On the Stability of Superheavy Nuclei against Fission, Alpha Decay and Electron Capture *

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The potential energy surface has been calculated by two methods which are compared with respect to spontaneous fission. In the first one essentially the sum of the single particle energies is computed as was done in a previous paper [3] while in the second one the Strutinsky technique of renormalizing to a liquid drop model has been applied. Also the half-lives for electron capture are investigated together with the predictions of the half-lives for spontaneous fission and α-decay. The results support the existence of superheavy nuclei in the regions around $Z=114$ and $Z=164$.

I. Introduction

The stability of superheavy elements has been previously investigated [1–3]. Islands of stability were found around $Z=114$, $N=196$ and around $Z=164$, $N=318$. The latter one occurs far beyond the known stable nuclei. The fusion reactions, which lead into the vicinity of these islands of stability mostly result in neutron deficient nuclei.

In this paper we investigate again these two regions of stability with an additional decay channel taken into consideration: The possibility of successive electron capture to pass from neutron poor nuclei to the neutron rich (and thus more stable) nuclei in the islands of stability is investigated. The calculational procedure for electron capture is presented in Section V, while the results are given in the 6-th section. It turns out, that this electron capture process is too slow compared with α-particle- and fission decay to improve the reaction conditions for the upper island at $Z=164$.

We present, furthermore, a comparison and discussion of two methods for the determination of the collective potential energy surface (PES) (Section III). The first one consists in the simple summation of the single particle energies (with corrections for pairing and Coulomb forces) while the second one uses the shell correction method of Strutinsky. Special attention has been given to the possibility of oblate fission [3] of superheavy nuclei. It is found, that both methods of computation agree reasonably well on the predictions of the prolate barriers. In the case of the oblate barriers the development of a more center liquid drop model is necessary in order to apply the renormalization method.

We have also studied the influence of variations in the extrapolation of the semi empirical mass formula on the locations of the beta stable vally, in the region of the quasi stable islands. This is discussed in Section IV and, together with the lifetimes of the various decay channnels, in the last Section VI.

II. Potential Energy Surface

In the discussion of the stability of nuclei against spontaneous fission the potential energy surface (PES) plays a central role. We shall sketch briefly two methods which are used in this paper for computing the PES. More details and an extensive discussion of the method can be found in references [1–3].

One method starts from an anisotropic three-dimensional oscillator of the form [4]

$$H = T + \frac{m}{2}\{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\} + C\,\boldsymbol{l}\cdot\boldsymbol{s} + D(\boldsymbol{l} - \langle \boldsymbol{l}^2\rangle_N)\,. \tag{1}$$

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[1] U. MOSEL and W. GREINER, Z. Phys. **217**, 256 [1968]. — U. MOSEL and W. GREINER, Z. Phys. **222**, 261 [1969].

[2] S. G. NILSSON, S. G. THOMPSON, and C. F. TSANG, Phys. Letters **28 B**, No. 7, 458 [1969].

[3] J. GRUMANN, U. MOSEL, B. FINK, and W. GREINER, Z. Phys. **228**, 371 [1969].

[4] S. G. NILSSON, UCRL-18355, Berkeley 1968. — G. GUSTAVSON, I. L. LAMM, B. NILSSON, and S. G. NILSSON, Arkiv Fysik **36**, No. 69, 613 [1967].

In the intrinsic system one obtains through a self-consistency argument the connection between the oscillator frequencies and the deformation parameters (a_0, a_2) which specify the axes of the ellipsoidal nuclear shape. The strength of the spin-orbit and l^2-terms are extrapolated with a formula given by SEEGER and PERISHO [5] into the region of superheavy nuclei.

$$\mu = 2\,D/C\,, \qquad \varkappa = -\frac{1}{2}\frac{C}{\hbar\,\omega_0}$$

$$\mu = \mu_0\,, \quad \varkappa = \varkappa_0\{\tfrac{1}{2}\,(N+1)\,(N+2)\}^{-1/3}, \tag{2}$$

	Z	N
$\varkappa_0$	0.180	0.210
μ_0	0.620	0.308

Up to now in the shell model ansatz (1) only the long range part of the nuclear force is taken into account. But a comparison with the moments of inertia and the stiffness of the nuclei against β- and γ-vibrations shows impressively, that the short range part cannot be neglected. This we have taken into account by a BCS-calculation, considering 24 levels symmetrically to the Fermi surface. The strength of the pairing force was adjusted to the even-odd-mass-differences of the actinides [6]. This yields

$G * A = 32$ MeV for protons,

$$\hbar\,\omega_0 = 41.0\,A^{-1/3}. \tag{3}$$

$G * A = 29$ MeV for neutrons,

We also took into account the Coulomb energy $E_c(a_0, a_2)$, for which we made the ansatz of a homogeneously charged ellipsoid [1, 3, 7]. The PES is then obtained in the form

$$E(a_0, a_2) = \sum_{Z,N}\{\sum_i \varepsilon_i(a_0, a_2)\,V_i^2 - \Delta^2/G\} + E_c(a_0, a_2) \tag{4}$$

where $\varepsilon_i(a_0, a_2)$ are the eigenvalues of the Hamiltonian (1), and V_i^2 and Δ^2 are respectively the occupation probability of the single particle level $|i\rangle$ and the gap os occuring in the BCS formalism. With this method the quadrupole degrees of freedom of the PES, which are most important in the deformation process between ground state and fission barrier have been treated. In particular we computed all quantities (fission frequency, zero point energy etc.) of the nuclei, which are needed for the investigation of the fission half-lives. The fission half-lives are obtained by a simple WKB calculation [1, 3].

