

PHYSICS LETTERS B

Physics Letters B 662 (2008) 37-42

www.elsevier.com/locate/physletb

# The heavy-quark potential in an anisotropic plasma

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Received 4 December 2007; received in revised form 21 January 2008; accepted 14 February 2008

Available online 29 February 2008

Editor: J.-P. Blaizot

### **Abstract**

We determine the hard-loop resummed propagator in an anisotropic QCD plasma in general covariant gauges and define a potential between heavy quarks from the Fourier transform of its static limit. We find that there is stronger attraction on distance scales on the order of the inverse Debye mass for quark pairs aligned along the direction of anisotropy than for transverse alignment.

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## 1. Introduction

Information on quarkonium spectral functions at high temperature has started to emerge from lattice-QCD simulations; we refer to Ref. [1] for recent work and for links to earlier studies. This has motivated a number of attempts to understand the lattice measurements within non-relativistic potential models including finite temperature effects such as screening [2]. A detailed discussion of the properties of the heavy-quark potential in the deconfined phase of QCD is given in Ref. [3], which also provides a comprehensive list of earlier work. Also, Laine et al. have recently derived a Schrödinger equation for the finite-temperature Wilson loop to leading order within "hard-thermal loop" (HTL) resummed perturbation theory by analytic continuation to real time [4]. Aside from the well-known screened Debye potential, their result includes an imaginary part due to Landau damping of low-frequency modes of the gauge field, corresponding to a finite life-time of quarkonium states.

The present Letter is a first attempt to consider the effects due to a local anisotropy of the plasma in momentum space on the heavy-quark potential. Such deviations from perfect isotropy are expected for a real plasma created in high-energy heavy-ion collisions, which undergoes expansion. The HTL propagator of an anisotropic plasma has been calculated in time-axial gauge in Ref. [5]. We derive the result for general covariant gauges, which allows us to define a non-relativistic potential via the Fourier transform of the propagator in the static limit.

## 2. Hard-thermal-loop self-energy in an anisotropic plasma

The retarded gauge-field self-energy in the hard-loop approximation is given by [6]

$$\Pi^{\mu\nu}(p) = g^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} v^{\mu} \frac{\partial f(\mathbf{k})}{\partial k^{\beta}} \left( g^{\nu\beta} - \frac{v^{\nu} p^{\beta}}{p \cdot v + i\epsilon} \right). \tag{1}$$

Here,  $v^{\mu} \equiv (1, \mathbf{k}/|\mathbf{k}|)$  is a light-like vector describing the propagation of a plasma particle in space–time. The self-energy is symmetric,  $\Pi^{\mu\nu}(p) = \Pi^{\nu\mu}(p)$ , and transverse,  $p_{\mu}\Pi^{\mu\nu}(p) = 0$ .

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In a suitable tensor basis the components of  $\Pi^{\mu\nu}$  can be determined explicitly. For anisotropic systems there are more independent projectors than for the standard equilibrium case [5]. Here, we extend the tensor basis used in [5] to a four-tensor basis appropriate for use in general covariant gauges. Specifically,

$$A^{\mu\nu} = -g^{\mu\nu} + \frac{p^{\mu}p^{\nu}}{p^2} + \frac{\tilde{m}^{\mu}\tilde{m}^{\nu}}{\tilde{m}^2},\tag{2}$$

$$B^{\mu\nu} = -\frac{p^2}{(m \cdot p)^2} \frac{\tilde{m}^{\mu} \tilde{m}^{\nu}}{\tilde{m}^2},\tag{3}$$

$$C^{\mu\nu} = \frac{\tilde{m}^2 p^2}{\tilde{m}^2 p^2 + (n \cdot p)^2} \left[ \tilde{n}^{\mu} \tilde{n}^{\nu} - \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \left( \tilde{m}^{\mu} \tilde{n}^{\nu} + \tilde{m}^{\nu} \tilde{n}^{\mu} \right) + \frac{(\tilde{m} \cdot \tilde{n})^2}{\tilde{m}^4} \tilde{m}^{\mu} \tilde{m}^{\nu} \right],\tag{4}$$

$$D^{\mu\nu} = \frac{p^2}{m \cdot p} \left[ 2 \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \tilde{m}^{\mu} \tilde{m}^{\nu} - \left( \tilde{n}^{\mu} \tilde{m}^{\nu} + \tilde{m}^{\mu} \tilde{n}^{\nu} \right) \right]. \tag{5}$$

Here,  $m^{\mu}$  is the heat-bath vector, which in the local rest frame is given by  $m^{\mu} = (1, 0, 0, 0)$ , and

$$\tilde{m}^{\mu} = m^{\mu} - \frac{m \cdot p}{p^2} p^{\mu} \tag{6}$$

is the part that is orthogonal to  $p^{\mu}$ .

