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Optimal Investment Strategies for Insurance Companies when Capital Requirements are imposed by a Standard Formula

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Abstract

The Solvency II standard formula employs an approximate Value-at-Risk approach to define risk-based capital requirements. This paper investigates how the standard formula's stock risk calibration influences the equity position and investment strategy of a shareholder-value-maximizing insurer with limited liability. The capital requirement for stock risks is determined by multiplying a regulation-defined stock risk parameter by the value of the insurer's stock portfolio. Intuitively, a higher stock risk parameter should reduce risky investments as well as insolvency risk. However, we find that the default probability does not necessarily decrease when reducing the investment risk (by increasing the stock investment risk parameter). We also find that depending on the precise interaction between assets and liabilities, some insurers will invest conservatively, whereas others will prefer a very risky investment strategy, and a slight change of the stock risk parameter may lead from a conservative to a high risk asset allocation.

Keywords: Solvency Regulation, Capital Requirements, Asset Allocation, Insurer Default Risk

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1 Introduction

Solvency II, the future framework for insurance regulation in the European Union, is a significant move away from rules-based regulation towards principle-based regulation. In particular, the new standard will not impose any direct limitations on an insurer's asset allocation and explicitly accords "freedom of investment" in Article 133.[2] Instead of relying on static rules, future solvency regulation will build on a three-pillar framework. Pillar I contains quantitative risk-based capital requirements, which are supposed to ensure that the insurer's annual default probability is below 0.5%. Capital requirements can be determined either by using an internal risk capital model developed by the insurer and approved by the regulator, or by means of a standard formula defined in the Solvency II framework. In addition, Solvency II will provide qualitative requirements for governance and risk management in Pillar II and disclosure and transparency requirements in Pillar III.

Despite the promised "freedom of investment" principle, some researchers and practitioners have expressed fear that Solvency II could lead to lower stock positions of insurance companies, which in turn could create financing problems for the real industry and possibly endanger insurance returns. The standard formula is said to demand excessive capital for stock holdings, as opposed to government bonds. Existing articles have also pointed out weaknesses and arbitrariness in the statistical calibration and claim that the standard formula does not account for long-term effects such as mean-reversion of stock returns.[3]

To investigate the question of whether the standard formula requires excessive capital from a theoretical perspective, one must distinguish between two possible cases. On the one hand, it is possible that insurers have sufficient self-interest to hold capital and that the Solvency II capital requirements or regulation in general is not necessary. As shown by Rees et al. (1999), insurers will raise sufficient equity funds to ensure perfect solvency if policyholders can observe the insurer's default risk level and regulators do not impose restrictions on their asset allocation. In this case, which is known as "market discipline", insurers will invest a portion of their assets into risky stocks in order to attain an efficient portfolio allocation. On the other hand, there could be an information asymmetry or a commitment problem between the insurer and the policyholders, implying that policyholders' willingness-to-pay does not sufficiently reflect the insurer's default risk.

In this article, we investigate the insurer's shareholder-value-maximising strategy, focusing on the case in which market discipline is not effective; the impact of market discipline is also discussed (cf. section 4.4). Shareholders are protected by limited liability, which could be interpreted as a put option on the future equity. The limited liability protection incentivises the insurer to employ a high-risk strategy, i.e. to aim for high investment risk and low equity capital.[4] However, the standard formula translates more investment risk into more capital and the insurer needs to find the optimal trade-off in order to maximize shareholder value.

The standard formula's capital requirement for stock risks is determined by multiplying a regulation-defined stock risk parameter by the value of the insurer's stock portfolio. We demonstrate that the insurer reduces its risky stock investments when being confronted to a higher stock risk parameter in the standard formula. If the stock risk parameter is lax (low), the insurer can take high investment risk without facing significant consequences for the capital requirement. Therefore, shareholder value is maximised by investing a large amount of assets into risky stocks. If, in turn, the stock risk parameter is conservative (high), taking investment risks is significantly penalised by higher capital requirements and the insurer will decide on a conservative investment strategy.

Intuitively, more investment risk might lead to a higher default probability. However, we find that the default probability is not necessarily minimised by the most conservative stock risk parameter, but may reach its minimum through a rather moderate one. For a higher stock risk parameter, the insurer reduces its capital requirement by de-risking the portfolio. At the same time, diversification effects between investment and liability risks become weaker. The likelihood thereby increases that the capital is not sufficient to cover underwriting losses, meaning that the default probability increases. We discuss these results in light of different assumptions as to the correlation between asset and liability risks. If the insurer's actual correlation deviates from what is assumed in the standard formula, it is possible that the insurer's optimal asset allocation is always a corner solution. A slight change of the stock risk parameter could then lead from a fully conservative to a high-risk asset allocation, leading to a jump in default probability.

On the one hand, our results add some credence to the industry's fear of Solvency II by verifying the coherence between the strictness of the standard formula calibration and the insurer's optimal stock investments. On the other hand, our results point out that regulators should be very careful when calibrating the standard formula, since a reduction of the stock risk parameter that generates weaker capital requirements could lead to an excessive increase of insolvency risk.

The paper is organised as follows. Section 2 contains an overview of the relevant literature. Section 3 explains the modelling framework. Section 4 investigates the impact of the standard formula calibration on optimal investment strategies and default put option value, taking the probability of default into account. Section 5 concludes.

2 Literature Overview

The argument that risk-based solvency regulation with a one-year time horizon could have undesirable side effects on insurers' asset allocation is raised by van Bragt et al. (2010) as well as by Bec and Gollier (2009). Van Bragt et al. (2010) analyse how capital requirements based on a simplified standard formula influence a life insurer's risk-and-return profile. The authors investigate the capital requirements resulting from different investment policies and demonstrate that the standard formula induces a reduction in short-term risk that can drive down the long-term expected returns. Bec and Gollier (2009) investigate horizon effects for French data. Even though stocks have a higher risk in the short term, they have the advantage of being slightly mean-reverting. Bond and bill returns, however, are mean-averting. Therefore, given a longer investment horizon, stocks will be less risky than they appear in a short-term consideration.[5] Both of the above-mentioned articles argue that the one-year Value-at-Risk might make insurers focus too much on their one-year risk profile and might thus deter them from stock investments, which could be superior in a long-term analysis.

The theoretical model of Filipović et al. (2014) shows that capital requirements can reduce the insurer's potential to invest riskily. The authors argue that the feasibility of risky investments causes a commitment problem between the insurer and its shareholders, and that capital requirements may help to mitigate this problem. However, the article assumes that stock risk can be measured directly under the Value-at-Risk, and hence does not investigate the influence of the calibration of an approximate risk measurement as the standard formula. Also, the insurer's initial equity capital is an exogenous variable in their framework.

The appropriateness of the risk measure Value-at-Risk, that is underlying the Solvency II capital requirements, has been extensively discussed in prior research. One part of the debate relates to the question in how far a risk measure should acknowledge diversification effects. Artzner et al (1999) consider a risk measure to be coherent only if it, amongst other requirements, is sub-additive: the risk measure should assign a lower (or equal) amount to a portfolio of risks than to the included risks in sum. In general, the Value-at-Risk is not sub-additive.[6] Nevertheless, Dhaene et al (2008) show that formulating capital requirements based on the Value-at-Risk is appropriate to avoid mergers that would lead to a more risky situation. Another critical property of the Value-at-Risk is that it does not capture the risk distribution's tail and therefore could encourage firms to take more risk by accepting distributions with heavy but very seldom outcomes (cf. Dowd and Blake 2006; Campbell 2012). Gatzert and Schmeiser (2008) demonstrate that the Tail-Value-at-Risk with a 99% confidence level (which is the capital requirement of the Swiss Solvency Test) imposes stricter capital requirements than the Value-at-Risk with a 99.5% confidence level, as the latter allows for a higher default put option value. In our numerical examples it turns out that the Value-at-Risk incentivises the insurer to employ a high-risk investment strategy and adjust its capital level in order to take additional risk in the tail.

Using a simulation study with a multi-period time horizon, Wiehenkamp (2010) analyses how a life insurance company will adjust its investment strategy to different regulatory regimes. He shows that both the standard model and the internal model under Solvency II reduce the insurer's optimal risky asset allocation and thus lower insolvency risk. The author also points out that the parameters of the standard formula (i.e. shocks and correlation coefficients, see section 3.1) could fail to restrict the default probability to the desired 99.5 % confidence level.

