite strategy: they fare much better if they delay claiming and use retirement assets to bridge their consumption needs, versus buying DIAs. This is because the least educated have a higher Social Security replacement rate and a higher mortality risk, whereas the better educated receive relatively lower Social Security benefits and can anticipate longer lifetimes. Finally, we also document that providing access to variable deferred annuities with some equity exposure further enhances retiree wellbeing in most cases, compared to having access only to fixed annuities. While current regulatory policy stipulates that variable annuities are disallowed as QLACs in US retirement plans, we find that well-designed variable deferred income annuities in retirement plan portfolios can markedly enhance retiree financial wellbeing.

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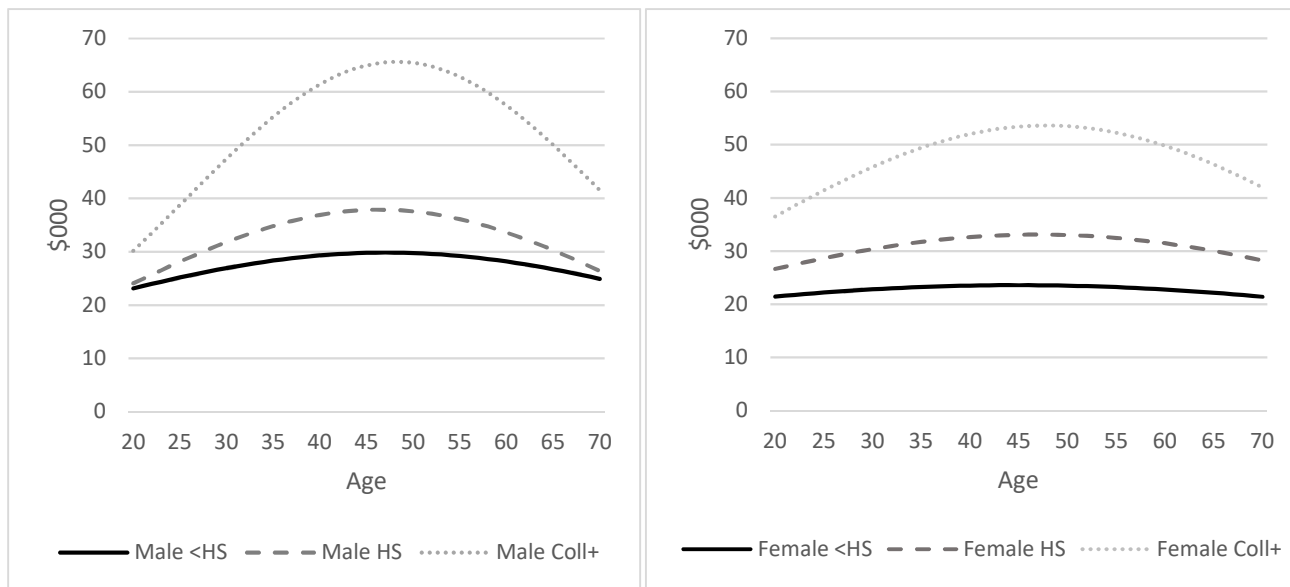
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Figure 1: Estimated average income profiles for females and males by educational level