The direction of anisotropy in momentum space is determined by the vector

$$n^{\mu} = (0, \mathbf{n}),\tag{7}$$

where **n** is a three-dimensional unit vector. As before,  $\tilde{n}^{\mu}$  is the part of  $n^{\mu}$  orthogonal to  $p^{\mu}$ .

The self-energy can now be written as

$$\Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu}. \tag{8}$$

In order to determine the four structure functions explicitly we need to specify the phase-space distribution function. We employ the following *ansatz*:

$$f(\mathbf{p}) = f_{\rm iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p} \cdot \mathbf{n})^2}\right). \tag{9}$$

Thus,  $f(\mathbf{p})$  is obtained from an isotropic distribution  $f_{\rm iso}(|\mathbf{p}|)$  by removing particles with a large momentum component along  $\mathbf{n}$ . The function  $f_{\rm iso}(|\mathbf{p}|)$  should decrease monotonically with  $|\mathbf{p}|$ , so that the square of the Debye mass defined in Eq. (13) is guaranteed to be positive; however, in the real-time approach employed here, the distribution  $f_{\rm iso}$  need not necessarily be thermal.

The parameter  $\xi$  determines the degree of anisotropy,  $\xi = (1/2)\langle \mathbf{p}_{\perp}^2 \rangle / \langle p_z^2 \rangle - 1$ , where  $p_z \equiv \mathbf{p} \cdot \mathbf{n}$  and  $\mathbf{p}_{\perp} \equiv \mathbf{p} - \mathbf{n}(\mathbf{p} \cdot \mathbf{n})$  denote the particle momentum along and perpendicular to the direction  $\mathbf{n}$  of anisotropy, respectively. If  $f_{\text{iso}}$  is a thermal ideal-gas distribution and  $\xi$  is small then  $\xi$  is also related to the shear viscosity of the plasma; for example, for one-dimensional Bjorken expansion [7]

$$\xi = \frac{10}{T\tau} \frac{\eta}{s},\tag{10}$$

where T is the temperature,  $\tau$  is proper time, and  $\eta/s$  is the ratio of shear viscosity to entropy density. In an expanding system, non-vanishing viscosity implies finite momentum relaxation rate and therefore an anisotropy of the particle momenta.

Since the self-energy tensor is symmetric and transverse, not all of its components are independent. We can therefore restrict our considerations to the spatial part of  $\Pi^{\mu\nu}$ ,

$$\Pi^{ij}(p,\xi) = m_D^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v} \cdot \mathbf{n}) n^l}{(1 + \xi(\mathbf{v} \cdot \mathbf{n})^2)^2} \left( \delta^{jl} + \frac{v^j p^l}{p \cdot v + i\epsilon} \right), \tag{11}$$

and employ the following contractions:

$$p^{i}\Pi^{ij}p^{j} = \mathbf{p}^{2}\beta,$$

$$A^{il}n^{l}\Pi^{ij}p^{j} = (\mathbf{p}^{2} - (n \cdot p)^{2})\delta,$$

$$A^{il}n^{l}\Pi^{ij}A^{jk}n^{k} = \frac{\mathbf{p}^{2} - (n \cdot p)^{2}}{\mathbf{p}^{2}}(\alpha + \gamma),$$

$$\operatorname{Tr}\Pi^{ij} = 2\alpha + \beta + \gamma.$$
(12)

The Debye mass  $m_D$  appearing in Eq. (11) is given by

$$m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty d\rho \, \rho^2 \frac{df_{\rm iso}(\rho)}{d\rho},$$
 (13)

where  $\rho \equiv |\mathbf{p}|$ . We do not list the rather cumbersome explicit expressions for the four structure functions  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  here since they have already been determined in Ref. [5].

