A physics-based neural network reconstruction of the dense matter equation of state from neutron star observables

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Abstract. We introduce a novel technique that utilizes a physics-driven deep learning method to reconstruct the dense matter equation of state from neutron star observables, particularly the masses and radii. The proposed framework involves two neural networks: one to optimize the EoS using Automatic Differentiation in the unsupervised learning scheme; and a pre-trained network to solve the Tolman–Oppenheimer–Volkoff (TOV) equations. The gradient-based optimization process incorporates a Bayesian picture into the proposed framework. The reconstructed EoS is proven to be consistent with the results from conventional methods. Furthermore, the resulting tidal deformation is in agreement with the limits obtained from the gravitational wave event, GW170817.

1 Introduction

Neutron stars (NSs) are unique laboratories for studying the properties of strongly interacting dense matter. NS observables like masses and radii are direct probes to the underlying equation of state (EoS). The recent increase in data from electromagnetic and gravitational wave observations of neutron stars and their mergers has resulted in an immense wave of studies aimed at constraining the dense matter EoS [1–10]. Past attempts to reconstruct the neutron star EoS include conventional methods like the Bayesian inference [11, 13, 20], or the more recent supervised machine learning inference [14–17]. In this work, we introduce a novel physics-based deep learning approach to constrain the neutron star EoS. The proposed method involves an unsupervised learning algorithm in the Automatic Differentiation (AD) framework. We adopt a deep neural network representation for the EoS. This allows for a versatile and unbiased characterization of the EoS, e.g., the occurrence of a strong first order phase transition.

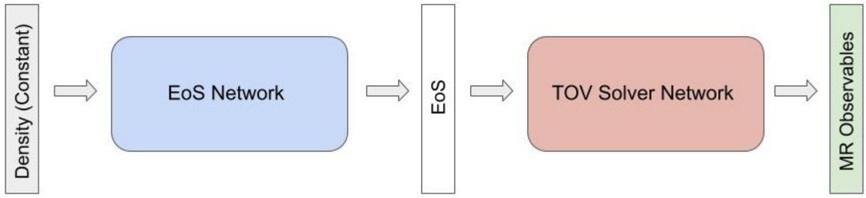

Figure 1. A schematic representation of the proposed algorithm for reconstructing the neutron star EoS via automatic differentiation.

2 Automatic Differentiation

We present a schematic illustration of the proposed mechanism in Fig. 1. The `EoS Network`, as the name suggests, is used to flexibly represent an EoS. It uses density (ρ_i) as the input, and outputs the corresponding pressure (P_i). The `TOV Solver Network`, on the other hand operates as an efficient emulator for the TOV equations. It is a pre-trained network that outputs the mass-radius (M-R) curve, given any input EoS (further details on the network structure, its training procedure, etc, can be found in [18]). The `EoS Network` is combined with the well-trained `TOV-Solver Network` network and optimized in an unsupervised learning scheme. As a result, the trainable parameters in the optimization procedure are the weights of the `EoS Network` alone. Furthermore, we constrain the weights of the `EoS Network` to be non-negative. This ensures a monotonically non-decreasing output for the EoS, so as to fulfill thermodynamic stability. We therefore optimize the `EoS Network` to output an EoS, which in turn reproduces an M-R curve that best fits the NS observations. In order to account for the observational uncertainty, we sample several sets of M-R curves from a Gaussian distribution of the given observations and train the network correspondingly. The optimization process uses a gradient-based algorithm within the AD framework to minimize the loss function, χ^2,

$$\chi^2 = \sum_{i=1}^{N_{\rm obs}} \frac{(M_i - M_{{\rm obs},i})^2}{\Delta M^2_{{\rm obs},i}} + \frac{(R_i - R_{{\rm obs},i})^2}{\Delta R^2_{{\rm obs},i}}. \tag{1}$$