Focusing on stock risk, Mittnik (2011) identifies several weaknesses in the calibration of the Solvency II standard formula. He points out that annualising daily returns to calculate the one-year Value-at-Risk makes the Value-at-Risk estimates, which form the basis for the standard formula, highly unstable and arbitrary. Also, correlations might be implied even if the data are independent, whereas truly existing dependencies suggested in the historical data might be lost. With regard to the specifications of the fifth Quantitative Impact Study (QIS 5), Hampel and Pfeifer (2011) identify a bias in the quantile estimation regarding premium and reserve risk. This bias results from the QIS 5 assumption of an expected loss ratio of 1. The authors suggest a formula for calculating an undertaking-specific standard deviation that can be used in the Solvency Capital Requirement calculation to help overcome the bias.

Thus, the literature identifies several problems with measuring risk based on the Value-at-Risk and an approximate standard formula; such ambiguity in the set-up of capital requirements is not considered in the existing models on investment decisions under capital requirements. To the best of our knowledge, there is no analysis addressing the question how different calibrations of the standard formula would impact the insurance company's risk-taking and the resulting default probability. By employing option pricing theory as the measurement for an insurer's shareholder value, we investigate the consequences of different standard formula parameterisations for the capital and investment strategy of a value-maximising insurance company.

At first glance, it might be questionable as to whether there is room for different parameterisations, since the stock risk parameter is calibrated using a broad market index. However, the regulator could, for example, reduce the stock risk parameter in order to mitigate pro-cyclical effects in case of adverse capital market conditions.[7] In QIS 5, the stock risk parameter was therefore reduced by 9 percentage points, for example from 39% to 30% for "global equity". In addition, transitional measures can be applied that generate relatively weak capital requirements for the first years when Solvency II comes into force. In this regard, the most recent impact assessment (the 2013 Long Term Guarantee Assessment) reduced the stock risk parameter for "global equity" and "other equity" to 22%.[8] Gatzert and Martin (2012) demonstrate that the standard formula underestimates the stock risk at the 99.5% confidence level if the countercyclical adjustment from QIS 5 is applied.

Finally, the findings of our article can be linked with the literature dealing with model risk. The literature has studied how model uncertainty should be recognised, e.g., in derivative pricing (Cont, 2004) or insurers' investment-reinsurance decisions (Zhang and Siu, 2009). Kerkhof et al (2010) propose a framework to quantify model risk and highlight that significant amounts of capital should be set aside. Model uncertainty might also be an issue when determining capital requirements by using the Solvency II standard formula. Gatzert and Martin (2012) highlight that capital requirements calculated according to the QIS5 specifications are too lax if stock risk is represented by Heston's (1993) stochastic volatility model instead of geometric Brownian motions. In this context, our results emphasize that regulators should not respond to such uncertainty by setting the stock risk parameter too high, as this could cause an increase in default risk. Furthermore, our results point out that the regulator might incorrectly forecast the insurer's reaction to a regulatory restriction if he misestimates the insurer's risk profile (as demonstrated in section 4.3 with regard to the correlation between asset and liability risks). To close this gap, the "Own Risk and Solvency Assessment" that will be required by Solvency II might be an important tool to provide the regulator with necessary information about the risk profile.

3. **Model Framework**

We consider an insurance company in a one-period setting. At time 0, the insurer receives premiums in the amount of Π. Shareholders endow the company with equity in the amount of *K* and, hence, the insurer's initial assets are given by

$$A_0 = \Pi + K \,. \tag{1}$$

The percentage share of assets invested in a risky stock *M* is denoted by α.; the other assets are invested at the risk-free interest rate r_f. At time 1, policyholders file their claims to the amount of L_1. The time-0-value of the liabilities is denoted by L_0.

The insurer decides on an optimal combination of equity capital and investment risk under the objective of maximising shareholder value (*SHV*). The regulatory standard formula acts as a restriction in the optimisation problem.

3.1 The Regulatory Standard Formula

We incorporate capital requirements in line with the Solvency II framework. Under Solvency II, insurers need to evaluate their assets and liabilities on a market-consistent balance sheet. The difference between the assets and liabilities is called the Basic Own Funds (*BOF*).[9] The Solvency Capital Requirement (*SCR*) is defined by the Value-at-Risk of the change in Basic Own Funds over one year at a confidence level of 99.5%.[10] The standard formula determines the SCR as an approximation.

In the following section, we provide an overview of the design of the Solvency II standard formula and how we incorporate it. The standard formula is based on a modular approach which consists of several risk modules (such as market risk) and sub-modules (such as stock risk). In a first step, individual *SCR*s for sub-modules are determined. In the case of the stock risk, it is calculated as to how the insurer's *BOF* would change in a regulation-specified scenario, i.e. the value of the insurer's investment portfolio is "shocked" by a drop in stock prices of 30%, for example.[11] In a second step, the individual *SCR*s for sub-modules are aggregated by a certain aggregation formula to company level. The result is denoted as the Basic Solvency Capital Requirement (*BSCR*). In a third step, a capital requirement for operational risk and an adjustment position accounting for the fact that technical provisions and deferred taxes can absorb losses are added.[12]

In the simplified version of the standard formula that we employ in this article, the market risk comprises only stock risk. In line with the current proposal for the Solvency II standard formula,[13] the regulatory scenario for stock risk assumes that the value of the insurer's equity portfolio decreases by the factor $shock_{st}$. As our model insurer holds equities to the amount of $\alpha \cdot (\Pi + K)$, the *SCR* for stock risks is determined as follows:

$$SCR_{stock}(\alpha) = shock_{st} \cdot \alpha \cdot (\Pi + K) \quad (2)$$

Apparently, the *SCR* for stock risks increases in the portion α of assets that the insurer invests into stocks. Since this relation is crucial for our analysis, we denote the capital requirement SCR_{stock} as a function in α.

With regard to the liability risk module, we focus on the premium risk in non-life insurance. In line with the latest draft of the Solvency II standard formula, we determine the corresponding *SCR* for liability risks as follows:[14]

$$SCR_{liab} = 3 \cdot \sigma_L^{cr} \cdot L_0 \quad (3)$$

where σ_L^{cr} denotes the standard deviation of the combined ratio (i.e. the ratio of claims and expenses over premiums). This formula is considered to approximate the Value-at-Risk with a confidence level of 99.5% for log-normally distributed risks.[15]

Next, the company-wide *SCR* is determined. Consistent with the Solvency II standard formula,[16] the capital requirements for market risk and liability risk are aggregated by the following formula:

$$SCR(\alpha) = \sqrt{\left(SCR_{stock}(\alpha)\right)^2 + 2 \cdot corr \cdot SCR_{stock}(\alpha) \cdot SCR_{liab} + \left(SCR_{liab}\right)^2} \, . \qquad (4)$$

This approximate aggregation formula would provide exact results, for example, if stock risks and liability risks were multivariate normally distributed. The parameter *corr* reflects the correlation between stock and liability risks. For simplicity, we neglect adjustments for operational risks or the loss-absorbing capacity of technical provisions and deferred taxes.

In total, the regulator demands that the insurer's Own Funds (*OF*)[17] cover the Solvency Capital Requirement, i.e. $OF = A_0 - L_0 \geq SCR(\alpha)$.

3.2 Modelling Asset and Liability Risks

We model the insurer's asset and liability risks via geometric Brownian motions.[18] Under the real-world measure *P*, the risky stock *M* evolves according to

$$dM_t = \mu_M M_t dt + \sigma_M M_t dW_{M,t}^P ,$$

with μ_M and σ_M the instantaneous drift and volatility of *M*, and $W_{M,t}^P$ denoting a standard Brownian motion under *P*. Therefore, the insurer's asset process is defined by

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}^P ,$$

with $\mu_A = (1-\alpha) \cdot r_f + \alpha \cdot \mu_M$ and $\sigma_A = \alpha \cdot \sigma_M$, and $W_{A,t}^P = W_{M,t}^P$.[19] The liability process is described by

$$dL_t = \mu_L L_t dt + \sigma_L L_t dW_{L,t}^P ,$$

with μ_L and σ_L the instantaneous drift and volatility of the liability process and $W_{L,t}^P$ denoting a standard Brownian motion under *P*. The geometric Brownian motions $W_{A,t}^P$ and $W_{L,t}^P$ are correlated with $\rho_{A,L}$.

