

Supplemental materials for “Determination of the pseudoscalar decay constant $f_{D_s^+}$
via $D_s^+ \rightarrow \mu^+ \nu_\mu$ ”

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Figure 1 shows the M_{BC} distributions of the ST D_s^- candidates from $e^+e^- \rightarrow D_s^- D_s^{*+}$, $e^+e^- \rightarrow D_s^+ D_s^{*-}$, and $e^+e^- \rightarrow D_s^+ D_s^-$ processes based on MC simulation. Both D_s^- mesons directly produced from e^+e^- annihilation and indirectly produced from D_s^{*-} decays are retained by our nominal M_{BC} requirement.

Table I summarizes the ST yield N_{ST} , the background yield N_{ST}^{bkg} in the M_{tag} signal regions, the DT yield N_{DT} , the signal efficiency $\varepsilon_{\gamma(\pi^0)\mu^+\nu_\mu}$ and the obtained $\mathcal{B}_{D_s^+ \rightarrow \mu^+\nu_\mu}$ for each ST mode. Although the background levels for various ST modes are much different, the BF's measured with individual ST modes are consistent with each other.

As an independent check, we further examine the μ^+ PID efficiencies of data and MC simulation, $\varepsilon_{\mu\text{PID}}^{\text{data}}$ and $\varepsilon_{\mu\text{PID}}^{\text{MC}}$, by analyzing $e^+e^- \rightarrow \gamma_{ISR}\psi(3686), \psi(3686) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \mu^+\mu^-$ events (sample I) and corresponding 2D reweighted efficiencies based on $e^+e^- \rightarrow \gamma\mu^+\mu^-$ samples (sample II). Two samples with much different topologies give consistent $\varepsilon_{\mu\text{PID}}^{\text{data}}$, $\varepsilon_{\mu\text{PID}}^{\text{MC}}$, and $f_{\mu\text{PID}}^{\text{cor}} = \varepsilon_{\mu\text{PID}}^{\text{data}}/\varepsilon_{\mu\text{PID}}^{\text{MC}}$, as shown in Table II. The obtained $f_{\mu\text{PID}}^{\text{cor}}$ in these two samples are different with that in $D_s^+ \rightarrow \mu^+\nu_\mu$ mainly due to much higher muon momentum.

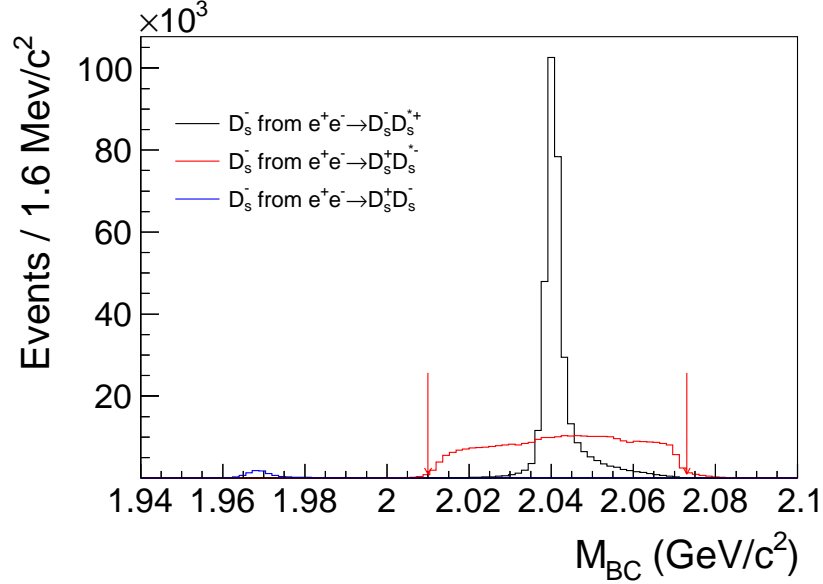


Fig. 1: The M_{BC} distributions of the ST D_s^- candidates from $e^+e^- \rightarrow D_s^+ D_s^{*-}$, $e^+e^- \rightarrow D_s^- D_s^{*+}$, and $e^+e^- \rightarrow D_s^+ D_s^-$ processes. The red arrows give our nominal M_{BC} window for the ST D_s^- candidates.

