Da zwischen der Durchflußmenge und dem Druckgradienten nach [12] ein linearer Zusammenhang besteht, scheint der Einfluß der Turbulenz auf unsere Ergebnisse nur gering zu sein.

Die Frage, wie stark die nur ungenau bekannte Wandtemperatur unsere Ergebnisse beeinflußt, wurde dadurch beantwortet, daß einmal nach Formel (5) mit drei verschiedenen T_W-Werten gerechnet wurde, zum anderen T_W für alle Stromstärken konstant gesetzt wurde. Die Ergebnisse sind in Abb. 3 wiedergegeben. Die Abhängigkeit der mittleren kinematischen Zähigkeit von der Wandtemperatur nimmt wie erwartet mit zunehmendem Bogenstrom ab. Die berechneten mittleren kinematischen Zähigkeitswerte, die verschiedenen Wandtemperaturen zugeordnet sind, liegen innerhalb der Meßgenauigkeit.

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Non-Local Shell Model Parameters for Nuclear Bound States

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By fitting shell model parameters to the mass defects of the nuclei the model is found to be strongly non-local, $\beta = 1.30 \ldots 1.50$ fm, $V_0 = 120 \ldots 150$ MeV.

We made the attempt to determine the parameters of a non-local shell model by fitting the binding energies of the protons and the charge radius of the nucleus, together with its mass defect. The latter has been strongly underestimated in calculations using a local shell model. The model is the same as in [1].

From the charge distribution

$$\varrho(r) = \frac{1}{4\pi} \sum_{\substack{\text{proton} \\ \text{levels}}} n_i \, \psi_i(r)^2 ,$$

n_i being the number of nucleons in the i-th level ε_i, ψ_i the corresponding radial wave function, we get the equivalent radius

$$r_{eq} = \sqrt{\frac{5}{3}} \left[\frac{4\pi}{Z} \int_0^\infty \mathrm{d}r \, r^4 \varrho(r) \right]^{1/2} \cdot A^{-1/3} .$$

The mass defect is obtained from the formula

$$D = -\tfrac{1}{2} \sum_{\substack{\text{all nucleon} \\ \text{levels}}} n_i (\varepsilon_i + t_i) ;$$

$$t_i \equiv \langle i \,|\, t \,|\, i \rangle = \frac{\hbar^2}{2m} \int_0^\infty \mathrm{d}r \, [l_i(l_i+1) \, \psi_i(r)^2 + r^2 \psi_i'(r)^2]$$

is the mean value of the kinetic energy in the single particle state i.

The medium weight and heavy nuclei the equivalent radii of which are known from electron scattering experiments [2] are ^{32}S, ^{40}Ca, ^{51}V, ^{59}Co, ^{115}In, $^{121, 123}$Sb, ^{197}Au, ^{208}Pb, and ^{209}Bi. We know their mass defects [3], and for $A < 60$ the binding energies of the 2s protons are determined by (p,2p)-experiments [4]. For the other nuclei we estimated the magnitude of the binding energy of the last proton from its separation energy. The last information is given only for odd-A nuclei, therefore we have not studied ^{208}Pb.

In order to fit the experimental values we had to determine six parameters: V_0, σ, τ, β, r_0, and a. The spin-orbit coupling constant σ was roughly fixed to 0.55 from the splitting of the d-levels in ^{40}Ca. A variation of σ has approximately no effect on the values of the mass defect and of the equivalent radius. The isospin parameter τ was kept [1] equal to 2. The parameters V_0, β, and r_0 were determined for several a, and it turned out that if a was too high or too low, the position of the d-center relative to the s-level in ^{40}Ca or the level order versus mass number for ^{121}Sb and ^{123}Sb was wrong. Thus we chose a to be about 0.65 to 0.70 fm; this choice agrees with fits to nucleon-nucleus scattering [5].

This method did not work for ^{51}V and ^{59}Co: with $a = 0.65$ fm, the radius parameter r_0 went to 0.90 fm. Thus we tried it the other way round and determined a, β, and V_0 for $r_0 = 1.07$ fm ($= r_0$ of the charge distribution). We do not think this had much success.

The results are given in table 1.

Compared with the parameters found in optical model studies [5] ($\beta = 0.85$ fm, $V_0 = 71$ MeV) our values essentially indicate a remarkably larger non-locality β of 1.30 to 1.50 fm and correlated with it a potential depth of 110 to 140 MeV. These parameters are consistent with those determined in a similar attempt [6], using the equivalent radii, the proton knock-out reaction data, and the electron scattering results for light and medium weight nuclei as the data to be fitted.

Our values need not be a concentration to the optical model parameters: they indicate a certain dependence of β on the energy. In Fig. 1 we see the energy dependence of the equivalent local potential [5, 7] using

[1] H. MELDNER, G. SÜSSMANN, and W. ULRICI, Z. Naturforschg. **20 a**, 1217 [1965].
[2] R. HOFSTADTER, Rev. Mod. Phys. **28**, 214 [1956].
[3] J. H. E. MATTAUCH, W. THIELE, and A. H. WAPSTRA, Nucl. Phys. **67**, 1 [1965].
[4] M. RIOU, Rev. Mod. Phys. **37**, 375 [1965].
[5] F. PEREY and B. BUCK, Nucl. Phys. **32**, 353 [1962].
[6] A. SWIFT and L. R. B. ELTON, to be published, and L. R. B. ELTON, private communication.
[7] W. E. FRAHN, Nucl. Phys. **66**, 358 [1965].