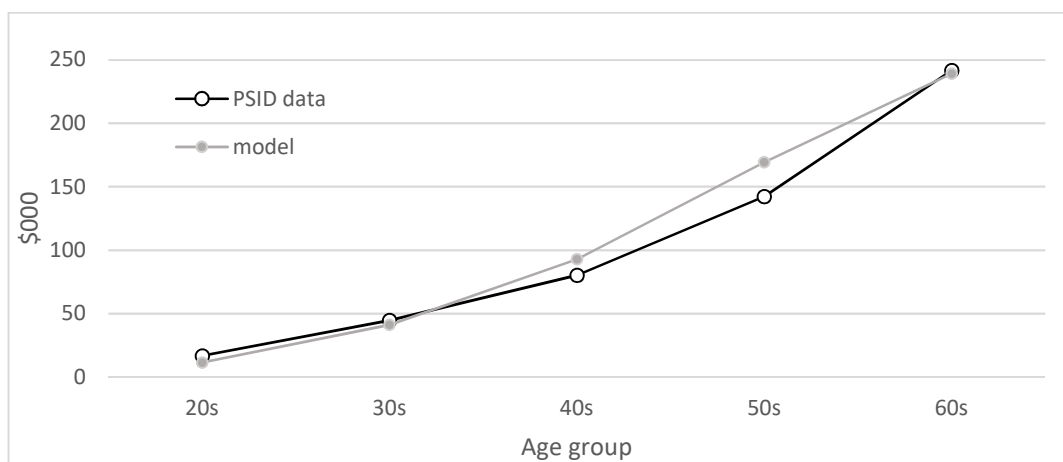
Panel A. Male expected income profiles

Panel B. Female expected income profiles



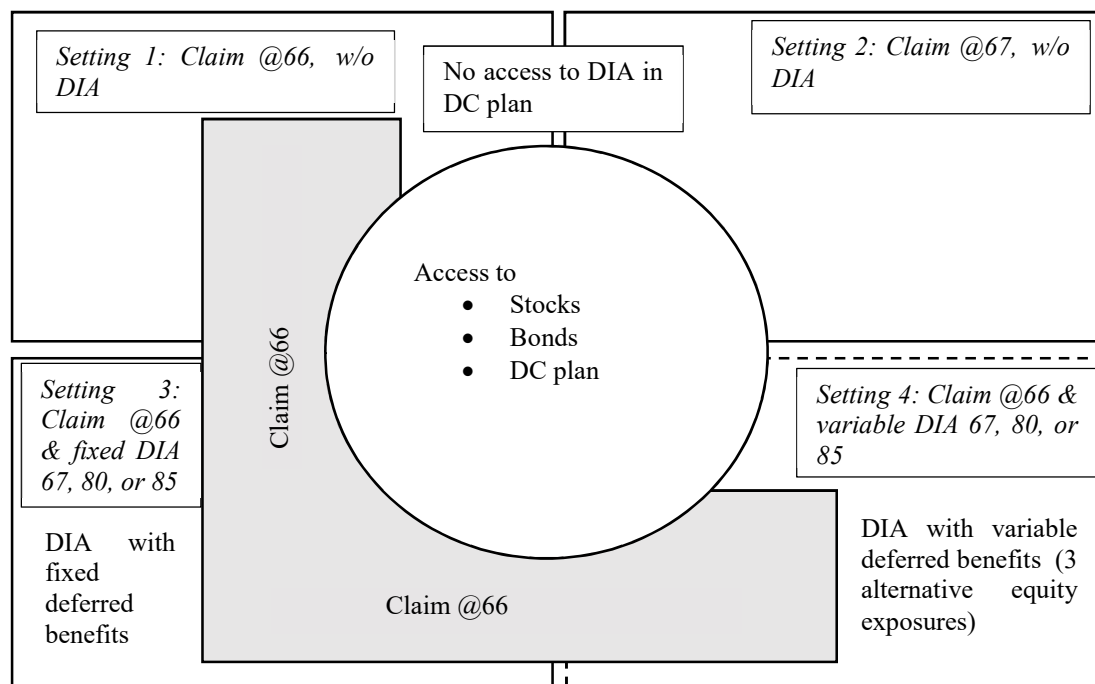
Note: The average income profiles are based on wage rate regressions using PSID data (see Appendix B), assuming a 40 hour work-week and 52 weeks of employment per year. Educational groupings are <High School, High School graduate, and at least some college (<HS, HS, Coll+). Source: Authors' calculations.

Figure 2: Average simulated versus empirical defined contribution plan asset values (\$000)



Note: The figure compares empirical DC tax-qualified account balances across the US population by age, using PSID data (in \$000), with our life cycle model simulations where workers lack access to DIAs. Model simulations are based on average defined contribution asset levels over 200,000 simulated life cycles of employees. The number of simulations by sex/education are: male <HS 13,000, HS 30,000, Coll+ 57,000; females <HS 11,000, HS 28,000, Coll+ 61,000 as per the National Center on Education Statistics (2016). Source: Authors' calculations.