Table I: Summary of N_{ST} , N_{ST}^{bkg} , N_{DT} , $\varepsilon_{\gamma(\pi^0)\mu^+\nu_\mu}$, and the obtained $\mathcal{B}_{D_s^+ \rightarrow \mu^+\nu_\mu}$ with various ST modes. The uncertainties are only statistical. The signal efficiencies have been corrected by $f_{\mu\text{PID}}^{\text{cor}}$ as described in manuscript. The variations of the signal efficiencies are mainly due to different multiplicities of the tag sides.

ST mode	N_{ST}	N_{ST}^{bkg}	N_{DT}	$\varepsilon_{\gamma(\pi^0)\mu^+\nu_\mu}$ (%)	$\mathcal{B}_{D_s^+ \rightarrow \mu^+\nu_\mu}$ ($\times 10^{-3}$)
$K^+ K^- \pi^+$	133959 ± 633	173160	373.3 ± 18.9	49.73 ± 0.24	5.55 ± 0.28
$K^+ K^- \pi^+ \pi^0$	41377 ± 916	221099	123.1 ± 10.7	57.32 ± 0.85	5.14 ± 0.46
$\pi^+ \pi^+ \pi^-$	35966 ± 913	300499	90.0 ± 9.9	51.21 ± 0.53	4.84 ± 0.55
$K_S^0 K^+$	32039 ± 291	18776	79.7 ± 9.0	49.77 ± 0.36	4.95 ± 0.56
$K_S^0 K^+ \pi^0$	11294 ± 433	52788	38.4 ± 6.1	56.71 ± 2.34	5.94 ± 0.97
$K^+ \pi^+ \pi^-$	15877 ± 872	246528	45.6 ± 7.2	51.21 ± 1.30	5.55 ± 0.93
$K_S^0 K_S^0 \pi^+$	4832 ± 180	11274	20.2 ± 4.4	50.55 ± 1.25	8.19 ± 1.82
$K_S^0 K^- \pi^+ \pi^+$	14046 ± 240	26873	44.1 ± 6.5	51.91 ± 0.91	5.98 ± 0.89
$K_S^0 K^+ \pi^+ \pi^-$	7171 ± 292	37456	24.7 ± 4.9	54.14 ± 1.21	6.29 ± 1.28
$\eta_{\gamma\gamma} \pi^+$	19323 ± 725	53701	63.5 ± 8.1	52.72 ± 0.62	6.17 ± 0.82
$\eta_{\pi^+ \pi^- \pi^0} \pi^+$	5508 ± 202	11225	20.2 ± 4.5	54.00 ± 1.13	6.73 ± 1.51
$\eta'_{\pi^+ \pi^- \eta_{\gamma\gamma}} \pi^+$	9242 ± 155	5002	33.0 ± 5.7	56.30 ± 0.54	6.27 ± 1.09
$\eta'_{\gamma\rho^0} \pi^+$	25191 ± 695	152363	75.1 ± 8.6	53.74 ± 0.72	5.49 ± 0.65
$\eta_{\gamma\gamma} \rho^+$	32835 ± 1537	166324	108.4 ± 10.5	60.70 ± 0.91	5.38 ± 0.58

Table II: Summary of $\varepsilon_{\mu\text{PID}}^{\text{data}}$, $\varepsilon_{\mu\text{PID}}^{\text{MC}}$, and $f_{\mu\text{PID}}^{\text{cor}}$ obtained from samples I and II.

Samples	$\varepsilon_{\mu\text{PID}}^{\text{data}}$ (%)	$\varepsilon_{\mu\text{PID}}^{\text{MC}}$ (%)	$f_{\mu\text{PID}}^{\text{cor}}$
I	76.64 ± 0.68	81.04 ± 0.21	0.946 ± 0.009
II	76.85 ± 0.30	81.66 ± 0.11	0.941 ± 0.004