The stochastic differential equations are solved by (cf. Merton, 1976)

$$A_t = A_0 \cdot \exp\left(\left(\mu_A - \sigma_A^2/2\right)t + \sigma_A W_{A,t}^P\right)$$

$$L_t = L_0 \cdot \exp\left(\left(\mu_L - \sigma_L^2/2\right)t + \sigma_L W_{L,t}^P\right).$$

We assume that the assets and liabilities can be replicated with instruments that are traded on an arbitrage-free and complete market. Policyholders, to purchase coverage for their risks, do not have access to the financial market (or only at higher costs than the insurer);[20] their willingness-to-pay is described by the premium function that is introduced in the next section. There is thus a unique equivalent risk-neutral measure Q, under which we obtain

$$A_t = A_0 \cdot \exp\left(\left(r_f - \sigma_A^2/2\right)t + \sigma_A W_{A,t}^Q\right)$$
$$L_t = L_0 \cdot \exp\left(\left(r_f - \sigma_L^2/2\right)t + \sigma_L W_{L,t}^Q\right),$$

with $W_{A,t}^Q$ and $W_{L,t}^Q$ denoting standard Brownian motions under Q. Since insurance claims are usually not perfectly correlated with financial instruments, the assumption about their replicability is a short-cut that allows us to determine the shareholder value in a relatively simple analytical form (cf. Eq. 10 and 11).[21] Nevertheless, the valuation of liabilities according to the market value of a replicating portfolio (if such a portfolio exists) is in general in line with Solvency II specifications.[22]

3.3 The Insurer's Target Function and the Premium Function

The insurer's target is to maximise shareholder value. Shareholders receive at time 1 the difference of assets and liabilities if this value is positive. Otherwise, they stay on their limited liability, the insurer defaults and policyholders' claims are indemnified only to the amount of the insurer's existing assets. In total, the time-1 cash flow to shareholders is given by $\max\{A_1 - L_1;0\}$.

Formally, the insurer's target function is the time-0-value of the (net) shareholder value which is defined by

$$SHV = \exp(-r_f)\mathrm{E}_Q\left[\max\{A_1 - L_1;0\}\right] - K \tag{5}$$

The *SHV* can be rewritten as follows:

$$\begin{aligned} SHV &= \exp(-r_f)\mathrm{E}_Q\left[A_1 - L_1 + \max\{L_1 - A_1;0\}\right] - K \\ &= A_0 - L_0 + \exp(-r_f)\mathrm{E}_Q\left[\max\{L_1 - A_1;0\}\right] - K \end{aligned} \tag{6}$$

The third summand in the last equation is the time-0-value of the insurer's option to default (commonly referred to as default put option):

$$DPO = \exp(-r_f)\mathrm{E}_Q\left[\max\{L_1 - A_1;0\}\right]. \tag{7}$$

Thus, the *DPO* is a component of the *SHV*. The *DPO* can also be interpreted as the time-0-value of policyholders' loss due to insurer default risk. Using Equation 1, it follows that

$$SHV = \Pi - L_0 + DPO. \tag{8}$$

We model the insurance premium using the following function:

$$\Pi = L_0 - \lambda \cdot DPO + \tau. \tag{9}$$

Here, the factor $\lambda \geq 0$ reflects how strongly the insurance market penalises the insurer's default risk, measured by the *DPO* value; τ is a premium loading. If $\lambda = 0$, the insurance premium does not react to the insurer's safety level, meaning that there is no market discipline. The combination of $\lambda = 1$ and $\tau = 0$ represents the so-called fair premium.[23] $\lambda > 1$ reflects a situation with market discipline: here, policyholders accept the insurer's additional risk-taking only if the premium is discounted more strongly than in the fair premium. The parameters λ and τ do not necessarily need to be constants, but λ may vary in DPO and τ may depend on L_0.

Inserting Equation 9 into 8, we can rewrite *SHV* as follows

$$SHV = (1 - \lambda) \cdot DPO + \tau \tag{10}$$

For the subsequent procedure in this article, it is important to note that *DPO*, and thus *SHV*, depend on the asset allocation parameter α and the Own Funds $OF = A_0 - L_0$. Given that risks evolve according to geometric Brownian motions, we can incorporate this relation using Margrabe's (1978, pp. 177–179) formula, thus

$$DPO = L_0 \cdot N(z) - A_0 \cdot N(z - \sigma) \tag{11}$$

Here, N denotes the cumulative standard normal distribution function, while $z = \ln\left(\frac{L_0}{A_0}\right) \cdot \frac{1}{\sigma} + \frac{\sigma}{2}$ and $\sigma = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho_{AL}\sigma_A\sigma_L}$. σ corresponds to the volatility of the insurer's liability-to-asset ratio L_1/A_1.

On the one hand, α influences σ, as $\sigma_A = \alpha \cdot \sigma_M$; the relation between α and σ is positive if $\alpha \cdot \sigma_M > \rho_{AL} \cdot \sigma_L$.[24] This case applies if $\rho_{AL} < 0$ (i.e. high liabilities more likely coincide with low asset values) or if $\alpha \cdot \sigma_M$ is sufficiently large. Then, a higher portion of risky stocks α increases the riskiness of the asset liability portfolio and raises the DPO. Conversely, if $\rho_{AL} > 0$ and $\alpha \cdot \sigma_M$ is sufficiently small, additional stock risk helps to diversify the portfolio and reduces the *DPO*.

On the other hand, OF determines the initial liability-to-asset ratio L_0/A_0. Under the assumption that L_0 stays constant, a higher value for OF implies a lower liability-to-asset ratio L_0/A_0, which lowers DPO.[25]

4 Optimal Investment Strategies

We next employ a simulation study that allows us to explore the insurer's optimal investment strategy under different calibrations of the standard formula and to represent their interconnectedness graphically.

4.1 Model Calibration (Base Scenario)

In the base scenario, the model insurer is parameterised as follows: the time-0 value of insurance liabilities is set to $L_0 = 2500$. The drift and volatility of the risky asset process are set to $\mu_M = 8\%$ and $\sigma_M = 15\%$. For the liability process, the parameters are set to $\mu_L = 1\%$ (reflecting inflation) and $\sigma_L = 15\%$ (cf. Gatzert and Schmeiser, 2008; Yow and Sherris, 2008, pp. 307–309). For simplicity, we set $r_f = 0\%$. In the first part of our analysis, we set the correlation coefficient between asset and liability risks $\rho_{AL} = -0.25$. Later, we consider a sensitivity analysis for this parameter.

To calibrate the premium function, we set in the base case $\lambda = \tau = 0$, i.e. $\Pi = L_0$, with no market discipline and no premium loading. As can be seen in Equation 10, the insurer will, in this situation, aim for a high *DPO* value to maximise *SHV*. We will discuss the impact of the premium function in section 4.4.

The regulatory standard formula is calibrated as follows. In the base scenario, we set $shock_{st} = 49\%$, which is the highest value for this parameter that has been proposed during the latest impact studies for the Solvency II standard formula.[26] Regarding the liability module, the standard deviation of the combined ratio is $\sigma_L^{cr} = 15.24\%$.[27] Therefore, $SCR_{liab} = 3 \cdot \sigma_L^{cr} \cdot L_0 = 1143$.

Finally, we set the regulatory correlation coefficient, which is used in Equation 4. Setting $corr = 0.25$ matches the Solvency II parameterisation[28] as well as the actual correlation between the assets and liabilities of our model insurer. The different signs of ρ_{AL} and *corr* only result from the design of the formulas: the parameter *corr* in the standard formula defines the relation between asset losses and liability increases, while the model parameter ρ_{AL} defines the relation between the final asset value A_1 and the liability value L_1.

4.2 Influence of the standard formula's stock risk parameter $shock_{st}$

In a first step, we derive the insurer's shareholder-value-maximising stock investment in the base scenario (i.e. $shock_{st}$ = 49%). Given that there is no market discipline ($\lambda = 0$), we aim to reveal which investment strategy enables the insurer to achieve a higher *DPO*, and thus a higher *SHV*. The optimal strategy can be understood graphically by looking at Figure 1. The black solid line in Figure 1 is the capital curve according to the standard formula. This line depicts all those combinations of α and *OF* that just meet the capital requirement according to the standard formula, i.e. $OF = SCR^{SF}(\alpha)$, where $SCR^{SF}(\alpha)$ is determined in Equation 4 and $OF = A_0 - L_0$. As the premium is set to $\Pi = L_0$, Equation 1 implies that $OF = K$.

In Figure 1, $SCR^{SF}(\alpha)$ is an increasing function in α, meaning that the standard formula requires additional capital for a higher stock risk. In addition to the capital curve, Figure 1 shows several iso-*DPO* curves in the diagram. These curves depict those combinations of (α, OF) that lead to a constant *DPO* value, according to Equation 7.

