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Can supercooling explain the HBT puzzle?

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Abstract

Possible hadronization of supercooled QGP, created in heavy ion collisions at RHIC and SPS, is discussed within a Bjorken hydrodynamic model. Such a hadronization is expected to be a very fast shock-like process, what, if hadronization coincides or shortly followed by freeze out, could explain a part of the HBT puzzle, i.e., the flash-like particle emission ($R_{out}/R_{side} \approx 1$). HBT data also show that the expansion time before freeze out is very short (~ 6–10 fm/c). In this Letter we discuss the question of supercooled QGP and the timescale of the reaction.

1. Introduction

Two-particle interferometry has become a powerful tool for studying the size and duration of particle production from elementary collisions (e^+e^-, pp and $p\bar{p}$) to heavy ions like Au + Au at RHIC or Pb + Pb at SPS [1,2]. For the case of nuclear collisions, the interest mainly focuses on the possible transient formation of a deconfined state of matter. This could affect the size of the region from where the hadrons (mostly pions) are emitted as well as the time for particle production.

Comparing recent data [3] from RHIC with SPS data one finds a "puzzle" [4]: all the HBT radii are pretty similar although the center of mass energy is changed by an order of magnitude. Discussions at "Quark Matter 2002" [5] lead to the conclusion that the duration of particle emission, as well as the lifetime of the system before freeze out, appear to be shorter than the predictions of most of the model at the physics market.

It was demonstrated that a strong first-order QCD phase transition within continuous hydrodynamical

expansion would lead to long lifetimes of the particle source [2,6,7],[1] which would manifest itself as a large R_{out}/R_{side} ratio. Now this type of hadronization is excluded by experimental data.

An alternative possibility, discussed in Refs. [9–13], is the hadronization from the supercooled QGP (sQGP). This is expected to be a very fast shock-like process. If the hadronization from sQGP coincides with freeze out, like it was assumed in Ref. [12], then this could explain a part of the HBT puzzle, i.e., the flash-like particle emission ($R_{out}/R_{side} \approx 1$). In this work we are asking the following question—can the hadronization from sQGP explain also the another part of the HBT puzzle, i.e., a very short (~ 6 [14]–10 [5,15] fm/c) expansion time before freeze out?

2. Shock hadronization of the sQGP

Relativistic shock phenomena were widely discussed with respect to their connection to high-energy heavy ion collisions (see, for example, [16]). In thermal equilibrium by admitting the existence of the sQGP and the superheated hadronic matter (HM) we have essentially richer picture of discontinuity-like transitions than in standard compression and rarefaction shocks. The system evolution in relativistic hydrodynamics is governed by the energy–momentum tensor $T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}$ and conserved charge currents (in our applications to heavy ion collisions we consider only the baryonic current nu^{μ}). They consist of local thermodynamical fluid quantities (the energy density ϵ, pressure p, baryonic density n) and the collective four-velocity $u^{\mu} = \sqrt{1 - \mathbf{v}^2}\,(1, \mathbf{v})$. Continuous flows are the solutions of the hydrodynamical equations:

$$\partial_{\mu}T^{\mu\nu} = 0, \qquad \partial_{\mu}nu^{\mu} = 0, \tag{1}$$

with specified initial and boundary conditions. These equations are nothing more than the differential form of the energy–momentum and baryonic number conservation laws. Along with these continuous flows, the conservation laws can also be realized in the form of

discontinuous hydrodynamical flows which are called shock waves and satisfy the following equations:

$$T_0^{\mu\nu}d\sigma_{\nu} = T^{\mu\nu}d\sigma_{\nu}, \qquad n_0u_0^{\mu}d\sigma_{\mu} = nu^{\mu}d\sigma_{\mu}, \tag{2}$$

where $d\sigma^{\mu}$ is the unit 4-vector normal to the discontinuity hypersurface. In Eq. (2) the zero index corresponds to the initial state ahead of the shock front and quantities without an index are the final state values behind it. A general derivation of the shock equations (valid for both space-like and time-like normal vectors $d\sigma^{\mu}$) was given in Ref. [17].