For the purpose of comparison with the outlined calculations we also computed the PES using the Strutinsky renormalization technique [2, 8]. This technique starts with the semiempirical mass formula of the liquid drop model [9], which reproduces quite well the nuclear binding energies as a function of the mass- and charge-number. One achieves a better agreement with the experiment by introducing empirical shell corrections, which are a function of the deformation of the drop. The success of this classical model – if applied to fission theory – seems to indicate, that if shell effects are by some means, averaged out from the microscopically calculated PES, this "averaged" PES can be identified with the PES of the LDM model. One is thus lead to the procedure of removing the average trend from the single particle calculations of the PES [calculated according to Eq. (4)] and replacing it by the PES obtained from the liquid drop model. In other words, the local structure (shell effects) is determined by the shell model and added to the general smooth trend given by the liquid drop model. For the averaging of the single particle energies we used the method of STRUTINSKY [8]. Thus, in this second model, the PES has the form

$$E(a_0, a_2) = E_{\mathrm{LD}} + \sum_{N,Z}\{E_{\mathrm{shell}} - E_\gamma\} \tag{5}$$

where $$E_{\mathrm{shell}} = E(a_0, a_2) - E_c(a_0, a_2) \tag{6}$$

and $$E_{\mathrm{LD}} = -c_1(1-\varkappa I^2)\,A + c_2\,f_s(1-\varkappa I^2)\,A^{2/3} + E_c(a_0, a_2) \tag{7}$$

with the parameters of the mass formula [9]

$$c_1 = 15.677 \qquad \varkappa = 1.79\,,$$
$$c_2 = 18.56\,, \qquad I = (N-Z)/A\,. \tag{8}$$

The averaged energy $E(\gamma)$ depends on the width of the gaussian functions, which smear out the single particle energies. They are adjusted for each region:

$\gamma = 0.88\,\hbar\,\omega_0$ for nuclei around $Z = 114$,

$\gamma = 1.10\,\hbar\,\omega_0$ for nuclei around $Z = 164$. (9)

[5] P. W. SEEGER and R. C. PERISHO, LA 3751, Los Alamos 1967.

[6] J. GRUMANN, Diplomarbeit, Frankfurt (Main) Januar 1969.

[7] B. C. CARLSON, J. Math. Phys. **2**, 441 [1961].

[8] V. M. STRUTINSKY, Nucl. Phys. **A 96**, 420 [1967]; **A 122**, 1 [1968].

[9] W. D. MYERS and W. J. SWIATECKI, Nucl. Phys. **81**, 1 [1966]; Ark. Fys. **36**, 36 [1966].

To look for the sensitivity of the surface energy on small changes of the geometrical form of the drop we compare simple ($a_2 = 0.0$) ellipsoidal and quadrupole shapes. By assuming an ellipsoidal shape for the LDM the correction functions of the surface "f_s" and the Coulomb energy can be integrated analytically, while for quadrupole shapes numerical methods are necessary. For the Coulomb energy we took an expansion [10] up to the 8-th order in the deformation parameter a_0 [11]

$$\frac{E_c\,(\text{Quadrupole})}{E_c\,(\text{Sphere})} = 1 + k_2\,a_0^2 + k_3\,a_0^3 + \ \dots + k_n\,a_0^n,$$

$$\begin{aligned} k_2 &= -0.79577472\cdot 10^{-1}, & & \\ k_3 &= -0.95611666\cdot 10^{-2}, & k_6 &= -0.11512560\cdot 10^{-1}, \\ k_4 &= 0.32955231\cdot 10^{-1}, & k_7 &= -0.16423437\cdot 10^{-2}, \\ k_5 &= 0.46441703\cdot 10^{-2}, & k_8 &= 0.35659242\cdot 10^{-2}. \end{aligned} \tag{10}$$

The investigation of the PES (4) shows, that many superheavy nuclei prefer energetically oblate fission [1,3]. Now the question arises whether the model (5) for the PES, which is based on the liquid drop model with shell corrections does show the same effect?

III. Comparison of the Methods

In comparing these two methods of computation, we have to keep in mind that in both models we restrict ourselves in the Hamiltonian (1) to pure ellipsoidal surfaces and the influence of higher order effects (that means: more complicated surfaces) becomes more important, if the deformation is greater than $|a_0| = 0.4$. This region is, in the case of superheavy nuclei, already "behind" the saddlepoint where the PES (4) calculated in the pure single particle model increases rapidly to infinity for increasing $|a_0|$. This is so, because the sum of the zero point energies increases in the Nilsson model with the deformation to infinity. In the Strutinsky method this effect of the Nilsson shell model is avoided because of the renormalization procedure. However, as the general trend is given by the LDM, the surface energy causes the energies to increase with deformation, in the interval $|a_0| < 0.4$, up to nuclei around $Z = 114$. Since the surface energy continues to increase for ellipsoidal shapes if the deformation increases further, the PES will increase in both procedures of calculation and, therefore, ellipsoidal shapes fail to describe the behaviour of the nuclei if $|a_0| \gg 0.4$. For the influence of the vibrational degree of freedom (a_2) on the fission barriers is small [1], relative to that of a_0, we set a_2 to zero in the Hamiltonian (1) and in the surface- and Coulomb energy (ellipsoidal and quadrupole shaped) of the mass formula.

Indeed, some calculations of the PES according to the method (5), in which the same parameters for calculation of the shell correction were used as in the model (4), reveal the following: For sur-