If the insurer invests completely risk-free $(\alpha = 0\%)$, the standard formula requires an initial equity capital $SCR^{SF}(0\%) = 1143$, resulting in $DPO(0\%, 1143) = 0.88$. The dashed line in Figure 1 depicts the iso-*DPO* curve that corresponds to $DPO(\alpha, OF) \equiv 0.88$. If the insurer invests all assets in risky stocks (α =100%), we have $SCR^{SF}(100\%) = 3333$, resulting in $DPO(100\%, 3333) = 0.04$. The dotted line marks those combinations of α and *OF* that result in $DPO(\alpha, OF) \equiv 0.04$.

Since the regulator strongly penalises the asset risk in the standard formula in the base scenario ($shock_{st}$ = 49%), the adoption of a risk-free asset allocation allows for a relatively strong reduction in required capital, which in turn leads to the highest *DPO*. Visually, this result occurs because the slope of the capital curve is lower than that of the *DPO* curves over the entire range of α = 0% to α = 100%. The insurer will choose no investment risk (α = 0%) to raise the *DPO*.

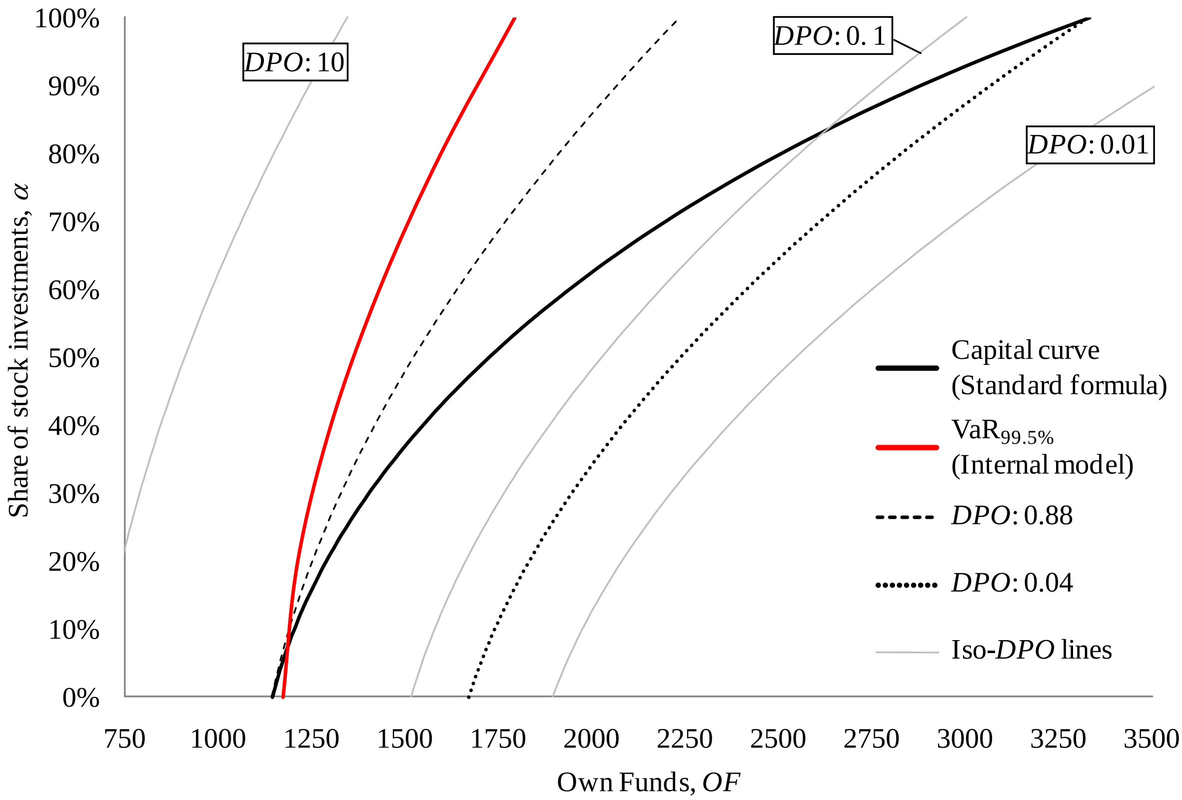
The red solid line in Figure 1 presents the required capital according to the Value-at-Risk with a confidence level of 99.5%. This line could be interpreted as the capital requirement according to an internal risk capital model that is able to determine the Value-at-Risk exactly.

Technically, this line depicts all combinations of α and *OF* that lead to a real-world default probability of $P(L_1 > A_1) = 0.5\%$. As long as α is not too high (i.e. below about 10%), the internal model requires little additional capital for the stock risk. This is because a small portion of stock risk diversifies well with the liability risk and therefore causes only a small change in the default probability.

Comparing the solid black against the red line, it can be seen that the standard formula is slightly too lax for the optimal stock investment α = 0% as it requires less capital than the internal model and therefore leads to a default probability above 0.5%. An explanation for this is that the capital requirement for premium risks is based on an approximation of the Value-at-Risk (cf. section 3.1). For the purpose of the simplified analysis in this article, several components of the standard formula are omitted (such as reserve risk, lapse risk, CAT risks or operational risk) which likely include safety margins and make the standard formula potentially conservative. For values of α above 7%, the standard formula requires more capital than the internal model. Due to the standard formula's strict stock risk parameter in this example, the internal model becomes more favorable for the insurer for higher portions of risky investments. For α = 100%, the standard formula requires 86% more capital than the internal model (SCR^{SF} / SCR^{IM} = 3333 / 1794 = 1.86).

The internal model leads to a different optimal investment strategy than the standard formula with $shock_{st}$ = 49%. When using the internal model, the insurer maximises the *DPO* by choosing the highest investment risk possible. Even though the internal model's capital requirement ensures a default probability of 0.5% for all investment strategies, additional investment risk enables the insurer to attain more risk in the distribution tail. Therefore, the high-investment-risk strategy allows for the highest *DPO* (amounting to 2.99) and thus maximal *SHV*.

Figure 1 Combinations of Own Funds and shares of stock investments in the base scenario with $shock_{st}$ = 49%.



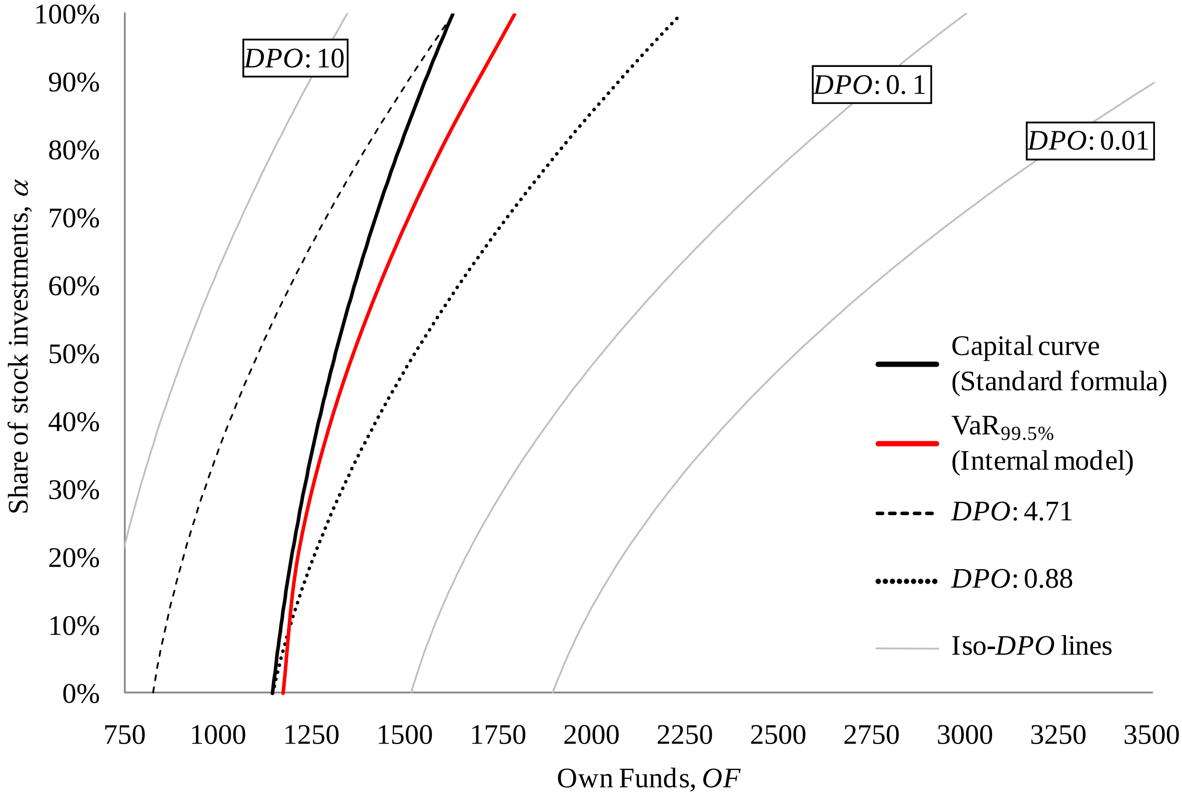
We now modify the base scenario by changing the stock risk parameter to $shock_{st}$ = 22%, which is the lowest value for this parameter that has been recently proposed.[29] Being confronted by the low value, the insurer needs to hold less additional capital when investing more assets into the risky stock. The black solid line in Figure 2 shows the new capital curve according to the standard formula. The lower stock risk parameter translates into a higher slope of the capital curve as compared to Figure 1.

