Absolute Measurements of Branching Fractions of Cabibbo-Suppressed Hadronic $D^{0(+)}$ Decays Involving Multiple Pions

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By analyzing e^+e^- annihilation data with an integrated luminosity of 2.93 fb⁻¹ collected at the center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector, we present the first absolute measurements of the branching fractions of twenty Cabibbo-suppressed hadronic $D^{0(+)}$ decays involving multiple pions. The largest four branching fractions obtained are $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-\pi^0) = (1.343 \pm 0.013_{\text{stat}} \pm 0.016_{\text{syst}})\%$, $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-2\pi^0) = (0.998 \pm 0.019_{\text{stat}} \pm 0.024_{\text{syst}})\%$, $\mathcal{B}(D^+ \rightarrow 2\pi^+\pi^-\pi^0) = (1.174 \pm 0.021_{\text{stat}} \pm 0.021_{\text{syst}})\%$, and $\mathcal{B}(D^+ \rightarrow 2\pi^+\pi^-2\pi^0) = (1.074 \pm 0.040_{\text{stat}} \pm 0.030_{\text{syst}})\%$. The *CP* asymmetries for the six decays with highest event yields are also determined.

Investigations of hadronic $D^{0(+)}$ decays are of general importance for both charm and bottom physics. For example, Ref. [1] suggests that hadronic $D^{0(+)}$ decays involving three charged pions are crucial backgrounds for the tests of lepton flavor universality (LFU) in semileptonic *B* decays. However, many Cabibbo-suppressed hadronic $D^{0(+)}$ decays with three charged pions are unexplored mainly due to low detection efficiencies and high background contamination. Precision and comprehensive measurements of the absolute branching fractions (BFs) of these decays provide necessary inputs to unravel the hints of LFU violation observed in semileptonic *B* decays.

According to theoretical predictions, the CP violation in charmed hadron decays is expected to be at the order of 10^{-3} for singly Cabibbo-suppressed processes, and much smaller for Cabibbo-favored and doubly Cabibbosuppressed processes [2–9]. The LHCb experiment reported the first observation of CP violation in $D^0 \rightarrow$ K^+K^- and $\pi^+\pi^-$ with an asymmetry difference of $\Delta A_{CP} = (1.54 \pm 0.29) \times 10^{-3}$ [10]. A similar magnitude of CP asymmetry is predicted in $D^0 \rightarrow K^+K^{*-}$ and $D^0 \rightarrow \rho^+\pi^-$ [8]. Refs. [2, 3] suggest that the CP asymmetries in $D \rightarrow \rho \pi$ decays are in the range of $(0.3 \sim 5) \times 10^{-4}$. Therefore, searching for CP violation in Cabibbo-suppressed $D^{0(+)}$ decays into three pions is an interesting pursuit.

The Cabibbo-suppressed hadronic neutral D decays into $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\pi^0\pi^0$ also provide a promising way to extract the CKM angle γ in $B^+ \to D^{(*)0}K^{(*)+}$ due to the similar magnitude of interference amplitudes between D^0 and \bar{D}^0 decays into $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0$ [11]. More precise measurements of these BFs can improve estimations of the measurement precision for the CPviolation phase angle γ with these modes.

This Letter reports the first absolute measurements of the BFs of the Cabibbo-suppressed hadronic decays $D^0 \rightarrow \pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, $\pi^+\pi^-2\eta$, $4\pi^0$, $3\pi^0\eta$, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $\pi^+\pi^-3\pi^0$, $2\pi^+2\pi^-2\pi^0$, and $D^+ \rightarrow 2\pi^+\pi^-$, $\pi^+2\pi^0$, $2\pi^+\pi^-\pi^0$, $\pi^+3\pi^0$, $3\pi^+2\pi^-$, $2\pi^+\pi^-2\pi^0$, $2\pi^+\pi^-\pi^0\eta$, $\pi^+4\pi^0$, $\pi^+3\pi^0\eta$, $3\pi^+2\pi^-\pi^0$, $2\pi^+\pi^-3\pi^0$ based on 2.93 fb⁻¹ e^+e^- annihilation data taken at the center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector [12]. Moreover, CP asymmetries for the six decays with the highest yields are determined. To date, only the BFs of seven of these decay modes have been measured relative to reference modes and only the CP asymmetries in $D^0(\bar{D}^0) \rightarrow \pi^+\pi^-\pi^0$ and $D^{\pm} \rightarrow \pi^+\pi^-\pi^{\pm}$ have been measured by various experiments [13]. Throughout this Letter, chargeconjugated processes are implied except when discussing CP asymmetries.

