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Decay of charmonium states into a scalar and a pseudoscalar glueball

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Abstract. In the framework of a chiral symmetric model, we expand a $U(4)_R \times U(4)_L$ symmetric linear sigma model with (axial-)vector mesons by including a dilaton field, a scalar glueball, and the pseudoscalar glueball. We compute the decay width of the scalar charmonium state $\chi_{C0}(IP)$ into a predominantly scalar glueball $f_0(1710)$. We calculate the decay width of the pseudoscalar charmonium states $\eta_C(IS)$ into a predominantly scalar glueball $f_0(1710)$ as well as into a pseudoscalar glueball with a mass of 2.6 GeV (as predicted by Lattice-QCD simulations) and with a mass of 2.37 GeV (corresponding to the mass of the resonance $X(2370)$). This study is interesting for the upcoming PANDA experiment at the FAIR facility and BESIII experiment. Moreover, we obtain the mixing angle between a pseudoscalar glueball, with a mass of 2.6 GeV, and the charmonium state η_C.

1 Introduction

Charmonium is a bound state of a charm and an anti-charm quark. It is one of the simplest bound states of Quantum Chromodynamic (QCD), the strong interaction theory of quarks and gluons, like positronium in Quantum Electrodynamic (QED). The first charmonium state has been discovered (J/ψ) with quantum number $J^{PC} = 1^{--}$ in November 1974 at BNL [1] and SLAC [2]. Since that time great scientific experimental [3] and theoretical process [4] have been achieved for charmonium spectroscopy. All charmonium states have been observed below the open-charm threshold [5]. The study of the charmonium spectroscopy is very neccessary for understanding hadronic dynamics as the hydrogen atom [6] and the strong interaction physics.

The colourless bound states of gluons are called glueballs. There is an active experimental and theoretical research program, searching for states that are (predominantly) glueballs as seen in Refs.[7]. These efforts are important for understanding glueball properties which gives insight into the non-perturbative behavior of QCD. By Lattice QCD [8], the glueball spectrum has been predicted where the third lightest glueball is a pseudoscalar state ($J^{PC} = 0^{-+}$), denoted as $\widetilde{G}$, with a mass of 2.6 GeV. The mass of a pseudoscalar glueball $\widetilde{G}$ could be also lower than the lattice-QCD prediction and might have already been observed in the BESIII experiment. At that experiment the pseudoscalar resonances have been investigated in J/ψ decays [9]. Particularly, the resonance $X(2370)$ is a good candidate because it has been clearly observed in the $\pi^+\pi^-\eta'$ channel and is

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quite narrow ($\sim$ 80 MeV). There is a good candidate for the scalar glueball which is the resonance $f_0(1710)$, because its mass is very close to lattice-QCD predictions, and because it is produced in the gluon-rich decay of the J/ψ, as seen in Refs. [10–12]. Moreover, in Ref. [13], the three physical resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are assigned predominantly to the nonstrange meson $\sigma_N \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, the hidden-strange meson $\sigma_S \equiv s\bar{s}$, and the scalar glueball $G \equiv gg$, respectively, as a result of solving the mixing between the three bare fields (σ_N, σ_S, G).

The extended Linear Sigma Model (eLSM), includes (axial-)vector and (pseudo)scalar charmed mesons, which was introduced in Refs. [14, 15] as a result of the extension from $N_f = 3$ to $N_f = 4$ case. The symmetry properties of the eLSM have been discussed in Ref. [16]. The eLSM has four charmonium states, which are the ground-state (axial-)vector $J/\psi(1S)$ and $\chi_{c1}(1P)$ with quantum numbers $J^{PC} = 1^{-+}$ and $J^{PC} = 1^{++}$ as well as (pseudo-)scalar ground states $\eta_c(1S)$ and $\chi_{c0}(1P)$ with quantum numbers $J^{PC} = 0^{-+}$ and $J^{PC} = 0^{++}$, respectively, [14–16]. In the present work, we include the scalar glueball to be a dynamical field in the study of charmonum states, whereas it was a frozen field the $N_f = 4$ study of masses of open and hidden charmed mesons as well as of the decay of open charmed mesons [14, 15]. Including the scalar glueball in the eLSM is important to guarantee dilation invariance of the model. Furthermore, the pseudoscalar glueball has first been included into the eLSM via the interaction of the pseudoscalar glueball and (pseudo)scalar mesons in the case of $N_f = 3$ as seen in Ref.[17].