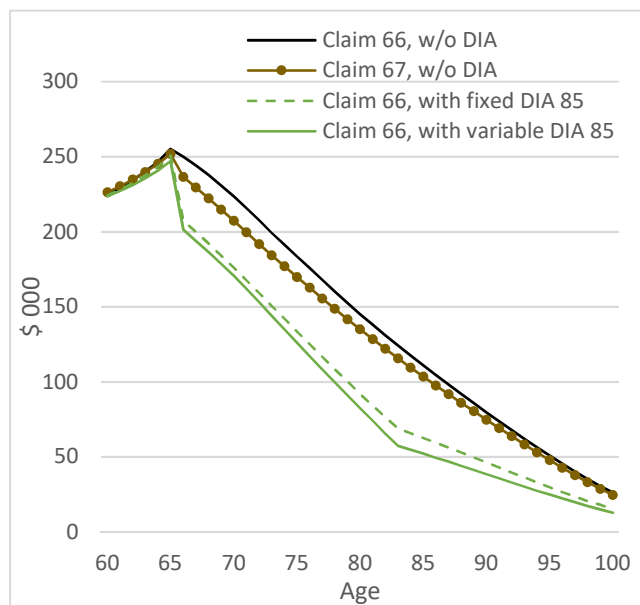
Figure 3: Overview of the settings compared



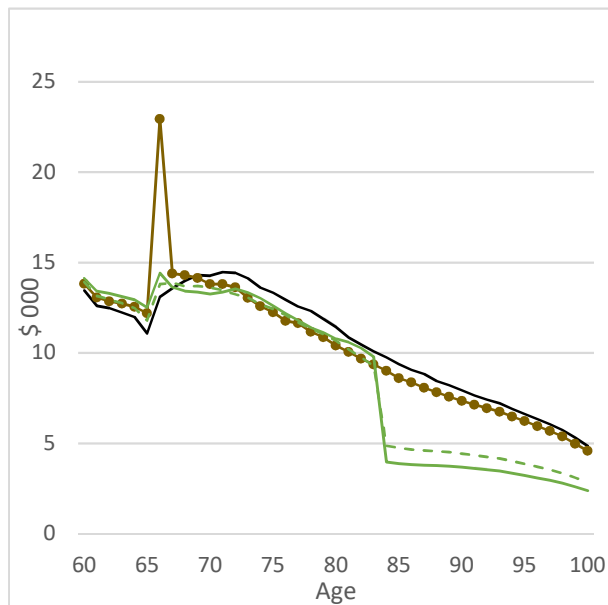
Note: This figure outlines the model settings examined here with respect to two claiming ages (66 or 67) and with or without access to a fixed or variable income annuity deferred to age 67, 80, or 85 held in a qualified defined contribution plan. Setting 4 embeds variable annuities with three patterns of equity exposure: 20%, 50%, and according to a life cycle glide path. Settings outlined in solid lines are currently permitted under QLAC regulations, whereas the cases outlined in dashes are not. Source: Authors' calculations.

Figure 4: Expected life cycle profiles without vs with access to a Deferred Income Annuity (DIA) with fixed or variable payouts

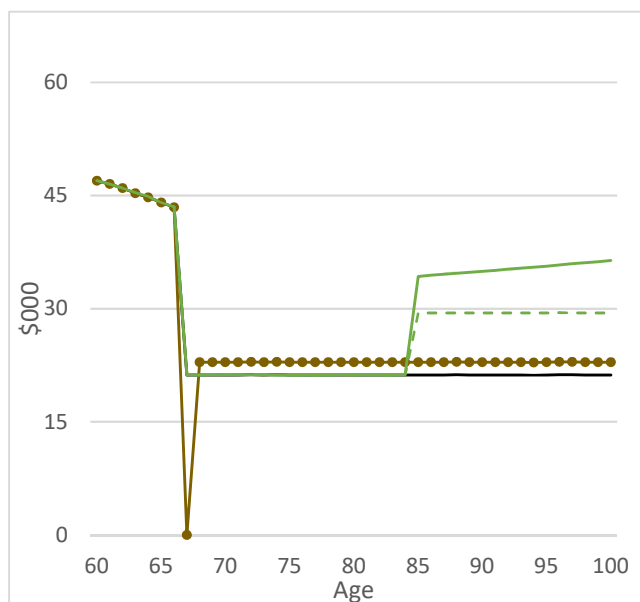
Panel A. Defined contribution asset values