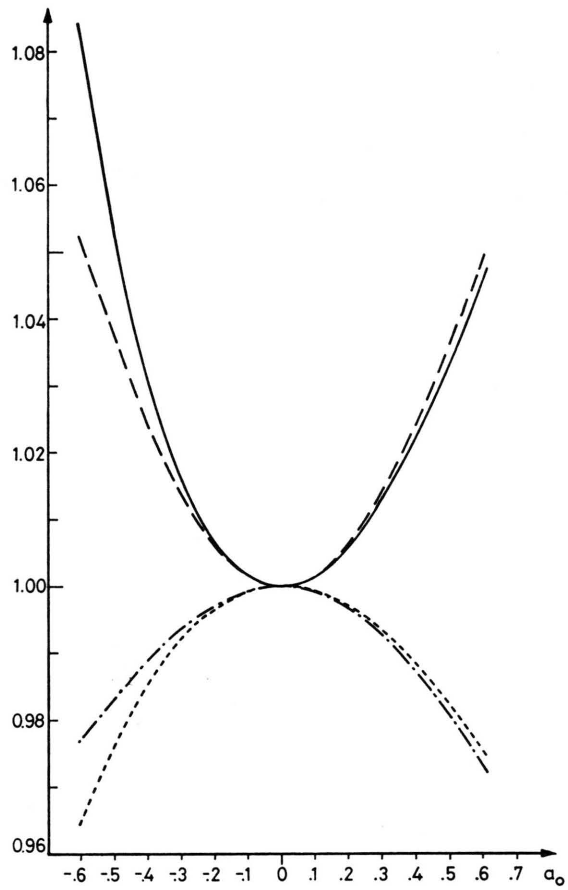

Fig. 1. The correction functions (normalized to a sphere), which involve the total deformation dependence of the LDM [9]. The two rising curves belong to the surface term, for an ellipsoidal (full line) and quadrupole surface (dashed line). The dot-dashed (quadrupole surface) and dotted line (ellipsoidal surface) show the dependence of the Coulomb energy on the deformation. There is nearly a congruence for both shapes by prolate deformations, but remarkable differences for strongly oblate values.

[10] W. J. Swiatecki, Second United Nations International Conference on Peaceful Uses of Atomic Energy, Geneva 1. Sep.—13. Sep. 1958, Vol. 15, page 248.

[11] We thank R. Fraser for computing the coefficients.

faces which belong to the intervall $-0.2 < a_0 < 0.2$ both models agree very well in their trends. Some differences in the absolute values (for example the stiffness against vibrations etc.) can be eliminated by a new fit. The prolate fission barriers appear at the same deformations and show the same trend. But in the LDM-based calculations there are no oblate barriers for all the nuclei up to $Z \sim 114$. Only in the region around $Z = 164$ the oblate barrier occurs in both models (see Figs. 2 and 3).

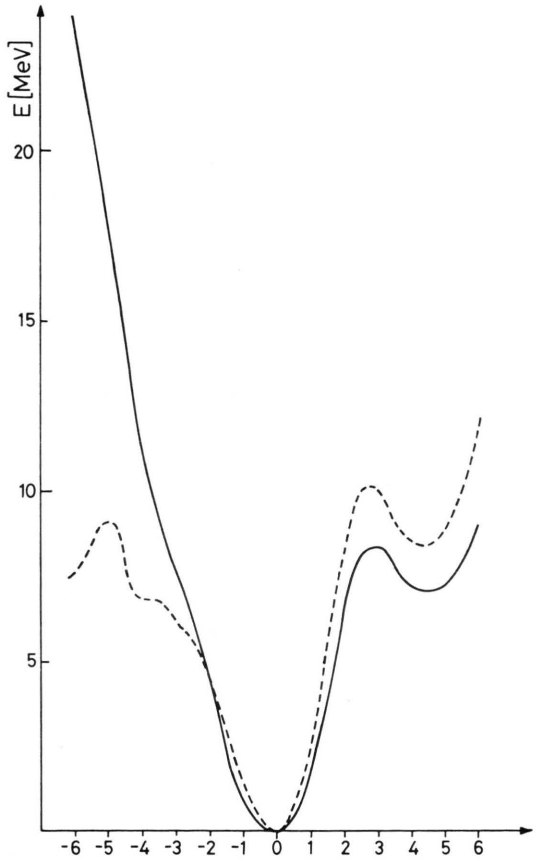

Fig. 2. A cut of the PES along the a_{20}-axis ($a_{22} = 0$) calculated for $^{194}X_{114}$ with an ellipsoidal shape. The dashed line depends on the pure single particle model, whereas the full line shows the normalized PEC. Both functions are set zero for the spherical shape $[E(a_{20} = 0,\ a_{22} = 0) = 0.0]$.

This lack of the oblate barrier for the nuclei up to $Z \sim 114$ is only due the surface energy terms of the LDM because the shell correction of the Strutinsky method shows this oblate effect. In Figure 1 are drawn the functions which involve the deformation dependent part of the LDM. We see that especially for oblate deformations the surface term rises ex-

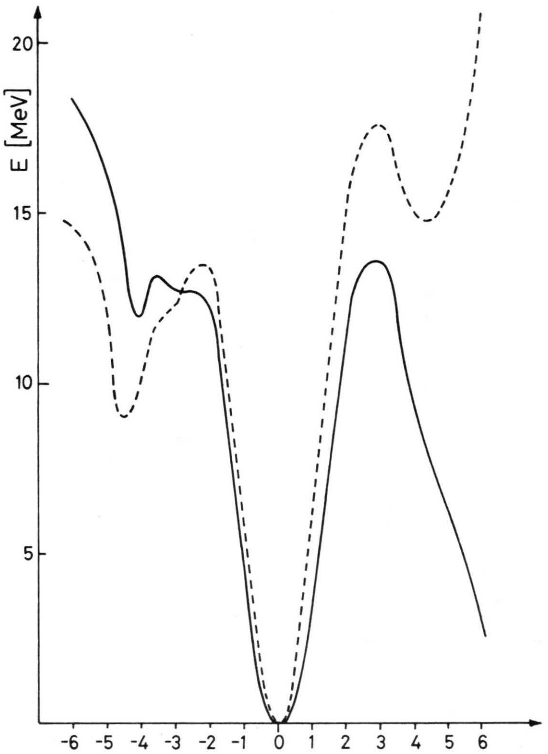

Fig. 3. The same as in Fig. 2 for the nucleus $^{318}X_{164}$.

tremely rapidly with increasing deformation and, therefore, the oblate shell effects are compensated and washed out. This peculiar trend depends not on the approximation of an ellipsoidal surface, since a quadrupole shape shows a similar effect. A comparison of these resembling forms shows (see Figs. 4 and 5), that the PES is hardly altered for prolate deformation up to $a_0 \sim 0.5$ but on the oblate side the quadrupole shape shows an even weaker indication of a saddle.

