Supplemental: Observation of the Doubly Cabibbo-Suppressed Decay $D^+ \to K^+\pi^+\pi^-\pi^0$ and Evidence for $D^+ \to K^+\omega$

M. Ablikim[1], M. N. Achasov[10,c], P. Adlarson[64], S. Ahmed[15], M. Albrecht[4], A. Amoroso[63A,63C], Q. An[60,48], Anita[21], X. H. Bai[54], Y. Bai[47], O. Bakina[29], R. Baldini Ferroli[23A], I. Balossino[24A], Y. Ban[38,k], K. Begzsuren[26], J. V. Bennett[5], N. Berger[28], M. Bertani[23A], D. Bettoni[24A], F. Bianchi[63A,63C], J Biernat[64], J. Bloms[57], A. Bortone[63A,63C], I. Boyko[29], R. A. Briere[5], H. Cai[65], X. Cai[1,48], A. Calcaterra[23A], G. F. Cao[1,52], N. Cao[1,52], S. A. Cetin[51B], J. F. Chang[1,48], W. L. Chang[1,52], G. Chelkov[29,b], D. Y. Chen[6], G. Chen[1], H. S. Chen[1,52], M. L. Chen[1,48], S. J. Chen[36], X. R. Chen[25], Y. B. Chen[1,48], Z. J Chen[20,l], W. S. Cheng[63C], G. Cibinetto[24A], F. Cossio[63C], X. F. Cui[37], H. L. Dai[1,48], J. P. Dai[42,g], X. C. Dai[1,52], A. Dbeyssi[15], R. B. de Boer[4], D. Dedovich[29], Z. Y. Deng[1], A. Denig[28], I. Denysenko[29], M. Destefanis[63A,63C], F. De Mori[63A,63C], Y. Ding[34], C. Dong[37], J. Dong[1,48], L. Y. Dong[1,52], M. Y. Dong[1,48,52], S. X. Du[68], J. Fang[1,48], S. S. Fang[1,52], Y. Fang[1], R. Farinelli[24A], L. Fava[63B,63C], F. Feldbauer[4], G. Felici[23A], C. Q. Feng[60,48], M. Fritsch[4], C. D. Fu[1], Y. Fu[1], X. L. Gao[60,48], Y. Gao[61], Y. Gao[38,k], Y. G. Gao[6], I. Garzia[24A,24B], E. M. Gersabeck[55], A. Gilman[56], K. Goetzen[11], L. Gong[37], W. X. Gong[1,48], W. Gradl[28], M. Greco[63A,63C], L. M. Gu[36], M. H. Gu[1,48], S. Gu[2], Y. T. Gu[13], C. Y Guan[1,52], A. Q. Guo[22], L. B. Guo[35], R. P. Guo[40], Y. P. Guo[28], Y. P. Guo[9,h], A. Guskov[29], S. Han[65], T. T. Han[41], T. Z. Han[9,h], X. Q. Hao[16], F. A. Harris[53], K. L. He[1,52], F. H. Heinsius[4], T. Held[4], Y. K. Heng[1,48,52], M. Himmelreich[11,f], T. Holtmann[4], Y. R. Hou[52], Z. L. Hou[1], H. M. Hu[1,52], J. F. Hu[42,g], T. Hu[1,48,52], Y. Hu[1], G. S. Huang[60,48], L. Q. Huang[61], X. T. Huang[41], Y. P. Huang[1], Z. Huang[38,k], N. Huesken[57], T. Hussain[62], W. Ikegami Andersson[64], W. Imoehl[22], M. Irshad[60,48], S. Jaeger[4], S. Janchiv[26,j], Q. Ji[1], Q. P. Ji[16], X. B. Ji[1,52], X. L. Ji[1,48], H. B. Jiang[41], X. S. Jiang[1,48,52], X. Y. Jiang[37], J. B. Jiao[41], Z. Jiao[18], S. Jin[36], Y. Jin[54], T. Johansson[64], N. Kalantar-Nayestanaki[31], X. S. Kang[34], R. Kappert[31], M. Kavatsyuk[31], B. C. Ke[43,1], I. K. Keshk[4], A. Khoukaz[57], P. Kiese[28], R. Kiuchi[1], R. Kliemt[11], L. Koch[30], O. B. Kolcu[51B,e], B. Kopf[4], M. Kuemmel[4], M. Kuessner[4], A. Kupsc[64], M. G. Kurth[1,52], W. Kühn[30], J. J. Lane[55], J. S. Lange[30], P. Larin[15], L. Lavezzi[63A,63C], H. Leithoff[28], M. Lellmann[28], T. Lenz[28], C. Li[39], C. H. Li[33], Cheng Li[60,48], D. M. Li[68], F. Li[1,48], G. Li[1], H. B. Li[1,52], H. J. Li[9,h], J. L. Li[41], J. Q. Li[4], Ke Li[1], L. K. Li[1], Lei