If the insurer decides on a risk-free asset allocation (α = 0%), the standard formula's capital requirement is unchanged at the amount of $SCR^{SF}(0\%) = 1143$, resulting in $DPO(0\%, 1143) = 0.88$. The dotted line depicts those combinations of α and OF that lead to $DPO(\alpha, OF) \equiv 0.88$. If the insurer invests only in risky stocks (α = 100%), we have $SCR^{SF}(100\%) = 1628$, and $DPO(100\%, 1628) = 4.71$. The combinations (α, OF) with $DPO(\alpha, OF) \equiv 4.71$ are depicted by the dashed line in Figure 2. Since

$DPO(0\%, 1143) < DPO(100\%, 1628)$, the insurer maximises its *SHV* by engaging in a high-risk investment strategy.

Comparing the capital curve of the standard formula against that of the internal model, we see that the standard formula is now less conservative than the internal model and leads to a default probability above 0.5% for any value of α. The standard formula underestimates the capital requirement in particular for high shares of stock investments.

Figure 2 Combinations of Own Funds and shares of stock investments for $shock_{st}$ = 22%.



In both of the above cases, corner solutions are optimal, i.e. the insurer invests all assets either in risk-free securities or in risky stocks. However, it may also turn out that an interior solution is optimal. To illustrate such a case, we set $shock_{st}$ = 39%, which is to be applied under Solvency II for the so-called "global equities" which are listed in stock exchanges located in the EEA and OECD when no countercyclical adjustment or transitional measure is applied.[30]

Table 1 Standard formula's solvency capital requirement SCR^{SF}, *DPO* and default probability for different investment strategies and given $shock_{st}$ = 39%

Portion of stock investment, α	0%	. . .	16%	17%	18%	. . .	100%
SCR^{SF}	1143		1222	1228	1235		2492
DPO	0.87848		0.89625	0.89630	0.89622		0.42771
Default probability	0.59%		0.43%	0.42%	0.41%		0.05%

Given that $shock_{st}$ = 39%, Table 1 shows the standard formula's capital requirement for different portions of stock investments. The default probability decreases in α on the whole interval $\alpha \in [0\%, 100\%]$, meaning that the capital requirement for stock risk is relatively strict in terms of the default probability. In contrast, the *DPO* increases with higher portions of risky assets until α = 17%, where the *DPO* reaches its maximum. For values of α above 17%, the additional capital required by the standard formula causes the *DPO* to decrease.

We next generalise the analysis of optimal investment strategies for different values of $shock_{st}$, and consider the whole interval $shock_{st} \in [22\%, 49\%]$. In the light of Figures 1 and 2, the capital curve of the standard formula becomes flatter for higher values of $shock_{st}$; it starts at the same point for $(\alpha, OF) = (0\%, 1143)$ irrespective of $shock_{st}$.

Figure 3.a depicts the insurer's optimal share of risky investments depending on the regulation-assumed shock for stock risk. Figure 3.b presents the resulting default put option value according to the optimal investment strategy, and Figure 3.c shows the corresponding default probability. Table 2 provides the results in numbers. To analyse the influences of the stock risk parameter $shock_{st}$, we distinguish between four different cases.

In the first case— $shock_{st}$ smaller than 30%—, the standard formula does not sufficiently reflect and penalise the stock risk and the insurer thus aims at investing all assets into the risky stock (cf. the results shown in Figure 2). While higher values for $shock_{st}$ do not affect the insurer's investment risk-taking, they require the insurer to hold additional Own Funds and thereby cause a reduction of the *DPO* and the default probability (cf. Table 2 as well as Figure 3.b and c.

In the second case— $shock_{st}$ between about 30% and 33%—the insurer will no longer prefer to invest all assets into the risky stock, but will choose $\alpha^* < 100\%$. Interestingly, the de-risking is so significant that the insurer needs to hold fewer Own Funds the higher the parameter $shock_{st}$ is (cf. Table 2). In total, the *DPO* as well as the default probability decrease, reflecting the reduction of the investment risk.

In the third case— $shock_{st}$ between about 33% and 44%—the insurer further lowers the portion of risky investments α when being confronted with a higher parameter of $shock_{st}$ and at the same time needs to hold fewer Own Funds. In contrast to the second case, the *DPO* and the default probability move in different directions for higher values of $shock_{st}$: while the *DPO* decreases in $shock_{st}$, the default probability increases. The reason behind the latter effect lies in a reduced amount of Own Funds the insurer needs to hold when investing more conservatively. At the same time, there are less diversification effects between the insurer's asset and liability risks, as the risk profile is more strongly dominated by liability risks. In total, it thereby becomes more likely that the time-1-value of liabilities exceeds that of the assets.

In the fourth case— $shock_{st}$ higher than 44%—the insurer decides to hold no stocks any more. Therefore, any values of $shock_{st}$ in this area have the same consequences for the required Own Funds, the *DPO* and the default probability.

It is notable that within this relatively narrow interval of stock risk parameters, say between 32% and 42%, the optimal stock investment decreases rapidly from 88% to 3%. Therefore, a relatively slight change of the stock risk parameter can have significant consequences for the investment strategy, the default put option value and the default. Also, it is worth mentioning that the default probability is above Solvency II's target of 0.5% for very high (above 41%) as well as very low values (below 25%) of $shock_{st}$ (cf. Figure 3.c).

Figure 3 Optimal fraction of risky stock investments, *DPO* and default probability depending on standard formula parameter $shock_{st}$

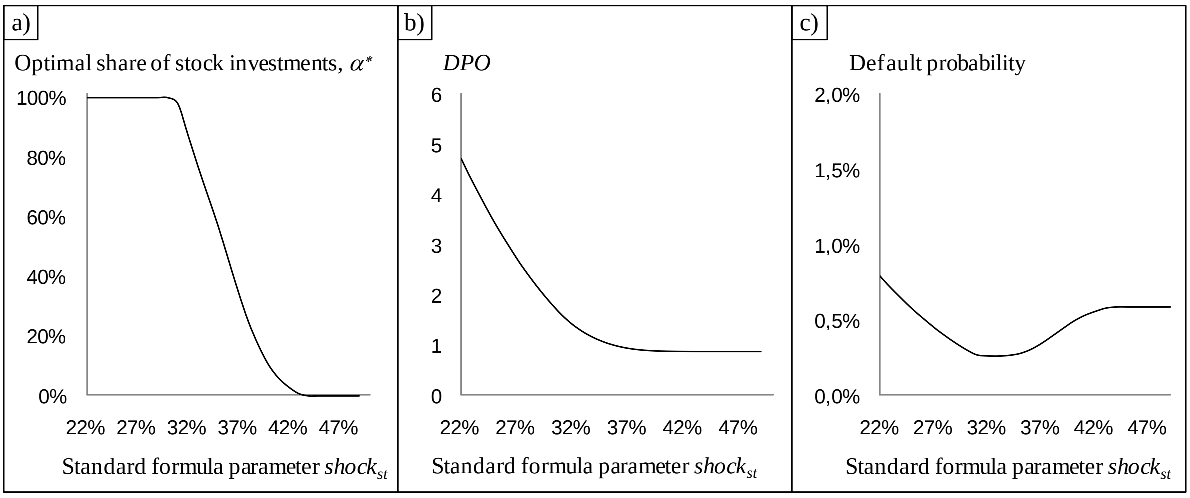


Table 2 Optimal fraction of risky stock investments, capital requirement (standard formula), *DPO* and default probability depending on standard formula parameter $shock_{st}$.

