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Consistency of field-theoretical and kinetic calculations of viscous transport coefficients for a relativistic fluid

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ABSTRACT

We investigate the ratios $\beta_\eta \equiv \eta/\tau_\pi$ and $\beta_\zeta \equiv \zeta/\tau_\Pi$, i.e., the ratios of shear, η, and bulk, ζ, viscosities to the relaxation times τ_π, τ_Π of the shear stress tensor and bulk viscous pressure, respectively, in the framework of causal relativistic dissipative fluid dynamics. These viscous transport coefficients are computed both in a field-theoretical and a kinetic approach based on the Boltzmann equation. Our results differ from those of the traditional Boltzmann calculation by Israel and Stewart. The new expressions for the viscous transport coefficients agree with the results obtained in the field-theoretical approach when the contributions from pair annihilation and creation (PAC) are neglected. The latter induce non-negligible corrections to the viscous transport coefficients.

1. Relativistic fluid dynamics is an important model to understand various collective phenomena in astrophysics and heavy-ion collisions. However, the relativistically covariant extension of the Navier–Stokes equations is acausal and unstable [1,2]. The reason is that the irreversible currents (the shear stress tensor $\pi^{\mu\nu}$, the bulk viscous pressure Π, etc.) are linearly proportional to the thermodynamic forces (the shear tensor $\sigma^{\mu\nu}$, the expansion scalar θ, etc.), with the constant of proportionality being the shear viscosity coefficient η, the bulk viscosity coefficient ζ, etc. Thus, the forces have an instantaneous influence on the currents, which obviously violates causality and leads to instabilities. These problems are solved by introducing retardation into the definitions of the irreversible currents, leading to equations of motion for these currents which thus become independent dynamical variables. Theories of this type are called causal relativistic dissipative fluid dynamics (CRDF) [2].

With the retardation, in general the irreversible currents and the thermodynamic forces are no longer linearly proportional to each other. As a consequence, it is not clear whether the transport coefficients for CRDF can be computed with methods commonly used for Navier–Stokes fluids, such as the Green–Kubo–Nakano (GKN) formula. So far, there are several approaches to derive the transport coefficients of CRDF [3–10]. They were first calculated by Israel and Stewart (IS) applying the so-called 14-moment approximation to the Boltzmann equation [3]. However, the Boltzmann equation is applicable only in the dilute limit and hence we cannot expect these results to describe the behavior of dense fluids. Recently, a new microscopic formula to calculate the transport coefficients of CRDF from time-correlation functions was proposed [4–7]. This formula is the analogue of the GKN formula in Navier–Stokes fluids. Since this formula is derived from quantum field theory, it will be applicable even to dense fluids.

However, in a leading-order perturbative calculation which should apply in the dilute limit, i.e., the regime of applicability of the Boltzmann equation, the field-theoretical formula gives results which are different from those of the IS calculation. Is this inconsistency due to a problem with the field-theoretical formula or with the calculation based on the Boltzmann equation? Or is there simply no correspondence between the field-theoretical and the Boltzmann derivation of the transport coefficients of CRDF? This would come as a surprise, as this correspondence does exist in the case of Navier–Stokes fluids [11].

In this Letter, we show that the field-theoretical and the Boltzmann calculations are indeed consistent, even for CRDF. The key point is that the 14-moment approximation employed by Israel and Stewart is not unique. Recently, a new method to obtain equations of motion for the irreversible currents from the Boltzmann equation was suggested [12], leading to expressions for the transport coefficients which are different from the IS results. As we demonstrate in this Letter, the new transport coefficients turn out to be consistent with those obtained from the field-theoretical formula.

