# Supplemental Material for “First study of reaction $\Xi^0 n \to \Xi^- p$ using $\Xi^0$-nucleus scattering at an electron-positron collider”

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## I. DERIVATION OF THE FORMULA FOR THE CROSS SECTION $\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be})$

The formula for the cross section $\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be})$ is:

$$\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be}) = \frac{N^{\mathrm{sig}}}{\epsilon \mathcal{B} \mathcal{L}_{\mathrm{eff}}}, \tag{1}$$

where $\epsilon$ is the selection efficiency, $\mathcal{B}$ is the product of the branching ratios of all intermediate resonances, defined as $\mathcal{B} \equiv \mathcal{B}(\bar{\Xi}^0 \to \bar{\Lambda}\pi^0)\mathcal{B}(\bar{\Lambda} \to \bar{p}\pi^+)\mathcal{B}(\pi^0 \to \gamma\gamma)\mathcal{B}(\Xi^- \to \Lambda\pi^-)\mathcal{B}(\Lambda \to p\pi^-)$, and $\mathcal{L}_{\mathrm{eff}}$ is the effective luminosity of the $\Xi^0$ flux and target materials. In the formula for the effective luminosity, the angular distribution of the $\Xi^0$ flux, the attenuation of the $\Xi^0$ flux, the number of target nuclei, and the weight of different target materials are considered in turn. In the next step, each component in the formula will be introduced respectively.

The measured angular distribution of the process $J/\psi \to \Xi^0\bar{\Xi}^0$ [1] is:

$$\frac{\mathrm{d}N(\theta)}{\mathrm{d}(\cos\theta)} \propto (1 + \alpha\cos^2\theta), \tag{2}$$

where $\alpha$ is the parameter of the angular distribution of $J/\psi \to \Xi^0\bar{\Xi}^0$, $\theta$ is the angle between the $\Xi^0$ and the beam direction, as shown in Fig. 1, and $\cos\theta$ is from $-1$ to $+1$. According to the integral formula:

$$\int_{-1}^{1} \frac{\mathrm{d}N(\theta)}{\mathrm{d}(\cos\theta)} \mathrm{d}(\cos\theta) = N_{J/\psi}\mathcal{B}_{J/\psi}, \tag{3}$$

where $N_{J/\psi}$ is the number of $J/\psi$ events, $\mathcal{B}_{J/\psi}$ is the branching fraction of $J/\psi \to \Xi^0\bar{\Xi}^0$, we get

$$\frac{\mathrm{d}N(\theta)}{\mathrm{d}(\cos\theta)} = \frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{\int_{-1}^{1}(1+\alpha\cos^2\theta)\mathrm{d}(\cos\theta)}(1+\alpha\cos^2\theta) = \frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{2+\frac{2}{3}\alpha}(1+\alpha\cos^2\theta). \tag{4}$$

Therefore, the following formula can be obtained:

$$\frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta} = \frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{2+\frac{2}{3}\alpha}(1+\alpha\cos^2\theta)\sin\theta, \tag{5}$$

where $\theta$ goes from 0 to $\pi$.

The beam pipe is composed of layers of composite material, which is composed of gold (${}^{197}$Au), beryllium (${}^9$Be) and oil (${}^{12}$C :${}^1$H= 1 : 2.13), as shown in Fig. 1, and more details can be found in Ref. [2]. The distance from a position to the $z$-axis is defined as $x$, so the number of nuclei (${}^{197}$Au, ${}^9$Be, ${}^{12}$C) per unit volume $N(x)$ is:

$$N(x) = \begin{cases} \frac{\rho_{\mathrm{Au}}}{A_{\mathrm{Au}}\cdot 1\mathrm{u}} = \frac{19.32\mathrm{g/cm}^3}{197\cdot 1.6605\times10^{-27}\mathrm{kg}} = 5.91\times10^{22}\ \mathrm{cm}^{-3}, & 3.148564 \le x \le 3.15\ \mathrm{cm} \\ \frac{\rho_{\mathrm{Be}}}{A_{\mathrm{Be}}\cdot 1\mathrm{u}} = \frac{1.85\mathrm{g/cm}^3}{9\cdot 1.6605\times10^{-27}\mathrm{kg}} = 1.24\times10^{23}\ \mathrm{cm}^{-3}, & 3.15 < x \le 3.23\ \mathrm{cm} \\ \frac{\rho_{\mathrm{Oil}}}{(A_{\mathrm{C}}+A_{\mathrm{H}}\cdot 2.13)\cdot 1\mathrm{u}} = \frac{0.81\mathrm{g/cm}^3}{(12+1\cdot 2.13)\cdot 1\mathrm{u}} = 3.45\times10^{22}\ \mathrm{cm}^{-3}, & 3.23 < x \le 3.31\ \mathrm{cm} \\ \frac{\rho_{\mathrm{Be}}}{A_{\mathrm{Be}}\cdot 1\mathrm{u}} = \frac{1.85\mathrm{g/cm}^3}{9\cdot 1.6605\times10^{-27}\mathrm{kg}} = 1.24\times10^{23}\ \mathrm{cm}^{-3}, & 3.31 < x \le 3.37\ \mathrm{cm} \end{cases} \tag{6}$$

where $\rho$ is the volume density, $A$ is the number of nucleons in the nucleus and u is the atomic mass unit.

There is no definite conclusion about the cross section ratios between the reaction processes $\Xi^0 + {}^A\mathrm{X} \to \Xi^- + p + {}^{A-1}\mathrm{X}$. Generally, from the measurements of other particle interactions with nuclei, the cross section is proportional to $A^{\alpha'}$, where $A$ is the number of nucleons in the nucleus and $\alpha'$ is an exponential coefficient in the range $\frac{2}{3}$ to 1 [3–7]. $\alpha' = \frac{2}{3}$ is the most common situation, which corresponds to a pure surface process and the reaction is due to the interaction with single nucleons on the nucleus surface. For the reaction process $\Xi^0 + {}^A\mathrm{X} \to \Xi^- + p + {}^{A-1}\mathrm{X}$, we assume $\alpha' = \frac{2}{3}$ to get the nominal result of the ratio of cross sections for ${}^9$Be, ${}^{12}$C and ${}^{197}$Au nuclei, and take $\alpha' = 1$ to get the systematic uncertainty. Hence, the cross section with neutron is proportional to $A^{\frac{2}{3}} \times \frac{N}{A} = \frac{N}{A^{\frac{1}{3}}}$, where $A$ and $N$ are the numbers of nucleons and neutrons in the nucleus. Then we can get the cross section ratios

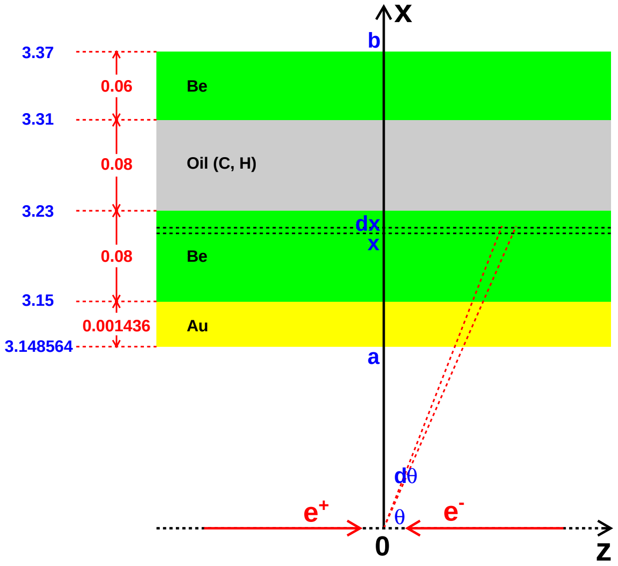

FIG. 1. The schematic diagram of the beam pipe, the length units are centimeter (cm). The $z$-axis is along the $e^+e^-$ beam direction, and the $x$-axis is perpendicular to the $e^+e^-$ beam direction. "$a$" and "$b$" are the distances from the inner surface and the outer surface of the beam pipe to the $z$-axis, which are $a = 3.148564$ cm and $b = 3.37$ cm.

