Amplitude analysis and branching fraction measurement of $D_s^+ \to K^- K^+ \pi^+ \pi^0$

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We present the first amplitude analysis of the decay $D_s^+ \to K^-K^+\pi^+\pi^0$ using data samples of 6.32 fb^{-1} recorded with the BESIII detector between 4.178 and 4.226 GeV. More than 3000 events selected with a purity of 97.5% are used to perform the amplitude analysis, and nine components are found necessary to describe the data. Relative fractions and phases of the intermediate decays are determined. With the detection efficiency determined by the results of the amplitude analysis, we measure the branching fraction of $D_s^+ \to K^-K^+\pi^+\pi^0$ decay to be $(5.42 \pm 0.10_{\rm stat.} \pm 0.17_{\rm syst.})\%$.

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I. INTRODUCTION

Accurate measurements of D_s^+ decays are important for the study of other decay processes that are dominated by final states involving D_s^+ mesons, particularly for those of B_s^0 decays [1]. The decay $D_s^+ \to K^-K^+\pi^+\pi^0$ is a Cabibbo-favored decay (the inclusion of charge conjugate reactions is implied throughout this paper). Due to its large branching fraction (BF), it is usually selected as a "tag mode" for the measurement of other decays of the D_s^+ meson [2–7]. However, the BF of the $D_s^+ \to K^-K^+\pi^+\pi^0$ decay has a large systematic uncertainty due to the poor knowledge of intermediate state processes [8, 9]. An amplitude analysis of this decay is expected to provide a detailed understanding of its intermediate structures and significantly improve the experimental precision of its decay BF.

The four-body hadronic decays of D_s^+ mesons are dominated by two-body intermediate processes, for example $D_s^+ \to VV$ or $D_s^+ \to AP$ decays, where V, A, and P denote vector, axial-vector, and pseudoscalar mesons, respectively. Measurements of the BFs of these two-body decays are important to test theoretical calculations [10–13] and to better understand the decay mechanisms of the D_s^+ meson. In recent years, many measurements of $D_s^+ \to PP$ and $D_s^+ \to VP$ decays have been reported [14]. However, there are few studies focusing on $D_s^+ \to AP$ and $D_s^+ \to VV$ decays. The amplitude analysis of $D_s^+ \to AP$ decay allows the study of substructures involving $K_1(1270)$, $K_1(1400)$, and $f_1(1420)$ mesons. The measurements of the intermediate resonances $K_1(1270)$ and $K_1(1400)$ are also useful to understand the mixing of these two axial-vector kaons [15]. For $D_s^+ \to VV$, two processes, namely $D_s^+ \to \phi\rho^+$ and $D_s^+ \to \bar{K}^{*0}K^{*+}$, which are represented by the decay diagrams in Fig. 1, can be studied in the $D_s^+ \to K^-K^+\pi^+\pi^0$ decay. The BF of the decay $D_s^+ \to \phi\rho^+$ was measured to be $(8.4^{+1.9}_{-2.3})\%$ [16] by the CLEO experiment based on a data sample corresponding to an integrated luminosity of 689 pb^{-1} at the $\Upsilon(3S)$ and $\Upsilon(4S)$ resonances and at e^+e^- enter-of-mass energies ($E_{\rm cm}$) just below and above the $\Upsilon(4S)$ resonance. The previous most precise determination of the BF of $D_s^+ \to \bar{K}^{*0}K^{*+}$ decay, $(7.2 \pm 2.6)\%$ [17], was performed by the ARGUS experiment using a data sample of 432 pb^{-1} collected at $E_{\rm cm} = 10.4$ GeV. The goal of the present analysis is to improve the precision of these measurements.

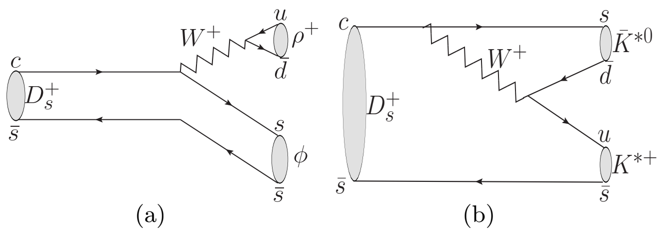

Figure 1. Decay diagrams of (a) $D_s^+ \to \phi\rho^+$ and (b) $D_s^+ \to \bar{K}^{*0}K^{*+}$ decays.

Moreover, a recent study [18] points out that the measured values of the ratio of $K_1(1270)$ decay ($R_{K_1(1270)} \equiv \frac{\mathcal{B}(K_1(1270)\to K^*\pi)}{\mathcal{B}(K_1(1270)\to K\rho)}$), which are listed in Table I, are inconsistent between different experiments [19–24]. They are expected to be identical under the narrow width approximation for the $K_1(1270)$ meson and assuming CP conservation in strong decays [18].

Table I. Values of $R_{K_1(1270)}$ determined by different experiments. Fit1 and Fit2 refer to amplitude analyses performed with the mass and width of the $K_1(1270)^+$ meson fixed or left free in the fit, respectively.

$R_{K_1(1270)}$	Process	Experiment
1.18 ± 0.43	$D^0 \to K^- K_1^+(1270)$	CLEO [19]
0.11 ± 0.06	$D^0 \to K^+ K_1^-(1270)$	CLEO [19]
0.19 ± 0.10	$D^0 \to K^-\pi^+\pi^+\pi^-$	BESIII [20]
0.24 ± 0.04	$D^0 \to K^-\pi^+\pi^+\pi^-$	LHCb [21]
0.45 ± 0.05	$B^+ \to J/\psi K^+\pi^+\pi^-$	Belle [22] (Fit 1)
0.30 ± 0.04	$B^+ \to J/\psi K^+\pi^+\pi^-$	Belle [22] (Fit 2)
0.38 ± 0.13	$K^-p \to K^-\pi^-\pi^+p$	ACCMOR [23]
0.45 ± 0.14	$D^0 \to K^- K_1^+(1270)$	CLEO [24]

In this paper, we present the first amplitude analysis of the decay $D_s^+ \to K^-K^+\pi^+\pi^0$ using a data sample of 6.32 fb^{-1} collected with the BESIII detector at center-of-mass energies between 4.178 GeV and 4.226 GeV. The amplitude model is constructed with the covariant tensor formalism [25] and described in Section IV. The BF measurement is presented in Section V.

II. DETECTOR AND DATA SETS

The BESIII detector is a magnetic spectrometer [26, 27] located at the Beijing Electron Position Collider (BEPCII) [28]. The cylindrical core of the BESIII detector consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI (Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive-plate counter muon identifier modules interleaved with steel. The acceptance of charged particles and photons is 93% over the 4π solid angle. The resolution of charged-particle momentum at 1 GeV/c is 0.5% while that of the specific ionization energy loss (dE/dx) is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution of the TOF barrel part is 68 ps. The end-cap TOF system was upgraded in 2015 with multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [29, 30].