We thus see, that with this simple LDM we cannot reproduce and study the oblate barriers suggested by the model of Eq. (4). Indeed, one expects that an oblate nucleus fissions by sneaking in the φ-direction, where φ is the azimuthal angle defined around the symmetry axis. One therefore needs to include the $Y_{2\pm 2}$, $Y_{3\pm m}$ $(m = 1, 2, 3)$ $Y_{4\pm m}$ $(m = 1, 2, 3, 4)$ etc. deformations as well in the liquid drop as in the shell model calculations. This has been done recently by FRASER et al. [12, 13] who have found that the $Y_{3\pm 3}$-modes, for example, be-

[12] R. FRASER and W. GREINER, to be published.

[13] R. FRASER, J. GRUMANN, and W. GREINER, submitted to Phys. Rev. Lett.

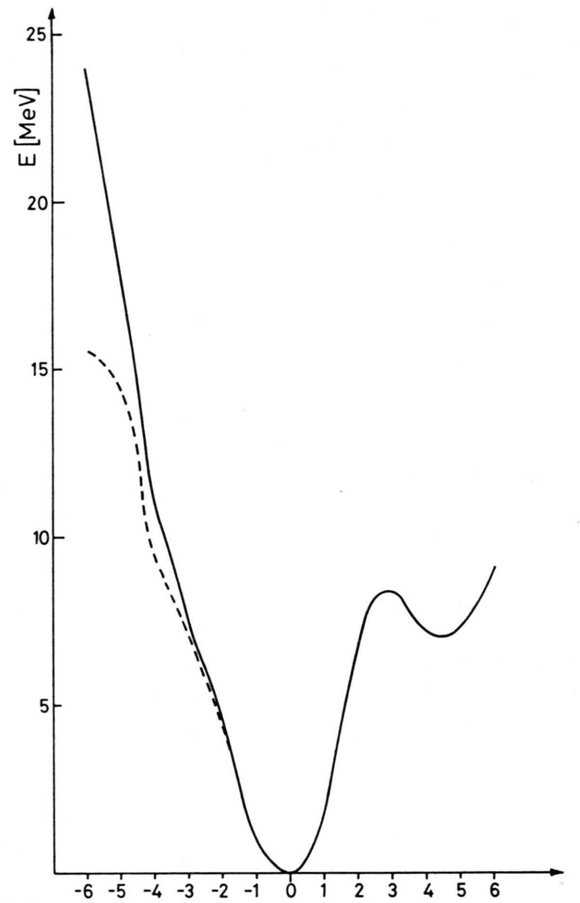

Fig. 4. A comparison of the Strutinsky renormalized PEC for $^{184}X_{114}$ between ellipsoidal (full line) and quadrupole (dashed line) shapes. Unit of the abscissa is $a_0 * 10$.

come unstable at the oblate saddle point thus leading to ternary fission. One thus can conclude that in order to investigate the possibility of oblate fission further the asymmetric (φ-dependent) shell models have to be studied and — more correctly — the more-center-shell models have to be developed especially in connection with multiple fission processes [14].

IV. Mass Formual and Binding Energies

For the calculation of radioactive decays, the nuclear binding energies are needed in order to obtain Q-values for the various break-ups. There are various mass formulas, phenomenological or

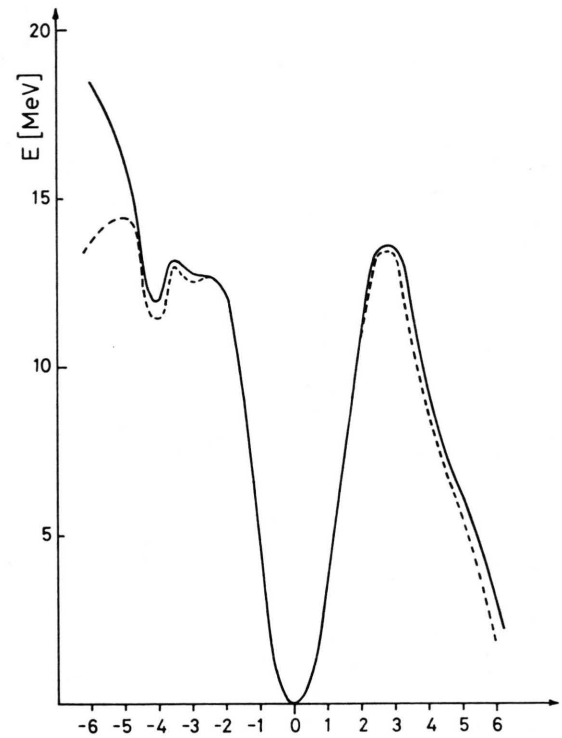

Fig. 5. The same as in Fig. 4 for $^{318}X_{164}$. In both Figs. 4 and 5 the different shapes influence only oblate deformations and it seems, that the quadrupole shape tends to give an oblate saddle point.

model based, that predict the binding energies. We tried to use that one, which shows the best agreement with the experimental data, but keeps the computing time as low as possible, namely the mass-formula of MYERS and SWIATECKI [9]. It keeps the maximal difference between "theoretical" and "experimental" data within 5 MeV [15]. It also contains some kind of shell correction due to the nuclear shell structure and incorporates nuclear deformations in a crude manner. So one can say that the formula contains a great amount of our knowledge about nuclear matter. For axially symmetric nuclei the formula has the form:

$$B(N,Z;\alpha) = c_1 A - c_2 A^{2/3}(1 + \tfrac{2}{5}\alpha^2 - \tfrac{4}{105}\alpha^3) - c_3 \frac{Z^2}{A^{1/3}} (1 - \tfrac{1}{5}\alpha^2 - \tfrac{4}{105}\alpha^3) + c_4 \frac{Z^2}{A} - S(N,Z)(1-\Theta^2)\exp(-\Theta^2) - \delta \quad (11)$$

where α is the nuclear deformation and

$$c_1 \text{ stands for } a_1[1 - \varkappa((N-Z)/A)^2], \quad (12)$$

[14] Such work is in progress by H. DIEHL, P. BERGMANN, and H. J. SCHEFFER, Institut für Theoretische Physik der Universität Frankfurt (Main).