The important constraint on the transitions (2) (thermodynamical stability condition) is the requirement of non-decreasing entropy (s is the entropy density):

$$su^{\mu}d\sigma_{\mu} \geqslant s_0u_0^{\mu}d\sigma_{\mu}. \tag{3}$$

To simplify our consideration and make our arguments more transparent we consider only one-dimensional hydrodynamical motion. To study the shock transitions at the surface with space-like (s.l.) normal vector (we call them s.l. shocks) one can always choose the Lorentz frame where the shock front is at rest. Then $d\sigma^{\mu} = (0, 1)$ at the surface of shock discontinuity, and Eq. (2) in this (standard) case becomes:

$$T_0^{01} = T^{01}, \qquad T_0^{11} = T^{11}, \qquad n_0u_0^1 = nu^1. \tag{4}$$

Solving Eq. (4) one obtains

$$\begin{aligned} v_0^2 &= \frac{(p - p_0)(\epsilon + p_0)}{(\epsilon - \epsilon_0)(\epsilon_0 + p)}, \\ v^2 &= \frac{(p - p_0)(\epsilon_0 + p)}{(\epsilon - \epsilon_0)(\epsilon + p_0)}, \end{aligned} \tag{5}$$

and the well-known Taub adiabat (TA) [18]

$$n^2X^2 - n_0^2X_0^2 - (p - p_0)(X + X_0) = 0, \tag{6}$$

where $X \equiv (\epsilon + p)/n^2$.[2]

For discontinuities on a hypersurface with a time-like (t.l.) normal vector $d\sigma^{\mu}$ (we call them t.l. shocks) one can always choose another convenient Lorentz frame ("simultaneous system") where $d\sigma^{\mu} = (1, 0)$.

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[1] If we use some microscopic model for hadronization, for example, nucleation of relativistic first-order phase transition [8], the lifetime is even longer—it was estimated to be about 50–100 fm/c in Ref. [8].

[2] It has been shown in a series of works [19], that freeze out through the space-like hypersurface leads to non-equilibrium post FO distribution.
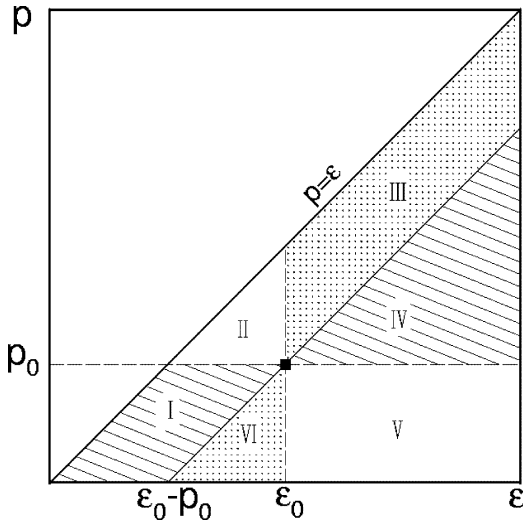
Fig. 1. Possible final states in the (energy density–pressure)-plane for shock transitions from the initial state (ϵ_0, p_0). I and IV are the physical regions for s.l. shocks, III and VI for t.l. shocks. II and V are unphysical regions for both types of shocks. Note, that only states with $p \leqslant \epsilon$ are possible for any physical equation of state in the relativistic theory.

Eq. (2) is then

$$T_0^{00} = T^{00}, \qquad T_0^{10} = T^{10}, \qquad n_0 u_0^0 = n u^0. \tag{7}$$

Solving Eq. (7) we find

$$\tilde{v}_0^2 = \frac{(\epsilon - \epsilon_0)(\epsilon_0 + p)}{(p - p_0)(\epsilon + p_0)},$$

$$\tilde{v}^2 = \frac{(\epsilon - \epsilon_0)(\epsilon + p_0)}{(p - p_0)(\epsilon_0 + p)}, \tag{8}$$

where we use the "$\sim$" sign to distinguish the t.l. shock case (8) from the standard s.l. shocks of (5). Another relation contains only the thermodynamical variables. It appears to be identical to the TA of Eq. (6). Eqs. (8) and (5) are connected to each other by simple relations [11]:

$$\tilde{v}_0^2 = \frac{1}{v_0^2}, \qquad \tilde{v}^2 = \frac{1}{v^2}. \tag{9}$$

These relations show that only one kind of transition can be realized for a given initial state and final state. The physical regions $[0, 1)$ for v_0^2, v^2 (5) and for $\tilde{v}_0^2, \tilde{v}^2$ (8) can be easily found in $(\epsilon - p)$-plane [11]. For a given initial state (ϵ_0, p_0) they are shown in Fig. 1. For supercooled initial QGP states the TA no longer passes through the point (ϵ_0, p_0) and new possibilities of t.l.

shock hadronization transitions to regions III and VI in Fig. 1 appear.