The data samples used in this analysis contain a total of 6.32 fb^{-1} collected at center-of-mass energies between $E_{\rm cm} = 4.178$ and 4.226 GeV with the BESIII detector. The integrated luminosity of each data sample is shown in Table II. In this energy region, pairs of $D_s^{\pm}D_s^{*\mp}$ mesons are produced. The $D_s^{*\pm}$ meson predominantly decays to $D_s^{\pm}\gamma$ (93.5%), and only a small fraction decays to $D_s\pi^0$ (5.8%) [14]. A double-tag (DT) technique is employed to measure the absolute BF of the D_s^+ decays [31]. First, the D_s^- meson is fully reconstructed in one of the following decay modes: $D_s^- \to K_S^0K^-$, $D_s^- \to K^+K^-\pi^-$, $D_s^- \to K_S^0K^-\pi^0$, $D_s^- \to K_S^0K^-\pi^+\pi^-$, $D_s^- \to K_S^0K^+\pi^-\pi^-$, $D_s^- \to \pi^-\eta_{\gamma\gamma}$, $D_s^- \to \pi^-\eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$, and $D_s^- \to K^-\pi^+\pi^-$. These are referred to as single-tag (ST) events. Second, the $D_s^+ \to K^-K^+\pi^+\pi^0$ decay events are selected.

Generic Monte Carlo (MC) simulated event samples are produced with the GEANT4-based [32, 33] software at $E_{\rm cm} = 4.178 - 4.226$ GeV. The samples include all known open charm decays; the continuum process ($e^+e^- \to q\bar{q}$, $q = u$, d, and s); Bhabha scattering; the $\mu^+\mu^-$, $\tau^+\tau^-$, and diphoton processes; and the $c\bar{c}$ resonances (J/ψ, $\psi(3686)$, and $\psi(3770)$) via initial-state radiation (ISR). The open charm processes are generated using CONEXC [34], and their subsequent decays are modeled by EVTGEN [35] with known BFs from the Particle Data Group (PDG) [14]. The simulation of ISR production of J/ψ, $\psi(3686)$, and $\psi(3770)$ mesons is performed with KKMC [36]. The effects of final-state radiation (FSR) from charged tracks are simulated by PHOTOS [37]. The remaining unknown decays are generated with the LUNDCHARM model [38]. The generic MC sample is used to study backgrounds and determine the efficiencies of tag modes and the signal mode for the BF measurement, in which our amplitude analysis model is used to generate the signal mode events.

A phase-space (PHSP) MC sample is produced with the D_s^+ meson decaying to $K^-K^+\pi^+\pi^0$ generated with a uniform distribution and D_s^- meson decaying to the tag modes. Initially, the PHSP MC sample is used to calculate the normalization integral used in the determination of the amplitude model parameters in the fit to data.

Then we re-generate the signal MC sample with D_s^+ meson decaying to $K^-K^+\pi^+\pi^0$ using the amplitude model and D_s^- meson decaying to the tag modes. The normalization integral performed with signal MC samples results in more accurate fit parameters of magnitudes and phases and improves the computational efficiency of the MC integration. The signal MC sample is also used to calculate the goodness of the fit in this analysis. The PHSP MC sample is used to determine the efficiency mentioned in Section IV A.

III. EVENT SELECTION

Charged tracks except for those from K_S^0 decays are required to have a distance of closest approach to the interaction point (IP) within 1 cm in the transverse plane and within 10 cm along the MDC axis (z axis). The polar angle of the charged track with respect to the z axis θ is required to satisfy $|\cos\theta| < 0.93$. Kaons and pions are identified by combining the dE/dx information in the MDC and the time-of-flight from the TOF. If the probability of the kaon hypothesis is larger than that of the pion hypothesis, the track is identified as a kaon. Otherwise, the track is identified as a pion. Particle identification (PID) is not performed for the π^+ or π^- from K_S^0 decays.

The π^0 and η candidates are reconstructed via diphoton decays. The timing of the electromagnetic showers in the EMC is required to be within [0,700] ns of the trigger time, and the deposited energy must be greater than 25 (50) MeV in the barrel (endcap) region of the EMC. Good showers must satisfy $|\cos\theta| < 0.80$ ($0.86 < |\cos\theta < 0.92$) in the barrel (endcap) and be more than 20° from the nearest charged track. The unconstrained invariant masses of π^0, η and η' ($\eta' \to \pi^+\pi^-\eta_{\gamma\gamma}$) are required to be within [115, 150] MeV/c^2, [500, 570] MeV/c^2, and [946, 970] MeV/c^2, respectively. A kinematic fit is performed to constrain $M_{\gamma\gamma}$ to the known π^0 (η) mass, and the χ^2 of the corresponding fit is required to be less than 30 (20) for π^0 (η) candidates.

The K_S^0 candidates are reconstructed in the decay $K_S^0 \to \pi^+\pi^-$. Two oppositely charged tracks with distances of closest approach to the IP less than 20 cm along the z axis are assigned as $\pi^+\pi^-$ without further PID requirements. A constrained vertex fit of each pair of tracks is performed. We select K_S^0 candidates with a χ^2 of the vertex fit less than 100 and an invariant mass of the $\pi^+\pi^-$ pair ($M_{\pi^+\pi^-}$) obtained with the vertex fit in the range [487, 511] MeV/c^2. In the case of the decay modes $D_s^- \to K_S^0K^-\pi^0$, $D_s^- \to K_S^0K^-\pi^+\pi^-$ and $D_s^- \to K_S^0K^+\pi^+\pi^-$, the decay length of the K_S^0 candidates obtained with the secondary vertex fit [39] must be at least two times its fit uncertainty. For the $D_s^- \to K^-\pi^+\pi^-$ process, to remove possible mis-identified events of $D_s^- \to K_S^0K^-$, we require $M_{\pi^+\pi^-}$ to be outside of the range [487,511] MeV/c^2.

To identify the process $e^+e^- \to D_s^{*-}D_s^+$, we define the recoil mass $M_{\rm rec}$ of D_s^- candidates as

$$M_{\rm rec} = \sqrt{\left(E_{\rm cm} - \sqrt{|\vec{p}_{D_s^-}|^2 + m_{D_s^-}^2}\right)^2 - |\vec{p}_{D_s^-}|^2}\,, \quad (1)$$

where $m_{D_s^-}$ is the nominal D_s^- mass [14] and $\vec{p}_{D_s^-}$ is the momentum of the D_s^- candidate. The values of $M_{\rm rec}$ are required to be in the regions depending on the center-of-mass energy as listed in Table II. The D_s^- mass windows for the eight tag modes are shown in Table III.

Table II. The integrated luminosities ($\mathcal{L}_{\rm int}$) and the requirements on $M_{\rm rec}$ for various energies. $M_{\rm rec}$ is defined in Eq. 1.

$E_{\rm cm}$ (GeV)	$\mathcal{L}_{\rm int}$ (pb^{-1})	$M_{\rm rec}$ (GeV/c^2)
4.178	3189.0	[2.050, 2.180]
4.189	526.7	[2.048, 2.190]
4.199	526.0	[2.046, 2.200]
4.209	517.1	[2.044, 2.210]
4.219	514.6	[2.042, 2.220]
4.226	1091.7	[2.040, 2.220]

Table III. The D_s^- mass requirements for the eight tag modes.