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2. In the following, we work in the Landau–Lifshitz frame where there is no energy flow in the rest frame of the fluid. In the original IS calculation, the evolution equations of the shear stress tensor and the bulk viscous pressure are obtained from the second moment of the Boltzmann equation,

$$\partial_\mu \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} K^\mu K^\nu K^\rho f = \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} K^\nu K^\rho C[f], \tag{1}$$

where f is the single-particle distribution function, $K^\mu = (E_\mathbf{k}, \mathbf{k})$ with $E_\mathbf{k} = \sqrt{\mathbf{k}^2 + m^2}$, g is the degeneracy factor, and $C[f]$ is the collision term. In order to obtain a closed set of equations, one assumes a specific form for f,

$$f = f_0 + f_0(1 - af_0)\big(e + e_\mu K^\mu + e_{\mu\nu} K^\mu K^\nu\big), \tag{2}$$

where e, e^μ, and $e^{\mu\nu}$ constitute a set of 14 independent parameters related to the irreversible currents by matching conditions and $f_0 = (e^{\beta u_\mu K^\mu} + a)^{-1}$ is the single-particle distribution function in local equilibrium, with $a = \pm 1$ for fermions/bosons; $\beta \equiv 1/T$ is the inverse temperature and u_μ is the fluid 4-velocity normalized as $u^\mu u_\mu = 1$. Then Eq. (1) is decomposed into scalar, vector, and tensor parts which are interpreted as the evolution equations of the bulk viscous pressure, the particle diffusion (heat conduction) current, and the shear stress tensor, respectively.

The idea of the new derivation based on the Boltzmann equation is as follows [12]. In the Boltzmann approach, the shear stress tensor and the bulk viscous pressure are always defined by

$$\pi^{\mu\nu} = \Delta^{\mu\nu}{}_{\alpha\beta} \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} K^\alpha K^\beta (f - f_0), \tag{3}$$

$$\Pi = -\frac{m^2}{3} \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} (f - f_0), \tag{4}$$

where the tensor $\Delta^{\mu\nu\alpha\beta} = (\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha} - (2/3)\Delta^{\mu\nu}\Delta^{\alpha\beta})/2$, with $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. The evolution equations for $\pi^{\mu\nu}$ and Π are now obtained directly by applying the comoving time derivative to the above definitions, and substituting the Boltzmann equation together with Eq. (2), for details see Ref. [12].

The final result for the evolution equations is

$$\Delta^{\mu\nu}{}_{\alpha\beta} u^\rho \partial_\rho \pi^{\alpha\beta} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2(\beta_\eta + \eta_{\pi\Pi}\Pi)\sigma^{\mu\nu} - 2\eta_{\pi\pi}\Delta^{\mu\nu}_{\alpha\beta}\pi^\alpha_\lambda \sigma^{\beta\lambda} + 2\Delta^{\mu\nu}_{\alpha\beta}\pi^\alpha_\lambda \omega^{\beta\lambda} + \eta_{\pi\theta}\pi^{\mu\nu}\theta, \tag{5}$$

$$u^\rho \partial_\rho \Pi = -\frac{\Pi}{\tau_\Pi} - (\beta_\zeta + \zeta_{\Pi\Pi}\Pi)\theta + \zeta_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}, \tag{6}$$

where $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$, $\theta = \partial_\mu u^\mu$ is the expansion scalar, $\sigma^{\mu\nu} = \Delta^{\mu\nu}{}_{\alpha\beta}\nabla^\alpha u^\beta$ is the shear tensor, and $\omega^{\mu\nu} = (\nabla^\mu u^\nu - \nabla^\nu u^\mu)/2$ is the vorticity. The relaxation times for the shear stress tensor and the bulk viscous pressure are τ_π and τ_Π, respectively, and the transport coefficients $\beta_\eta \equiv \eta/\tau_\pi$ and $\beta_\zeta \equiv \zeta/\tau_\Pi$. The other transport coefficients ($\eta_{\pi\Pi}, \eta_{\pi\pi}, \eta_{\pi\theta}, \zeta_{\Pi\Pi}$, and $\zeta_{\Pi\pi}$) appearing in Eqs. (5) and (6) play no role in the following discussion. Note that the form of these equations is the same as for the previous Boltzmann calculations [3].

In general, the values of the transport coefficients depend on the collision term. However, because of the approximation (2), the contributions from the collision term factorize out in a similar way in η (ζ) and τ_π (τ_Π). Thus, the ratios β_η and β_ζ are simply thermodynamic functions. The other coefficients appearing in Eqs. (5) and (6) will not be discussed in this work.