Here, $(M_{{\rm obs},i}, R_{{\rm obs},i})$ are the M-R observations, $(\Delta M_{{\rm obs},i}, \Delta R_{{\rm obs},i})$ are their respective uncertainties, and (M_i, R_i) represent the output of the `TOV-Solver Network`. However, the observational data is limited and not uniformly distributed across the M-R curve. Moreover, the uncertainties of the observations used in this study vary by large factors due to different measurement techniques. As a consequence, there is a possibility of a disordering in the M-R pairs sampled from the normal distribution of the respective observations. To tackle the induced complication, we implement the 'closest approach' optimization [20]. Therefore, the loss in each iteration during the training is,

$$\chi^2 = \sum_{i=1}^{N_{\rm obs}} \frac{(M(\rho_{ci}) - M_{{\rm obs},i})^2}{\Delta M^2_{{\rm obs},i}} + \frac{(R(\rho_{ci}) - R_{{\rm obs},i})^2}{\Delta R^2_{{\rm obs},i}}\,, \tag{2}$$

where the central density, ρ_{ci}, for every $i^{\rm th}$ observation is updated as,

$$\rho_{ci} = \arg\min_{\rho_c} \frac{(M(\rho_c) - M_{{\rm obs},i})^2}{\Delta M^2_{{\rm obs},i}} + \frac{(R(\rho_c) - R_{{\rm obs},i})^2}{\Delta R^2_{{\rm obs},i}}\,. \tag{3}$$

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We implement Eq. (3) to determine the central densities of $(M_{\rm obs}, R_{\rm obs})$, such that they result in the least distance between the M-R observations and the M-R curve obtained from the `TOV-Solver Network`. We further discard any reconstructed EoS that does not comply with the causal condition or fails to support a $1.9 M_\odot$ star. The procedure described above was first tested on several mock M-R data and has been proven to work efficiently [18]. In the next section, we present the results of the reconstructed EoS using the current NS observational data.

3 Results

In order to reconstruct the NS EoS, we use the existent M-R observational data [8, 10, 21–23], which also include the recent NICER measurements. We approximate the uncertainty of each M-R observational point to fit a 1D normal distribution for both mass and radius, individually [15]. We sample 1000 M-R curves from the fitted distributions and reconstruct the EoS from these 18 M-R observations. The resulting EoS is plotted in the left panel of Fig. 2. The pink shaded curve depicts the 95% confidence interval (CI) of the reconstructed EoS from the method discussed above. The mean of the reconstructed EoS curve is marked by the dashed red line. Furthermore, the results are compatible with EoS reconstructions from previous works that include alternate methods like Bayesian inference [24, 25] and machine learning inference [15]. The M-R curve corresponding to the EoS in Fig. 2 is plotted in the right panel of the same figure. Furthermore, the deduced tidal deformability of a $1.4 M_\odot$ neutron star, $\Lambda_{1.4 M_\odot}$, from the reconstructed EoS, using the present method is evaluated at $\Lambda_{1.4} = 224^{+107.3}_{-107.3}$ (95% CI). This value falls within the estimated range of $\Lambda_{1.4 M_\odot} = 190^{+390}_{-120}$, obtained from the gravitational wave event, GW170817 [26].

The next generation telescopes and gravitational-wave detectors provide scope for higher precision on NS observables. With the findings of this work, we therefore conclude that in future, there is hope for a finer reconstruction of the dense matter EoS.