Tag mode	Mass window (GeV/c^2)
$D_s^- \to K_S^0K^-$	[1.948, 1.991]
$D_s^- \to K^+K^-\pi^-$	[1.950, 1.986]
$D_s^- \to K_S^0K^-\pi^0$	[1.946, 1.987]
$D_s^- \to K_S^0K^-\pi^+\pi^-$	[1.958, 1.980]
$D_s^- \to K_S^0K^+\pi^-\pi^-$	[1.953, 1.983]
$D_s^- \to \pi^-\eta_{\gamma\gamma}$	[1.930, 2.000]
$D_s^- \to \pi^-\eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$	[1.940, 1.996]
$D_s^- \to K^-\pi^+\pi^-$	[1.953, 1.983]

D_s^+ meson decays with invariant masses $M_{D_s^+}$ in the region [1.87, 2.06] GeV/c^2 are selected. Good vertex fits of all charged tracks on both the signal and the tag side are required. A multi-constraint kinematic fit of $e^+e^- \to D_s^{*\pm}D_s^{\mp} \to \gamma D_s^{\pm}D_s^{\mp}$ with D_s^- decaying to one of the tag modes and D_s^+ decaying to the signal mode is performed. The set of constraints including four-momentum conservation in the e^+e^- system and the mass constraints of the π^0 meson, D_s^+ meson, D_s^- meson and $D_s^{*\pm}$ meson is labeled C_1. Based on the requirements of C_1, a set of constraints C_2 is defined by excluding the signal $M_{D_s^+}$ constraint, and C_3 by excluding the mass constraints of the $D_s^{\pm}$ meson on both the signal and tag side.

If there are multiple candidate combinations in an event, the candidate with the minimum χ^2 of the C_2 kinematic fit ($\chi^2_{C_2}$) is chosen. A good C_1 kinematic fit is required. To reduce the background while avoiding peaking background which is caused by constraining the mass of $D_s^{\pm}$ meson ($M_{D_s^{\pm}}$), we require the χ^2 of the C_3 kinematic fit ($\chi^2_{C_3}$) less than 250.

We reject classes of background events which are listed in Table IV. For backgrounds categorized as (a), (b) and (c), a π^0 from the D_s^- decay is wrongly associated to the D_s^+ meson on the opposite side. These are vetoed if the χ^2 of the C_1 kinematic fit ($\chi^2_{C_1}$) of the reconfigured combination is better than that of the original. For backgrounds categorized as (d), the events with $D^+ \to K^-\pi^+\pi^+$ decay versus $D^- \to K^+K^-\pi^-\pi^0$ decay are wrongly reconstructed as $D_s^- \to K^-\pi^+\pi^-$ decay versus $D_s^+ \to K^+K^-\pi^+\pi^0$ decay, when a π^- meson from D^- decay is exchanged with a π^+ meson from D^+ decay. If the reconstructed $D^\pm$ masses of the signal and the tag modes fall in the region within 0.055 GeV/c^2 of the nominal $M_{D^\pm}$, the events are rejected. For background categories (e) and (f), events with $K_S^0K^+K^-$ satisfying $|M_{K_S^0K^+K^-} - M_{D^0}^{\rm PDG}| < 0.045$ GeV/c^2 are rejected, where $M_{D^0}^{\rm PDG}$ is the nominal mass of D^0 [14]. For background category (g), the wrong signal combination survives due to exchanging the π^0 meson from D^0 decay and the π^- meson from $\bar{D}^0$ decay and misidentifying the π^- meson as a K^- meson. They are suppressed by rejecting events satisfying $|M_{K^-\pi^+\pi^0} - M_{D^0}^{\rm PDG}| < 0.055$ GeV/c^2 and $|M_{K^+\pi^-\pi^+\pi^-} - M_{D^0}^{\rm PDG}| < 0.055$ GeV/c^2. Background type (h) events are suppressed by applying a veto on events with $|M_{K^-\pi^+\pi^0} - M_{D^0}^{\rm PDG}| < 0.045$ GeV/c^2.

Table IV. Misreconstructed background processes.

Category	Background
(a)	$D_s^+ \to K^+K^-\pi^+,\ D_s^- \to \pi^-\pi^0\eta$
(b)	$D_s^+ \to K^+K^-\pi^+,\ D_s^- \to \pi^-\pi^0\eta'$
(c)	$D_s^+ \to K^+K^-\pi^+,\ D_s^- \to K^-\pi^-\pi^+\pi^0$
(d)	$D^+ \to K^-\pi^+\pi^+,\ D^- \to K^+K^-\pi^-\pi^0$
(e)	$\bar{D}^0 \to K_S^0K^+K^-,\ D^0 \to K^-\pi^+\pi^0$
(f)	$\bar{D}^0 \to K_S^0K^+K^-,\ D^0 \to K^-\pi^+\pi^0\pi^0$
(g)	$D^0 \to K^-\pi^+\pi^0,\ \bar{D}^0 \to K^+\pi^-\pi^+\pi^-$
(h)	$\bar{D}^0 \to K^+\pi^-\pi^0,\ D^0 \to K^-\pi^+\pi^0$

Events containing a possible mis-formed π^0 meson on the signal side are also rejected. Events in which the invariant mass of the higher-energy photon from the signal side combined with a photon from the $D_s^* \to D_s\gamma$ decay is within [0.12, 0.15] GeV/c^2 and with $|dM_{\rm recombined}| < |dM|$ are rejected, where dM is the mass difference between the signal D_s^+ meson and the tagged D_s^- meson, and $dM_{\rm recombined}$ is the corresponding mass difference with the signal π^0 replaced by the recombined π^0. We also check the recombined mass of the higher-energy photon from signal side and the photon from tag side and reject events with recombined masses within [0.12, 0.15] GeV/c^2.

After the full selection, the invariant mass spectra of the signal D_s^+ candidates for data samples collected at center-of-mass energies 4.178-4.226 GeV are shown in Fig. 2, together with fits to the mass spectra. The selected events have a high purity of about 97.5% in the signal region [1.935, 1.99] GeV/c^2. Studies of the generic MC samples show that peaking background is negligible. The background description by the generic MC has been verified by comparisons of data with the generic MC samples in the sideband regions [1.88, 1.92] GeV/c^2 and [2.00, 2.04] GeV/c^2. A good agreement is found, and the generic MC samples are used to model the residual background contamination in the signal region. The four-momenta of the final state particles after a two-constraint kinematic fit to the signal candidate, constraining the D_s^+ mass and π^0 mass to their known values [14], are used to perform the amplitude analysis.

IV. AMPLITUDE ANALYSIS

The amplitude analysis of $D_s^+ \to K^-K^+\pi^+\pi^0$ decay is performed by using an unbinned maximum likelihood fit. The likelihood function is constructed with the probability density function (PDF) described in the following, in which the momenta of the four daughter particles are used as inputs.

A. Likelihood Function Construction

The PDF used to construct the likelihood of the amplitude is given by

$$f_S(p_j) = \frac{\epsilon(p_j)|M(p_j)|^2 R_4(p_j)}{\int \epsilon(p_j)|M(p_j)|^2 R_4(p_j) dp_j}\,, \tag{2}$$

where p_j is the set of the final state particles' four momenta, and $\epsilon(p_j)$ is the detection efficiency parameterized in terms of the final state particles' four momenta. The PDF $f_S(p_j)$ is normalized by the integration. The standard element of the four-body PHSP [25] is defined as

$$R_4(p_j)dp_j = \delta^4\Big(p_{D_s^+} - \sum_{j=1}^{4} p_j\Big) \prod_{j=1}^{4} \frac{d^3p_j}{(2\pi)^3 2E_j}\,, \tag{3}$$

where j runs over the four daughter particles and E_j is the energy of particle j.