Li[3], P. L. Li[60,48], P. R. Li[32], S. Y. Li[50], W. D. Li[1,52], W. G. Li[1], X. H. Li[60,48], X. L. Li[41], Z. B. Li[49], Z. Y. Li[49], H. Liang[1,52], H. Liang[60,48], Y. F. Liang[45], Y. T. Liang[25], L. Z. Liao[1,52], J. Libby[21], C. X. Lin[49], B. Liu[42,g], B. J. Liu[1], C. X. Liu[1], D. Liu[60,48], D. Y. Liu[42,g], F. H. Liu[44], Fang Liu[1], Feng Liu[6], H. B. Liu[13], H. M. Liu[1,52], Huanhuan Liu[1], Huihui Liu[17], J. B. Liu[60,48], J. Y. Liu[1,52], K. Liu[1], K. Y. Liu[34], Ke Liu[6], L. Liu[60,48], Q. Liu[52], S. B. Liu[60,48], Shuai Liu[46], T. Liu[1,52], X. Liu[32], Y. B. Liu[37], Z. A. Liu[1,48,52], Z. Q. Liu[41], Y. F. Long[38,k], X. C. Lou[1,48,52], F. X. Lu[16], H. J. Lu[18], J. D. Lu[1,52], J. G. Lu[1,48], X. L. Lu[1], Y. Lu[1], Y. P. Lu[1,48], C. L. Luo[35], M. X. Luo[67], P. W. Luo[49], T. Luo[9,h], X. L. Luo[1,48], S. Lusso[63C], X. R. Lyu[52], F. C. Ma[34], H. L. Ma[1], L. L. Ma[41], M. M. Ma[1,52], Q. M. Ma[1], R. Q. Ma[1,52], R. T. Ma[52], X. N. Ma[37], X. X. Ma[1,52], X. Y. Ma[1,48], Y. M. Ma[41], F. E. Maas[15], M. Maggiora[63A,63C], S. Maldaner[28], S. Malde[58], Q. A. Malik[62], A. Mangoni[23B], Y. J. Mao[38,k], Z. P. Mao[1], S. Marcello[63A,63C], Z. X. Meng[54], J. G. Messchendorp[31], G. Mezzadri[24A], T. J. Min[36], R. E. Mitchell[22], X. H. Mo[1,48,52], Y. J. Mo[6], N. Yu. Muchnoi[10,c], H. Muramatsu[56], S. Nakhoul[11,f], Y. Nefedov[29], F. Nerling[11,f], I. B. Nikolaev[10,c], Z. Ning[1,48], S. Nisar[8,i], S. L. Olsen[52], Q. Ouyang[1,48,52], S. Pacetti[23B,23C], X. Pan[9,h], Y. Pan[55], A. Pathak[1], P. Patteri[23A], M. Pelizaeus[4], H. P. Peng[60,48], K. Peters[11,f], J. Pettersson[64], J. L. Ping[35], R. G. Ping[1,52], A. Pitka[4], R. Poling[56], V. Prasad[60,48], H. Qi[60,48], H. R. Qi[50], M. Qi[36], T. Y. Qi[2], T. Y. Qi[9], S. Qian[1,48], W.-B. Qian[52], Z. Qian[49], C. F. Qiao[52], L. Q. Qin[12], X. S. Qin[4], Z. H. Qin[1,48], J. F. Qiu[1], S. Q. Qu[37], K. H. Rashid[62], K. Ravindran[21], C. F. Redmer[28], A. Rivetti[63C], V. Rodin[31], M. Rolo[63C], G. Rong[1,52], Ch. Rosner[15], M. Rump[57], A. Sarantsev[29,d], Y. Schelhaas[28], C. Schnier[4], K. Schoenning[64], D. C. Shan[46], W. Shan[19], X. Y. Shan[60,48], M. Shao[60,48], C. P. Shen[9], P. X. Shen[37], X. Y. Shen[1,52], H. C. Shi[60,48], R. S. Shi[1,52], X. Shi[1,48], X D Shi[60,48], J. J. Song[41], Q. Q. Song[60,48], W. M. Song[27,1], Y. X. Song[38,k], S. Sosio[63A,63C], S. Spataro[63A,63C], F. F. Sui[41], G. X. Sun[1], J. F. Sun[16], L. Sun[65], S. S. Sun[1,52], T. Sun[1,52], W. Y. Sun[35], X Sun[20,l], Y. J. Sun[60,48], Y. K. Sun[60,48], Y. Z. Sun[1], Z. T. Sun[1], Y. H. Tan[65], Y. X. Tan[60,48], C. J. Tang[45], G. Y. Tang[1], J. Tang[49], V. Thoren[64], I. Uman[51D], B. Wang[1], B. L. Wang[52], C. W. Wang[36], D. Y. Wang[38,k], H. P. Wang[1,52], K. Wang[1,48], L. L. Wang[1], M. Wang[41], M. Z. Wang[38,k], Meng Wang[1,52], W. H. Wang[65], W. P. Wang[60,48], X. Wang[38,k], X. F. Wang[32], X. L. Wang[9,h], Y. Wang[49], Y. Wang[60,48], Y. D. Wang[15], Y. F. Wang[1,48,52], Y. Q. Wang[1], Z. Wang[1,48], Z. Y. Wang[1], Ziyi Wang[52], Zongyuan Wang[1,52], D. H. Wei[12], P. Weidenkaff[28], F. Weidner[57], S. P. Wen[1], D. J. White[55], U. Wiedner[4],