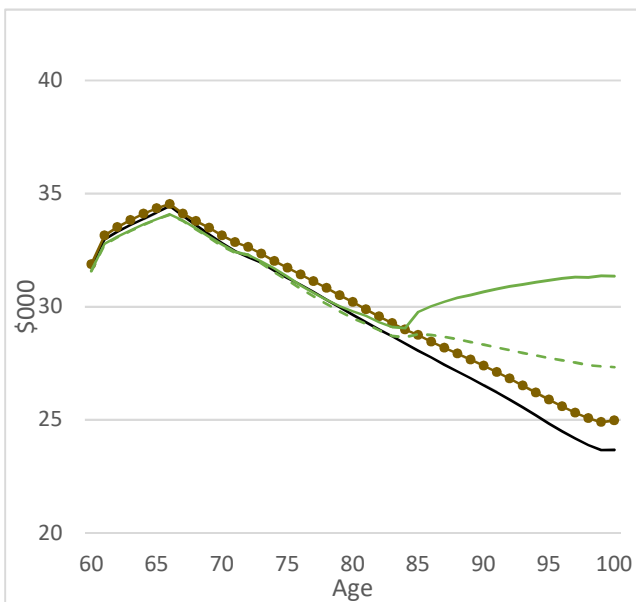
Panel B. Defined contribution plan withdrawals