Nucleus	r_0 (fm)	a (fm)	β (fm)	V_0 (MeV)	Proton binding energies (MeV) State	calc.	exp.	Mass defect (MeV) calc.	exp.	Equivalent radius (fm) calc.	exp.
$^{32}_{16}$S	1.04	0.65	1.38	137	2 s 1/2	8.7	8.8	280	272	1.298	1.30
					1 d 5/2	16.5	16.1				
					1 p 1/2	35.0	33.5?				
					1 p 3/2	43.5					
					1 s 1/2	76.0	70−80				
$^{40}_{20}$Ca	1.07	0.70	1.53	137	1 d 3/2	8.9	8.4	339	342	1.320	1.32
					2 s 1/2	11.0	11.1				
					1 d 5/2	19.1	19.0				
					1 p 1/2	38.2	36.8?				
					1 p 3/2	45.4					
					1 s 1/2	76.8	70−80?				
$^{51}_{23}$V	1.07	0.50	1.73	157	1 f 7/2	2.8	8.1	449	446	1.251	1.25
					2 s 1/2	14.6	14.7				
$^{59}_{27}$Co	1.07	0.47	1.78	157	1 f 7/2	4.5	9.5	475	517	1.254	1.27
					2 s 1/2	15.8	13,2				
$^{115}_{49}$In	1.07	0.65	1.56	149	1 g 9/2	9.2	9.3	970	979	1.188	1.19
$^{121}_{51}$Sb	1.15	0.65	1.30	114	1 g 7/2	5.0		1011	1026	1.199	1.20
					2 d 5/2	5.5	5.8				
$^{123}_{51}$Sb	1.17	0.65	1.30	112	2 d 5/2	6.7		1036	1042	1.202	1.20
					1 g 7/2	6.8	6.6				
$^{197}_{79}$Au	1.08	0.65	1.67	157	2 d 3/2	5.2	5.8	1510	1559	1.179	1.18
$^{209}_{83}$Bi	1.17	0.65	1.41	118	1 h 9/2	3.8	3.8	1647	1640	1.203	1.20

Table 1. Non-local shell model parameters, and comparison of calculated quantities with experimental ones. τ and σ were kept fixed at 2.0 and 0.55, respectively.

a GAUSSian non-locality distribution. The true energy dependence seems to be somewhat like the $\beta = 1.30$ fm one for low energies and like the $\beta = 0.90$ fm one for high energies. Non-locality distributions other than GAUSSian should be able to reproduce this behaviour.

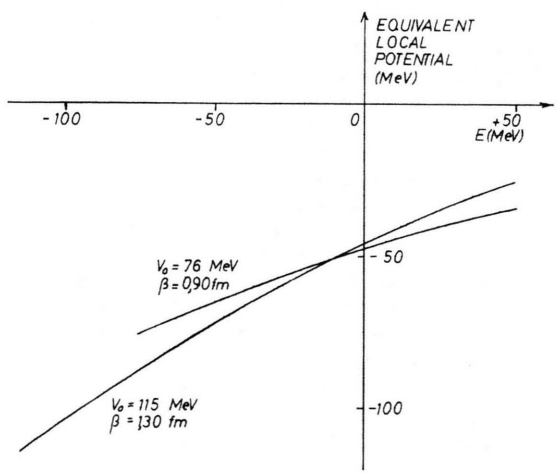

Fig. 1. Approximate energy dependence of the local potential equivalent to a non-local one [7] using a GAUSSian non-locality distribution.

For ^{51}V, ^{59}Co, ^{115}In, and ^{197}Au, the parameters β and V_0 are extremely high. A possible explanation is most easily seen for ^{51}V, where the f 7/2 level is poorly fitted: The experimental one lies much deeper than the calculated one. This is a characteristic of a spherical model treating non-spherical nuclei. Accordingly we should conclude that these nuclei are non-spherical. The model seems to be rather sensitive for deviations from sphericity. In our calculations we found it necessary to increase V_0 and β in order to keep the mass defect if the separation energy had to be lowered.

The model explains the (e,e'p)-data of the Rome-Frascati group [8] with ^{32}S and ^{40}Ca in a natural way. It is not able to predict the energies of the levels above the FERMI energy. The reason may be seen from Fig. 1: The level distances in that region are too large because of the strong energy dependence of the potential. Again this indicates an energy dependence of the parameter β.

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[8] U. AMALDI, G. CAMPOS VENUTI, G. CORTELLESSA, C. FRONTEROTTA, E. REALE, P. SALVADORI, and P. HILLMAN, Phys. Rev. Letters **8**, 171 [1964], and private communication on preliminary results with ^{40}Ca.