[15] A. M. WAPSTRA, Atomic Masses of Nucleides, Handbuch der Physik, Bd. **38**, Sect. 12, p. 1.

while

$$c_2 \text{ stands for } a_2[1-\varkappa((N-Z)/A)^2] \,. \qquad (13)$$

$S(N,Z)$ is the nuclear shell correction function defined in Ref. [9]. Θ denotes the deformation magnitude α/α_0. In our calculations we started with the two original sets of the parameters as given in Ref. [9]. They are:

	Set I		Set II		
a_1	15.677	MeV	15.4941	MeV	
a_2	18.56	MeV	17.9439	MeV	
c_3	0.717	MeV	0.7053	MeV	
c_4	1.21129	MeV	1.1526	MeV	(14)
$\varkappa$	1.79		1.7826		
α_0^2	$0.3645/A^{2/3}$		$0.98568/A^{2/3}$		
C	5.8	MeV	5.8	MeV	
c	0.2		0.325		

c and C being the constants occuring in the shell correction $S(N,Z)$. Later in Sect. VI we will calculate the extrapolation of the beta stable valley as well with this parameter set and with slightly modified parameters in order to see how sensitive the extrapolation of the valley is to the various changes (uncertainties) in the parameters.

V. Electron Capture in Superheavy Nuclei

The usual electron-capture (EC) rate [16] is given by

$$\lambda_{\mathrm{EC}} = \frac{2\pi}{\hbar} \sum_{n,\bar{n}} |\langle f | H_\beta | i \rangle|^2 \qquad (15)$$

where the index n stands for all the quantum numbers of a bound electron state and the index $\bar{n}$ for all the quantum numbers of the outgoing neutrino. The rate can be evaluated in some type of multipole expansion [16, 17]. We use the well known multipole expansion by BRYSK and ROSE [17] i. e. we take only the transition matrix elements with the largest contribution to each multipole expansion term. In the vicinity of the magic numbers the single particle shell model is supposed to be a good approximation for the wave functions of the nucleons. The nuclear wave function need not to be relativistic, as the velocity of the nucleons reach only about 1/10 of the velocity of light even for the heaviest nuclei. The shell model wave functions of the Hamiltonian (1) satisfy these conditions.

The transition energies $W(Z,N)$ are calculated from the binding energies $B(Z,N)$ of the decaying nuclei in the usual manner:

$$W(Z,N) = B(Z-1,N+1) - B(Z,N) - 1.294 + B_{\mathrm{n}}(\mathrm{e})\,; \qquad (16)$$

$B_{\mathrm{n}}(\mathrm{e})$ is the total energy of the bound electron including the electron rest mass in the n-shell. The binding energies have been calculated as above, but in the region around $Z=114$ we used also the binding energies calculated by SEEGER and PERISHO [5].

The electrons of such heavy nuclei have to be treated relativisticly and thus the Dirac equation for bound electrons has to be solved exactly. This was done, as by FRICKE et al. [18], by a numerical integration of the Dirac equation, with a Thomas-Fermi potential approximating for the many electron problems. All the difficulties with the screening effects due to using the hydrogen-like wave function do not arise in this type of calculation. If $R=1.2\,A^{1/3}$ fm is the usual radius of the spherical nuclei and

$$\varrho_{\mathrm{F}}(r) = \frac{\varrho_0}{1+\exp[(r-b)/a]} \qquad (17)$$

the Fermi-type proton distribution, we define the constants a and b as

$$a = R/10\,, \qquad b = R - a/2\,. \qquad (18)$$

The values of the "large" and "small" components of the electron spinor are taken at the average process radius defined as b. The whole calculation has been done up to $Z=171$ where the 1 s electron reaches the lower (positron) continuum [18]. Only the 1 s, 2 s and 2 $\mathrm{p}_{1/2}$ electrons are considered in this calculation, because they show the largest overlap with the nucleus [18]. The EC-rate has been calculated for all even-odd and odd-even nuclei in both superheavy regions around $Z=114$ and $Z=164$. As shown in the Table 1 the calculation reproduces

[16] M. F. SCHOPPER, Weak Interactions and Nuclear Beta Decay, North Holland Publ. Co., Amsterdam 1966, or J. E. KONOPINSKI, The Theory of Beta Radioactivity, Clarendon Press, Oxford 1966.

[17] H. BRYSK and M. E. ROSE, Rev. Mod. Phys. **30**, 1169 [1968]. — T. MOROVIĆ, Diplomarbeit, Frankfurt (Main) 1969. — T. MOROVIĆ, Nucelar Reactions Induced by Heavy Ions, North Holland Publ. Co., Amsterdam 1969, p. 799.

[18] B. FRICKE and W. GREINER, Phys. Lett. **30 B**, No. 5, 317 [1969]; Physik in unserer Zeit **1**, 21 [1970]. — B. FRICKE, W. GREINER, and J. WABER, submitted to Theor. Chem. Acta.

the experimental half-lives of the known nuclei quite well.

Table 1. Experimental and calculated EC half-lives in the region of known nuclei. The state X* represent the decays into excited states of the daughter nuclei. Experimental data has been taken from the Nuclear Data Sheets.

Decay	$\log f_0 t$ (exp)	Half-Lives [sec] Exp.	Calc.
$^{58}Ce_{81} \rightarrow {}^{57}La_{82}$	5.3	$5 \cdot 10^6$	$4.8 \cdot 10^6$
$^{60}Nd_{81} \rightarrow {}^{59}Pr_{82}$	5.7	$5 \cdot 10^6$	$1.4 \cdot 10^6$
$^{81}Tl_{118} \rightarrow {}^{80}Hg_{119}$	5.9	$5 \cdot 10^5$	$7.7 \cdot 10^5$
$^{81}Tl_{118} \rightarrow {}^{80}Hg_{119}$	6.3	$6 \cdot 10^5$	$3.2 \cdot 10^6$
$^{81}Tl_{118} \rightarrow {}^{80}Hg_{119}$	6.2	$3 \cdot 10^5$	$5.4 \cdot 10^5$
$^{83}Bi_{122} \rightarrow {}^{82}Pb_{123}$	5.6	$1 \cdot 10^7$	$2.9 \cdot 10^7$
$^{83}Bi_{124} \rightarrow {}^{82}Pb_{125}$	7.4	$4 \cdot 10^9$	$5.1 \cdot 10^{10}$

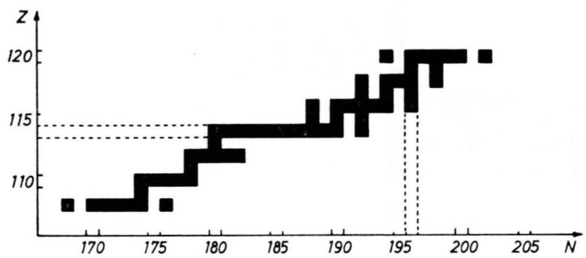

Fig. 6 a.