3. Hadronization of the sQGP within Bjorken hydrodynamics

For a study of the expanding QGP we have chosen a framework of the one-dimensional Bjorken model [20] (actually our principal results will not change if we use 3D Bjorken model). Within the Bjorken model all the thermodynamical quantities are constant along constant proper time curves, $\tau = \sqrt{t^2 - z^2} = \text{const}$. The important result of Bjorken hydrodynamics (which assumes a perfect fluid) is that the evolution of the entropy density, is independent of the Equation of State (EoS), namely,

$$s(\tau) = \frac{s(\tau_{\text{init}})\tau_{\text{init}}}{\tau}. \tag{10}$$

In Bjorken model the natural choice of the freeze out hypersurface is $\tau = \text{const}$ hypersurface, where normal vector is parallel to the Bjorken flow velocity, $v = z/t$. Thus, $d\sigma^\nu = (1, 0)$ in the rest frames of each fluid element. This leads to the simple solution of the t.l. shock equations (7):

$$\tilde{v}^2 = \tilde{v}_0^2 = 0, \qquad \epsilon = \epsilon_0,$$

$$n = n_0, \qquad p \neq p_0. \tag{11}$$

The entropy condition (3) is reduced to

$$s \geqslant s_0. \tag{12}$$

Now let us try to answer the main question of this Letter—can QGP expansion with t.l. shock hadronization of supercooled state be faster than the hadronization through the mixed phase? The initial state is given at the proper time $\tau_{\text{init}} \equiv \tau_Q$, when the local thermal equilibrium is achieved in the QGP state $Q \equiv (\epsilon_Q, p_Q, s_Q)$. The final equilibrium hadron state is also fixed, by experiment or otherwise, as $H \equiv (\epsilon_H, p_H, s_H)$. For the continuous expansion given by Eq. (10) the proper time for the $Q \rightarrow H$ transition is (see Fig. 2(a)):

$$\tau_H = \frac{s_Q \tau_Q}{s_H}. \tag{13}$$

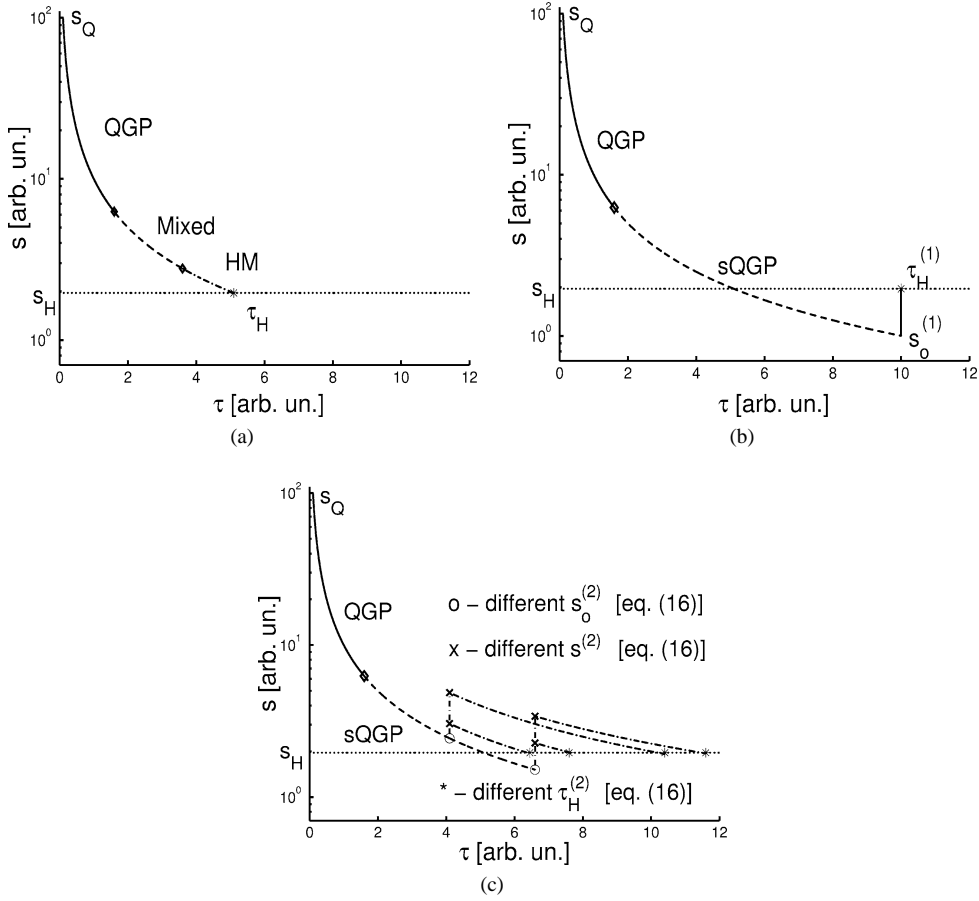If our system enters the sQGP phase and the particle emission is flash-like, i.e., the system hadronizes