The BESIII detector is a magnetic spectrometer [14] located at the Beijing Electron Positron Collider (BEPCII) [15]. Simulated samples produced with a GEANT4-based [16] Monte Carlo (MC) package including the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiencies and to estimate backgrounds. The simulation of e^+e^- annihilations modeled with the generator KKMC [17] includes the beam-energy spread and initialstate radiation. The inclusive MC samples consist of the production of $D\bar{D}$ pairs with consideration of quantum coherence for all neutral D modes, the non- $D\bar{D}$ decays of the $\psi(3770)$, the initial-state radiation production of the J/ψ and $\psi(3686)$ states, and continuum processes. Known decay modes are modeled with EVTGEN [18] using the BFs taken from Ref. [13], and the remaining unknown decays of the charmonium states are modeled with LUNDCHARM [19, 20]. Final-state radiation from charged final-state particles is incorporated using PHOTOS [21].

At $\sqrt{s} = 3.773$ GeV, $D^0 \bar{D}^0$ or $D^+ D^-$ pairs are produced without accompanying hadron(s), thereby offering a clean environment to investigate hadronic Ddecays with double-tag (DT) method [22]. The singletag (ST) \bar{D} candidates are selected by reconstructing a \bar{D}^0 or D^- in the hadronic decay modes: $\bar{D}^0 \rightarrow K^+\pi^-$, $K^+\pi^-\pi^0$, and $K^+\pi^-\pi^-\pi^+$, and $D^- \rightarrow K^+\pi^-\pi^-$, $K^0_S\pi^-$, $K^+\pi^-\pi^-\pi^0$, $K^0_S\pi^-\pi^0$, $K^0_S\pi^+\pi^-\pi^-$, and $K^+K^-\pi^-$. Events in which a signal candidate is reconstructed in the presence of an ST \bar{D} meson are referred to as DT events. The BF of the signal decay is determined by [23]

$$\mathcal{B}_{\rm sig} = N_{\rm DT} / (N_{\rm ST}^{\rm tot} \cdot \epsilon_{\rm sig}), \tag{1}$$

where $N_{\rm ST}^{\rm tot} = \sum_i N_{\rm ST}^i$ and $N_{\rm DT}$ are the total yields of the ST and DT candidates in data, respectively. The ST yield for the tag mode *i* is $N_{\rm ST}^i$, and the efficiency $\epsilon_{\rm sig}$ for detecting the signal *D* decay is averaged over the tag modes *i*.

The selection criteria of K^{\pm} , π^{\pm} , K_S^0 , π^0 , and η , are the same as those used in the analyses presented in Refs. [23, 24]. For $\bar{D}^0 \to K^+\pi^-$, the backgrounds from cosmic rays and Bhabha events are rejected by using the same requirements described in Ref. [25]. For $\bar{D}^0 \to$ $K^+\pi^-\pi^-\pi^+$, the $\bar{D}^0 \to K_S^0 K^{\pm}\pi^{\mp}$ decays are suppressed by requiring the invariant masses of all $\pi^+\pi^-$ pairs to be outside the mass window (0.483, 0.513) GeV/ c^2 .

Tagged \overline{D} mesons are identified using two variables: the energy difference $\Delta E_{\text{tag}} \equiv E_{\text{tag}} - E_{\text{b}}$ and the beamconstrained mass $M_{\text{BC}}^{\text{tag}} \equiv \sqrt{E_{\text{b}}^2 - |\vec{p}_{\text{tag}}|^2}$. Here, E_{b} is the beam energy, $\vec{p}_{\rm tag}$ and $E_{\rm tag}$ are the momentum and energy of \bar{D} in the rest frame of e^+e^- system, respectively. The $\Delta E_{\rm tag}$ of ST \bar{D} candidates must be in the range (-55, 40) MeV for the tag modes involving π^0 and (-25, 25) MeV for the other tag modes, due to differing resolutions. For each tag mode, if there are multiple candidates in an event, only the one yielding the smallest $|\Delta E_{\rm tag}|$ is accepted.

To extract the yields of ST \bar{D} candidates for individual tag modes, binned maximum-likelihood fits are performed to the corresponding $M_{\rm BC}^{\rm tag}$ distributions of the accepted ST candidates following Ref. [24]. The \bar{D} signal is modeled by an MC-simulated shape convolved with a double-Gaussian function describing the resolution difference between the data and MC simulation. The combinatorial background shape is described by an ARGUS function [26]. The total yields of the ST \bar{D}^0 and D^- candidates in data are (232.8 ± $0.2_{\rm stat}) \times 10^4$ and (155.8 ± $0.2_{\rm stat}) \times 10^4$, respectively.

The signal D decays are selected from the remaining tracks and showers recoiling against the tagged \overline{D} candidates. To reject the main backgrounds from Cabibbo-suppressed decays containing K_S^0 , the $\pi^+\pi^-$ and $\pi^0\pi^0$ combinations are required to not fall in the mass windows (0.468, 0.528) GeV/ c^2 and (0.428, 0.548) GeV/ c^2 , respectively. These mass windows correspond to at least 4σ of resolution.