Panel C. Labor, Social Security, and annuity income

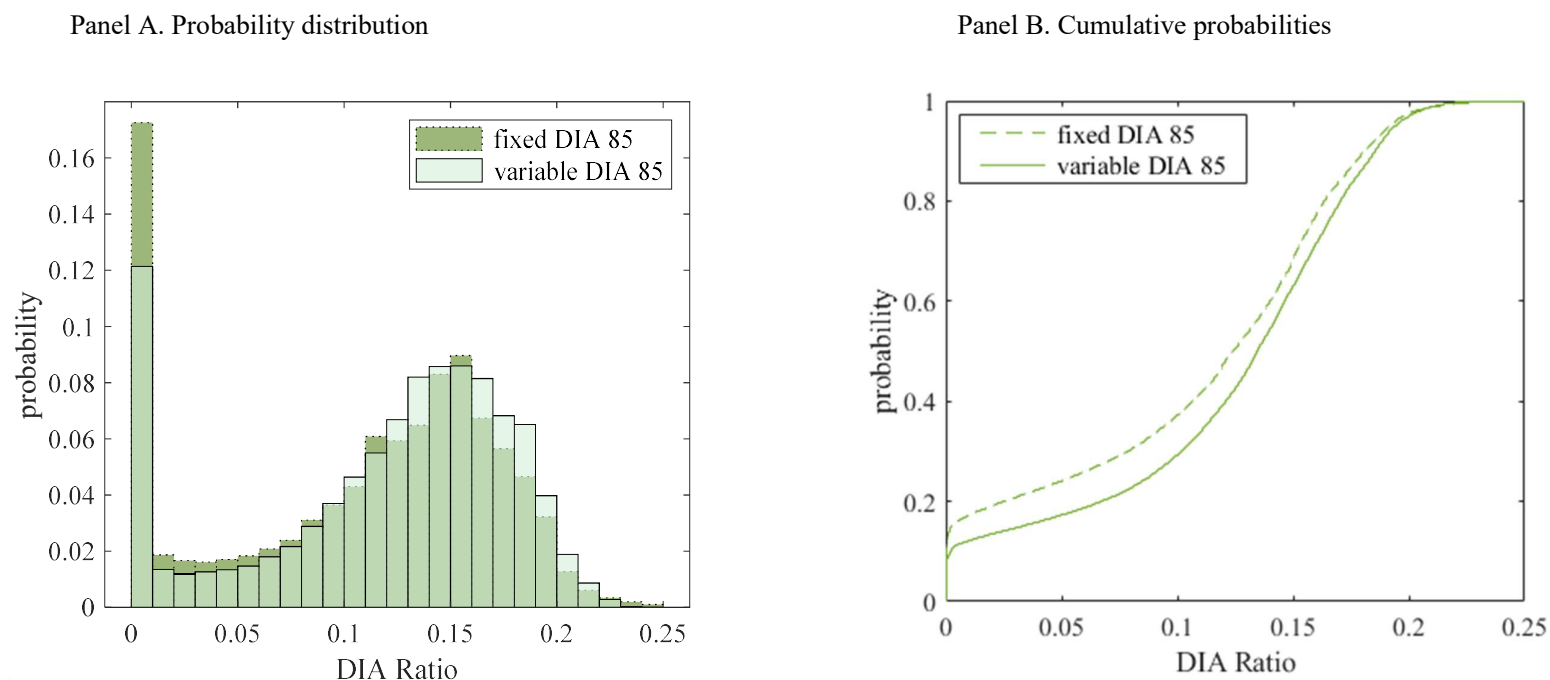


Panel D. Consumption



Note: These figures show expected values from 200,000 simulated lifecycles by age for US workers having access to tax-qualified defined contribution plans. Panel A shows average DC asset values; Panel B depicts average DC withdrawals; Panel C shows labor, Social Security, and DIA payouts (fixed versus variable with a 50/50 stock bond portfolio); and Panel D reports consumption. For additional detail see Figure 2. Source: Authors' calculations

Figure 5: Distribution of optimal DIA ratios with fixed vs variable deferred annuities



Note: The *DIA Ratio* indicates the optimal percentage of DC plan assets that the retiree converts to a DIA at age 66, payable from age 85. The dark green bars in Panel A indicate the relative frequency of DIA Ratios purchased at age 66, generated from 200,000 simulated lifecycles for US workers having access to a deferred fixed DIA in their defined contribution plans; the light green bars indicate the demand for variable DIAs having a 50/50 stock bond portfolio for the same simulated workers. Panel B shows the corresponding cumulative probability distribution of optimal DIA ratios: the solid line refers to fixed DIAs and the dotted line to variable DIAs. For additional details see Figure 3, settings 3 and 4. Source: Authors' calculations

Table 1: Optimal DIA ratios by sex/education subgroups and alternative deferral ages

Sex	Education	Fixed DIA			Variable DIA		
		Age 85	Age 80	Age 67	Age 85	Age 80	Age 67
Female	Coll+	0.12	0.18	0.21	0.13	0.20	0.22
	HS	0.09	0.16	0.20	0.10	0.18	0.22
	<HS	0.04	0.08	0.15	0.05	0.11	0.18
Male	Coll+	0.13	0.18	0.21	0.14	0.20	0.22
	HS	0.09	0.15	0.20	0.11	0.18	0.21
	<HS	0.06	0.10	0.17	0.07	0.13	0.20

Note: The DIA Ratio refers to the fraction of the individual's DC plan assets used to purchase the specified DIA at age 66, for alternative payout start dates. The variable DIA is invested 50/50 in equities/bonds; see text. Source: Authors' calculations.

Table 2: Welfare Analysis I: Alternative Ways to Annuitize Defined Contribution Assets, Fixed vs Variable DIA and Delay Claiming (Reference case Setting 1, Figure 3)

Sex	Education	Claim @66		Claim @67
		Fixed DIA	Variable DIA	w/o DIA
Female	Coll+	38,804	41,305	20,594
	HS	14,528	16,264	11,560
	<HS	3,410	4,295	7,725
Male	Coll+	46,870	50,207	26,809
	HS	16,215	18,635	14,767
	<HS	6,360	7,939	10,641

Note: The reference case in this table is “claim @66, w/o DIA.” The values given refer to the additional amounts that must be paid into the DC plan that would yield the same utility to the individual who claims her Social Security benefits at age 66 and has no access to a DIA, versus the three settings indicated. Both DIAs (columns 1 and 2) start payouts from age 85, while the Variable DIA uses a 50/50 stock bond portfolio. In column 3, the individual delays claiming Social Security to age 67, and withdraws from her DC plan to finance consumption that year. Source: Authors’ calculations.