In this work, we study the decay properties of the charmonium state χ_{C0} into a scalar glueball and scalar-isoscalar states as well as of the charmonium state η_C into a scalar glueball, a pseudoscalar glueball, and a scalar-isoscalar states in the eLSM . We compute also the mixing angle between charmonium state η_C and a pseudoscalar glueball.

2 A $U(4)_R \times U(4)_L$ Linear Sigma Model with Glueballs

The dilaton field, a scalar glueball G, and a pseudoscalar glueball $\tilde{G}$ are included in the extended Linear Sigma model (eLSM) with (pseudo)scalar and (axial-)vector mesons as seen in Refs. [17–19]. The corresponding Lagrangian describing only the interaction of a scalar glueball and a pseudoscalar glueball with mesons in the eLSM and relevant for the decay of charmonium states into glueballs reads

$$\mathcal{L} = \mathcal{L}_{dil}(G) - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}(\Phi^\dagger\Phi) + \mathrm{Tr}\left\{\left[\left(\frac{G}{G_0}\right)^2 \frac{m_1^2}{2} + \Delta\right]\left[(L^\mu)^2 + (R^\mu)^2\right]\right\} + c(\det\Phi - \det\Phi^\dagger)^2 + ic_{\tilde{G}\Phi}\tilde{G}(\det\Phi - \det\Phi^\dagger) + \dots , \tag{1}$$

where

$$\mathcal{L}_{dil}(G) = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left[G^4 \ln\left(\frac{\mathrm{G}}{\Lambda_\mathrm{G}}\right) - \frac{\mathrm{G}^4}{4}\right], \tag{2}$$

is a dilation Lagrangian which describes the scalar dilaton filed that is represented by a scalar glueball $G \equiv |gg>$ with quantum number $J^{PC} = 0^{++}$, and emulates the trace anomaly of pure Yang-Mills QCD [19, 20]. The dimensionful parameter $\Lambda_G \sim N_C\,\Lambda_{QCD}$ sets the energy scale of low-energy QCD and is identical to the minimum of the dilaton potential G_0, $G_0 = \Lambda_G$ [18]. Lattice QCD gives the mass of a scalar glueball m_G of about (1.5 – 1.7) GeV [21]. As a result of Ref. [13], the resonance $f_0(1710)$ is predominantly a scalar glueball with a 86 percent admixture. At the classical level of the Yang-Mills sector of QCD, the dilatation symmetry, $x^\mu \to \lambda^{-1}x^\mu$, is realized and explicitly broken due to the logarithmic term of the dilaton field potential [22]. This breaking leads to the divergence of the corresponding current [18]:

$$\partial_\mu J^\mu_{dil} = T^\mu_{dil,\mu} = -\frac{1}{4} m_G^2 \Lambda_G^2. \tag{3}$$

Note that we include the scalar glueball in the eLSM to combine the dilatation invariance with the meson mass terms. A further breaking of dilatation and chiral symmetry are represented by the chiral anomaly term $c(det\Phi - det\Phi^\dagger)^2$ with a dimensionful constant c. The quark-mass term $\mathrm{Tr}\left[\Delta(L^{\mu 2} + R^{\mu 2})\right]$ contributes to the masses of the (axial-)vector mesons and also breaks the dilatation and chiral symmetry due to nonzero current quark masses [14], with Δ defined as

$$\Delta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_S & 0 \\ 0 & 0 & 0 & \delta_C \end{pmatrix}, \tag{4}$$

where $\delta_S \sim m_S^2$ and $\delta_C \sim m_C^2$. The left-handed L^μ and right-handed R^μ represent the (axial-)vector multiples, see Ref.[14] for the case of $N_f = 4$. Furthermore, the last field included in the eLSM is the pseudoscalar glueball $\widetilde{G} \equiv |gg\rangle$, with quantum numbers $J^{pc} = 0^{-+}$, via $ic_{\tilde{G}\Phi}\tilde{G}(\det\Phi - \det\Phi^\dagger)$. This term couples the pseudoscalar glueball with (pseudo)scalar mesons. where $c_{\tilde{G}\phi}$ is a dimensionless coupling constant. This term has been successfully studied in the decay of a pseudoscalar glueball into scalar and pseudoscalar mesons in the case of $N_f = 3$ as seen in Ref.[17]. In eLSM Lagrangian (1) the dots refer to further chirally invariant terms, which are irrelevant for the present work, see Refs.[14, 15].