In principle, the tensor basis (2)–(5) could be chosen differently, such that the individual tensors have a simpler structure. For example, one could choose

$$C^{\mu\nu} = \tilde{n}^{\mu}\tilde{n}^{\nu} - \frac{\tilde{m} \cdot \tilde{n}}{2\tilde{m}^2} (\tilde{m}^{\mu}\tilde{n}^{\nu} + \tilde{m}^{\nu}\tilde{n}^{\mu}), \tag{14}$$

$$D^{\mu\nu} = \frac{(\tilde{m} \cdot \tilde{n})^2}{\tilde{m}^4} \tilde{m}^\mu \tilde{m}^\nu - \tilde{n}^\mu \tilde{n}^\nu. \tag{15}$$

However, in the basis (2)–(5) the spatial components of  $\Pi^{\mu\nu}$  are identical to those from Ref. [5] and so we can avoid the rather tedious re-evaluation of the four structure functions.

## 3. Propagator in covariant gauge in an anisotropic plasma

From the above result for the gluon self-energy one can obtain the propagator  $i\Delta_{ab}^{\mu\nu}$ . It is diagonal in color and so color indices will be suppressed. In covariant gauge, its inverse is given by

$$(\Delta^{-1})^{\mu\nu}(p,\xi) = -p^2 g^{\mu\nu} + p^{\mu} p^{\nu} - \Pi^{\mu\nu}(p,\xi) - \frac{1}{\lambda} p^{\mu} p^{\nu}$$

$$= (p^2 - \alpha) A^{\mu\nu} + (\omega^2 - \beta) B^{\mu\nu} - \gamma C^{\mu\nu} - \delta D^{\mu\nu} - \frac{1}{\lambda} p^{\mu} p^{\nu},$$
(16)

where  $\omega \equiv p \cdot m$  and  $\lambda$  is the gauge parameter. Upon inversion, the propagator is written as

$$\Delta^{\mu\nu}(p,\xi) = \alpha' A^{\mu\nu} + \beta' B^{\mu\nu} + \gamma' C^{\mu\nu} + \delta' D^{\mu\nu} + \eta p^{\mu} p^{\nu}. \tag{17}$$

Using  $(\Delta^{-1})^{\mu\sigma}\Delta_{\sigma}{}^{\nu}=g^{\mu\nu}$  it follows that the coefficient of  $g^{\mu\nu}$  in  $(\Delta^{-1})^{\mu\sigma}\Delta_{\sigma}{}^{\nu}$  should equal 1 while the coefficients of the other tensor structures, for example of  $n^{\mu}n^{\nu}$ ,  $n^{\mu}p^{\nu}$  and  $p^{\mu}p^{\nu}$ , should vanish. Hence, we can determine the coefficients in the propagator from the following equations

$$\alpha' = \frac{1}{p^2 - \alpha},\tag{18}$$

$$(p^2 - \alpha - \gamma)\gamma' - \delta\delta' \frac{p^2(\mathbf{p}^2 - (n \cdot p)^2)}{\omega^2} = \frac{\gamma}{p^2 - \alpha},\tag{19}$$

$$(p^2 - \alpha - \gamma)\delta' = \delta\beta' \frac{p^2}{\omega^2},\tag{20}$$

$$\frac{\delta}{p^2 - \alpha} + \delta \gamma' = (\omega^2 - \beta) \delta' \frac{p^2}{\omega^2},\tag{21}$$

$$\frac{1}{n^2} + \frac{\eta}{\lambda} p^2 = 0. {22}$$

Hence, we find that in covariant gauge the propagator in an anisotropic plasma is given by

$$\Delta^{\mu\nu} = \frac{1}{p^2 - \alpha} \left[ A^{\mu\nu} - C^{\mu\nu} \right] + \Delta_G \left[ \left( p^2 - \alpha - \gamma \right) \frac{\omega^4}{p^4} B^{\mu\nu} + \left( \omega^2 - \beta \right) C^{\mu\nu} + \delta \frac{\omega^2}{p^2} D^{\mu\nu} \right] - \frac{\lambda}{p^4} p^{\mu} p^{\nu}, \tag{23}$$

where

$$\Delta_G^{-1} = (p^2 - \alpha - \gamma)(\omega^2 - \beta) - \delta^2[\mathbf{p}^2 - (n \cdot p)^2]. \tag{24}$$

For  $\xi = 0$ , we recover the isotropic propagator in covariant gauge

$$\Delta_{\rm iso}^{\mu\nu} = \frac{1}{p^2 - \alpha} A^{\mu\nu} + \frac{1}{(\omega^2 - \beta)} \frac{\omega^4}{p^4} B^{\mu\nu} - \frac{\lambda}{p^4} p^{\mu} p^{\nu}. \tag{25}$$