Table 3: Welfare Analysis II: Fixed DIAs for Alternative Claiming Ages (Reference case Setting 2, Figure 3)

Sex	Education	Age 85	Age 80	Age 67
Female	Coll+	17,367	20,989	7,926
	HS	2,832	6,020	1,916
	<HS	-4,056	-2,690	-2,779
Male	Coll+	19,129	21,729	6,449
	HS	1,368	4,104	-0,664
	<HS	-4,021	-2,077	-2,620

Note: The reference case in this table is “claim @67, w/o DIA.” The values given refer to the additional amounts that must be paid into the DC plan that would yield the same utility to the individual who claims her Social Security benefits at age 67 and has no access to a DIA, versus the three deferral ages indicated for a fixed DIA purchased at age 66. Source: Authors’ calculations.

Table 4: Welfare Analysis III: Variable DIA Designs (Reference case Setting 2, Figure 3)

Sex	Education	20% Stocks			50% Stocks			(125-age) % Stocks		
		Age 85	Age 80	Age 67	Age 85	Age 80	Age 67	Age 85	Age 80	Age 67
Female	Coll+	19,426	23,316	7,526	19,753	24,074	8,053	19,584	23,667	7,507
	HS	3,923	7,545	1,963	4,489	8,425	2,638	4,469	8,273	2,377
	<HS	-3,642	-2,018	-2,345	-3,224	-1,546	-1,887	-3,267	-1,607	-1,966
Male	Coll+	20,862	24,611	4,907	22,316	26,492	7,562	22,381	26,728	9,517
	HS	2,801	5,954	0,774	3,654	7,695	1,641	3,554	7,114	1,626
	<HS	-3,223	-1,045	-1,814	-2,538	-80	-1,066	-2,681	-178	-1,398

Note: The reference case in this table is “claim @67, w/o DIA.” The values refer to the additional amounts that must be paid into the DC plan to would yield the same utility to the individual who claims her Social Security benefits at age 67 and has no access to a DIA, versus an individual who claims at age 66 and can purchase variable DIAs having different equity exposures and payouts starting at the three deferral ages indicated. Source: Authors’ calculations

Appendix A: Tax treatment in our model

We integrate a US-type progressive tax system in our model to explore the impact of having access to a qualified (tax-sheltered) defined contribution account; all dollar values are in \$2017. The worker pays taxes on labor income and on capital gains from investments in bonds and stocks. During her work life, she invests A_t in her tax-qualified pension account which reduces taxable income to an annual maximum amount $D_t = \$18,000$ (additional catch-up contributions of \$6,000 per year are permitted from age 51). Correspondingly, withdrawals W_t from the tax-qualified account increase taxable income. The worker's taxable income is reduced by a general standardized deduction GD equal to \$6,350 per year for a single individual. Consequently, taxable income in working age is given by:

$$Y_{t+1}^{tax} = \max[\max(S_t \cdot (R_{t+1} - 1) + B_t \cdot (R_f - 1); 0) + Y_{t+1}(1 - h_t) + W_t - \min(A_t; D_t) - GD; 0]. \quad (A1)$$

For Social Security (Y_{t+1}) taxation up to age 66, we use the following rules: when the *combined income*²¹ is between \$25,000 and \$34,000 (over \$34,000), 50% (85%) of benefits are taxed.²² After age 66, we set $A_t = 0$, i.e. no further contributions to defined contribution retirement plans occur since the individual stops working.