as $\sigma^{^9\text{Be}} : \sigma^{^{12}\text{C}} : \sigma^{^{197}\text{Au}} = \frac{5}{9^{\frac{1}{3}}} : \frac{6}{12^{\frac{1}{3}}} : \frac{118}{197^{\frac{1}{3}}} = 2.4037 : 2.6207 : 20.2796 = 1.000 : 1.090 : 8.437$. We define $\sigma(x) = C(x)\sigma^{^9\text{Be}} = C(x)\sigma(\Xi^0 + {}^9\text{Be} \to \Xi^- + p + {}^8\text{Be})$, where $C(x)$ is:

$$C(x) = \begin{cases} 8.437, & 3.148564\ \text{cm} \le x \le 3.15\ \text{cm} \\ 1.000, & 3.15\ \text{cm} < x \le 3.23\ \text{cm} \\ 1.090, & 3.23\ \text{cm} < x \le 3.31\ \text{cm} \\ 1.000, & 3.31\ \text{cm} < x \le 3.37\ \text{cm} \end{cases} \tag{7}$$

As shown in Fig. 1, at the position of $\theta$ and within the range of $\mathrm{d}\theta$, the number of $\Xi^0$ that can reach the position of $x$ is:

$$\frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta e^{-\frac{t}{\tau}\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta e^{-\frac{x}{\sin\theta v\tau}\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta e^{-\frac{x}{\sin\theta\frac{P_{\Xi^0}}{m_{\Xi^0}}\tau}} = \frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta e^{-\frac{x}{\sin\theta\beta\gamma L}}, \tag{8}$$

where $v$ is the speed of $\Xi^0$, $E_{\text{beam}}$ is the $e^+$ or $e^-$ beam energy for data, $\tau$ is mean lifetime of $\Xi^0$, $c$ is speed of light in vacuum, $\beta\gamma \equiv \frac{P_{\Xi^0}}{m_{\Xi^0}c} = \frac{\sqrt{E^2_{\text{beam}} - m^2_{\Xi^0}c^4}}{m_{\Xi^0}c^2}$, and $L \equiv c\tau$ [8].

Then these $\Xi^0$ particles can interact with neutrons in the material of the beam pipe in the range $\mathrm{d}x$ at position $x$ to produce the reaction process $\Xi^0 n \to \Xi^- p$. So according to the definition of cross section, we get the number of surviving signal events for the reaction process as:

$$\frac{\mathrm{d}N(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta e^{-\frac{x}{\sin\theta\beta\gamma L}}\sigma(x)N(x)\frac{\mathrm{d}x}{\sin\theta}\epsilon\mathcal{B} = \frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{2+\frac{2}{3}\alpha}(1+\alpha\cos^2\theta)e^{-\frac{x}{\sin\theta\beta\gamma L}}\sigma(x)N(x)\epsilon\mathcal{B}\mathrm{d}\theta\mathrm{d}x, \tag{9}$$

where $\epsilon$ is the selection efficiency, and $\mathcal{B} = \mathcal{B}(\bar{\Xi}^0 \to \bar{\Lambda}\pi^0)\mathcal{B}(\bar{\Lambda} \to \bar{p}\pi^+)\mathcal{B}(\pi^0 \to \gamma\gamma)\mathcal{B}(\Xi^- \to \Lambda\pi^-)\mathcal{B}(\Lambda \to p\pi^-)$.