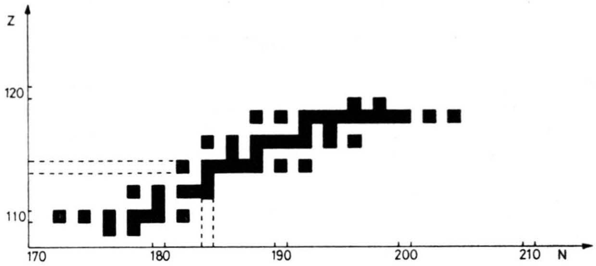

Fig. 6 b.

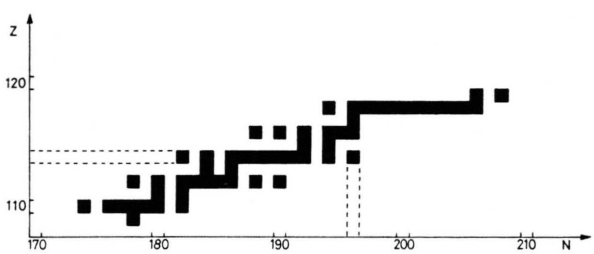

Fig. 6 c.

Fig. 6. Beta stable valley for the region around $Z=114$; a) with the results of the Ref. [5]; b) using the formula (11) with the set II and assuming the magic neutron number $N=184$; c) the same as b) but $N=196$.

VI. The Lifetimes of Superheavy Elements

The various extrapolations of the LD mass formulas induce an uncertainty of the beta stable valley. However, the variations are not too large so that the various extrapolations include always the regions around $Z=114$, $N=196$ (184) and $Z=164$, $N=318$. The typical stairway behaviour of the valley remains conserved due to the shell corrections applied to the LDM (see Figs. 6 and 7). The electron capture half-lives in the superheavy region around $Z=114$ are shown for various mass-formulas in Fig. 10. They do not exceed the values of the known elements. For the nuclei of the island around $Z=164$, however, we find much shorter half-lives (see Fig. 13), as the overlap of the nuclear wave functions and the overlap of the electron wave function with the nucleus are relatively larger in this region due to the large proton numbers.

These more detailed calculations on the stability of all even-even nuclei around $Z=114$, $N=196$ and $Z=164$, $N=318$ against spontaneous fission, show only small differences from the recent calculations of Ref. [3]. We used the same WKB approximations for the estimates of the half-lives against spontaneous fission:

$$^{10}\log T_{1/2}\,[\text{years}] = -28.04 - {}^{10}\log E_{\text{vib}} + 0.434\,\text{K}\,. \quad (19)$$

The notation is the same as in Ref. [3].

The half-lives against α-decay has been calculated with the crude formula:

$$^{10}\log T_{1/2}\,[\text{years}] = C_1[Z\,Q^{-1/2} - Z^{2/3}] - C_2 \quad (20)$$

where again the notation of Ref. [3] has been used.

In order to predict possible stable superheavy elements we have to look for the different decay channels. In the region around $Z=114$ the contour lines of the half-lives (Fig. 8) show a large area

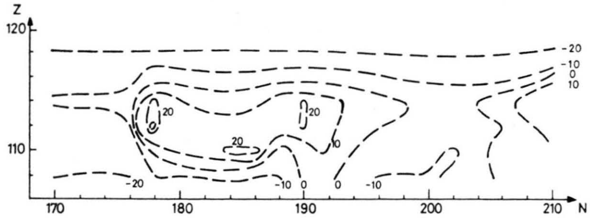

Fig. 8. The fision half-lives for the region $Z=114$. Units $^{10}\log$ (years).

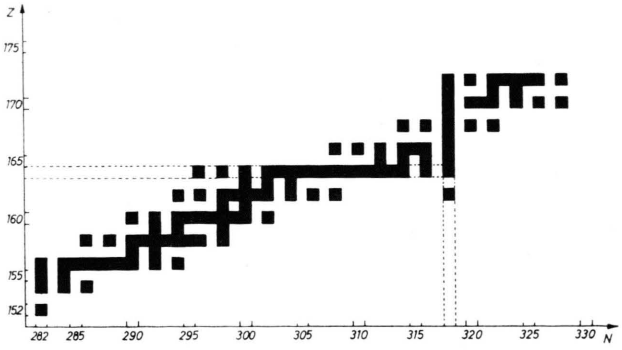

Fig. 7 a.

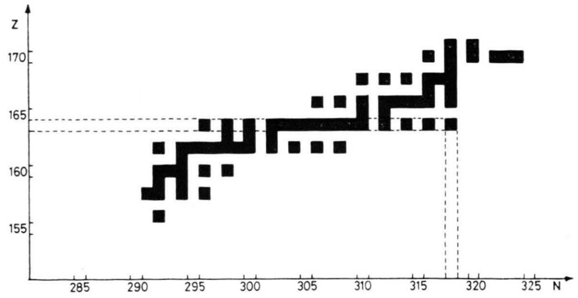

Fig. 7 b.