The ratios β_η, β_ζ are expressed as

$$\beta_\eta = \frac{1}{15}\left[9P + \varepsilon - m^4 \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E^3_\mathbf{k}} f_0(\mathbf{k})\right], \tag{7}$$

$$\beta_\zeta = \left(\frac{1}{3} - c_s^2\right)(\varepsilon + P) - \frac{2}{9}(\varepsilon - 3P) - \frac{m^4}{9} \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E^3_\mathbf{k}} f_0(\mathbf{k}). \tag{8}$$

Here, ε, P, and $c_s^2 = dP/d\varepsilon$ are the energy density, the pressure, and the velocity of sound squared, respectively. In the following, we shall prove that the values (7) and (8) obtained in this new derivation are consistent with those from the field-theoretical approach.

For the sake of comparison, we also quote the values of β_η and β_ζ as computed within the approach of Israel and Stewart:

$$\beta_\eta = -\frac{J_{42}}{J_{52}} I_{31}, \tag{9}$$

$$\beta_\zeta = \frac{I_{10}[J_{40}J_{20} - (J_{30})^2] + (\varepsilon + P)(J_{30}J_{20} - J_{40}J_{10}) - \beta J_{41}[J_{30}J_{10} - (J_{20})^2]}{[J_{30}J_{10} - (J_{20})^2]\psi}, \tag{10}$$

where we defined the thermodynamical functions,

$$I_{nq} = \frac{1}{(2q+1)!!} \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} E_\mathbf{k}^{n-2q} \big(\Delta^{\mu\nu} k_\mu k_\nu\big)^q f_0(\mathbf{k}), \tag{11}$$

$$J_{nq} = \frac{1}{(2q+1)!!} \int \frac{g\, d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} E_\mathbf{k}^{n-2q} \big(\Delta^{\mu\nu} k_\mu k_\nu\big)^q f_0(\mathbf{k})\big[1 - af_0(\mathbf{k})\big], \tag{12}$$

$$\psi = -\frac{J_{30}(J_{31}J_{30} - J_{40}J_{20}) + J_{40}(J_{41}J_{10} - J_{31}J_{20}) - J_{51}[J_{30}J_{10} - (J_{20})^2]}{J_{21}(J_{31}J_{30} - J_{41}J_{20}) + J_{31}(J_{41}J_{10} - J_{31}J_{20}) - 5J_{42}[J_{30}J_{10} - (J_{20})^2]/3}. \tag{13}$$

3. The field-theoretical approach uses the projection operator method [4–7,13]. First, we discuss the shear viscosity. The expression for β_η is

$$\beta_{\eta,\mathrm{ft}} = (\varepsilon + P)\frac{\int d^3\mathbf{x}\,(T^{yx}(\mathbf{x}), T^{yx}(\mathbf{0}))}{\int d^3\mathbf{x}\,(T^{0x}(\mathbf{x}), T^{0x}(\mathbf{0}))}, \tag{14}$$

where $T^{\mu\nu}$ is the energy–momentum tensor and the inner product is defined by Kubo's canonical correlation,

$$(X, Y) = \int_0^\beta d\lambda\, \beta^{-1}\langle e^{\lambda H} X e^{-\lambda H} Y\rangle_{\mathrm{eq}}. \tag{15}$$

Here, H is the Hamiltonian and $\langle\cdots\rangle_{\mathrm{eq}}$ indicates the thermal expectation value.

In order to compare with the Boltzmann calculation, it is sufficient to calculate the correlation functions appearing in Eq. (14) in the free-gas approximation. Using

$$\int d^3\mathbf{x}\left(T^{0x}(\mathbf{x}), T^{0x}(\mathbf{0})\right) = \frac{\varepsilon + P}{\beta}, \tag{16}$$

we obtain for bosons

$$\beta_{\eta,\mathrm{ft}} = -\beta\frac{\partial}{\partial\beta}\int\frac{g\,d^3\mathbf{k}}{(2\pi)^3}\frac{(\mathbf{k}^2)^2}{15E_{\mathbf{k}}^3} f_0(\mathbf{k}) + \int\frac{g\,d^3\mathbf{k}}{(2\pi)^3}\frac{(\mathbf{k}^2)^2}{30E_{\mathbf{k}}^3}\left[1 + 2f_0(\mathbf{k})\right]. \tag{17}$$