This analysis uses an isobar model formulation, where the signal decay amplitude, $M(p_j)$, is represented as a coherent sum of many two-body amplitude modes

$$M(p_j) = \sum_n c_n A_n(p_j)\,, \tag{4}$$

where c_n is written in the polar form as $\rho_n e^{i\phi_n}$ (ρ_n and ϕ_n are the magnitude and phase for the $n^{\rm th}$ amplitude, respectively). $A_n(p_j)$ is the $n^{\rm th}$ amplitude function modeled as

$$A_n(p_j) = P_n^1(m_1)P_n^2(m_2)S_n(p_j)X_n^1(p_j)X_n^2(p_j)X_n^{D_s^+}(p_j)\,, \tag{5}$$

where the indices 1 and 2 correspond to the two intermediate resonances, respectively. $X_n^{D_s^+}(p_j)$ is the

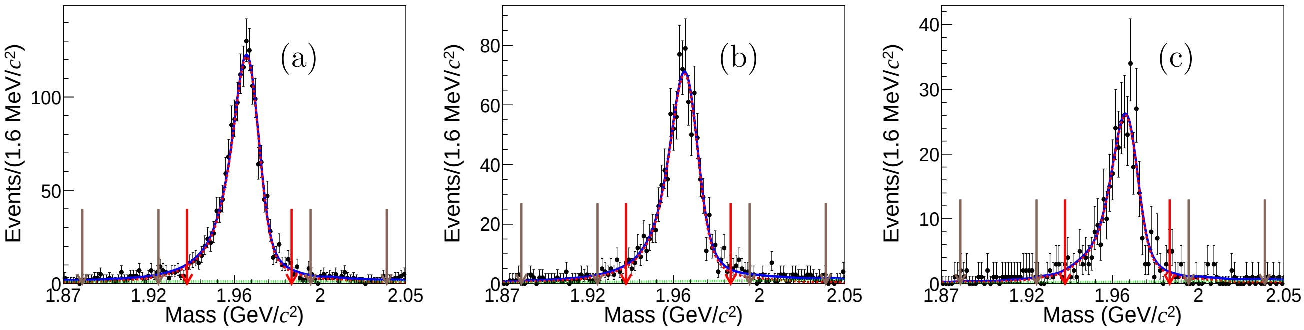

Figure 2. Fits to the invariant mass spectra of the signal D_s^+ candidates for data samples collected at center-of-mass energies (a) 4.178 GeV, (b) 4.189-4.219 GeV and (c) 4.226 GeV. The black dots with error bars represent data. The red dotted line represents the MC-simulated shape convolved with a Gaussian function. The green dashed lines are the fitted backgrounds. The blue solid line represents the total fitted shape. The red arrows represent the requirements applied in the amplitude analysis and the brown arrows represent the sideband region. We obtain 1708, 1024, and 356 events in the signal regions of $M_{D_s^+}$ at 4.178 GeV, 4.189-4.219 GeV and 4.226 GeV with purities ($w_{\rm sig}$) of 97.7%, 97.3% and 97.4%, respectively.

Blatt-Weisskopf barrier factor [40–42] for the D_s^+ meson, while $P_n^{1,2}(m_1, m_2)$ and $X_n^{1,2}(p_j)$ are the propagators and Blatt-Weisskopf barrier factors of the intermediate resonances 1 and 2, respectively. For non-resonant states, we set the propagator to unity, which can be regarded as a very broad resonance. $S_n(p_j)$ is the spin factor which is constructed with the covariant tensor formalism [25].

The 2.5% background contribution is described by the background PDF:

$$f_{\mathcal{B}}(p_j) = \frac{\mathcal{B}(p_j)R_4(p_j)}{\int \mathcal{B}(p_j)R_4(p_j)dp_j}\,, \tag{6}$$

where $\mathcal{B}(p_j)$ is the five-dimensional ($M_{K^-K^+}, M_{\pi^+\pi^0}, M_{K^-\pi^0}, M_{K^-\pi^+}, M_{K^-K^+\pi^0}$) background distribution with a shape determined from generic MC events in the signal region.

The combined PDF describing signal and background is

$$\begin{aligned} f_T(p_j) &= w_{\rm sig} f_S(p_j) + (1-w_{\rm sig}) f_{\mathcal{B}}(p_j) \\ &= w_{\rm sig}\frac{\epsilon(p_j)|M(p_j)|^2R_4(p_j)}{\int \epsilon(p_j)|M(p_j)|^2R_4(p_j)dp_j} \\ &\quad + (1-w_{\rm sig})\frac{\mathcal{B}(p_j)R_4(p_j)}{\int \mathcal{B}(p_j)R_4(p_j)dp_j}\,, \end{aligned} \tag{7}$$

where the factor $\epsilon(p_j)$ in the numerator can be taken out as in Eq. 8. In this way, the $\epsilon(p_j)$ term, which is independent of the fitted variables, is a constant and can be dropped in the likelihood fit. For the determination of $\epsilon(p_j)$, we generate 3×10^8 PHSP MC events at 4.178 GeV, 4.189-4.219 GeV and 4.226 GeV, and about $3\times(4\times10^6)$ events are selected with the event selection. The background shape is determined from the selected generic MC events, hence one has to divide the background function by the efficiency, $\mathcal{B}_\epsilon \equiv \mathcal{B}/\epsilon$. The value $\epsilon(p_j)$ is calculated as the fraction of selected PHSP MC events in the five-dimensional space ($M_{K^-K^+}, M_{\pi^+\pi^0}, M_{K^-\pi^0}, M_{K^-\pi^+}, M_{K^-K^+\pi^0}$) with $10\times10\times10\times10\times10$ bins.

The combined PDF becomes

$$\begin{aligned} f_T(p_j) = \epsilon(p_j)R_4(p_j)\Bigg[& w_{\rm sig}\frac{|M(p_j)|^2}{\int \epsilon(p_j)|M(p_j)|^2R_4(p_j)dp_j} \\ &+ (1-w_{\rm sig})\frac{\mathcal{B}_\epsilon(p_j)}{\int \epsilon(p_j)\mathcal{B}_\epsilon(p_j)R_4(p_j)dp_j}\Bigg]\,. \end{aligned} \tag{8}$$

The corresponding likelihood function is defined as

$$L_i = \prod_{k_i=1}^{N^i_{\rm data}} f_T^{k_i}(p_j)\,, \tag{9}$$

where i denotes the data sample, k_i runs over each event and $N^i_{\rm data}$ is the number of events in data sample i. The log-likelihood is used to perform the max-likelihood calculation.

Because we divide the whole data set taken at 4.178-4.226 GeV into three parts, 4.178 GeV, 4.189-4.219 GeV, and 4.226 GeV, the PDFs of the background and the efficiency are considered separately. Therefore, we sum the log-likelihood functions for the three samples

$$\ln\mathcal{L} = \sum_{i=1}^{N=3} \ln L_i\,. \tag{10}$$

The normalization integrals in the denominator of Eq. 8 are obtained by summing over an MC sample

$$\int \epsilon(p_j)|M(p_j)|^2R_4(p_j)dp_j \approx \frac{1}{N_{\rm MC}}\sum_{k=1}^{N_{\rm MC}}\frac{|M(p_j^k)|^2}{|M^{\rm gen}(p_j^k)|^2}\,, \tag{11}$$

$$\int \epsilon(p_j)\mathcal{B}_\epsilon(p_j)R_4(p_j)dp_j \approx \frac{1}{N_{\rm MC}}\sum_{k=1}^{N_{\rm MC}}\frac{\mathcal{B}_\epsilon(p_j^k)}{|M^{\rm gen}(p_j^k)|^2}\,, \tag{12}$$

where $N_{\rm MC}$ is the number of the selected MC events and $M^{\rm gen}(p_j)$ is the amplitude that is set with the parameters used to generate the signal MC sample, which are initially obtained from the results using the PHSP MC integration. $M^{\rm gen}(p_j)$ is a constant over the whole PHSP. Then with the results obtained from the fit to data, the signal MC sample is generated and used in MC integration. We generate equal MC samples at the three different center-of-mass energy points. For PHSP MC samples, a total of about twelve million events are generated. For the signal MC samples, about ten million signal MC events are generated in total.