Fig. 2. Different ways for a system to go from Q state (s_Q) to H state (s_H) are presented on $\{s, \tau\}$ plane. (a) Shows continuous expansion, which takes time τ_H, Eq. (13). (b) Presents flash-like particle emission, i.e., simultaneous hadronization and freeze out; which takes time $\tau_H^{(1)}$, Eq. (14). (c) Shows several possibilities according to scenario 2 with shock-like hadronization into superheated HM. Time $\tau_H^{(2)}$ (16) can be smaller or larger than $\tau_H^{(1)}$, depending on details of the EoS, but always larger than τ_H.

and freezes out at the same time, then Eq. (10) is also valid all the time with final t.l. shock transition to the same H state. We call this as a scenario number one (see Fig. 2(a)). Our system should go into supercooled phase to the point where $\epsilon_0^{(1)} = \epsilon_H, n_0^{(1)} = n_H$, as it is required by Eq. (11). At this point our sQGP has entropy density $s_0^{(1)}$. It is value depend on the EoS, but t.l. shock transition is only possible if $s_0^{(1)} \leqslant s_H$ according to Eq. (12). Thus, for the proper time of $Q \to H$ transition according to first scenario we have:

$$\tau_H^{(1)} = \frac{s_Q \tau_Q}{s_0^{(1)}} \geqslant \tau_H. \tag{14}$$

We can also study a scenario number two when our system supercools to the state $(\epsilon_0^{(2)}, p_0^{(2)}, s_0^{(2)})$, then hadronizes to a superheated HM state $(\epsilon^{(2)}, p^{(2)}, s^{(2)})$, and then this HM state expands to the same freeze out state $H \equiv (\epsilon_H, p_H, s_H)$. (see Fig. 2(c)). At the point of the shock transition one has:

$$\tau_0^{(2)} = \frac{s_Q \tau_Q}{s_0^{(2)}}. \tag{15}$$

Then we have a t.l. shock transition satisfying Eq. (11), and following the HM branch of the hydrodynamical

expansion we find:

$$\tau_H^{(2)} = \frac{s^{(2)}\tau_0^{(2)}}{s_H} = \frac{s_Q \tau_Q}{s_H}\frac{s^{(2)}}{s_0^{(2)}} \geqslant \tau_H, \qquad (16)$$

since $s^{(2)} \geqslant s_0^{(2)}$ due to non-decreasing entropy condition (12). In this second scenario the value of the of entropy density $s_0^{(2)}$ of sQGP can be both smaller and larger than HM final value s_H. Depending on details of the EoS the proper time $\tau_H^{(2)}$ (16) of the $Q \to H$ transition can also be smaller as well as larger than $\tau_H^{(1)}$ (14), but always larger than τ_H.

4. Conclusions

The conclusion of our analysis seems to be a rather general one: the system's evolution through a supercooled phase and time-like shock hadronization cannot be shorter than a continuous expansion within the perfect fluid hydrodynamics independently of the details of EoS and the parameter values of the initial, Q, and final, H, states. Although in such a way we may achieve flash-like particle emission, supported by the HBT data, the expansion time becomes longer, making it harder to reproduce the experimental HBT radii.

So, how can we achieve shorter freeze out time than the minimal one coming from (very fast) Bjorken expansion via thermal and phase equilibrium? Any delay in the phase equilibration (see assignment 9 in Ref. [21]) or/and any dissipative process in our system lead to the entropy production, what increases the time needed to reduce entropy density to $s_0 \leqslant s_H$ (for the flash-like particle emission).

The system may, nevertheless, freeze out and hadronize into a non-equilibrated hadron gas well before τ_H. This is possible, e.g., through a dominantly s.l. hypersurface with non-decreasing entropy condition, Eq. (12) [19], at earlier times from a slightly supercooled QGP. On the other hand, a dominantly s.l. hypersurface gives a finite duration of the particle emission, making it harder to reproduce experimental $R_{\text{out}}/R_{\text{side}}$ ratio.

The construction of a full reaction model, which simultaneously describes data on two particle interferometry, hadron spectra and hadron abundances is a formidable task which is still ahead of us.

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