In line with US rules for federal income taxes, our progressive tax system has seven income tax brackets (IRS 2017). These brackets $i = 1, \dots, 7$ are defined by a lower and an upper bound of taxable income $Y_{t+1}^{tax} \in [lb_i, ub_i]$ and determine a marginal tax rate r_i^{tax} . In the year 2017, the marginal taxes rates for a single person are 10% from \$0 to \$9,325, 15% from \$9,326 to \$37,950, 25% from \$37,951 to 90,900, 28% from \$90,901 to \$191,650, 33% from \$191,651 to \$416,700, 35% from \$416,701 to \$418,400 and 39.6% above \$418,401 (see IRS 2017). Based on these tax brackets, the dollar amount of taxes payable is given by:²³

$$\begin{aligned} IT_{t+1}^{tax} = & (Y_{t+1}^{tax} - lb_7) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \cdot r_7^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_6) \cdot 1_{\{lb_7 > Y_{t+1}^{tax} \geq lb_7\}} + (ub_6 - lb_6) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \right) \\ & \cdot r_6^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_5) \cdot 1_{\{lb_6 > Y_{t+1}^{tax} \geq lb_5\}} + (ub_5 - lb_5) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_6\}} \right) \\ & \cdot r_5^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_4) \cdot 1_{\{lb_5 > Y_{t+1}^{tax} \geq lb_4\}} + (ub_4 - lb_4) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_5\}} \right) \\ & \cdot r_4^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_3) \cdot 1_{\{lb_4 > Y_{t+1}^{tax} \geq lb_3\}} + (ub_3 - lb_3) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_4\}} \right) \\ & \cdot r_3^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_2) \cdot 1_{\{lb_3 > Y_{t+1}^{tax} \geq lb_2\}} + (ub_2 - lb_2) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_3\}} \right) \\ & \cdot r_2^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_1) \cdot 1_{\{lb_2 > Y_{t+1}^{tax} \geq lb_1\}} + (ub_1 - lb_1) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_2\}} \right) \\ & \cdot r_1^{tax} \end{aligned} \quad (A2)$$

²¹ Combined income is sum of adjusted gross income, nontaxable interest, and half of her Social Security benefits.

²² See <https://www.ssa.gov/planners/taxes.html>

²³ We assume that capital gains are taxed at the same rate as labor income, so we abstract from the possibility that long-term investments may be taxed at a lower rate.

where, for $A \subseteq X$, the indicator function $1_A \rightarrow \{0, 1\}$ is defined as:

$$1_A(x) = \begin{cases} 1 & | x \in A \\ 0 & | x \notin A. \end{cases} \quad (\text{A3})$$

In line with US regulation, the individual must pay an additional penalty tax of 10% on early withdrawals prior to age $59 \frac{1}{2}$ ($t = 36$).

Appendix B: Wage rate process modeling

We calibrate the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2017 from age 25 to 69. During the work life, the individual's labor income profile has deterministic, permanent, and transitory components. The shocks are uncorrelated and normally distributed according to $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$ and $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$. The wage rate values are expressed in \$2017. These are estimated separately by sex and by educational level. The educational groupings are: less than High School (<HS), High School graduate (HS), and those with at least some college (Coll+). Extreme observations below \$5 per hour and above the 99th percentile are dropped. We use a second order polynomial in age. The regression function is:

$$\ln(w_{i,y}) = \beta_1 * age_{i,y} + \beta_2 * age_{i,y}^2 + \beta_{waves} * wave \text{ dummies}, \quad (\text{B1})$$

where $\log(w_{i,y})$ is the natural log of wage at time y for individual i , age is the age of the individual divided by 100. OLS regression results for the wage rate process equations appear in Table B1.

To estimate the variances of the permanent and transitory components, we follow Carroll and Samwick (henceforth CS, 1997) and Hubener et al. (2016). We calculate the difference of the observed log wage and our regression results, and we take the difference of these differences across different lengths of time d . For individual i , the residual is:

$$r_{i,d} = \sum_{s=0}^{d-1} (N_{t+s}) + U_{i,t+d} - U_{i,t} \quad (\text{B2})$$

We then regress the $v_{id} = \overline{r_{i,d}^2}$ on the lengths of time d between waves and a constant:

$$v_{id} = \beta_1 \cdot d + \beta_2 \cdot 2 + e_{id}, \quad (\text{B3})$$

where the variance of the permanent factor $\sigma_N^2 = \beta_1$ and the $\sigma_U^2 = \beta_2$ represents the variance of the transitory shocks.