In this framework, we use the multiplets of scalar and pseudoscalar mesons, Φ, in the case of $N_f = 4$, as seen in Ref.[15] as

$$\Phi = (S^a + iP^a)t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ & D_0^{*0} + iD^0 \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 & D_0^{*-} + iD^- \\ K_0^{*-} + iK^- & \overline{K}_0^{*0} + i\overline{K}^0 & \sigma_S + i\eta_S & D_{S0}^{*-} + iD_S^- \\ \overline{D}_0^{*0} + i\overline{D}^0 & D_0^{*+} + iD^+ & D_{S0}^{*+} + iD_S^+ & \chi_{C0} + i\eta_C \end{pmatrix}, \tag{5}$$

where S and P refer to scalar and pseudoscalar mesons, respectively, whereas t^a are the generators of the group $U(N_f)$. The multiplet Φ transforms as $\Phi \to U_L \Phi U_R^\dagger$ under $U_L(4) \times U_R(4)$ chiral transformations, while $U_{L(R)} = e^{-i\theta^a_{L(R)} t^a}$ is an element of $U(4)_{R(L)}$, under parity which $\Phi(t, \overrightarrow{x}) \to \Phi^\dagger(t, -\overrightarrow{x})$, and under charge conjugate $\Phi \to \Phi^\dagger$. The determinant of Φ is invariant under $SU(4)_L \times SU(4)_R$, but not under $U(1)_A$ because $\det\Phi \to \det U_A \Phi U_A = e^{-i\theta_A^0 \sqrt{2N_f}} \det\Phi \neq \det\Phi$. The pseudoscalar glueball $\widetilde{G}$ is also chirally invariant, which is invariant under parity ($\widetilde{G} \to -\widetilde{G}$), but not invariant under the axial $U_A(1)$ transformation.

In the Lagrangian (1), all states are assigned as physical resonances to light quark-antiquark states with mass $\lesssim$ 2 GeV [17] and heavy quark-antiquark states [14]. In the present work, we need to deal only with the scalar and pseudoscalar mesons. For the scalar sector S, the isotriplet field $\overrightarrow{a_0}$ and kaonic field K^{*0} are assigned to the physical states $a_0(1450)$ and $K_0^*(1430)$, respectively. In the scalar-isoscalar sector, the identification of the scalar glueball G is still uncertain, the two most likely candidates are $f_0(1500)$ and $f_0(1710)$ and/or admixtures of them. The bare nonstrange field

$\sigma_N \equiv \left|\bar{u}u + \bar{d}d\right\rangle / \sqrt{2}$, the bare strange field $\sigma_S \equiv |\bar{s}s\rangle$ and the scalar glueball (G) are assigned from the following mixing matrix which is constructed in Ref. [13]:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.94 & -0.17 & 0.29 \\ 0.21 & 0.97 & -0.12 \\ -0.26 & 0.18 & 0.95 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv (\overline{u}u + \overline{d}d)/\sqrt{2} \\ \sigma_S \equiv \overline{s}s \\ G \equiv gg \end{pmatrix} . \tag{6}$$

As a conclusion of Ref.[13], we obtain that the resonances $f_0(1370)$ and $f_0(1500)$ are predominantly $\left|\bar{u}u + \bar{d}d\right\rangle / \sqrt{2}$ and $|\bar{s}s\rangle$, respectively. However the resonance $f_0(1710)$ is predominantly a scalar glueball G. In the scalar charm sector, open charmed meson D_0^*, strange-cahrm meson D_{S0}^* and scalar-isoscalar state χ_{C0} are assigned to the resonances $D_0^*(2400)$, $D_{S0}^*(2317)$ and the graund charm-anticharm resonance $\chi_{C0}(IP)$, respectively. Now let us turn to the pseudoscalar sector P, the light quark-antiquark present the pion isotriplet $\vec{\pi}$ and the kaon isodoublet K [19], the isoscalar fields nonstrange $\eta_N \equiv |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and strange $\eta_S \equiv \bar{s}s\rangle$ which are mixed in the physical fields η and η' as

$$\eta = \eta_N \cos\varphi + \eta_S \sin\varphi, \;\; \eta' = -\eta_N \sin\varphi + \eta_S \cos\varphi, \tag{7}$$

where the mixing angle is $\varphi \simeq 44.6°$ [19]. In the charm sector, there are the well-established D resonance, the open strange-charmed resonance D_S, and charmonium ground state $\eta_C(IS)$ [14].