Signal D mesons are identified using the energy difference $\Delta E_{\rm sig}$ and the beam-constrained mass $M_{\rm BC}^{\rm sig}$, calculated similarly to the ST side. For each signal mode, if there are multiple candidates in an event, only the one with the minimum $|\Delta E_{\rm sig}|$ is chosen. Signal decays are required to satisfy the $\Delta E_{\rm sig}$ requirements shown in Table 1.

For each signal decay mode, the signal yield $(N_{\rm DT})$ is obtained from a two-dimensional (2D) unbinned maximum-likelihood fit [27] to the $M_{\rm BC}^{\rm tag}$ versus $M_{\rm BC}^{\rm sig}$ distribution of the accepted DT candidates. See Fig. 2 in the Supplemental Material [28] for an illustration of the 2D distribution. The signal events concentrate around $M_{\rm BC}^{\rm tag} = M_{\rm BC}^{\rm sig} = M_D$, where M_D is the averaged mass of D [13]. The events with correctly reconstructed D (\bar{D}) and incorrectly reconstructed \bar{D} (D), referred to as BKGI, spread along the lines around $M_{\rm BC}^{\rm sig} = M_D$ ($M_{\rm BC}^{\rm tag} = M_D$). Events smeared along the diagonal (BKGII) are mainly from the $e^+e^- \rightarrow q\bar{q}$ processes. Events with uncorrelated and incorrectly reconstructed D and \bar{D} (BKGIII) disperse across the whole allowed kinematic region.

In the fit, the probability density functions (PDFs) for signal, BKGI, BKGII, and BKGIII contributions are constructed as a(x,y), $b_x(x) \cdot c_y(y; M_{\rm BC}^{\rm end}, \xi_y) + b_y(y) \cdot c_x(x; M_{\rm BC}^{\rm end}, \xi_x), c_z(z; \sqrt{2}M_{\rm BC}^{\rm end}, \xi_z) \cdot g(k; 0, \sigma_k)$, and $c_x(x; M_{\rm BC}^{\rm end}, \xi_x) \cdot c_y(y; M_{\rm BC}^{\rm end}, \xi_y)$, respectively. Here, $x = M_{\rm BC}^{\rm sig}$, $y = M_{\rm BC}^{\rm tag}$, $z = (x + y)/\sqrt{2}$, and $k = (x - M_{\rm BC}^{\rm tag})$.

 $y)/\sqrt{2}$. The PDFs of signal, a(x,y), $b_x(x)$, and $b_y(y)$, are described by the MC-simulated shapes smeared with individual Gaussian resolution function with parameters derived from the corresponding one-dimensional $M_{\rm BC}$ fits, to consider resolution difference between data and MC simulation. $c_f(f; M_{\rm BC}^{\rm end}, \xi_f)$ is an ARGUS function [26] (f denotes x, y, or z), where the endpoint $M_{\rm BC}^{\rm end} = 1.8865 \ {\rm GeV}/c^2$ is fixed in the fit. The Gaussian function $g(k; 0, \sigma_k)$ has a mean of zero and a standard deviation parameterized by $\sigma_k = \sigma_0 \cdot (\sqrt{2}M_{\rm BC}^{\rm end} - z)^p$, where σ_0 and p are fit parameters. In addition, the yields and shapes of the peaking background (PBKG) components, which are mainly from D decays into the same final state as a signal mode but involve $K_S^0 \to \pi^+\pi^$ or $\pi^0\pi^0$ and $K^- \rightarrow \pi^-\pi^0$ decay, are fixed based on MC simulations and the known BFs of various PBKG components [13, 29]. Relative to the corresponding signal yields, the PBKG components are 9.5%, 18.2%, and 36.2% for $D^+ \to \pi^+ 4\pi^0$, $D^0 \to 2\pi^+ 2\pi^- \pi^0$, and $D^0\,\rightarrow\,\pi^+\pi^-3\pi^0,$ respectively, and range from 0.1% to 6.3% for the other signal decays.

The $M_{\rm BC}^{\rm tag}$ and $M_{\rm BC}^{\rm sig}$ projections of the 2D fits of the DT candidates reconstructed from data are shown in Fig. 1 and the fitted DT yields are summarized in Table 1.

To determine the signal efficiencies (ϵ_{sig}), the threebody decays are simulated with a modified data-driven generator BODY3 [18], which was developed to simulate different intermediate states in data for a given threebody final state. The Dalitz plot from data, corrected for backgrounds and efficiencies, is taken as input for the BODY3 generator. The efficiencies across phase space are obtained with MC samples generated according to a phase space distribution. Each of the four-body and five-body decays is simulated with a mixed signal MC sample. These decays generated with phase space models including contributions from η , ω , η' , $\rho(770)$, $a_0(980)$, $a_1(1260), b_1(1235)$ and ϕ intermediate states are mixed with fractions obtained by examining the corresponding invariant mass spectra. The data distributions for momenta and $\cos \theta$ (where θ is the polar angle of particle in the e^+e^- rest frame) of the daughter particles, and the invariant masses of each of the two- and multi-body particle combinations agree with the MC simulations. See Figs. 3-11 in the Supplemental Material [28] for explicit comparisons.