The first term in Eq. (17) contains the derivative with respect to β, which is re-expressed using

$$\frac{\partial f_0(\mathbf{k})}{\partial\beta} = \frac{E_{\mathbf{k}}}{\beta}\lim_{\mathbf{p}\to 0}\frac{f_0(\mathbf{k}+\mathbf{p})[1 + f_0(\mathbf{k})] - f_0(\mathbf{k})[1 + f_0(\mathbf{k}+\mathbf{p})]}{E_{\mathbf{k}+\mathbf{p}} - E_{\mathbf{k}}}. \tag{18}$$

In many-body physics [15], this term can be interpreted as the contribution from the collisions of bosons. The second term in Eq. (17) contains an ultraviolet divergent term due to the vacuum self-energy and is re-expressed as

$$1 + 2f_0(\mathbf{k}) = \lim_{\mathbf{p}\to 0}\left\{\left[1 + f_0(\mathbf{k}+\mathbf{p})\right]\left[1 + f_0(\mathbf{k})\right] - f_0(\mathbf{k}+\mathbf{p}) f_0(\mathbf{k})\right\}. \tag{19}$$

This term corresponds to the pair annihilation–creation (PAC) part.

Since the Boltzmann approach employed by Israel and Stewart and by us does not include the PAC part, for the sake of consistency we compare only the collision part in Eq. (17) with the kinetic result. A straightforward integration by parts gives the result (7), i.e., it is identical with the result of the new calculation based on the Boltzmann equation. On the other hand, the full result, i.e., including the PAC part, is $\beta_{\eta,\mathrm{ft}} = P$, and has already been quoted in Ref. [6]. Here, the vacuum contribution has been neglected. We emphasize again that the collision term does not affect the calculation of the ratios β_η, β_ζ. While this is true for one-component systems, this might not be the case for multi-component systems. We also mention that there have been attempts to include PAC terms into the Boltzmann equation, see e.g. Ref. [14], but we do not consider this in this work.

The temperature dependence of $\beta_\eta/(\varepsilon + P)$ is shown in Fig. 1.[1] One observes that the values obtained from the field-theoretical formula are larger than those for the new Boltzmann and the old IS calculations, because of the non-negligible contribution of the PAC part. Also, the new Boltzmann calculation gives values which are above the original IS result. Since $\beta_\eta/(\varepsilon + P)$ is related to the signal propagation speed in CRDF, we expect effects on the collective behavior of relativistic fluids [1]. The same transport coefficient of CRDF was calculated for weakly coupled quantum field theories in Ref. [8]. A direct comparison is, however, not possible, because these results were not extrapolated to zero coupling.

The difference between the new kinetic derivation and the original one by Israel and Stewart arises from the choice of the moments used to derive the evolution equations for $\pi^{\mu\nu}$ and Π. It can be shown that this difference disappears in the non-relativistic (i.e., low-temperature) limit, and the two kinetic results coincide, see Fig. 1. Note that the non-relativistic limit of the IS result is equivalent to the result of Grad's original 13-moment method applied to the non-relativistic Boltzmann equation [3].

Next, we consider β_ζ. In the field-theoretical approach, this coefficient is given by

$$\beta_{\zeta,\mathrm{ft}} = (\varepsilon + P)\frac{\int d^3\mathbf{x}\,(\hat{\Pi}(\mathbf{x}), \hat{\Pi}(\mathbf{0}))}{\int d^3\mathbf{x}\,(T^{0x}(\mathbf{x}), T^{0x}(\mathbf{0}))}, \tag{20}$$

where $\hat{\Pi} = [(1 - 3c_s^2)(T^{00} - \langle T^{00}\rangle_{eq}) - (T^\mu{}_\mu - \langle T^\mu{}_\mu\rangle_{eq})]/3$ [7]. For bosons in the free-gas approximation, we obtain

$$\beta_{\zeta,\mathrm{ft}} = -\beta\int\frac{g\,d^3\mathbf{k}}{(2\pi)^3}\frac{1}{9E_{\mathbf{k}}^3}\left(\mathbf{k}^2 - 3c_s^2 E_{\mathbf{k}}^2\right)^2\frac{\partial}{\partial\beta} f_0(\mathbf{k}) + \frac{m^4}{18}\int\frac{g\,d^3\mathbf{k}}{(2\pi)^3 E_{\mathbf{k}}^3}\left[1 + 2f_0(\mathbf{k})\right]. \tag{21}$$