G. Wilkinson[58], M. Wolke[64], L. Wollenberg[4], J. F. Wu[1,52], L. H. Wu[1], L. J. Wu[1,52], X. Wu[9,h], Z. Wu[1,48], L. Xia[60,48], H. Xiao[9,h], S. Y. Xiao[1], Y. J. Xiao[1,52], Z. J. Xiao[35], X. H. Xie[38,k], Y. G. Xie[1,48], Y. H. Xie[6], T. Y. Xing[1,52], X. A. Xiong[1,52], G. F. Xu[1], J. J. Xu[36], Q. J. Xu[14], W. Xu[1,52], X. P. Xu[46], F. Yan[9,h], L. Yan[63A,63C], L. Yan[9,h], W. B. Yan[60,48], W. C. Yan[68], Xu Yan[46], H. J. Yang[42,g], H. X. Yang[1], L. Yang[65], R. X. Yang[60,48], S. L. Yang[1,52], Y. H. Yang[36], Y. X. Yang[12], Yifan Yang[1,52], Zhi Yang[25], M. Ye[1,48], M. H. Ye[7], J. H. Yin[1], Z. Y. You[49], B. X. Yu[1,48,52], C. X. Yu[37], G. Yu[1,52], J. S. Yu[20,l], T. Yu[61], C. Z. Yuan[1,52], W. Yuan[63A,63C], X. Q. Yuan[38,k], Y. Yuan[1], Z. Y. Yuan[49], C. X. Yue[33], A. Yuncu[51B,a], A. A. Zafar[62], Y. Zeng[20,l], B. X. Zhang[1], Guangyi Zhang[16], H. H. Zhang[49], H. Y. Zhang[1,48], J. L. Zhang[66], J. Q. Zhang[4], J. W. Zhang[1,48,52], J. Y. Zhang[1], J. Z. Zhang[1,52], Jianyu Zhang[1,52], Jiawei Zhang[1,52], L. Zhang[1], Lei Zhang[36], S. Zhang[49], S. F. Zhang[36], T. J. Zhang[42,g], X. Y. Zhang[41], Y. Zhang[58], Y. H. Zhang[1,48], Y. T. Zhang[60,48], Yan Zhang[60,48], Yao Zhang[1], Yi Zhang[9,h], Z. H. Zhang[6], Z. Y. Zhang[65], G. Zhao[1], J. Zhao[33], J. Y. Zhao[1,52], J. Z. Zhao[1,48], Lei Zhao[60,48], Ling Zhao[1], M. G. Zhao[37], Q. Zhao[1], S. J. Zhao[68], Y. B. Zhao[1,48], Y. X. Zhao[25], Z. G. Zhao[60,48], A. Zhemchugov[29,b], B. Zheng[61], J. P. Zheng[1,48], Y. Zheng[38,k], Y. H. Zheng[52], B. Zhong[35], C. Zhong[61], L. P. Zhou[1,52], Q. Zhou[1,52], X. Zhou[65], X. K. Zhou[52], X. R. Zhou[60,48], A. N. Zhu[1,52], J. Zhu[37], K. Zhu[1], K. J. Zhu[1,48,52], S. H. Zhu[59], W. J. Zhu[37], X. L. Zhu[50], Y. C. Zhu[60,48], Z. A. Zhu[1,52], B. S. Zou[1], J. H. Zou[1]