The results for $N_{\rm DT}$, $\epsilon_{\rm sig}$, and the extracted BFs are summarized in Table 1. The smallest statistical significance is 6.8 standard deviations for $D^+ \to \pi^+ 4\pi^0$ mode. The signal efficiencies have been corrected by the data-MC differences in the selection efficiencies of π^{\pm} tracking, particle identification (PID) procedures and the reconstruction of π^0 or η .

The systematic uncertainties relative to the obtained BFs are discussed below. In the BF determinations using Eq. (1), all uncertainties from selecting the ST \overline{D} candidates are cancelled in the ratio. Systematic

uncertainties in the total yields of ST \overline{D} mesons related to the $M_{\rm BC}^{\rm tag}$ fits to the ST \overline{D} candidates, were previously estimated to be 0.5% for both neutral and charged \overline{D} [30– 32].

The tracking and PID efficiencies of π^{\pm} are investigated using other DT $D\bar{D}$ hadronic events. The differences of efficiencies between data and MC simulations are weighted by the corresponding π^{\pm} momentum spectra of signal MC events. The systematic uncertainties due to tracking and PID are assigned to be (0.2-0.4)% per π^{\pm} , based on the residual statistical uncertainties of the measured data-MC differences.

The systematic uncertainty of the π^0 reconstruction is assigned as (0.4-0.9)% per π^0 from studies of DT $D\bar{D}$ hadronic decay samples of $D^0 \to K^-\pi^+$, $K^-\pi^+\pi^+\pi^$ versus $\bar{D}^0 \to K^+\pi^-\pi^0$, $K^0_S\pi^0$ [30, 31]. Due to limited η statistics, the systematic uncertainty for η reconstruction is assigned by referring to that of π^0 .

The systematic uncertainty in the 2D fit to the $M_{\rm BC}^{\rm tag}$ versus $M_{\rm BC}^{\rm sig}$ distribution is examined by varying the smeared Gaussian function $(\pm 1\sigma)$, the endpoint of the ARGUS function $(\pm 0.2 \,{\rm MeV}/c^2)$, and the fixed PBKG yields $(\pm 1\sigma)$ of the quoted BF). Adding the changes from these three sources in quadrature yields the corresponding systematic uncertainties.

The systematic uncertainty due to the ΔE_{sig} requirement ranges from (0.1-1.3)% depending on the signal mode. They are evaluated from the efficiency differences obtained with and without smearing the ΔE_{sig} distributions for signal MC events with parameters derived from $D^0 \rightarrow \pi^+\pi^-\pi^0$, $D^0 \rightarrow \pi^+\pi^-2\pi^0$, $D^+ \rightarrow$ $\pi^+2\pi^0$, and $D^+ \rightarrow 2\pi^+\pi^-\pi^0$ to get the data-MC differences.

The systematic uncertainty due to the BODY3 generator is considered by varying the number of bins by $\pm 20\%$ and the systematic uncertainty in the mixed MC model is assigned by varying the fractions of various components by $\pm 1\sigma$ of the quoted BF, when available. Unmeasured components are varied by $\pm 25\%$, beyond which comparisons with observed mass spectra are unsatisfactory. The largest change of the signal efficiencies, (0.2-5.7)% for various signal modes, are assigned as the corresponding systematic uncertainties.

The systematic uncertainties due to the mass window applied to reject K_S^0 events are crosschecked by examining the changes of the BFs by varying the corresponding boundaries of the window by $\pm 5 \text{ MeV}/c^2$. If the difference of the BF is larger than 1 time the statistical uncertainty on the difference (taking the correlated samples into account), it is assigned as the corresponding systematic uncertainty. Otherwise, it is neglected.

The measurements of the BFs of the neutral D decays are affected by quantum correlation effect [33]. To take this effect into account, the CP-even fractions in various decays are needed. The $D^0 \rightarrow 4\pi^0$ and $D^0 \rightarrow 3\pi^0\eta$ final



Fig. 1. Projections of $M_{\rm BC}^{\rm tag}$ (left) and $M_{\rm BC}^{\rm sig}$ (right) of the 2D fits to the DT candidate events. Points with error bars are data and blue solid curves are the fit results. Black dotted, cyan blue solid, blue dotted, red dotted, pink long-dashed correspond to fitted signal, fixed peaking background, BKGI, BKGII, and BKGIII components, respectively.

states are both CP-even eigenstates. For $D^0 \to \pi^+\pi^-\pi^0$, its CP-even fraction has been determined to be 0.973 ± 0.017 [5]. For $D^0 \to \pi^+\pi^-2\pi^0$, $D^0 \to 2\pi^+2\pi^-\pi^0$, $D^0 \to \pi^+\pi^-3\pi^0$, and $D^0 \to 2\pi^+2\pi^-2\pi^0$, the CP-even fractions are estimated by the CP-even tag $D^0 \to K^+K^-$ and the CP-odd tag $D^0 \to K_S^0\pi^0$. Using the same method as described in Ref. [1] and the necessary parameters quoted from Refs. [2–4], we obtain the correction factors to account for the quantum correlation effect on the measured BFs; the results are summarized in Table 3 of the Supplemental Material [28]. After correcting the signal efficiencies by the individual factors, the residual uncertainties are assigned as systematic uncertainties.