[1] In Ref. [9], the fluid-dynamical equation is obtained by expanding the energy–momentum tensor in terms of gradients of the fluid velocity up to second order. Fluid dynamics derived in this way corresponds to the relativistic Burnett equation and the corresponding transport coefficients are not those of CRDF, although it is usually assumed that they are equal.

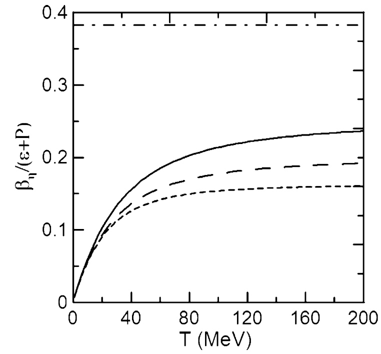

Fig. 1. The temperature dependence of $\beta_\eta/(\varepsilon+P)$ for pions, $m=140$ MeV. The solid, long-dashed, and short-dashed lines represent the results of the field-theoretical approach including the PAC part, the new Boltzmann calculation, and the IS calculation, respectively. For the sake of comparison, the result from an AdS/CFT calculation, $\beta_\eta/(\varepsilon+P)=1/[2(2-\ln 2)]$ [9], is shown by the dash-dotted line.

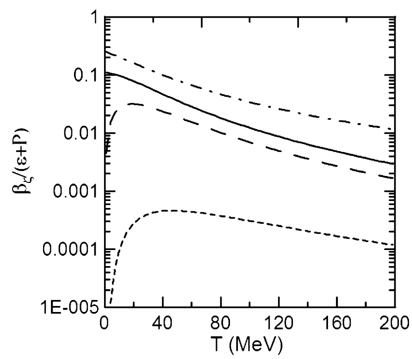

Fig. 2. The temperature dependence of $\beta_\zeta/(\varepsilon+P)$ for pions, $m=140$ MeV. The solid, long-dashed, and short-dashed lines represent the results of the field-theoretical approach including the PAC part, the new Boltzmann calculation, and the IS calculation, respectively. For the sake of comparison, the result from a string-theoretical calculation [10] is shown by the dash-dotted line.

Similarly to the case of $\beta_{\eta,\mathrm{ft}}$, the first and second terms are interpreted as the collision part and the PAC part, respectively. The collision part can be re-expressed using integration by parts, thermodynamic relationships, and the definition of c_s^2. It is then found to be exactly the same as the new Boltzmann result (8).

By taking the PAC part into account, the field-theoretical approach yields

$$\beta_{\zeta,\mathrm{ft}}=\left(\frac{1}{3}-c_s^2\right)(\varepsilon+P)-\frac{2}{9}(\varepsilon-3P). \tag{22}$$

The detailed derivation is given in Ref. [7].

The temperature dependence of $\beta_\zeta/(\varepsilon+P)$ is shown in Fig. 2. Similarly to the case of the shear viscosity, the new Boltzmann calculation and the field-theoretical formula predict larger values than the IS calculation. However, the difference is now more than an order of magnitude.

For β_ζ, the PAC part qualitatively changes the behavior, in particular, at low temperatures. If we neglect the PAC part, β_ζ increases rapidly in the low-temperature region, and starts to decrease at high temperature. This behavior is the same in all kinetic calculations. However, when we consider the PAC part, $\beta_{\zeta,\mathrm{ft}}$ becomes a monotonically decreasing function of temperature. This is the same as the behavior predicted by a string-theoretical calculation [10].