$shock_{st}$	28%	30%	32%	33%	34%	36%	38%	40%	42%	44%
α^*	100%	100%	88%	77%	67%	46%	25%	11%	3%	0%
SCR^{SF}	1872	1966	1873	1759	1656	1452	1281	1191	1155	1143
DPO	2.4161	1.8601	1.4217	1.2671	1.1476	0.9904	0.9139	0.8867	0.8795	0.8785
Default prob	0.399%	0.304%	0.261%	0.260%	0.265%	0.303%	0.392%	0.491%	0.556%	0.587%

4.3 Influence of Insurer's Asset-Liability Correlation

We next investigate a situation in which the insurer can change the correlation between its asset and liability risks without attracting the regulator's attention, i.e. the standard formula's parameter *corr* is assumed to remain unchanged. The insurer could attain a higher negative correlation, such as $\rho_{AL} = -0.5$, for example, by acquiring stocks of companies that suffer from risks that are also in the insurer's liability portfolio or by investing in insurance-linked securities. Moreover, as stock markets might turn down in the aftermath of catastrophic events, the correlation between asset and liability risks might also vary between different lines of business. In this section, we look at how the insurer will adjust its fraction of risky stock investments α in light of the new risk correlation. On the one hand, this allows us to discover how robust the standard formula is with regard to a change in the correlation. On the other hand, and even more importantly, we discover whether the insurer has an incentive to adjust its asset-liability strategy and take risks under a certain regulatory correlation coefficient.

Figure 4 illustrates the insurer's optimal stock investments for three different asset-liability correlations $\rho_{AL} \in \{-0.5, -0.25, 0, 0.1\}$. The standard case from above ($\rho_{AL} = -0.25$) is shown by the solid line. Here, high liability values are positively related to low asset values, meaning that these risks are rarely diversified. For $\rho_{AL} = -0.5$ (dashed line), the coincidence of high liability and low asset values is even more likely, meaning that the asset-liability profile has become more risky. Since the regulator cannot observe this change, capital requirements are relatively lax in this situation, and the insurer thus prefers a higher stock investment. As shown in Figure 4, the insurer's optimal stock investment decreases less rapidly in the standard formula's strictness parameter $shock_{st}$ and the insurer chooses high investment risk even for relatively high values of $shock_{st}$. Since the standard formula does not fully account for the higher correlation, $\rho_{AL} = -0.5$ allows for a higher *DPO* and *SHV* than $\rho_{AL} = -0.25$ (see Figure 5).

For $\rho_{AL} = 0$ (dotted line), the diversification between asset and liability risks is higher and thus the asset-liability portfolio is less risky than in the previous cases. The impact of $shock_{st}$ on the insurer's investment strategy is different now (cf. Figure 4): the insurer prefers to invest all assets into the risky stock until $shock_{st}$ reaches a critical value of about 28%. For higher values of $shock_{st}$, the insurer invests completely risk-free. Figure 7 provides a graphical explanation for this result: it can be seen that the iso-*DPO* curves are more concave than before, and also more concave than the standard formula capital curve. For $shock_{st}$ = 28%, the standard formula capital curve intersects the iso-*DPO* curve displaying *DPO* = 0.88 twice in about α = 0% and α = 100%. Therefore, there is no interim solution for α any more, but α moves directly from 100% to 0% when $shock_{st}$ increases. The dotted line in Figure 6 shows that the default probability takes a lower value if $shock_{st}$ is slightly below 28% (i.e. high investment risk) than if $shock_{st}$ is above 28% and the insurer invests risk-free. In contrast, the *DPO* decreases in $shock_{st}$. (cf. Figure 5). The reason behind this is that in the case of $shock_{st}$ < 28%, the insurer has more investment risks, but at the same time holds greater Own Funds and benefits from risk diversification between asset and liability risks. Analogously to the description in section 4.2 (third case), there are more states of the world at time 1 in which high liability values can be covered with the existing assets and a default can be avoided. This effect also causes that for $\rho_{AL} = -0.5$ the default probability is at a relatively low level if $shock_{st}$ is above 40% (cf. Figure 6): in comparison to $\rho_{AL} = 0$ and $\rho_{AL} = -0.25$ the insurer still invests a larger portion of assets in risky stocks. Therefore, there are some states of the world, in which the additional Own Funds and gains from stock investments help to cover high liabilities and prevent a default.

Finally, we consider the situation $\rho_{AL} = 0.1$, meaning that high asset values more likely coincide with high liability values (and vice versa). This situation may occur if a common risk driver, such as interest rate risk, influences the asset and liability values in the same direction. For example, consider a life insurer investing in a (rolling) bond portfolio that follows a geometric Brownian motion, and consider the liability value L_1 as the time-1-value of insurance benefits at later points in time. An increase (decrease) in interest rate would decrease (increase) the value of both the assets and liabilities at time 1.
The results for $\rho_{AL} = 0.1$ are depicted by the grey dash-dotted lines in Figures 4 – 6. It can be seen that they are similar to the situation with $\rho_{AL} = 0$: the insurer invests fully into the risky asset as long as $shock_{st}$ is below 25%; for higher values of $shock_{st}$, the insurer invests completely risk-free. This critical threshold is now lower than in the previous example, because the risky investment is less effective in raising *DPO* when assets and liabilities are better diversified. As discussed at the end of section 3.3, a small portion of investment risk reduces the *DPO* even if the own funds are held constant. If the correlation coefficient ρ_{AL} is increased further, the insurer will invest risk-free even for lower parameters $shock_{st}$, since investment risk becomes less attractive for the insurer to raise *DPO*.

In short, an insurer's optimal response to a certain stock risk parameter strongly depends on the interaction between its asset and liability risks. For some insurers, even a slight change in the stock risk parameter may lead to a completely different optimal investment strategy. Also, insurers have an incentive to seek opportunities that realise a positive relation between asset and liability risk (i.e. low asset values likely coincide with high liabilities and vice versa), as long as the regulator cannot observe this behaviour. This allows them to hold a relatively risky asset-liability portfolio without a corresponding capital add-on. The insurer's incentive to avoid diversification between asset and liability risks is particularly high if the stock risk parameter in the standard formula is low (see Figure 5).

Figure 4 **Influence of the correlation between asset and liability risks ρ_{AL} on the optimal stock investments α*.**

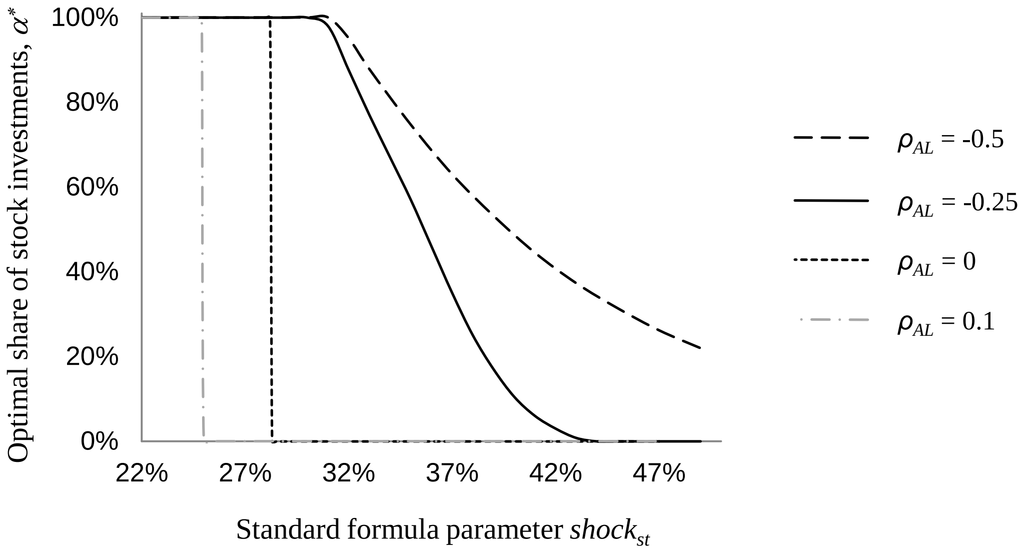


Figure 5 **Influence of the correlation between asset and liability risks ρ_{AL} on *DPO*.**

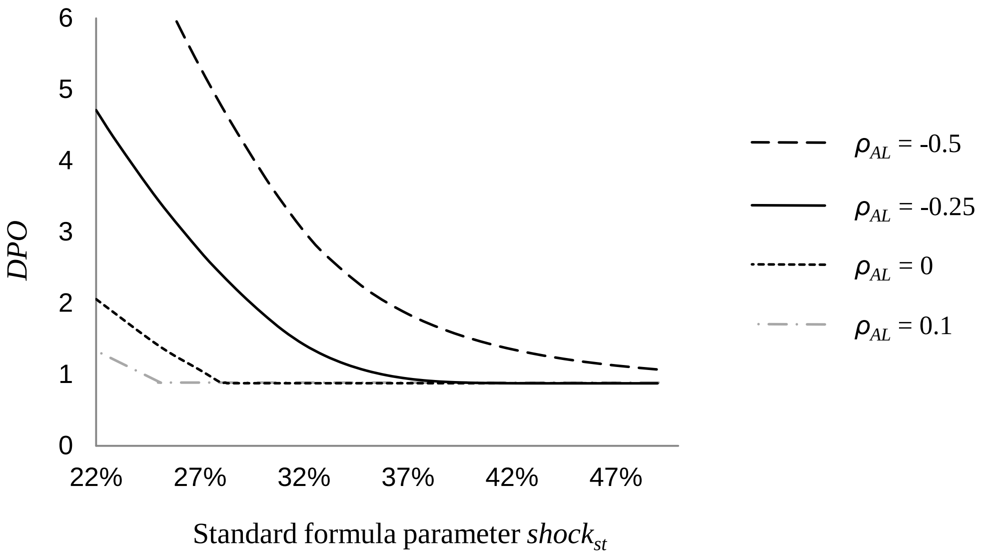


Figure 6 Influence of the correlation between asset and liability risks ρ_{AL} on the default probability.