## 4. Heavy quark potential in an anisotropic plasma

We determine the real part of the heavy-quark potential in the non-relativistic limit, at leading order, from the Fourier transform of the static gluon propagator,

$$V(\mathbf{r},\xi) = -g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \Delta^{00}(\omega = 0, \mathbf{p}, \xi)$$
(26)

$$=-g^{2}C_{F}\int\frac{d^{3}\mathbf{p}}{(2\pi)^{3}}e^{i\mathbf{p}\cdot\mathbf{r}}\frac{\mathbf{p}^{2}+m_{\alpha}^{2}+m_{\gamma}^{2}}{(\mathbf{p}^{2}+m_{\alpha}^{2}+m_{\gamma}^{2})(\mathbf{p}^{2}+m_{\beta}^{2})-m_{\delta}^{4}}.$$
(27)

The masses are given by

$$m_{\alpha}^{2} = -\frac{m_{D}^{2}}{2p_{\perp}^{2}\sqrt{\xi}} \left( p_{z}^{2} \arctan\sqrt{\xi} - \frac{p_{z}\mathbf{p}^{2}}{\sqrt{\mathbf{p}^{2} + \xi p_{\perp}^{2}}} \arctan\frac{\sqrt{\xi} p_{z}}{\sqrt{\mathbf{p}^{2} + \xi p_{\perp}^{2}}} \right), \tag{28}$$

$$(\sqrt{\xi} + (1+\xi)\arctan\sqrt{\xi})(\mathbf{p}^2 + \xi p_{\perp}^2) + \xi p_z(p_z\sqrt{\xi} + \frac{\mathbf{p}^2(1+\xi)}{\sqrt{\mathbf{p}^2 + \xi p_{\perp}^2}}\arctan\frac{\sqrt{\xi}p_z}{\sqrt{\mathbf{p}^2 + \xi p_{\perp}^2}})$$

$$m_{\beta}^2 = m_D^2 \frac{2\sqrt{\xi}(1+\xi)(\mathbf{p}^2 + \xi p_{\perp}^2)}{2\sqrt{\xi}(1+\xi)(\mathbf{p}^2 + \xi p_{\perp}^2)},$$
(29)

$$m_{\gamma}^{2} = -\frac{m_{D}^{2}}{2} \left( \frac{\mathbf{p}^{2}}{\xi p_{\perp}^{2} + \mathbf{p}^{2}} - \frac{1 + \frac{2p_{z}^{2}}{p_{\perp}^{2}}}{\sqrt{\xi}} \arctan \sqrt{\xi} + \frac{p_{z}\mathbf{p}^{2}(2\mathbf{p}^{2} + 3\xi p_{\perp}^{2})}{\sqrt{\xi}(\xi p_{\perp}^{2} + \mathbf{p}^{2})^{\frac{3}{2}}p_{\perp}^{2}} \arctan \frac{\sqrt{\xi} p_{z}}{\sqrt{\mathbf{p}^{2} + \xi p_{\perp}^{2}}} \right), \tag{30}$$

$$m_{\delta}^{2} = -\frac{\pi m_{D}^{2} \xi p_{z} p_{\perp} |\mathbf{p}|}{4(\xi p_{\perp}^{2} + \mathbf{p}^{2})^{\frac{3}{2}}}$$
(31)

and

$$\mathbf{p}^2 = p_\perp^2 + p_z^2. {32}$$

The above expressions apply when  $\mathbf{n} = (0, 0, 1)$  points along the z-axis; in the general case,  $p_z$  and  $\mathbf{p}_{\perp}$  get replaced by  $\mathbf{p} \cdot \mathbf{n}$  and  $\mathbf{p} - \mathbf{n}(\mathbf{p} \cdot \mathbf{n})$ , respectively.

We first check some limiting cases. When  $\xi = 0$  then  $m_{\beta} = m_D$  while all other mass scales in the static propagator vanish. Hence, we recover the isotropic Debye potential

$$V(\mathbf{r}, \xi = 0) = V_{\rm iso}(r) = -g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\mathbf{p}^2 + m_D^2} = -\frac{g^2 C_F}{4\pi r} e^{-\hat{r}},\tag{33}$$

where  $\hat{r} \equiv rm_D$ .