From the eLSM features, spontaneous symmetry breaking occurs when $m_0^2 < 0$, and consequently the scalar-isoscalar fields as well as the glueball field G condense [19, 23]. One has to shift the scalar-isoscalar fields by their vacuum expectation values to implement this breaking [14] as

$$\begin{aligned} G &\to G + G_0\,, & \sigma_N &\to \sigma_N + \phi_N\,, \\ \sigma_S &\to \sigma_S + \phi_S\,, & \chi_{C0} &\to \chi_{C0} + \phi_C\,. \end{aligned} \tag{8}$$

where G_0 is the gluon condensate equal to $\Lambda \approx 3.3$ GeV [13]. ϕ_N, ϕ_S, and ϕ_C represent the chiral-nonstrange, strange, and charm quark-antiquark condensates [14, 15], respectively.

3 Results

In the case of $N_f = 3$ most of the parameters are fixed, in the low-energy study [19]. We are left with only three new parameters related to the charm sector in the case of $N_f = 4$, δ_C, ϕ_c, ε_C, which are determined by a fit, with $\chi^2/\mathrm{d.o.f} \simeq 1$, including masses of open and hidden charmed mesons (see details in Ref. [14]).

In the present work, we have to change the value of the parameter c, which is the coefficient of the axial anomaly term, to fit the results of the decay widths of χ_{c0} and η_C. Therefore, for the determination of c, we use the decay widths of χ_{c0} into $\eta\eta$ and $\eta'\eta'$, which are [24].

$$\Gamma^{exp}_{\chi_{c0}\to\eta\eta} = (0.031 \pm 0.0039)\mathrm{MeV} \;\; \text{and} \;\; \Gamma^{exp}_{\chi_{c0}\to\eta'\eta'} = (0.02 \pm 0.0035)\mathrm{MeV}\,.$$

We then perform a fit by minimizing the χ^2-function,

$$\chi^2(c) \equiv \left(\frac{\Gamma^{th}_{\chi_{c0}\to\eta\eta}(c) - \Gamma^{exp}_{\chi_{c0}\to\eta\eta}}{\xi\Gamma^{exp}_{\chi_{c0}\to\eta\eta}}\right)^2 + \left(\frac{\Gamma^{th}_{\chi_{c0}\to\eta'\eta'}(c) - \Gamma^{exp}_{\chi_{c0}\to\eta'\eta'}}{\xi\Gamma^{exp}_{\chi_{c0}\to\eta'\eta'}}\right)^2 , \tag{9}$$

which gives $c = 7.178 \times 10^{-10}$ MeV^{-4} with $\chi^2/\mathrm{d.o.f} = 0.18$ where $\xi = 1$. The coupling constant $c_{\widetilde{G}\Phi}$ describes the coupling of the pseudoscalar glueball $\widetilde{G}$ and (pseudo)scalar mesons. By comparing the interaction Lagrangians of the pseudoscalar glueball with (pseudo)scalar mesons in the two cases $N_f = 3$ and $N_f = 4$ [16], we get

$$c_{\widetilde{G}\Phi} = \frac{\sqrt{2}\, c_{\widetilde{G}\Phi(N_f=3)}}{\phi_C} = 0.036. \tag{10}$$

The three-body decay of the charmonium state χ_{C0} into a scalar glueball and (pseudo)scalar mesons are reported in Table 1.

Table 1. The partial decay widths of χ_{c0}.

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta}$	4.10^{-4}	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta}$	3.10^{-3}	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta'\eta'}$	27.10^{-4}	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta'}$	89.10^{-6}	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta'}$	11.10^{-3}	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta}$	8.10^{-5}	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta'}$	3.10^{-5}	-

The decay widths of the charmonium state η_C into a scalar glueball and (pseudo)scalar mesons are presented in Table 2.

Table 2. The partial decay widths of η_c.

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\eta_c\to f_0(1370)\eta}$	0.00018	-
$\Gamma_{\eta_c\to f_0(1500)\eta}$	0.006	-
$\Gamma_{\eta_c\to f_0(1710)\eta}$	0.000032	-
$\Gamma_{\eta_c\to f_0(1370)\eta'}$	0.027	-
$\Gamma_{\eta_c\to f_0(1500)\eta'}$	0.024	-
$\Gamma_{\eta_c\to f_0(1710)\eta'}$	0.0006	-

There are no experimental data for these decays firm quoted in the PDG with which we can compare.