The uncertainties of MC statistics for various signal decays, (0.2-1.3)%, are considered as a systematic uncertainty. The uncertainties of the daughter BFs of $\eta \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ are 0.51% and 0.03%, respectively [13].

Adding all individual effects for each signal decay quadratically yields the total systematic uncertainties to be (1.2-11.9)% depending on the signal mode. The detailed systematic uncertainties are given in Table 5 of the Supplemental Material [28].

For the six decay modes with the highest yields, the BFs of D and \overline{D} decays, $\mathcal{B}^+_{\text{sig}}$ and $\mathcal{B}^-_{\text{sig}}$, are measured separately. Their asymmetry is determined by $\mathcal{A}_{CP}^{\text{sig}} = \frac{\mathcal{B}_{\text{sig}}^+ - \mathcal{B}_{\overline{\text{sig}}}^-}{\mathcal{B}_{\text{sig}}^+ + \mathcal{B}_{\overline{\text{sig}}}^-}$. The obtained BFs and asymmetries are summarized in Table 2. We find no statistically significant CP violation. Several systematic uncertainties cancel in the asymmetry: the tracking and PID of $\pi^+\pi^-$ pairs, π^0 and η reconstruction, daughter BFs, K_S^0 rejection windows, MC modeling, and strong phase of D^0 decays. The other systematic uncertainties are estimated separately as above.

To summarize, by analyzing $2.93 \,\mathrm{fb}^{-1}$ of $e^+e^$ annihilation data recorded at $\sqrt{s} = 3.773 \,\text{GeV}$ with the BESIII detector, we present the first absolute measurements of the BFs of twenty Cabibbo-suppressed hadronic $D^{0(+)}$ decays involving multiple pions. For $D^0 \rightarrow \pi^+\pi^-\pi^0, \ \pi^+\pi^-2\pi^0, \ 2\pi^+2\pi^-\pi^0, \ \text{and} \ D^+ \rightarrow$ $2\pi^+\pi^-, \pi^+2\pi^0, 2\pi^+\pi^-\pi^0, 3\pi^+2\pi^-$, the BF precisions are improved by factors of 1.2-2.9 compared to the world average values based on relative measurements. For the other 13 decay modes, the BFs are measured for the first time. The reported BFs offer important input for reliable estimations of potential background sources in the precision measurements of B and D decays, especially to properly evaluate the tensions found in the LFU tests with semileptonic B decays. Amplitude analyses of these multi-body decays with larger data samples available in the near future [39, 40] will open

Table 1. Requirements of ΔE_{sig} , DT yields in data (N_{DT}), detection efficiencies (ϵ_{sig} , including the BFs of η , and π^0 as well as correction factors described later), and the obtained BFs (\mathcal{B}_{sig}). The first nine modes are D^0 decays and the others are D^+ decays. For \mathcal{B}_{sig} , numbers in the first and second parentheses are last two digits of the statistical and systematic uncertainties, respectively. For N_{DT} , uncertainties are statistical only.