(BESIII Collaboration)

[1] *Institute of High Energy Physics, Beijing 100049, People's Republic of China*
[2] *Beihang University, Beijing 100191, People's Republic of China*
[3] *Beijing Institute of Petrochemical Technology, Beijing 102617, People's Republic of China*
[4] *Bochum Ruhr-University, D-44780 Bochum, Germany*
[5] *Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[6] *Central China Normal University, Wuhan 430079, People's Republic of China*
[7] *China Center of Advanced Science and Technology, Beijing 100190, People's Republic of China*
[8] *COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan*
[9] *Fudan University, Shanghai 200443, People's Republic of China*
[10] *G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia*
[11] *GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany*
[12] *Guangxi Normal University, Guilin 541004, People's Republic of China*
[13] *Guangxi University, Nanning 530004, People's Republic of China*
[14] *Hangzhou Normal University, Hangzhou 310036, People's Republic of China*
[15] *Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany*
[16] *Henan Normal University, Xinxiang 453007, People's Republic of China*
[17] *Henan University of Science and Technology, Luoyang 471003, People's Republic of China*
[18] *Huangshan College, Huangshan 245000, People's Republic of China*
[19] *Hunan Normal University, Changsha 410081, People's Republic of China*
[20] *Hunan University, Changsha 410082, People's Republic of China*
[21] *Indian Institute of Technology Madras, Chennai 600036, India*
[22] *Indiana University, Bloomington, Indiana 47405, USA*
[23] *(A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy*
[24] *(A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy*
[25] *Institute of Modern Physics, Lanzhou 730000, People's Republic of China*
[26] *Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia*
[27] *Jilin University, Changchun 130012, People's Republic of China*
[28] *Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany*
[29] *Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*
[30] *Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany*
[31] *KVI-CART, University of Groningen, NL-9747 AA Groningen, The Netherlands*
[32] *Lanzhou University, Lanzhou 730000, People's Republic of China*
[33] *Liaoning Normal University, Dalian 116029, People's Republic of China*
[34] *Liaoning University, Shenyang 110036, People's Republic of China*

[35] *Nanjing Normal University, Nanjing 210023, People's Republic of China*
[36] *Nanjing University, Nanjing 210093, People's Republic of China*
[37] *Nankai University, Tianjin 300071, People's Republic of China*
[38] *Peking University, Beijing 100871, People's Republic of China*
[39] *Qufu Normal University, Qufu 273165, People's Republic of China*
[40] *Shandong Normal University, Jinan 250014, People's Republic of China*
[41] *Shandong University, Jinan 250100, People's Republic of China*
[42] *Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*
[43] *Shanxi Normal University, Linfen 041004, People's Republic of China*
[44] *Shanxi University, Taiyuan 030006, People's Republic of China*
[45] *Sichuan University, Chengdu 610064, People's Republic of China*
[46] *Soochow University, Suzhou 215006, People's Republic of China*
[47] *Southeast University, Nanjing 211100, People's Republic of China*
[48] *State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People's Republic of China*
[49] *Sun Yat-Sen University, Guangzhou 510275, People's Republic of China*
[50] *Tsinghua University, Beijing 100084, People's Republic of China*
[51] *(A)Ankara University, 06100 Tandogan, Ankara, Turkey; (B)Istanbul Bilgi University, 34060 Eyup, Istanbul, Turkey; (C)Uludag University, 16059 Bursa, Turkey; (D)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey*
[52] *University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[53] *University of Hawaii, Honolulu, Hawaii 96822, USA*
[54] *University of Jinan, Jinan 250022, People's Republic of China*
[55] *University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom*
[56] *University of Minnesota, Minneapolis, Minnesota 55455, USA*
[57] *University of Muenster, Wilhelm-Klemm-Str. 9, 48149 Muenster, Germany*
[58] *University of Oxford, Keble Rd, Oxford, UK OX13RH*
[59] *University of Science and Technology Liaoning, Anshan 114051, People's Republic of China*
[60] *University of Science and Technology of China, Hefei 230026, People's Republic of China*
[61] *University of South China, Hengyang 421001, People's Republic of China*
[62] *University of the Punjab, Lahore-54590, Pakistan*
[63] *(A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy*
[64] *Uppsala University, Box 516, SE-75120 Uppsala, Sweden*
[65] *Wuhan University, Wuhan 430072, People's Republic of China*
[66] *Xinyang Normal University, Xinyang 464000, People's Republic of China*
[67] *Zhejiang University, Hangzhou 310027, People's Republic of China*
[68] *Zhengzhou University, Zhengzhou 450001, People's Republic of China*