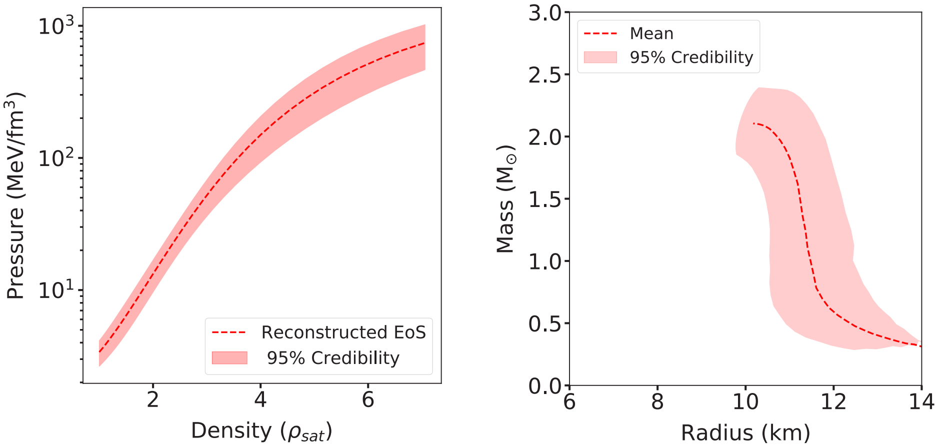

Figure 2. The left panel depicts the 95% CI of the reconstructed EoS from the proposed algorithm. The corresponding M-R curve is plotted in the right panel.

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References

[1] J. Antoniadis, P. C. C. Freire, N. Wex, *et al.* Science **340** (2013), 6131
[2] E. Fonseca, H. T. Cromartie, T. T. Pennucci, *et al.* Astrophys. J. Lett. **915** (2021) 1, L12
[3] R. W. Romani, D. Kandel, A. V. Filippenko, *et al.* Astrophys. J. Lett. **908** (2021) 2, L46
[4] B. P. Abbott *et al.* [LIGO Scientific and Virgo], Phys. Rev. X **9** (2019) 1, 011001
[5] L. Rezzolla, E. R. Most and L. R. Weih, Astrophys. J. Lett. **852** (2018) 2, L25
[6] E. R. Most, L. R. Weih, L. Rezzolla and J. Schaffner-Bielich, Phys. Rev. Lett. **120** (2018) 26, 261103
[7] M. Shibata, E. Zhou, K. Kiuchi and S. Fujibayashi, Phys. Rev. D **100** (2019) 2, 023015
[8] M. C. Miller, F. K. Lamb, A. J. Dittmann, *et al.* Astrophys. J. Lett. **887** (2019) 1, L24
[9] S. Soma and D. Bandyopadhyay, Astrophys. J. **890** (2020), 139
[10] T. E. Riley, A. L. Watts, P. S. Ray, *et al.* Astrophys. J. Lett. **918** (2021) 2, L27
[11] A. W. Steiner, J. M. Lattimer and E. F. Brown, Astrophys. J. **722** (2010), 33-54
[12] C. A. Raithel, F. Özel and D. Psaltis, Astrophys. J. **844** (2017) 2, 156
[13] S. Traversi, P. Char and G. Pagliara, Astrophys. J. **897** (2020), 165
[14] F. Morawski and M. Bejger, Astron. Astrophys. **642** (2020), A78
[15] Y. Fujimoto, K. Fukushima and K. Murase, JHEP **03** (2021), 273
[16] M. Ferreira and C. Providência, JCAP **07** (2021), 011
[17] P. G. Krastev, Galaxies **10** (2022) no.1, 16
[18] S. Soma, L. Wang, S. Shi, H. Stöcker and K. Zhou, JCAP **08** (2022), 071
[19] F. Özel and D. Psaltis, Phys. Rev. D **80** (2009), 103003
[20] C. A. Raithel, F. Özel and D. Psaltis, Astrophys. J. **844** (2017) 2, 156
[21] F. Özel, D. Psaltis, T. Güver, *et al.* Astrophys. J. **820** (2016) no.1, 28
[22] J. Nättilä, M. C. Miller, A. W. Steiner, *et al.* Astron. Astrophys. **608** (2017), A31
[23] D. Gonzalez-Caniulef, S. Guillot and A. Reisenegger, Mon. Not. Roy. Astron. Soc. **490** (2019) no.4, 58
[24] A. W. Steiner, J. M. Lattimer and E. F. Brown, Astrophys. J. Lett. **765** (2013), L5
[25] F. Özel and P. Freire, Ann. Rev. Astron. Astrophys. **54** (2016), 401-440
[26] B. P. Abbott *et al.* [LIGO Scientific and Virgo], Phys. Rev. Lett. **121** (2018) 16, 161101