Consider, on the other hand, the limit  $r \to 0$  for arbitrary  $\xi$ . The phase factor in (27) is essentially constant up to momenta of order  $|\mathbf{p}| \sim 1/r$  and since the masses are bounded as  $|\mathbf{p}| \to \infty$  they can be neglected. The potential then coincides with the vacuum Coulomb potential

$$V(\mathbf{r} \to 0, \xi) = V_{\text{vac}}(r) = -g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\mathbf{p}^2} = -\frac{g^2 C_F}{4\pi r}.$$
 (34)

The same potential emerges for extreme anisotropy since all  $m_i \to 0$  as  $\xi \to \infty$ :

$$V(\mathbf{r}, \xi = \infty) = -\frac{g^2 C_F}{4\pi r}.$$
(35)

This is due to the fact that at  $\xi = \infty$  the phase space density  $f(\mathbf{p})$  from Eq. (9) has support only in a two-dimensional plane orthogonal to the direction  $\mathbf{n}$  of anisotropy. As a consequence, the density of the medium vanishes in this limit.

For an anisotropic distribution, the potential depends on the angle between  $\mathbf{r}$  and  $\mathbf{n}$ . This can be seen analytically for small but non-zero  $\xi$ . To linear order in  $\xi$  the potential can be expressed as

$$V(\mathbf{r}, \xi \ll 1) = V_{\text{iso}}(r) - g^2 C_F \xi m_D^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{\frac{2}{3} - (\mathbf{p}\cdot\mathbf{n})^2/\mathbf{p}^2}{(\mathbf{p}^2 + m_D^2)^2}.$$
 (36)

For  $\mathbf{r}$  parallel to the direction  $\mathbf{n}$  of anisotropy,

$$V(\mathbf{r} \parallel \mathbf{n}, \xi \ll 1) = V_{\text{iso}}(r) \left[ 1 + \xi \left( 2 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{2}{\hat{r}} - 1 - \frac{\hat{r}}{6} \right) \right], \tag{37}$$

where  $\hat{r} \equiv rm_D$ , as before. This expression does not apply for  $\hat{r}$  much larger than 1, which is a shortcoming of the direct Taylor expansion of  $V(\mathbf{r}, \xi)$  in powers of  $\xi$ . However, for  $\hat{r} \simeq 1$  the coefficient of  $\xi$  is positive,  $(\cdots) = 0.27$  for  $\hat{r} = 1$ , and thus a slightly deeper potential than in an isotropic plasma emerges at distance scales  $r \sim 1/m_D$ .

When  $\mathbf{r}$  is perpendicular to  $\mathbf{n}$ ,

$$V(\mathbf{r} \perp \mathbf{n}, \xi \ll 1) = V_{\text{iso}}(r) \left[ 1 + \xi \left( \frac{1 - e^{\hat{r}}}{\hat{r}^2} + \frac{1}{\hat{r}} + \frac{1}{2} + \frac{\hat{r}}{3} \right) \right]. \tag{38}$$

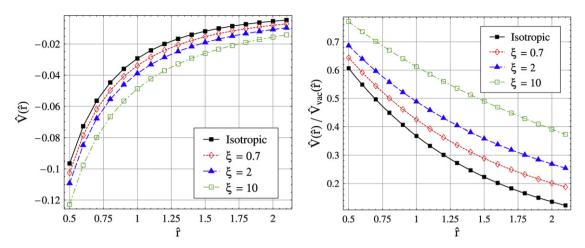


Fig. 1. Heavy-quark potential at leading order as a function of distance  $(\hat{r} = rm_D)$  for **r** parallel to the direction **n** of anisotropy. The anisotropy parameter of the plasma is denoted by  $\xi$ . Left: the potential divided by the Debye mass and by the coupling,  $\hat{V} = V/(g^2 C_F m_D)$ . Right: potential relative to that in vacuum.

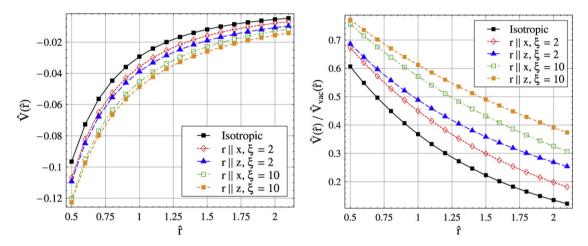


Fig. 2. Comparison of  $\hat{V}(\mathbf{r} \parallel \mathbf{n}, \xi)$  and  $\hat{V}(\mathbf{r} \perp \mathbf{n}, \xi)$ .