[a] *Also at Bogazici University, 34342 Istanbul, Turkey*
[b] *Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia*
[c] *Also at the Novosibirsk State University, Novosibirsk, 630090, Russia*
[d] *Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia*
[e] *Also at Istanbul Arel University, 34295 Istanbul, Turkey*
[f] *Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany*
[g] *Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China*
[h] *Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People's Republic of China*
[i] *Also at Harvard University, Department of Physics, Cambridge, MA, 02138, USA*
[j] *Currently at: Institute of Physics and Technology, Peace Ave.54B, Ulaanbaatar 13330, Mongolia*

[k] *Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China*
[l] *School of Physics and Electronics, Hunan University, Changsha 410082, China*

Figure 1 shows the comparison of two-body and three-body invariant mass distributions for the $D^+ \to K^+\pi^+\pi^-\pi^0$ candidate events.

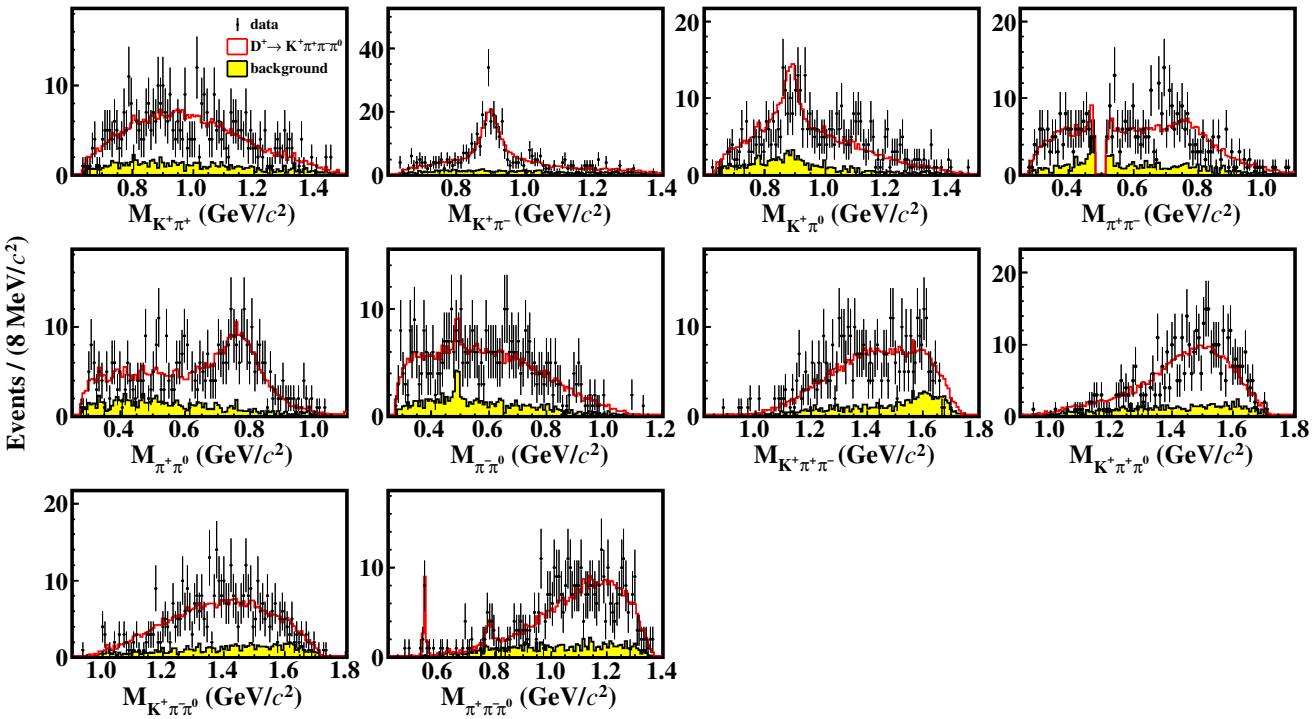

Fig. 1. Comparison of two-body and three-body mass distributions of the $D^+ \to K^+\pi^+\pi^-\pi^0$ candidate events between data (dots with error bars) and inclusive MC sample (red histograms). The yellow hatched histograms denote the MC-simulated backgrounds. The events have been required to be within $|M_{\rm BC}^{\rm tag\,(sig)} - M_{D^+}| < 0.006$ GeV/c^2 but without additional two ω requirements as in Fig. 3 in text, and the nominal signal MC events are used for comparison.