To save calculation time we discretize the CS-process. We simulate the exogenously-determined labor income process as $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$ with a deterministic trend $f(t)$, a permanent income component $P_{t+1} = P_t \cdot N_{t+1}$, and a transitory shock U_{t+1} . We divide the trajectories into 10 percentile groups and calculate the mean and the standard deviation (transitory shock) for each group. We work with the equivalent discrete Markov-switching process $Y_{t+1} = I_{t+1}^l \cdot U_{t+1}^{MS}$ with 10 levels (l) and a transitory shock for each level U_{t+1}^{MS} . The $\Pi_{ij,t} = Prob(l_{t+1} = i | l_t = j)$ is a time-dependent transition matrix representing the probability of moving from current (t) income level j to income level i one year later ($t+1$).

Table B1: Regression results for wage rates

Coefficient	Male <HS	Male HS	Male Coll+	Female <HS	Female HS	Female Coll+
Age/100	3.268*** (0.108)	6.035*** (0.0478)	9.382*** (0.0673)	1.381*** (0.110)	2.818*** (0.0457)	4.755*** (0.0689)
Age ² /10000	-3.466*** (0.129)	-6.502*** (0.0611)	-9.717*** (0.0864)	-1.540*** (0.131)	-3.001*** (0.0587)	-4.974*** (0.0894)
Constant	1.893*** (0.0339)	1.502*** (0.0119)	1.187*** (0.0147)	2.119*** (0.0292)	2.106*** (0.0105)	2.113*** (0.0143)
Permanent	0.00922*** (0.000447)	0.0132*** (0.000206)	0.0196*** (0.000302)	0.00811*** (0.000575)	0.0130*** (0.000198)	0.0208*** (0.000346)
Transitory	0.0276*** (0.00121)	0.0304*** (0.000584)	0.0380*** (0.000802)	0.0218*** (0.00151)	0.0266*** (0.000530)	0.0330*** (0.000920)
Observations	28,197	179,577	149,963	21,124	180,952	132,303
R-squared	0.224	0.286	0.309	0.149	0.262	0.260

Notes: Regression results for the natural logarithm of wage rates (in \$2017) are based on data from the Panel Study of Income Dynamics (PSID) for persons age 25-69 in waves 1975-2017. Independent variables include age and age-squared. Robust standard errors in parentheses. *** indicate the coefficient is significant at the 1% level. Source: Authors' calculations.

Appendix C: Population mortality tables by education and sex

A great deal of evidence shows that lower-educated individuals have lower life expectancies than their better-educated counterparts. This is relevant to the debate over whether and which workers would benefit from annuitization.

To explore the impact of these difference in mortality rates by educational levels, we follow Krueger et al. (2015) who calculated mortality rates by education and sex ($M_{sex}^{education}$) as below:

$$\begin{aligned}
 M_{male}^{average} &= 0.13M_{male}^{<HS} + 0.3M_{male}^{HS} + 0.57M_{male}^{Coll+} \\
 &= 0.13(M_{male}^{HS} \cdot 1.23) + 0.30M_{male}^{HS} + 0.57(M_{male}^{HS} \cdot 0.94) \\
 &= 0.9957 \cdot M_{male}^{HS}
 \end{aligned} \tag{C1}$$

Next we calculate the mortality for a male with a HS degree as follows:

$$M_{male}^{HS} = \frac{M_{male}^{average}}{0.9957} \tag{C2}$$

Mortality differentials for a male high school dropout or with Coll+ level education is as follows:

$$M_{male}^{<HS} = \frac{M_{male}^{average}}{0.9957} \cdot 1.23 \tag{C3}$$

$$M_{male}^{Coll+} = \frac{M_{male}^{average}}{0.9957} \cdot 0.94 \tag{C5}$$

Analogously, we calculate the following for females with different levels of education:

$$M_{female}^{<HS} = \frac{M_{female}^{average}}{0.9864} \cdot 1.32 \quad (C6)$$

$$M_{female}^{HS} = \frac{M_{female}^{average}}{0.9864} \quad (C7)$$

$$M_{female}^{Coll+} = \frac{M_{female}^{average}}{0.9864} \cdot 0.92 \quad (C8)$$

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