The interaction Lagrangian $[ic_{\tilde{G}\Phi}\tilde{G}(\det\Phi - \det\Phi^{\dagger})]$ contains only one decay process which describes the decay of the pseudoscalar charmonium η_c into a pseudoscalar glueball $\widetilde{G}$ through the channel $\eta_C \to \widetilde{G}\pi\pi$. By using the corresponding decay width for the three-body case [16], the decay width of the pseudoscalar charmonium state η_c into a pseudoscalar glueball with a mass of 2.6 GeV (as predicted by lattice QCD in the quenched approximation [21]) is

$$\Gamma_{\eta_C\to\pi\pi\widetilde{G}(2600)} = 0.124 \text{ MeV}, \tag{11}$$

and for a mass of the charmonium state η_c which is about of 2.37 GeV (corresponding to the mass of the resonance $X(2370)$ measured in the BESIII experiment [9])

$$\Gamma_{\eta_C\to\pi\pi\widetilde{G}(2370)} = 0.16 \text{ MeV} . \tag{12}$$

These results could be tested in the PANDA experiment at the upcoming FAIR facility.

The mixing between the charmonium state χ_{c0} and the scalar glueball G is neglected because it is expected to be small.

3.1 Mixing of a pseudoscalar glueball $\widetilde{G}$ and a charmonium state η_C

The full interaction Lagrangian of the mixing between the pseudoscalar glueball $\widetilde{G}$ and η_C has the form

$$\mathcal{L}_{\widetilde{G},\eta_C} = \frac{1}{2}(\partial_\mu \widetilde{G})^2 + \frac{1}{2}(\partial_\mu \eta_C)^2 - \frac{1}{2}m^2_{\widetilde{G}}\widetilde{G}^2 - \frac{1}{2}m^2_{\eta_c}\eta_c^2 + Z_{\widetilde{G}\eta_C}\widetilde{G}\,\eta_C \tag{13}$$

where

$$Z_{\widetilde{G}\eta_C} = \frac{-1}{4}\, c_{\widetilde{G}\Phi} Z_{\eta_C}\, \phi_N^2\, \phi_S\,. \tag{14}$$

The physical fileds η_C and $\widetilde{G}$ can be obtained through an SO(2) rotation

$$\begin{pmatrix} \widetilde{G}' \\ \eta_C' \end{pmatrix} = \begin{pmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{pmatrix} = \begin{pmatrix} \widetilde{G} \\ \eta_C \end{pmatrix}, \tag{15}$$

with

$$m^2_{\eta_C'} = m^2_{\widetilde{G}}\, sin^2\phi + m^2_{\eta_C} cos^2\phi + Z_{\widetilde{G}\eta_C} sin(2\phi), \tag{16}$$

$$m^2_{\widetilde{G}'} = m^2_{\widetilde{G}}\, cos^2\phi + m^2_{\eta_C} sin^2\phi - Z_{\widetilde{G}\eta_C} sin(2\phi), \tag{17}$$

where the mixing angle ϕ reads

$$\phi = \frac{1}{2}\arctan\left[\frac{-c_{\widetilde{G}\Phi}\, Z_{\eta_c}\phi_N^2\phi_S}{2(m^2_{\eta_C} - m^2_{\widetilde{G}})}\right]. \tag{18}$$

$c_{\widetilde{G}\Phi}$ is a dimensionless coupling constant between $\widetilde{G}\Phi$ determined by Eq.(10). We then obtain the mixing angle of the pseudoscalar glueball $\widetilde{G}$ and the pseudoscalar charm-anticharm meson η_c to be $-1°$, for a mass of the pseudoscalar glueball of 2.6 GeV, as predicted by lattice-QCD simulations [21].

4 Conclusion

In the present work we have represented a chirally invariant linear sigma model with (axial-)vector mesons in the four-flavour case, $N_f = 4$, by including a dilaton field and a scalar glueball field, and describing the interaction of the pseudoscalar glueball with (pseudo-)scalar mesons. We have calculated the decay widths of the hidden-charmed meson χ_{c0} into two and three scalar-isoscalar states as well as into a scalar glueball G (Tables 1), which is predominantly $f_0(1710)$. We have also computed the decay widths of the pseudoscalar charmonium state η_C into scalar-isoscalar states (Table 2) and into a pseudoscalar glueball $\widetilde{G}$, through the channel $\eta_C \to \pi\pi\widetilde{G}$. The latter is obtained from the interaction term of the pseudoscalar glueball. In addition, we have evaluated the mixing angle between the pseudoscalar glueball and η_c, which is very small and equal to $-1°$. The results presented in this work are interesting for the upcoming PANDA experiment at the FAIR facility as well as for the BESIII experiment.

Further applications of the described approach are to calculate the decay widths of hidden charmonium states into light mesons [25].

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