After integrating the above formula in the whole beam pipe region, the total number of surviving signal events $N^{\text{sig}}$ for the reaction process is:

$$N^{\text{sig}} = \int_a^b \int_0^\pi \frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{2+\frac{2}{3}\alpha}(1+\alpha\cos^2\theta)e^{-\frac{x}{\sin\theta\beta\gamma L}}\sigma(x)N(x)\epsilon\mathcal{B}\mathrm{d}\theta\mathrm{d}x. \tag{10}$$

Here, the beam pipe can be regarded as infinitely long with respect to the product $\beta\gamma L$ of $\Xi^0$.

Therefore, the cross section formula of $\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be}$ is:

$$\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be}) = \frac{N^{\mathrm{sig}}}{\epsilon\mathcal{B}\frac{N_{J/\psi}\mathcal{B}_{J/\psi}}{2+\frac{2}{3}\alpha}\int_a^b\int_0^\pi(1+\alpha\cos^2\theta)e^{-\frac{x}{\sin\theta\beta\gamma L}}N(x)C(x)\mathrm{d}\theta\mathrm{d}x}, \tag{11}$$

## II. SYSTEMATIC UNCERTAINTIES

The sources of systematic uncertainties related to the measured cross sections $\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be})$ and $\sigma(\Xi^0 + {}^{12}\mathrm{C} \to \Xi^- + p + {}^{11}\mathrm{C})$ are discussed below. The uncertainty in the tracking efficiency, photon efficiency, and PID efficiency is 1% per track or per photon [9]. The uncertainty from the track number requirement is studied by the control sample $J/\psi \to \Xi^-\bar{\Xi}^+ \to \Lambda\pi^-\bar{\Lambda}\pi^+ \to p\pi^-\pi^-\bar{p}\pi^+\pi^+$. We enlarge the nominal mass windows and $R_{xy}$ requirement by 20% to compare the difference of the result to the nominal one. The requirement on the $M_{\mathrm{recoil}}(\bar{\Xi}^0\Lambda)$ is changed to less than 0.1 GeV/$c^2$ to estimate the uncertainty.

In the MC simulation, we take the momentum of the neutron in the nucleus as zero, but due to the existence of the Fermi-momentum, there is a difference in the distribution of $(\Xi^- + p)$ momentum $P(\Xi^- + p)$ for data and MC. The monoenergetic momentum of the incident $\Xi^0$ is very high compared with the Fermi-momentum, according to the rule of momentum synthesis, the change of the $P(\Xi^- + p)$ for most events is within ±0.1 GeV/$c$. So to estimate the uncertainty from $P(\Xi^- + p)$, we vary the momentum of the free neutron by ±0.1 GeV/$c$ along the direction of the incident $\Xi^0$ in the generated signal MC, and take the larger difference as the uncertainty. We assume the distribution of $M(\Xi^- p)$ is flat in the nominal signal MC. To get the uncertainty from the $M(\Xi^- p)$ distribution, the difference in the efficiency between the nominal signal MC and the weighted MC according to the distribution of signal events in data is taken as systematic uncertainty. The reaction $\Xi^0 n \to \Xi^- p$ is simulated with a uniform angular distribution over the phase-space to estimate the nominal efficiency. The weighted efficiency of signal events is calculated based on real data, as shown in Fig. 2. The difference between the nominal efficiency and weighted efficiency is taken as the uncertainty from the angular distribution.

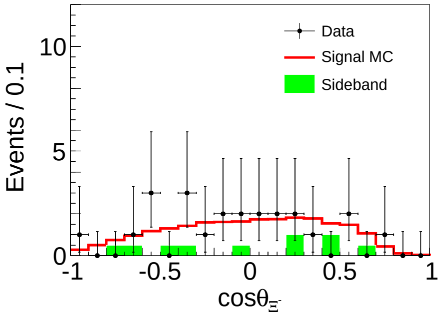

FIG. 2. Distribution of $\cos\theta_{\Xi^-}$ for data, where $\theta_{\Xi^-}$ is the scattering angle of the $\Xi^-$ in the $\Xi^- p$ rest frame. The green-shaded histogram corresponds to the normalized events from the $\Xi^-$ sideband region, and the signal MC distribution is normalized by the total number of events for data.