In the case of the Boltzmann calculations, β_η and β_ζ have the same forms for fermions as for bosons. This is true even for the field-theoretical approach, if the PAC part is neglected. Then, $\beta_{\eta,\mathrm{ft}}$ and $\beta_{\zeta,\mathrm{ft}}$ for fermions are given by Eqs. (7) and (8), respectively, i.e., the field-theoretical approach and the new Boltzmann calculation yield the same results. However, the contributions from the PAC terms depend on the statistics. For fermions, the full results of the field-theoretical calculation are

$$\beta_{\eta,\mathrm{ft}}=0, \tag{23}$$

$$\beta_{\zeta,\mathrm{ft}}=\left(\frac{1}{3}-c_s^2\right)(\varepsilon+P)-\frac{1}{3}(\varepsilon-3P). \tag{24}$$

The results of the field-theoretical calculation may be expressed in a unified way as

$$\beta_{\eta,\mathrm{ft}}=|3-\alpha|P, \tag{25}$$

$$\beta_{\zeta,\mathrm{ft}}=\left(\frac{s}{3}\frac{d}{ds}-\frac{\alpha}{9}\right)(\varepsilon-3P), \tag{26}$$

where $\alpha = 2$ for bosons and $\alpha = 3$ for fermions, and s is the entropy density. Note that, for a mixed system of bosons and fermions, the correlation functions which appear in the numerators and denominators of Eqs. (14) and (20) are the sum of both contributions, respectively.

Note that these calculations are leading-order results and will be modified by the effect of interactions. For example, the exact expression for τ_π/β is given by the ratio of the real and imaginary parts of the retarded Green's function of T^{yx} [7]. To leading order, the real part is approximated by the result for the free gas, while the imaginary part is not. The divergent τ_π for fermions, leading to a vanishing $\beta_{\eta,\mathrm{ft}}$ in Eq. (25), will be rendered finite by a more complete calculation.

As shown in Ref. [16], there is a sum rule for the bulk viscous pressure. There, the correlation function for bulk viscous pressure was calculated for interacting gauge bosons. In the weak-coupling limit, the result is reproduced by setting $\alpha = 4$ in Eq. (26). Similar correlation functions were studied in Lattice QCD [17].

4. We are now able to answer the question posed in the introduction. The inconsistency between the field-theoretical and Boltzmann calculations is due to an ambiguity in the IS calculation. To the best of our knowledge, this has not been realized before. An alternative derivation of fluid dynamics from the Boltzmann equation was suggested in Ref. [12]. In this Letter, we showed for the first time that the transport coefficients obtained in this new derivation are consistent with those from the field-theoretical approach. Thus, even in CRDF, there is a relation between the field-theoretical and Boltzmann calculations, just as for Navier–Stokes fluids.

At the same time, we found that, in the field-theoretical formula, there is a contribution from the PAC part which is not included in calculations using the standard Boltzmann equation. In a relativistic setting, particle annihilation and creation processes may occur, which re-distribute momenta as well as influence chemical equilibrium, thus affecting both the shear and the bulk viscosities. Therefore, one must not neglect the PAC part. When taking the PAC part into account, we have seen that $\beta_{\eta,\mathrm{ft}}$ and $\beta_{\zeta,\mathrm{ft}}$ can be expressed solely by thermodynamic quantities such as ε, P, and c_s as shown in Eqs. (25) and (26).

In this Letter, we simply dropped the temperature-independent vacuum contribution term in the calculation of the coefficients. The correct renormalization scheme for these quantities is unknown. It is very important to reliably determine the values of the transport coefficients. This should be studied more carefully in the future.

We have discussed the consistency of the field-theoretical and the Boltzmann calculations. We remark that $1+1$-dimensional scaling flow solutions were calculated using the new kinetic transport coefficients which were compared with a numerical simulation of the Boltzmann equation. We find better agreement with the new coefficients than with those of the IS calculation [12]. This may serve as another justification for the need to improve IS theory, corroborating the findings of this Letter.

In order to discuss particle diffusion (heat conduction), the Boltzmann approach should be generalized to a multi-component fluid, considering the flows of particles and anti-particles on an equal footing. For the field-theoretical approach, the appropriate projection operator to derive the corresponding formula for particle diffusion is not known and remains a challenge for future work.

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