We consider the effect from the PID, tracking and reconstruction efficiency differences between data and simulation by multiplying the weight of the MC event by a factor γ_ϵ, which is calculated as

$$\gamma_\epsilon(p_j) = \prod_j \frac{\epsilon_{j,\rm data}(p_j)}{\epsilon_{j,\rm MC}(p_j)}\,, \tag{13}$$

where $j = K^\mp$, $K^\pm$, $\pi^\pm$ and π^0. The signal MC integration becomes

$$\int \epsilon(p_j)|M(p_j)|^2 R_4(p_j)dp_j \approx \frac{1}{N_{\rm MC}} \sum_{k=1}^{N_{\rm MC}} \frac{|M(p_j^k)|^2\gamma_\epsilon(p_j^k)}{|M^{\rm gen}(p_j^k)|^2}\,. \tag{14}$$

We take the difference of efficiencies between data and MC as a systematic uncertainty for experimental effects.

1. Spin Factors

For a decay process of the form $a \to bc$, we use p_a, p_b, p_c to denote the momenta of the particles a, b, c, respectively. The spin projection operator $P^{(S)}_{\mu_1\ldots\mu_S\nu_1\ldots\nu_S}(a)$, for a resonance a with spin $S = 0, 1, 2$ and four-momentum p_a is given by

$$\begin{aligned} P^{(0)}(a) &= 1\,, \\ P^{(1)}_{\mu\mu'}(a) &= -g_{\mu\mu'} + \frac{p_{a,\mu}p_{a,\mu'}}{p_a^2}\,, \\ P^{(2)}_{\mu\nu\mu'\nu'}(a) &= \frac{1}{2}\Big(P^{(1)}_{\mu\mu'}(a)P^{(1)}_{\nu\nu'}(a) + P^{(1)}_{\mu\nu'}(a)P^{(1)}_{\nu\mu'}(a)\Big) \\ &\quad - \frac{1}{3}P^{(1)}_{\mu\nu}(a)P^{(1)}_{\mu'\nu'}(a)\,, \end{aligned} \tag{15}$$

where $g_{\mu\mu'}$ is the Minkowski metric.

The covariant tensors $\tilde{t}^{(L)}_{\mu_1\ldots\mu_l}(a)$ [25] for the final states of pure orbital angular momentum L are constructed from the relevant momenta p_a, p_b, p_c:

$$\tilde{t}^{(L)}_{\mu_1\ldots\mu_l}(a) = (-1)^L P^{(L)}_{\mu_1\ldots\mu^L\nu_1\ldots\nu_L}(a) r_a^{\nu_1}\cdots r_a^{\nu_L}\,, \tag{16}$$

where $r_a = p_b - p_c$. The corresponding covariant tensors with $L = 0, 1, 2$ are given as

$$\begin{aligned} \tilde{t}^{(0)}(a) &= 1\,, \\ \tilde{t}^{(1)}_\mu(a) &= -P^{(1)}_{\mu\mu'}(a)r_a^{\mu'}\,, \\ \tilde{t}^{(2)}_{\mu\nu}(a) &= P^{(2)}_{\mu\nu\mu'\nu'}(a)r_a^{\mu'}r_a^{\nu'}\,. \end{aligned} \tag{17}$$

The eleven types of decay modes used in this analysis are listed in Table V.

2. Blatt-Weisskopf Barrier Factors

The Blatt-Weisskopf barrier $X(p_j)$ [40–42] is a barrier function for a two-body decay process, $a \to bc$. The Blatt-Weisskopf barrier depends on the angular momenta and the momenta of the final state particles in the rest system of the parent particle. The definition is given by

$$\begin{aligned} X_{L=0}(q) &= 1, \\ X_{L=1}(q) &= z\sqrt{\frac{2}{z^2+1}}, \\ X_{L=2}(q) &= z^2\sqrt{\frac{13}{z^4+3z^2+9}}\,. \end{aligned} \tag{18}$$

where L denotes the angular momentum, $z = qR$, q is the magnitude of the momenta of the final state particles in the rest system of the parent particle, and R is the effective radius of the barrier. For a process $a \to bc$, we define $s_i = E_i^2 - p_i^2$, where i denotes a, b, c, and E_i, p_i are the particle's energy and momentum, such that

$$q^2 = \frac{(s_a + s_b - s_c)^2}{4s_a} - s_b\,, \tag{19}$$

while the values of R used in this analysis are 3.0 GeV^{-1} and 5.0 GeV^{-1} for intermediate resonances and the D_s^+ meson, respectively [43].

3. Propagators

We use the relativistic Breit-Wigner (RBW) function as the propagator for the resonances ϕ, $\bar{K}^{*0}$, $K^{*\pm}$, $\bar{K}_1^0(1400)$, $f_1(1510)$, $f_1(1420)$, and $\eta(1475)$, and fix their masses and widths to their PDG values [14], as listed in Table VI.

The RBW function is given by

$$P(m) = \frac{1}{(m_0^2 - m^2) - im_0\Gamma(m)}\,, \tag{20}$$

where $m = \sqrt{E^2 - p^2}$ and m_0 is the nominal mass of the resonance, and $\Gamma(m)$ is given by

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2L+1} \left(\frac{m_0}{m}\right) \left(\frac{X_L(q)}{X_L(q_0)}\right)^2\,, \tag{21}$$

where q_0 indicates the value of q when $s_a = m_0^2$.

Table V. Spin factor for each decay chain. All operators, i.e. $\tilde{t}$, have the same definitions as Ref. [25]. Scalar, pseudo-scalar, vector, and axial-vector states are denoted by S, P, V, and A, respectively. $[S]$, $[P]$, and $[D]$ indicate the orbital angular momenta $L=$ 0, 1, and 2 of the two-body final states, respectively.