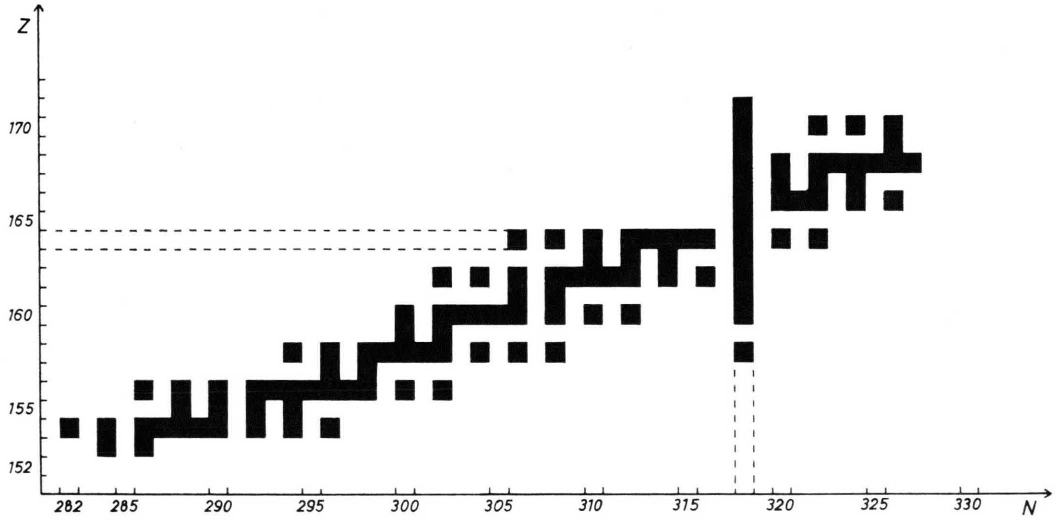

Fig. 7 c.

Fig. 7. Beta stable valley for the region around $Z=164$; a) using the formula (11) with the set I; b) the same as a) but using the set II; c) the same as a) but changing the parameters a_1 and a_2 as follows: increasing the asymmeary constant by 5 percent or increasing asymmetry and surface constant each by 5 percent or decreasing both by 5 percent.

where the nuclei are stable against spontaneous fission. But in comparison with the nuclei around $Z=164$ (Fig. 11) were the influence of the magic neutron number $N=318$ is evident, we cannot determine clearly whether $N=184$ or $N=196$ or both are magic numbers in the region around $Z=114$. Therefore, we used both as magic numbers by calculating the binding energies.

Taking all the decay channels into account, we see that around $Z=114$ electron capture does not have any importance as the half-lives are much longer than those of fission and alpha decay. Especially, at the neutron poor side, alpha half-lives become very short and in Figs. 9 a and 9 b we see that the general trend, to follow the beta-stable

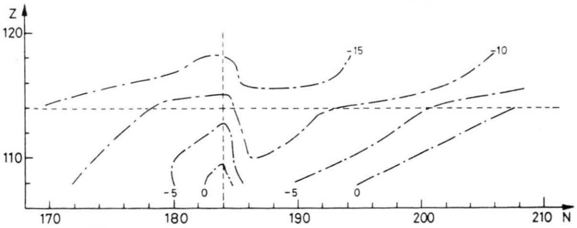

Fig. 9 a. The α-decay half-lives for the region $Z=114$ assuming, that $N=184$ being magic. Units $^{10}\log$ (years).

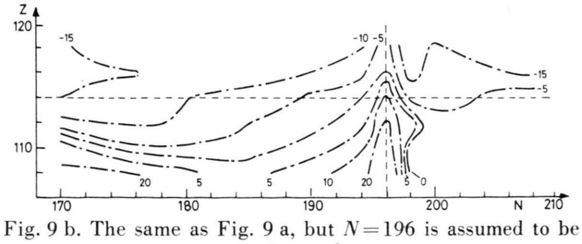

Fig. 9 b. The same as Fig. 9 a, but $N=196$ is assumed to be a magic number.

valley, is disturbed by the magic neutron numbers $N=184$, and $N=196$. The influence of these magic numbers on EC and beta-stability is small. There is, with respect to $N=184$ (Fig. 10 b), for $N=196$

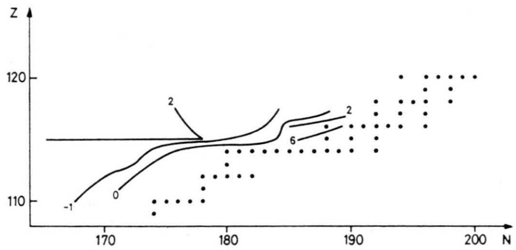

Fig. 10 a.

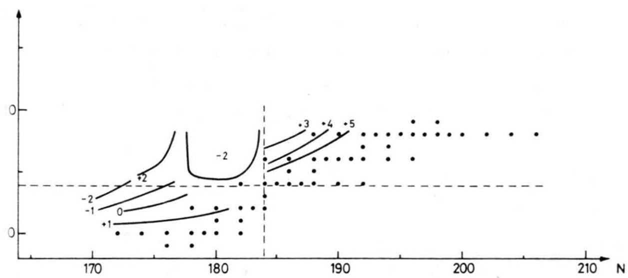

Fig. 10 b.

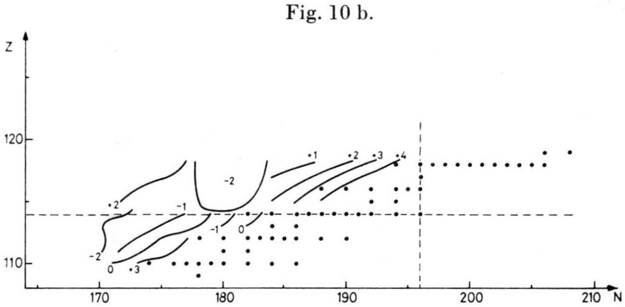

Fig. 10 c.

Fig. 10. EC half-lives for the region $Z=114$. Units ^{10}log (years). a) Using the results of the Ref. [5]; b) using (11) with the set II and assuming the magic number $N=184$; c) the same as b) but $N=196$. The black circles represent β-stable nuclei.

a regular shift to the neutron rich nuclei (Fig. 10 c). In order to determine which nuclei are stable we add the contours for spontaneous fission, electron capture, beta-stability and alpha-decay (Figs. 14 and 15) and note that for both magic numbers the nuclei around $Z=110$ and $N=184$ have a lifetime of almost a year.