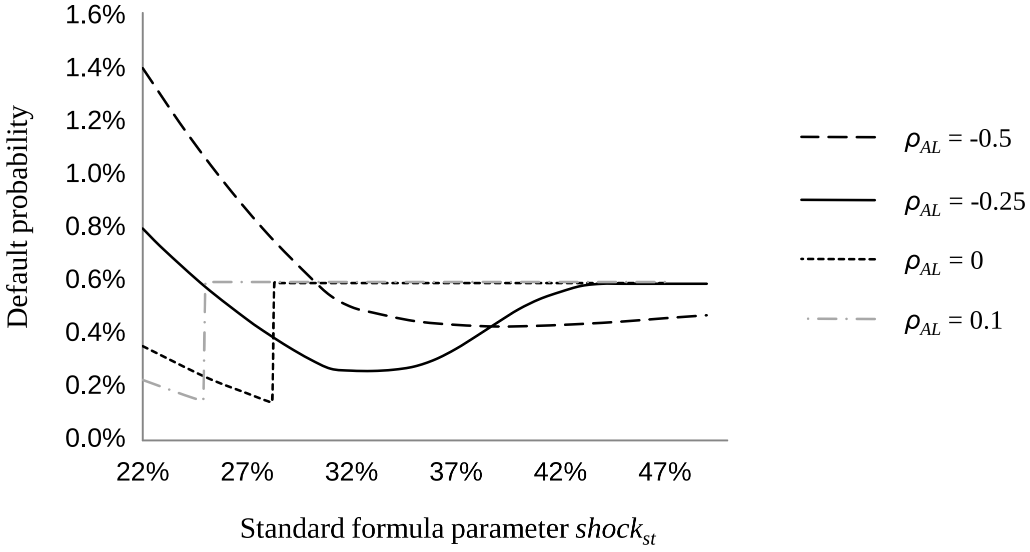
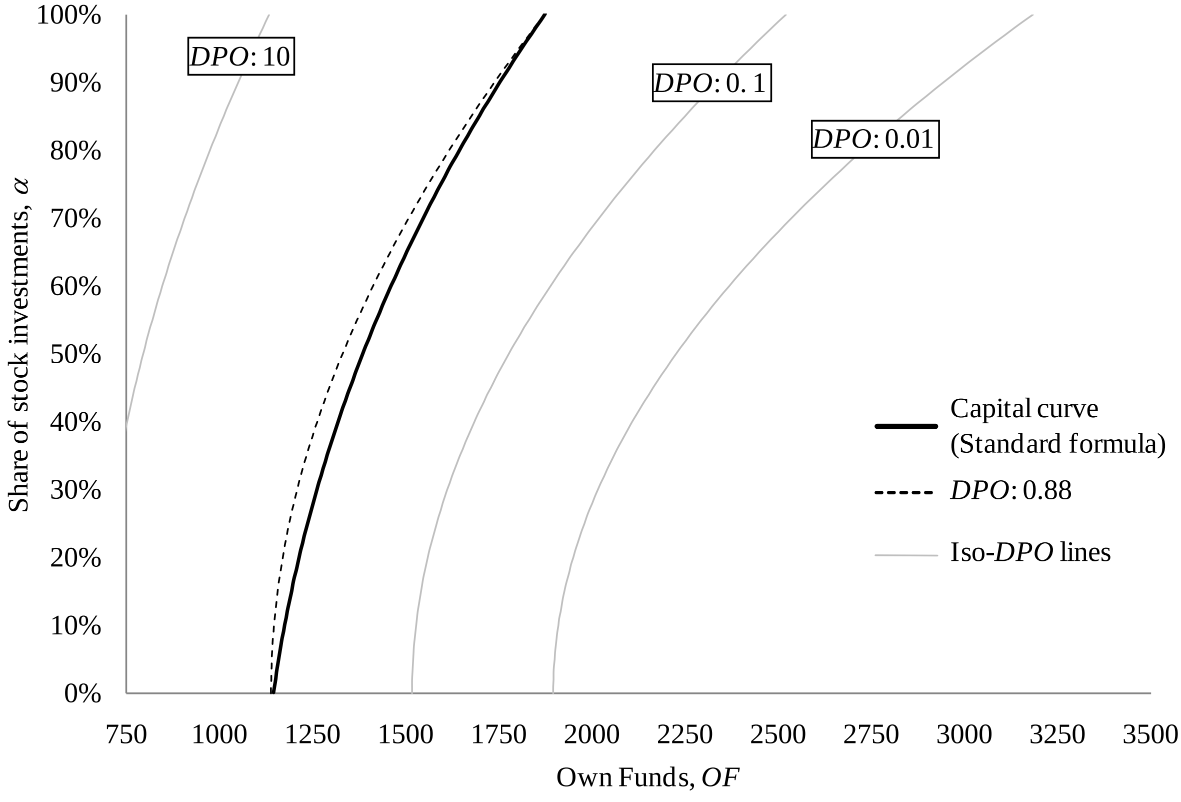


Figure 7 Combinations of Own Funds and shares of stock investments for $shock_{st}$ = 28% and ρ_{AL} = 0.



4.4 Influence of the Premium Function

We next investigate the impact of the premium function. The shape of the premium function determines whether a high or low *DPO* raises the insurer's *SHV*. The insurer will strive for a high *DPO* if the following inequality holds (cf. Equation 10):

$$\frac{\partial}{\partial\, DPO} SHV = -\frac{\partial\,\lambda}{\partial\, DPO} \cdot DPO + 1 - \lambda > 0 \,. \qquad (12)$$

Given that the premium function parameter λ is a constant, a high *DPO* is superior for the insurer if and only if $\lambda < 1$, meaning that the *DPO* has only a relatively small impact on the premium. In general, the first term on the right-hand side states that market discipline is also less likely if the parameter λ shrinks for higher values of *DPO*.

The results of sections 4.2 and 4.3 explain the insurer's optimal capital and investment strategy as long as market discipline is weak, i.e. Inequality 12 is fulfilled. Note that in the *OF*-α diagrams of Figures 1 and 2, neither the *DPO* curve nor the capital curves under the standard formula or the internal model would change with a different parameterisation of the premium function.

If $\lambda + \frac{\partial\,\lambda}{\partial\, DPO} \cdot DPO = 1$, the *SHV* is unaffected by the *DPO*. An important premium principle that falls in this case is the fair premium, i.e. $\lambda = 1$ and $\tau = 0$. Under the fair premium, an increase of the *DPO* is exactly compensated for by a lower premium income (and the other way around). Therefore, any combination of equity capital *OF* and risky investment α will lead to the same *SHV*. Looking at Figures 1 and 2, the insurer may attain any combination of *OF* and α on or below the capital curve, and is indifferent to the particular combination.

In the presence of market discipline, i.e. if $\lambda + \frac{\partial\,\lambda}{\partial\, DPO} \cdot DPO > 1$, the insurer creates SHV by implementing the safest possible strategy. In terms of Figures 1 and 2, a strategy now creates higher *SHV* the further it is towards the bottom right part of the diagram, i.e. it lies on the lowest possible *DPO* curve. Theoretically, the maximum *SHV* is achieved by setting the initial equity endowment to infinity, $K = \infty$, and therefore realising a *DPO* of zero. In the literature, existing articles explain the insurer's optimal safety level in the presence of market discipline often as a trade-off between the effects on the premium income on the one hand and transaction costs for risk management, such as costs of holding capital, on the other hand (cf., for example, Cummins and Danzon, 1997, or Zanjani 2002). Even though the standard formula might not impose a binding restriction in the presence of market discipline, it could be relevant, as the solvency ratio (i.e. OF / SCR) might be important information for policyholders to measure safety and could impact their willingness to pay.