Decay	$\Delta E_{\rm sig}$	$N_{\rm DT}$	$\epsilon_{\rm sig}$	$\mathcal{B}_{ m sig}$
	(MeV)		(%)	$(\times 10^{-4})$
$\pi^+\pi^-\pi^0$	(-62, 36)	12792.6(120.1)	40.91	134.3(13)(16)
$\pi^+\pi^-2\pi^0$	(-75, 37)	3783.7(70.5)	16.29	99.8(19)(24)
$\pi^+\pi^-2\eta$	(-37, 29)	42.5(6.7)	2.14	8.5(13)(04)
$4\pi^0$	(-105, 41)	96.0(11.5)	5.41	7.6(09)(07)
$3\pi^0\eta$	(-82, 40)	155.3(14.7)	2.83	23.6(22)(17)
$2\pi^+2\pi^-\pi^0$	(-52, 33)	942.4(40.0)	11.70	34.6(15)(15)
$2\pi^+ 2\pi^- \eta$	(-36, 28)	48.5(7.8)	3.46	6.0(10)(06)
$\pi^+\pi^-3\pi^0$	(-76, 39)	182.7(20.9)	5.13	15.3(17)(13)
$2\pi^+ 2\pi^- 2\pi^0$	(-64, 36)	350.0(22.9)	3.15	47.7(31)(21)
$2\pi^+\pi^-$	(-30, 28)	2614.3(58.0)	50.63	33.1(07)(05)
$\pi^+ 2\pi^0$	(-96, 44)	1968.0(51.7)	27.33	46.2(12)(09)
$2\pi^+\pi^-\pi^0$	(-59, 35)	4649.5(83.5)	25.42	117.4(21)(21)
$\pi^+ 3 \pi^0$	(-86, 39)	573.7(30.2)	8.83	41.7(22)(13)
$3\pi^+2\pi^-$	(-37, 33)	462.1(28.7)	16.26	18.2(11)(10)
$2\pi^+\pi^-2\pi^0$	(-74, 39)	1207.1(45.4)	7.21	107.4(40)(30)
$2\pi^+\pi^-\pi^0\eta$	(-51, 33)	191.4(15.9)	3.17	38.8(32)(12)
$\pi^+ 4\pi^0$	(-90, 41)	56.7(10.4)	1.87	19.5(36)(23)
$\pi^+ 3 \pi^0 \eta$	(-66, 37)	79.7(10.9)	1.77	28.9(40)(22)
$3\pi^+2\pi^-\pi^0$	(-49, 34)	182.8(17.3)	5.02	23.4(22)(15)
$2\pi^+\pi^-3\pi^0$	(-66, 37)	185.9(17.0)	3.49	34.2(31)(16)

Table 2. Charge-separated BFs $(\mathcal{B}_{sig}^+ \text{ and } \mathcal{B}_{\overline{sig}}^-)$, and their asymmetries (\mathcal{A}_{CP}^{sig}) . The first and second uncertainties are statistical and systematic, respectively, for \mathcal{A}_{CP}^{sig} ; while uncertainties for \mathcal{B}_{sig}^+ and $\mathcal{B}_{\overline{sig}}^-$ are only statistical.

		0	
Decay	$\mathcal{B}^+_{\rm sig}(\times 10^{-4})$	$\mathcal{B}_{\overline{\mathrm{sig}}}^{-}(\times 10^{-4})$	$\mathcal{A}_{CP}^{\mathrm{sig}}$ (%)
$\pi^+\pi^-\pi^0$	134.8 ± 1.8	133.3 ± 1.8	$+0.6 \pm 0.9 \pm 0.4$
$\pi^+\pi^-2\pi^0$	97.1 ± 2.6	102.3 ± 2.7	$-2.6\pm1.9\pm0.7$
$2\pi^+\pi^-$	33.5 ± 1.0	32.7 ± 1.0	$+1.2 \pm 2.1 \pm 0.6$
$\pi^+ 2\pi^0$	48.9 ± 1.8	43.4 ± 1.7	$+6.0 \pm 2.7 \pm 0.5$
$2\pi^+\pi^-\pi^0$	117.7 ± 3.0	116.8 ± 3.0	$+0.4\pm1.8\pm0.8$
$2\pi^+\pi^-2\pi^0$	102.7 ± 5.6	111.6 ± 5.8	$-4.2\pm3.8\pm1.3$

an opportunity to precisely extract more quasi-two-body hadronic $D^{0(+)}$ decay rates, e.g. $D^+ \rightarrow \rho^+ \pi^0$. Detailed knowledge of these hadronic $D^{0(+)}$ decays is essential to deeply explore quark U-spin and SU(3)-flavor symmetry breaking effects and thereby improve the predictions of CP violation in the charm sector [2, 41, 42]. Additionally, the asymmetries of the charge-conjugated BFs of the six $D^{0(+)}$ decays with largest yields are determined. No statistically significant CP violation is observed.

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SUPPLEMENTAL MATERIAL



Figure 2 shows the $M_{\rm BC}^{\rm tag}$ versus $M_{\rm BC}^{\rm sig}$ distribution of the accepted DT candidates in data. Detailed selection criteria can be found in context of the main paper.

Fig. 2. The $M_{\rm BC}^{\rm tag}$ versus $M_{\rm BC}^{\rm sig}$ distribution of the accepted DT candidates of $D^0 \to \pi^+\pi^-\pi^0$ in data. Here, ISR denotes initial state radiation, which spreads along the diagonal direction and extends to the higher $M_{\rm BC}$ sides. The $D_{\rm right}^{\rm sig}$ and $D_{\rm right}^{\rm tag}$ denotes the signal spreading around $M_{\rm BC}^{\rm sig} = M_D$ and $M_{\rm BC}^{\rm tag} = M_D$.

Figures 3 to 11 show comparisons between data and MC simulations for the distributions of invariant mass spectra of two-, three-, four- or five-body particle combinations, momenta and $\cos\theta$ of daughter particles for the signal DT candidates with more than 100 signal events. The candidates must satisfy additional requirements of $|M_{\rm BC}^{\rm tag(sig)} - M_D| < 0.006 \text{ GeV}/c^2$ and multiple possible combinations of daughter particles are all plotted when relevant (e.g., all four $\pi^+\pi^-$ combinations for $D^0 \rightarrow 2\pi^+2\pi^-\pi^0$, etc.)