In the region around $Z=164$ the beta-stability line lies above the fission and alpha-stable region (Fig. 16). Due to the uncertainty of the parameter extrapolation we used in this region two parameter

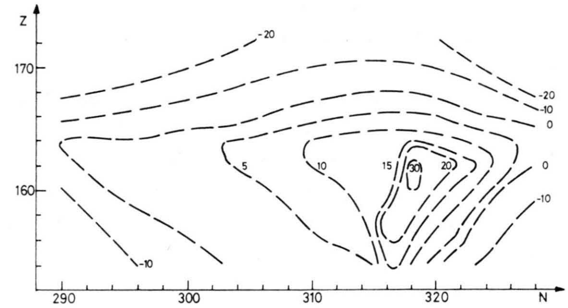

Fig. 11. The same as Fig. 8 for $Z=164$.

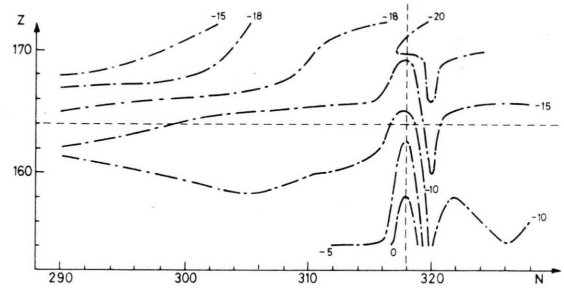

Fig. 12. The same as Fig. 9 for $Z=164$.

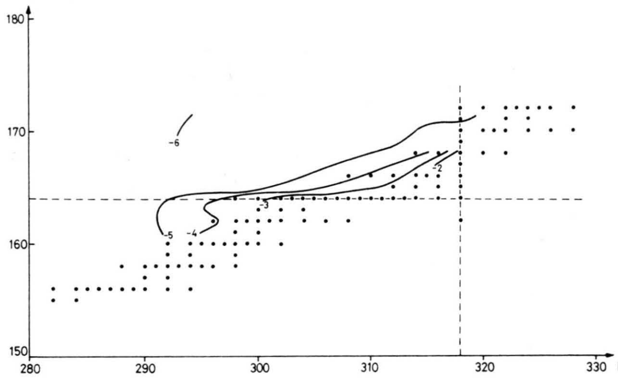

Fig. 13 a.

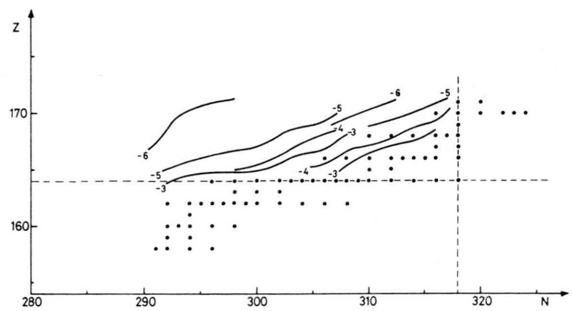

Fig. 13 b.

Fig. 13. The same as Fig. 10 for $Z=164$. a) Using (11) with the set I; b) using (11) with the set II.

sets for the binding energies and Figs. 13 a and 13 b show their small influence on EC and beta-stability.

If we assume that β^--decay gives similar half-lives to EC (see also Ref. [19]) we see that in the quasi-stable region the nuclei should have β^--half-lives of about 10^{-6} years. In fact, this quasi-stable region lies also on the neutron rich side of the beta stability line as in the region around $Z = 114$.

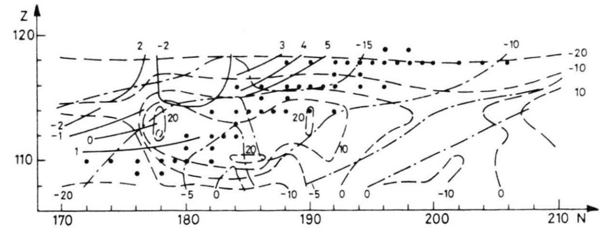

Fig. 14. Half-lives in the region $Z=114$. All three decay channels are open. The magic neutron number is assumed to be $N=184$. Units ^{10}log (years).

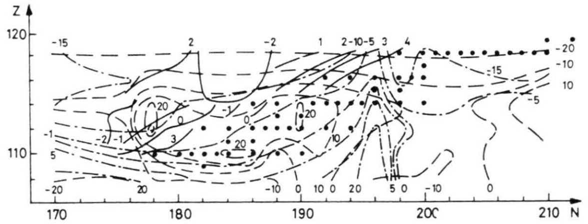

Fig. 15. The same as Fig. 14 but assuming $N=196$ as magic neutron number.

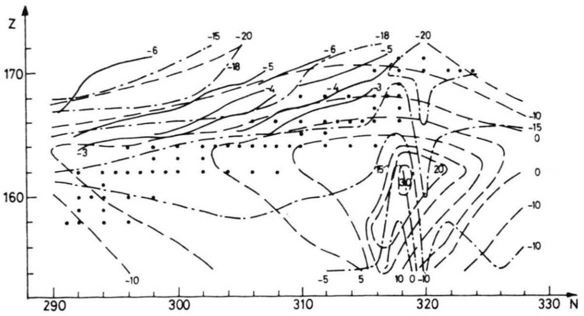

Fig. 16. Half-lives in the region $Z=164$. All three decay channels are open. Units ^{10}log (years).

We can also see that through the successive EC one cannot reach quite stable nuclei in the region $Z = 114$, because the α-half-lives are smaller by several orders of magnitude at the neutron poor side of the island. Furthermore, in the region $Z = 164$ the beta-stability line lies too far above the stability island. This, of course, relies on the far extrapolation of the mass formula and on the shell model. It may, therefore, be quite possible that these results can be changed favorably as well as more unfavorably.

[19] M. Yamada, K. Takahashi, and S. I. Koyama, Paper presented at Conference on the Properties of Nuclei Far from the Region of Beta-Stability in Leysin, Switzerland 1970.