Decay chain	$S(p)$
$D_s^+[S] \to V_1 V_2$	$\tilde{t}^{(1)\mu}(V_1)\ \tilde{t}^{(1)}_{\mu}(V_2)$
$D_s^+[P] \to V_1 V_2$	$\epsilon_{\mu\nu\lambda\sigma} p^{\mu}(D_s^+)\ \tilde{T}^{(1)\nu}(D_s^+)\ \tilde{t}^{(1)\lambda}(V_1)\ \tilde{t}^{(1)\sigma}(V_2)$
$D_s^+[D] \to V_1 V_2$	$\tilde{T}^{(2)\mu\nu}(D_s^+)\ \tilde{t}^{(1)}_{\mu}(V_1)\ \tilde{t}^{(1)}_{\nu}(V_2)$
$D_s^+ \to AP_1,\ A[S] \to VP_2$	$\tilde{T}^{(1)\mu}(D_s^+)\ P^{(1)}_{\mu\nu}(A)\ \tilde{t}^{(1)\nu}(V)$
$D_s^+ \to AP_1,\ A[D] \to VP_2$	$\tilde{T}^{(1)\mu}(D_s^+)\ \tilde{t}^{(2)}_{\mu\nu}(A)\ \tilde{t}^{(1)\nu}(V)$
$D_s^+ \to AP_1,\ A \to SP_2$	$\tilde{T}^{(1)\mu}(D_s^+)\ \tilde{t}^{(1)}_{\mu}(A)$
$D_s^+ \to VS$	$\tilde{T}^{(1)\mu}(D_s^+)\ \tilde{t}^{(1)}_{\mu}(V)$
$D_s^+ \to V_1P_1,\ V_1 \to V_2P_2$	$\epsilon_{\mu\nu\lambda\sigma}\ p^{\mu}_{V_1} r^{\nu}_{V_1}\ p^{\lambda}_{P_1}\ r^{\sigma}_{V_2}$
$D_s^+ \to PP_1,\ P \to VP_2$	$p^{\mu}(P_2)\ \tilde{t}^{(1)}_{\mu}(V)$
$D_s^+ \to PP_1,\ P \to SP_2$	1
$D_s^+ \to SS$	1

Table VI. The masses and widths of intermediate resonances used in this analysis.

Resonance	Mass (MeV/c^2)	Width (MeV)
ϕ	1019.461 ± 0.016	4.249 ± 0.013
ρ^+	775.11 ± 0.34	149.1 ± 0.8
$\bar{K}^{*0}$	895.55 ± 0.20	47.3 ± 0.5
$K^{*\pm}$	891.66 ± 0.26	50.8 ± 0.9
$\bar{K}_1^0(1270)$	1272 ± 7	87 ± 7
$\bar{K}_1^0(1400)$	1403 ± 7	174 ± 13
$f_1(1420)$	1426.3 ± 0.9	54.5 ± 2.6
$\eta(1475)$	1475 ± 4	90 ± 9
$a_0^0(980)$	990 ± 1 [47]	$g_{\eta\pi(K\bar{K})}$ (see text)

The $\bar{K}_1^0(1270)$ resonance is parameterized by a RBW but with a constant width $\Gamma(m) = \Gamma_0$, since this resonance is very close to the threshold of $K^-\rho^+$ and $\Gamma(m)$ varies very rapidly as m changes. Considering the obvious mass deviation, the mass and width of $\bar{K}_1^0(1270)$ are set to the average values (shown in Table VI) without including the results from Belle [44].

We parameterize the ρ^+ meson with the Gounaris-Sakurai lineshape (GS) [45], which is given by

$$P_{\rm GS}(m) = \frac{1 + d\frac{\Gamma_0}{m_0}}{(m_0^2 - m^2) + f(m) - i m_0 \Gamma(m)}\,. \quad (22)$$

The function $f(m)$ is given by

$$\begin{aligned} f(m) =& \Gamma_0 \frac{m_0^2}{q_0^3} \times \Bigg[q^2 (h(m) - h(m_0)) \\ & + (m_0^2 - m^2) q_0^2 \left. \frac{dh}{d(m^2)} \right|_{m_0^2} \Bigg]\,, \end{aligned} \quad (23)$$

where

$$h(m) = \frac{2q}{\pi m} \ln \left(\frac{m + 2q}{2 m_\pi} \right)\,, \quad (24)$$

$$\left. \frac{dh}{d(m^2)} \right|_{m_0^2} = h(m_0) \left[(8q_0^2)^{-1} - (2m_0^2)^{-1} \right] + (2\pi m_0^2)^{-1}\,, \quad (25)$$

and m_π is the charged pion mass.

The normalization condition at $P_{\rm GS}(0)$ fixes the parameter $d = f(0)/(\Gamma_0 m_0)$. It is found to be

$$d = \frac{3 m_\pi^2}{\pi q_0^2} \ln \left(\frac{m_0 + 2q_0}{2 m_\pi} \right) + \frac{m_0}{2\pi q_0} - \frac{m_\pi^2 m_0}{\pi q_0^3}\,. \quad (26)$$

The $a_0(980)$ meson lineshape is parameterized by the Flatté formula [46],

$$P_{a_0(980)} = \frac{1}{(m_0^2 - s_a) - i(g^2_{\eta\pi}\rho_{\eta\pi} + g^2_{K\bar{K}}\rho_{K\bar{K}})}\,, \quad (27)$$

where m_0 is the mass of $a_0(980)$ and $g^2_{\eta\pi(K\bar{K})}$ is the coupling constant. These parameters are fixed at the values given in Ref. [47], in which $m_0 = (0.990 \pm 0.001)\mathrm{GeV}/c^2$, $g^2_{\eta\pi} = (0.341 \pm 0.004)\mathrm{GeV}^2/c^4$ and $g^2_{K\bar{K}} = (0.892 \pm 0.022) g^2_{\eta\pi}$. The $\rho_{\eta\pi(K\bar{K})}$ is the PHSP factor and is given by $\rho_{\eta\pi(K\bar{K})} = 2q/\sqrt{s_a}$.

The $K\pi$ S-wave is modeled by a parameterization from scattering data [48], which is built from a Breit-Wigner shape for the $K_0^*(1430)$ resonance combined with an effective range parameterization for the non-resonant component with a phase shift given by

$$A(m) = F \sin\delta_F e^{i\delta_F} + R \sin\delta_R e^{i\delta_R} e^{i2\delta_F}\,, \quad (28)$$

with

$$\delta_F = \phi_F + \cot^{-1}\left[\frac{1}{aq} + \frac{rq}{2} \right]\,,$$

$$\delta_R = \phi_R + \tan^{-1}\left[\frac{M\Gamma(m_{K\pi})}{M^2 - m^2_{K\pi}} \right]\,,$$

where a and r are the scattering length and effective interaction length, respectively. The parameters $F(\phi_F)$ and $R(\phi_R)$ are the magnitude (phase) for the nonresonant state and resonance terms, respectively. The parameters M, F, ϕ_F, R, ϕ_R, a and r are fixed to the results of the $D^0 \to K_S^0\pi^+\pi^-$ analysis by the BABAR and Belle experiments [48] and are given in Table VII.

Table VII. The $K\pi$ S-wave parameters, obtained from a fit to the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot from the BABAR and Belle experiments [48]. The first uncertainties are statistical and the second systematic.

$M(\mathrm{GeV}/c^2)$	1.441 ± 0.002
$\Gamma(\mathrm{GeV})$	0.193 ± 0.004
F	0.96 ± 0.07
$\phi_F(\mathrm{deg})$	0.1 ± 0.3
R	1 (fixed)
$\phi_R(\mathrm{deg})$	-109.7 ± 2.6
a	0.113 ± 0.006
r	-33.8 ± 1.8

B. Fit Fractions and Statistical Uncertainty

The fit fractions of the individual components (amplitudes) are calculated according to the fit results. In the calculation, a large PHSP MC sample (twelve million events) with neither detector acceptance nor resolution included is used. The fit fraction (FF) for a component or an amplitude is defined as

$$\mathrm{FF}_n = \frac{\int |c_n A_n(p_j)|^2 R_4(p_j)dp_j}{\int |M(p_j)|^2 R_4(p_j)dp_j} \approx \frac{\sum_{k=1}^{N_{\mathrm{g,ph}}} |\tilde{A}_n(p_j^k)|^2}{\sum_{k=1}^{N_{\mathrm{g,ph}}} |M(p_j^k)|^2}, \tag{29}$$

where the integration is approximated by the PHSP MC summation at the generator level, $\tilde{A}_n(p_j^k)$ is either the n^{th} amplitude ($\tilde{A}_n(p_j^k) = c_n A_n(p_j^k)$) or the n^{th} component of a coherent sum of amplitudes ($\tilde{A}_n(p_j^k) = \sum c_{n_i} A_{n_i}(p_j^k)$), and $N_{\mathrm{g,ph}}$ is the number of PHSP MC events.