Table 3 summarizes the ST yields of $CP\pm$ tags from the fits to the $M_{\rm BC}^{\rm tag}$ distributions of the accepted ST candidates, the DT yields tagged by $CP\pm$ tags from the 2D fits to the $M_{\rm BC}^{\rm tag}$ versus $M_{\rm BC}^{\rm sig}$ distributions of the accepted DT candidates, and the quantum correlation (QC) factors obtained with the same method as described in Ref. [1] and the necessary parameters quoted from Refs. [2–4].

Table 4 summarizes the statistical significances of the decay modes.

Table 5 summarizes the systematic uncertainties for various sources in the measurements of BFs, which are assigned relative to the measured BFs. They are from the ST yield (N_{tag}) , π^{\pm} tracking efficiency, π^{\pm} PID efficiency, π^{0} and η reconstruction efficiency, daughter BFs, ΔE requirement, K_{S}^{0} rejection, MC statistics, MC generator, 2D fit, and strong phase. For each signal decay, the total uncertainty is obtained by quadratically adding all uncertainties.

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Fig. 3. Comparisons of the distributions of invariant masses of two-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^0 \rightarrow \pi^+ \pi^- \pi^0$ candidates between data (points with error bars) and the BODY3 signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

	$\bar{D}^0 \to K^+ K^- \ (CP+)$	$\bar{D}^0 \to K^0_S \pi^0 \ (CP-)$			
$CP \mod$	$S_{\rm measured}^-$	S_{measured}^+			
	57779 ± 287	70512 ± 311			
Decay mode	$M_{\rm measured}^{-}$	$M_{\rm measured}^+$	f_{CP+}	$f_{\rm QC}~(\%)$	Uncertainty (%)
$D^0 \to \pi^+ \pi^- \pi^0$	/	/	0.973 ± 0.017 [5]	93.5 ± 0.5	0.5
$D^0 \to \pi^+ \pi^- 2 \pi^0$	65.7 ± 11.1	169.8 ± 13.9	0.682 ± 0.077	97.4 ± 0.7	0.7
$D^0 \to 4\pi^0$	/	/	1	93.1 ± 0.5	0.5
$D^0 \to 3\pi^0 \eta$	/	/	1	93.1 ± 0.5	0.5
$D^0 \to 2\pi^+ 2\pi^- \pi^0$	37.8 ± 8.3	35.5 ± 6.6	0.438 ± 0.104	100.9 ± 0.9	0.9
$D^0 \to \pi^+ \pi^- 3 \pi^0$	$5.2^{+3.5}_{-2.8}$	$6.8^{+3.4}_{-2.7}$	$0.520^{+0.338}_{-0.269}$	$99.7^{+3.0}_{-2.4}$	3.0
$D^0\to 2\pi^+2\pi^-2\pi^0$	$3.5^{+2.8}_{-2.1}$	15.9 ± 3.7	$0.790^{+0.269}_{-0.255}$	$95.9^{+2.2}_{-2.1}$	2.2

Table 3. Summary of the ST yields of $CP \mp \text{tags}(S_{\text{measured}}^{\pm})$, the DT yields tagged by $CP \mp \text{tags}(M_{\text{measured}}^{\pm})$, the CP+ fraction (f_{CP+}) , and the QC factor (f_{QC}) . The uncertainties are statistical only. A "/" denotes unmeasured quantites, occuring for one mode with a high-precision extremal result and for the two CP-eigenstates.



Fig. 4. Comparisons of the distributions of invariant masses of two- or three-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^0 \rightarrow \pi^+ \pi^- 2\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

Decay mode	$\pi^+\pi^-\pi^0$	$\pi^+\pi^-2\pi^0$	$\pi^+\pi^-2\eta$	$4\pi^0$	$3\pi^0\eta$	$2\pi^+2\pi^-\pi^0$	$2\pi^+2\pi^-\eta$	$\pi^+\pi^-3\pi^0$	$2\pi^+2\pi^-2\pi^0$	$2\pi^+\pi^-$
Significance	> 10	> 10	8.7	> 10	> 10	> 10	9.1	> 10	> 10	> 10
Decay mode	$\pi^+ 2\pi^0$	$2\pi^+\pi^-\pi^0$	$\pi^+ 3 \pi^0$	$3\pi^+2\pi^-$	$2\pi^+\pi^-2\pi^0$	$2\pi^+\pi^-\pi^0\eta$	$\pi^+ 4\pi^0$	$\pi^+ 3 \pi^0 \eta$	$3\pi^+2\pi^-\pi^0$	$2\pi^+\pi^-3\pi^0$
Significance	> 10	> 10	> 10	> 10	> 10	> 10	6.8	9.9	> 10	> 10

Table 4. Statistical significances (σ) for various decay modes.