Finally, note that the existence of a premium loading τ is irrelevant for the question whether the insurer prefers a high or a low safety level. Therefore, the premium loading is also irrelevant for the analyses conducted in sections 4.2 and 4.3, whose relevance is only subject to Inequality 12.

5 Summary and outlook

This article explores an insurer's optimal capital and investment strategy when capital requirements are based on a standard formula. To that end, the shareholder value, which serves as the insurer's objective function, is evaluated using option pricing techniques. Since insurance buyers do not adjust willingness to pay to insurer risk-taking, the insurer aims at maximising the value of its default put option. This situation explains why at least some insurers aim to hold a significant share of risky investments without providing additional capital, which is a problem that Solvency II is intended to remedy.

We demonstrate that the calibration of the standard formula has a strong influence on the insurer's capital and investment strategy, and also on its default probability. Which stock risk parameter minimizes an insurer's default probability depends in particular on the interaction between its asset and liability risks. In all considered examples, the minimal default probability is achieved by a stock risk parameter that is associated with a significant level of investment risk. If the stock risk parameter is too low, the insurer will take additional investment risk without sufficiently increasing its equity capital. If the stock risk parameter is too high, the insurer invests more conservatively in order to reduce its capital requirement. The lower capital base, together with weaker diversification effects between asset and liability risks, makes it more likely that underwriting losses cannot be covered by the existing Own Funds. Altogether, we stress that in addition to potential problems of statistical errors (cf.

Mittnik, 2011), regulators and researchers should pay attention to the incentives that result from the formulation of capital requirements.

In our analysis, the influences of the standard formula parameter on the insurer's optimal strategy can be categorised into four cases. Which of these cases will manifest itself depends on each insurer's individual characteristics, such as the dependency between its asset and liability risks. Therefore, the standard formula may induce some insurers to purchase risky assets, while others invest completely risk-free. Monitoring the adjustment in insurers' investment strategies following first implementation of the standard formula, as well as parameter changes, will be a useful way for regulators to discover whether the standard formula assumptions actually fit an insurer's characteristics.

Our approach explains that the insurer's optimal strategy is driven by several discrepancies between the risk valuation in the standard formula and the risk valuation based on taking a shareholders' perspective. Firstly, the standard formula differs from the true Value-at-Risk measured by an exact internal model. In particular, we demonstrated that additional investment risk might be penalized differently.[31] Secondly, the Value-at-Risk does not capture all risks: under Solvency II, it is defined at a confidence level of 99.5%, meaning that any risks beyond this quantile do not impact the capital requirement. A high-investment-risk strategy allows the insurer to shift more risks into the tail and thereby make use of its limited liability. Finally, the shareholder value in our environment is determined under the assumption that the insurer's assets and liabilities can be replicated by instruments that are traded on an arbitrage-free and complete financial market. Changing this assumption might change the position and shape of the DPO curves in Figures 1, 2 and 7, and thus affect the insurer's specific optimal strategy for given standard formula parameterizations. However, the insurer's strategy can still be explained by making best use of the outlined discrepancies.

As we mentioned in the introduction, our approach focuses on the situation where the insurer's capital level is solely defined by the regulatory requirement. However, we believe that the approach could also work in the presence of market discipline, i.e. when demand provides insurers with incentives to hold more capital than the regulatory minimum. This is because insurers' safety levels are often reported as a percentage of the regulatory requirement and insurance buyers could well be influenced by this number. Furthermore, it seems likely that the regulatory requirements under Solvency II could serve as a benchmark for the internal modelling as well as the risk measurement processes of rating agencies (Mittnik, 2011, p. 2). Investigating the side effects of the standard formula calibration on insurance demand and insurers' optimal risk management strategy would be an important extension of the analysis conducted in this article. For example, one could determine those calibration parameters that optimise a certain regulatory objective function, such as the consumer surplus. It would be interesting to find out whether these "optimal" calibration factors coincide with the "objective" statistical estimates.

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[3] Bec and Gollier (2006, pp. 2–3, 18), van Bragt et al. (2010, p. 18), cf. Barnier (2011, pp. 2–3), Kelleher (24 April 2011), Dauer (18 July 2011) and Mittnik (2011).
[4] Merton (1976 and 1977).
[5] This argument can be traced back to Campbell and Viceira (2002, 2005), who provide empirical evidence for the United States.
[6] Danìelsson et al (2013) demonstrate theoretically and based on simulations that the Value-at-Risk satisfies the sub-additivity condition for important practical applications.
[7] European Parliament and Council of the European Union (2009), Article 106.
[8] EIOPA (2013), SCR.5.37, p. 139.
[9] EIOPA (2013), SCR.1.6, p. 116.
[10] European Parliament and Council of the European Union (2009), Article 101.
[11] EIOPA (2013), p. 116, SCR.1.5.
[12] European Parliament and Council of the European Union (2009), Art. 103.
[13] EIOPA (2013), p. 137 ff., SCR.5.6.
[14] EIOPA (2013), p. 226 f.
[15] European Commission (2010), p. 177
[16] European Parliament and Council of the European Union (2009), Annex IV.
[17] European Parliament and Council of the European Union (2009), Articles 87 – 91, define items other then the Basic Own Funds that are considered to absorb losses and therefore accepted as Own Funds. We may neglect those items for the purpose of our analysis.
[18] Similar model set-ups have been employed, e.g., by Cummins and Danzon (1997) and Gatzert and Schmeiser (2008).
[19] Korn and Korn (1999, pp. 65, 117–118).
[20] Ibragimov et al. (2010), p. 556.
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[23] The fair premium is used, e.g., by Doherty and Garven (1986) and Gatzert and Schmeiser (2008).
[24] Note that $\partial\sigma/\partial\alpha = \frac{1}{2}\sigma^{-1}\cdot\left(2\alpha\sigma_M^2 - 2\rho_{AL}\sigma_M\sigma_L\right)$.
[25] To verify this, set $x = L_0/A_0$, i.e. $DPO = L_0 \cdot \left[N(z) - x^{-1}\cdot N(z-\sigma)\right]$. We have $\partial DPO/\partial x = L_0 \cdot \left[\varphi(z)\partial z/\partial x + x^{-2}N(z-\sigma) - x^{-1}\cdot\varphi(z-\sigma)\partial z/\partial x\right]$, where φ denotes the density function of the standard normal distribution. The first and third term within the square brackets cancel out, since $\varphi(z)/\left(x^{-1}\cdot\varphi(z-\sigma)\right) = x\cdot\exp\left(-z^2/2 + (z-\sigma)^2/2\right) = x\cdot\exp\left(-(\ln x/\sigma + \sigma/2)^2/2 + (\ln x/\sigma - \sigma/2)^2/2\right)$ $= x\cdot\exp(-\ln(x)) = 1$. Thus, $\partial DPO/\partial x = L_0\cdot x^{-2}N(z-\sigma) > 0$.
[26] The values 39% and 49% are proposed for $shock_{st}$ depending on the type of equities. They reflect the original parameters without a countercyclical adjustment, as in QIS 5, or a transitional measure, as in the Long Term Guarantee Assessment. The value 49% is to be applied, for example, for hedge funds, equities which are not listed or equities which are listed in stock exchanges located outside the EEA and OECD. Cf. EIOPA (2013), SCR.5.36, p. 139.
[27] Under the real-world measure P, the combined ratio $L_1/\Pi = L_1/L_0$ is lognormally distributed with parameters (μ, σ) = (0.01 - 0.15^2/2, 0.15). Therefore, the standard deviation of L_1/Π is given by: $\sigma_L^{cr} = \sqrt{\exp\left(2\cdot\mu+\sigma^2\right)\cdot\left(\exp\left(\sigma^2\right)-1\right)} = \sqrt{\exp(0.02)\cdot\left(\exp\left(0.15^2\right)-1\right)} = 15.24\%$.
[28] Cf. European Parliament and Council of the European Union (2009), Annex IV.
[29] The value 22% was applied as a transitional measure in the Long Term Guarantee Assessment Cf. EIOPA (2013), SCR.5.37, p. 139.
[30] EIOPA, SCR.5.36, p. 226.
[31] This can be seen in Figure 2: the capital requirements are relatively close for small portions of stock investments, but distant for higher portions.