The uncertainty from the MC statistics is estimated according to the number of generated signal MC events. To estimate the uncertainty from the efficiency curve parameterization, we replace the constant with a first-order polynomial function to parameterize the efficiency curve in the beam pipe region, and the change in the results is taken as systematic uncertainty. The uncertainty from the fit procedure includes the signal shape, the fit range and the background shape. The uncertainty from the signal shape is estimated by using the MC-determined signal shape convolved with a free Gaussian function to instead and compare the difference, the uncertainty from the fit range is obtained by varying the limit of the fit range by ±10 MeV/$c^2$, and the uncertainty associated with the background shape is estimated by changing a first-order polynomial function to a second-order one or a constant.

The uncertainty from the number of $J/\psi$ events is estimated in Ref. [10], and the uncertainty of the branching fractions is taken from the PDG [8]. To estimate the uncertainties from the angular distribution of $J/\psi \to \Xi^0\bar{\Xi}^0$ and the $\Xi^0$ mean lifetime, we vary the angular distribution parameter $\alpha$ and the mean lifetime by $\pm 1\sigma$. The uncertainty from the position of the $e^+e^-$ interaction point is obtained by changing the integral range by ±0.1 cm, which is from $(a, b)$ to $(a + 0.1, b + 0.1)$ or $(a - 0.1, b - 0.1)$, and the larger difference in the result is taken as the uncertainty. Because the beam pipe is made up of composite material, to extract the cross section $\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be})$ or $\sigma(\Xi^0 + {}^{12}\mathrm{C} \to \Xi^- + p + {}^{11}\mathrm{C})$, we assume the reaction is due to the interaction with single neutrons on the nucleus surface. To estimate the uncertainty from the assumption of the cross section ratio, we choose an extreme assumption that the cross section is proportional to the number of neutrons in the nucleus ($\alpha' = 1$), and the difference in the result for the two different extreme assumptions is taken as the uncertainty.

A summary of the systematic uncertainties is presented in Table I, and the total systematic uncertainty is obtained by adding all the individual components in quadrature.

TABLE I. Summary of systematic uncertainties (in %).

| Source | $\sigma(\Xi^0 + {}^9\mathrm{Be} \to \Xi^- + p + {}^8\mathrm{Be})$ | $\sigma(\Xi^0 + {}^{12}\mathrm{C} \to \Xi^- + p + {}^{11}\mathrm{C})$ |
|---|---|---|
| Tracking efficiency | 6.0 | 6.0 |
| Photon efficiency | 2.0 | 2.0 |
| PID efficiency | 6.0 | 6.0 |
| Track number | 3.0 | 3.0 |
| Mass windows | 7.8 | 7.8 |
| $R_{xy}$ requirement | 6.6 | 6.6 |
| $M_{\mathrm{recoil}}(\bar{\Xi}^0\Lambda)$ requirement | 4.3 | 4.3 |
| $(\Xi^- + p)$ momentum | 10.0 | 10.0 |
| $M(\Xi^- p)$ distribution | 0.6 | 0.6 |
| Angular distribution of $\Xi^0 n \to \Xi^- p$ | 1.0 | 1.0 |
| MC statistics | 0.7 | 0.7 |
| Efficiency curve parameterization | 0.5 | 0.5 |
| Fit procedure | 5.0 | 5.0 |
| Number of $J/\psi$ | 0.4 | 0.4 |
| Branching fractions | 3.6 | 3.6 |
| Angular distribution of $J/\psi \to \Xi^0\bar{\Xi}^0$ | 0.1 | 0.1 |
| $\Xi^0$ mean lifetime | 2.7 | 2.7 |
| $e^+e^-$ interaction point | 2.7 | 2.7 |
| Cross section ratios | 7.2 | 2.1 |
| sum | 20.4 | 19.2 |

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