Fig. 5. Comparisons of the distributions of invariant masses of two-, three-, or four-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^0 \rightarrow 2\pi^+ 2\pi^- \pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).



Fig. 6. Comparisons of the distributions of invariant masses of two-, three-, four- or five-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^0 \rightarrow 2\pi^+ 2\pi^- 2\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).



Fig. 7. Comparisons of the distributions of invariant masses of two-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow 2\pi^+\pi^-$ candidates between data (points with error bars) and the BODY3 signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).



Fig. 8. Comparisons of the distributions of invariant masses of two-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow \pi^+ 2\pi^0$ candidates between data (points with error bars) and the BODY3 signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).



Fig. 9. Comparisons of the distributions of invariant masses of two- or three-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow 2\pi^+\pi^-\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).



Fig. 10. Comparisons of the distributions of invariant masses of two- ,three- or four-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow 3\pi^+ 2\pi^-$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

Source	$D^0 \rightarrow$								$D^+ \rightarrow$	
Source	$\pi^+\pi^-\pi^0$	$\pi^+\pi^-2\pi^0$	$\pi^+\pi^-2\eta$	$4\pi^{0}$	$3\pi^0\eta$	$2\pi^+2\pi^-\pi^0$	$2\pi^+2\pi^-\eta$	$\pi^+\pi^-3\pi^0$	$2\pi^+ 2\pi^- 2\pi^0$	$2\pi^+\pi^-$
N_{tag}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
π^{\pm} tracking	0.4	0.4	0.7	-	—	0.8	1.0	0.4	0.8	0.6
π^{\pm} PID	0.4	0.4	0.4	-	-	0.8	0.8	0.4	0.8	0.6
π^0/η reconstruction	0.4	1.3	1.6	2.3	2.5	0.6	0.7	1.9	1.3	—
2D fit	0.3	0.6	4.4	3.4	4.7	1.7	4.0	3.5	2.3	0.3
$\Delta E^{\rm sig}$ cut	0.1	0.1	1.3	0.1	0.1	0.2	0.8	0.1	0.1	0.3
MC generator	0.6	1.5	0.2	3.4	2.8	1.1	1.5	5.3	1.4	0.8
K_S^0 rejection	0.1	0.4	—	7.4	3.5	3.2	8.1	4.7	1.6	0.5
Strong phase	0.5	0.7	—	0.5	0.5	0.9	_	3.0	2.2	—
MC statistics	0.2	0.4	0.5	0.8	0.7	0.5	0.6	0.8	1.0	0.2
Daughter \mathcal{B}	0.03	0.07	1.02	0.14	0.52	0.03	0.51	0.10	0.07	—
Total	1.2	2.4	5.1	9.2	7.0	4.2	9.4	8.7	4.3	1.4
Source	$D^+ \rightarrow$									
Source	$\pi^+ 2\pi^0$	$2\pi^+\pi^-\pi^0$	$\pi^+ 3\pi^0$	$3\pi^+2\pi^-$	$2\pi^+\pi^-2\pi^0$	$2\pi^+\pi^-\pi^0\eta$	$\pi^{+}4\pi^{0}$	$\pi^+ 3\pi^0 \eta$	$3\pi^{+}2\pi^{-}\pi^{0}$	$2\pi^+\pi^-3\pi^0$
N_{tag}	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
π^{\pm} tracking	0.2	0.6	0.2	1.1	0.6	0.8	0.2	0.2	1.2	0.6
π^{\pm} PID	0.2	0.6	0.2	1.0	0.6	0.6	0.2	0.2	1.0	0.6
π^0/η reconstruction	1.0	0.6	1.8	-	1.4	1.4	2.7	3.4	0.8	2.5
2D fit	0.3	0.6	1.7	2.1	1.8	2.3	3.1	3.4	2.8	2.7
$\Delta E^{\rm sig}$ cut	0.1	0.1	0.1	0.2	0.1	0.5	0.1	0.1	0.4	0.1
MC generator	1.5	0.8	1.5	3.0	0.7	0.3	4.2	5.7	0.5	1.7
K_S^0 rejection	—	0.8	1.1	3.4	0.8	—	10.2	—	5.6	2.2
Strong phase	—	—	—	_	—	—	—	—	_	—
MC statistics	0.3	0.3	0.6	0.4	0.6	0.6	1.3	0.8	0.8	0.9
Daughter \mathcal{B}	0.07	0.03	0.10	-	0.07	0.51	0.14	0.52	0.03	0.10
Total	1.9	1.8	3.2	5.3	2.8	3.1	11.9	7.5	6.6	4.8

Table 5. Systematic uncertainties (%) in the measurements of the BFs for various decay modes.



Fig. 11. Comparisons of the distributions of invariant masses of two-, three-, four- or five-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow 3\pi^+ 2\pi^- \pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).