For the statistical uncertainty of FF, it is impractical to analytically propagate the uncertainties of the FFs from those of the magnitudes and phases. Instead, we randomly perturb the variables within their uncertainties obtained from the fit and calculate the FFs to determine the statistical uncertainties. We fit the distribution of each FF with a Gaussian function, and the width is reported as the uncertainty of the FF.

C. Results of the Amplitude Analysis

The amplitude of the $D_s^+[S] \to \phi\rho^+$ decay is expected to have the largest FF. Thus, this amplitude is chosen as the reference (its phase is fixed to 0, and the magnitude is fixed to 1). The notation $[S]$ denotes a relative orbital angular momentum $L = 0$ between daughters in a decay, and similarly for $[P]$ ($L = 1$), $[D]$ ($L = 2$). In addition, we fix some necessary physical relations which are shown in Appendix A.

We start the fit to the data with a baseline model including the amplitudes of $D_s^+ \to \phi\rho^+$, $D_s^+ \to \bar{K}^{*0}K^{*+}$, $D_s^+ \to \bar{K}_1^0(1270)K^+$ ($\bar{K}_1^0(1270) \to K^-\rho^+$ and $K^*\pi$) and $D_s^+ \to \bar{K}_1^0(1400)K^+$ ($\bar{K}_1^0(1400) \to K^*\pi$) decays, as the ϕ, ρ^+, $\bar{K}^{*0}$, K^{*+}, K^{*-}, $\bar{K}_1^0(1270)$, and $\bar{K}_1^0(1400)$ resonances are clearly observed in the corresponding invariant mass spectra. The statistical significances (SSs) of the above amplitudes, which are determined from the changes in log-likelihood and the numbers of degrees of freedom (NDOF) when the fits are performed with and without the amplitude included, are all much larger than 5σ.

Starting from the baseline model above, we add amplitudes of $f_1(1420)$, $f_1(1510)$, $\eta(1405)$, and $\eta(1475)$ resonances to try to improve the fit quality of the $K^-K^+\pi^0$ invariant mass spectrum. We keep amplitudes with significances larger than 5σ for the next iteration. The amplitudes of $D_s^+ \to f_1(1420)\pi^+$ ($f_1(1420) \to K^*K$) and $D_s^+ \to \eta(1475)\pi^+$ ($\eta(1475) \to a_0^0(980)\pi^0$) decays have significances larger than 5σ, and the amplitude of $D_s^+ \to f_1(1420)\pi^+$ ($f_1(1420) \to a_0^0(980)\pi^0$) decay improves the fit of the $K^-K^+\pi^0$ mass spectrum. We then try to add other amplitudes and find that $D_s^+ \to a_0^0(980)\rho^+$ decay has a significance of 6σ. Finally, we retain eighteen amplitudes in the nominal fit, which are categorized into nine processes (Table VIII). The amplitudes of the nominal fit are listed in Table IX. We have also tried other possible processes listed in Appendix B and find that their significances are all smaller than 3σ.

Table VIII. The nine components in the nominal amplitude model.

$D_s^+ \to \phi\rho^+$
$D_s^+ \to \bar{K}^{*0}K^{*+}$
$D_s^+ \to a_0^0(980)\rho^+$
$D_s^+ \to \bar{K}_1^0(1270)K^+(\bar{K}_1^0(1270) \to K^-\rho^+)$
$D_s^+ \to \bar{K}_1^0(1270)K^+(\bar{K}_1^0(1270) \to K^*\pi)$
$D_s^+ \to \bar{K}_1^0(1400)K^+(\bar{K}_1^0(1400) \to K^*\pi)$
$D_s^+ \to f_1(1420)\pi^+(f_1(1420) \to K^{*\mp}K^\pm)$
$D_s^+ \to f_1(1420)\pi^+(f_1(1420) \to a_0^0(980)\pi^0)$
$D_s^+ \to \eta(1475)\pi^+(\eta(1475) \to a_0^0(980)\pi^0)$

The fit results with phases, FFs and SSs for each amplitude are shown in Table IX. The ratio $\frac{\mathcal{B}(D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \to K^{*-}\pi^+)}{\mathcal{B}(D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \to K^-\rho^+)}$ is determined to be 0.17 ± 0.04 in this analysis, accounting for correlations.

The fit projections of three data samples on the invariant masses are shown in Fig. 3.

D. Goodness of Fit

To verify the five-dimensional fit, we determine the goodness of the fit. Since the D_s^+ meson and all four final particles have spin zero, the phase space of the decay $D_s^+ \to K^-K^+\pi^+\pi^0$ can be completely described by five linearly independent Lorentz invariant variables. The five invariant masses, $M_{K^-\pi^+}$, $M_{\pi^+\pi^0}$, $M_{K^-K^+}$, $M_{K^-\pi^+\pi^0}$ and $M_{K^-K^+\pi^0}$, are chosen as the five dimensions, which are divided into cells of equal size. When cells contain fewer than 10 events, adjacent cells are combined until the number of events in each cell is larger than 10. For each cell we calculate $\chi_p = \frac{N_p - N_p^{\rm exp}}{\sqrt{N_p^{\rm exp}}}$, and the goodness of the fit is given by $\chi^2 = \sum_{p=1}^{n} \chi_p^2$, where N_p and $N_p^{\rm exp}$ are the number of the observed events and the number determined by the fit results in the $p^{\rm th}$ cell, respectively, and n is the total number of cells. NDOF is given by $(n-1) - n_{\rm par}$, where $n_{\rm par}$ is the number of the free parameters in the fit. Overall, we find $\chi^2/{\rm NDOF} = 288.6/273$.

E. Systematic Uncertainties

Systematic uncertainties from the amplitude model, the background level, and the fit bias are considered. The systematic uncertainties of phases (ϕ) and FFs for different amplitudes are shown in Tables X and XI, respectively.

- *Amplitude model*

- (1) The uncertainties associated with the masses and widths of the intermediate resonances (ϕ, ρ^+, K^{*0}, $K^{*\pm}$, $K_1^0(1270)$, $K_1^0(1400)$, $f_1(1420)$, $\eta(1475)$) are estimated by varying the corresponding masses and widths listed in Table VI within 1σ.

- (2) For the lineshape of the ρ^+ meson, an alternative lineshape parameterization with RBW replacing GS is used.

- (3) The coupling constants and mass of $a_0(980)$ resonance are varied within the uncertainties given by Ref. [47].

- (4) We assume that the barrier effective radius (R) of the D_s^+ meson and other intermediate states have a uniform distribution. For the D_s^+ meson, the value of R is varied between 4.0 GeV^{-1} and 6.0 GeV^{-1}. For the intermediate states, R is varied between 2.0 GeV^{-1} and 4.0 GeV^{-1}.