The same limitations for  $\hat{r}$  apply as in Eq. (37). Here, too, the coefficient of the anisotropy parameter is positive,  $(\cdots) = 0.115$  for  $\hat{r} = 1$ , but smaller than for  $\mathbf{r} \parallel \mathbf{n}$ . Hence, a quark–antiquark pair aligned along the direction of momentum anisotropy and separated by a distance  $r \sim 1/m_D$  is expected to attract more strongly than a pair aligned in the transverse plane.

For general  $\xi$  and  $\hat{r}$ , the integral in (27) has to be performed numerically. The poles of the function are integrable. In Fig. 1 we show the potential in the region  $\hat{r} \sim 1$  for various degrees of plasma anisotropy. One observes that in general screening is reduced, i.e., that the potential at  $\xi > 0$  is deeper and closer to the vacuum potential than for an isotropic medium. This is partly caused by the lower density of the anisotropic plasma. However, the effect is not uniform in the polar angle, as shown in Fig. 2: the angular dependence disappears more rapidly at small  $\hat{r}$ , while at large  $\hat{r}$  there is stronger binding for  $\mathbf{r}$  parallel to the direction of anisotropy. Overall, one may therefore expect that quarkonium states whose wave-functions are sensitive to the regime  $\hat{r} \sim 1$  are bound more strongly in an anisotropic medium, in particular if the quark–antiquark pair is aligned along  $\mathbf{n}$ .

## 5. Discussion and outlook

We have determined the HTL gluon propagator in an anisotropic (viscous) plasma in covariant gauge. Its Fourier transform at vanishing frequency defines a non-relativistic potential for static sources. We find that, generically, screening is weaker than in isotropic media and so the potential is closer to that in vacuum, in particular if the  $Q\bar{Q}$  pair is aligned along the direction of anisotropy.

Our results are applicable when the momentum of the exchanged gluon is on the order of the Debye mass  $m_D$  or higher, i.e., for distances on the order of  $\lambda_D = 1/m_D$  or less. For realistic values of the coupling,  $\alpha_s \approx 0.3$ ,  $\lambda_D$  is approximately equal to the scale  $r_{\text{med}}(T) \approx 0.5(T_c/T)$  fm introduced in [3,8], where medium-induced effects appear.

<sup>&</sup>lt;sup>1</sup> They are simple first-order poles which can be evaluated using a principal part prescription.

Following the discussion in Ref. [3], at short distances,  $r < r_{\text{med}}(T)$ , the potential is given by

$$V(r) \simeq -\frac{\alpha}{r} + \sigma r,\tag{39}$$

where  $\sigma \simeq 1~{\rm GeV/fm}$  is the SU(3) string tension; color factors have been absorbed into the couplings. Since  $r_{\rm med}(T) \sim 1/T$ , it follows that at sufficiently high temperature  $r_{\rm med}(T)$  is smaller than  $\sqrt{\alpha/\sigma}$  and so the perturbative Coulomb contribution dominates over the linear confining potential at the length scale  $\lambda_D$ . Roughly, this holds for  $T \gtrsim 2T_c$ . In this case, our result is directly relevant for quarkonium states with wavefunctions which are sensitive to the length scale  $\lambda_D \simeq r_{\rm med}$ .

On the other hand, for lower T the scale  $r_{\rm med}(T)$  where medium-induced effects appear may grow larger than  $\simeq \sqrt{\alpha/\sigma}$ . In this regime, quarkonium states are either unaffected by the medium; namely, if the quark mass is very large and the typical momentum component in the wave function is  $\gg 1/r_{\rm med}(T)$ . Conversely, states with a root-mean square radius  $\gtrsim r_{\rm med}(T)$  do experience medium modifications. For such states, however, it is insufficient to consider only the (screened) Coulomb-part of the potential which arises from one-gluon exchange. Rather, one should then sum the medium-dependent contributions due to one-gluon exchange and due to the string [3]. We postpone detailed numerical solutions of the Schrödinger equation in our anisotropic potential to the future. It will also be interesting to understand how the width of quarkonium states [9] which arises in HTL resummed perturbation theory due to Landau damping of modes with low frequency [4] is affected by an anisotropy of the medium.

## Acknowledgements

We acknowledge helpful discussions with A. Mocsy and P. Petreczky. Y.G. thanks the Helmholtz Foundation and the Otto Stern School at Frankfurt University for their support. M.S. is supported by DFG project GR 1536/6-1.

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