- *Experimental effects*

- (5) These effects are related to the PID and tracking efficiency differences between data and MC, and are reflected in the factor γ_ϵ in Eq. 13. The PID efficiencies are studied using clean samples of $e^+e^- \to K^+K^-K^+K^-$, $K^+K^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-\pi^0$ decays, while a clean sample of $K^+K^-\pi^+\pi^-$ is used for the study of tracking efficiencies. These efficiencies are also used in the BF measurement (Section V C). The PID and tracking systematic uncertainties are taken as the efficiency differences between data and MC simulation. The uncertainties associated with γ_ϵ are obtained by performing alternative amplitude analyses varying PID and tracking efficiencies according to their uncertainties.

- *Background*

- (6) We vary the MC background yields within their uncertainties and take the largest difference from the fits as the uncertainty from the background level. In addition, we determine the background PDFs with another combination of five variables ($M_{K^-K^+}, M_{\pi^+\pi^0}, M_{K^+\pi^0}, M_{K^-\pi^+}, M_{K^-K^+\pi^0}$). The square root of the quadratic sum of these two uncertainties is taken as the background uncertainty.

- *Fit bias*

- (7) The uncertainty due to the fit procedure is evaluated by studying signal MC samples. An ensemble of 300 signal MC samples are generated according to the nominal result in this analysis. After applying the selection criteria, each of these samples has the same size as the data sample and is used to perform the same amplitude analysis. We define the pull of each parameter by $\frac{{\rm Out}(i)-{\rm In}(i)}{\sigma_{\rm stat.}(i)}$, where i denotes different parameters, ${\rm In}(i)$ denotes the input value as taken from the nominal fit to data, ${\rm Out}(i)$ is the value obtained from the fit to a signal MC sample and $\sigma_{\rm stat.}(i)$ is the corresponding statistical uncertainty. For each parameter, 300 pull values are obtained and the deviation of their average from zero is taken as the systematic uncertainty.

V. BRANCHING FRACTION MEASUREMENT

To determine the absolute BF of the decay $D_s^+ \to K^-K^+\pi^+\pi^0$, we reconstruct the eight decay ST modes (see Table III) and the DT events by fully reconstructing the tag channels and the signal channel.

The ST yields for each tag mode are given by

$$N_{\rm ST} = 2N_{D_s^+D_s^-}\mathcal{B}_{\rm tag}\varepsilon_{\rm ST}\,, \tag{30}$$

Table IX. Phase, FF, and SS for the different amplitudes, labeled as I, II..., XIV. Groups of related amplitudes are separated by horizontal lines. The last row of each group gives the total fit fraction of the above components with interferences considered. The amplitudes VIII, IX, X, and XII are constructed by two sub-amplitudes with fixed relations (see Appendix A). The ρ^+ resonance decays to $\pi^+\pi^0$. The ϕ and $a_0^0(980)$ resonances decay to K^-K^+. The $\bar{K}^{*0}$ resonance decays to $K^-\pi^+$, and the $K^{*\pm}$ resonance decays to $K^\pm\pi^0$. $K^*\pi$ indicates $\bar{K}^{*0}\pi^0$ and $K^{*-}\pi^+$. The uncertainties are statistical only.

Label	Amplitude	Phase (ϕ_n)	FF (%)	SS (σ)
I	$D_s^+[S] \to \phi\rho^+$	0.0 (fixed)	$42.64 \pm 1.30 \pm 0.77$	>20
II	$D_s^+[P] \to \phi\rho^+$	$1.64 \pm 0.05 \pm 0.02$	$8.58 \pm 0.69 \pm 0.37$	15.2
III	$D_s^+[D] \to \phi\rho^+$	$1.58 \pm 0.06 \pm 0.02$	$4.89 \pm 0.79 \pm 0.47$	8.4
	$D_s^+ \to \phi\rho^+$	$\cdots$	$56.17 \pm 1.05 \pm 1.24$	$\cdots$
IV	$D_s^+[S] \to \bar{K}^{*0}K^{*+}$	$1.13 \pm 0.06 \pm 0.03$	$15.49 \pm 0.81 \pm 0.36$	>20
V	$D_s^+[P] \to \bar{K}^{*0}K^{*+}$	$2.82 \pm 0.07 \pm 0.03$	$6.13 \pm 0.50 \pm 0.19$	16.2
VI	$D_s^+[D] \to \bar{K}^{*0}K^{*+}$	$1.76 \pm 0.07 \pm 0.03$	$4.00 \pm 0.47 \pm 0.34$	12.5
	$D_s^+ \to \bar{K}^{*0}K^{*+}$	$\cdots$	$22.44 \pm 0.81 \pm 0.32$	$\cdots$
VII	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270) \to K^-\rho^+$	$5.36 \pm 0.06 \pm 0.10$	$9.81 \pm 0.80 \pm 0.46$	>20
	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to \bar{K}^{*0}\pi^0$	$\cdots$	$0.69 \pm 0.13 \pm 0.12$	$\cdots$
	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^{*-}\pi^+$	$\cdots$	$1.27 \pm 0.27 \pm 0.25$	$\cdots$
VIII	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^*\pi$	$0.09 \pm 0.14 \pm 0.12$	$1.87 \pm 0.39 \pm 0.36$	7.2
	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to \bar{K}^{*0}\pi^0$	$\cdots$	$0.22 \pm 0.05 \pm 0.03$	$\cdots$
	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^{*-}\pi^+$	$\cdots$	$0.41 \pm 0.10 \pm 0.05$	$\cdots$
IX	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^*\pi$	$1.62 \pm 0.15 \pm 0.12$	$0.64 \pm 0.16 \pm 0.08$	5.5
	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270) \to K^*\pi$	$\cdots$	$2.57 \pm 0.42 \pm 0.42$	$\cdots$
	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to \bar{K}^{*0}\pi^0$	$\cdots$	$2.67 \pm 0.36 \pm 0.17$	$\cdots$
	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to K^{*-}\pi^+$	$\cdots$	$4.90 \pm 0.65 \pm 0.29$	$\cdots$
X	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to K^*\pi$	$5.66 \pm 0.08 \pm 0.05$	$7.23 \pm 0.95 \pm 0.41$	12.0
XI	$D_s^+ \to a_0^0(980)\rho^+$	$2.33 \pm 0.10 \pm 0.09$	$1.61 \pm 0.29 \pm 0.21$	6.0
	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*-}K^+$	$\cdots$	$0.87 \pm 0.17 \pm 0.07$	$\cdots$
	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*+}K^-$	$\cdots$	$0.87 \pm 0.17 \pm 0.07$	$\cdots$
XII	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*\mp}K^\pm$	$5.14 \pm 0.10 \pm 0.05$	$1.35 \pm 0.28 \pm 0.11$	6.5
XIII	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to a_0^0(980)\pi^0$	$5.77 \pm 0.14 \pm 0.07$	$0.65 \pm 0.24 \pm 0.12$	3.6
XIV	$D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to a_0^0(980)\pi^0$	$0.98 \pm 0.08 \pm 0.06$	$3.28 \pm 0.38 \pm 0.25$	9.7

and the DT yields are given by

$$N_{\mathrm{DT}} = 2N_{D_s^+D_s^-}\mathcal{B}_{\mathrm{tag}}\mathcal{B}_{\mathrm{sig}}\varepsilon_{\mathrm{DT}}\,, \tag{31}$$

where $N_{D_s^+D_s^-}$ is the total number of $D_s^+D_s^-$ pairs produced, $\mathcal{B}_{\mathrm{tag(sig)}}$ is the BF of the tag (signal) side, and $\varepsilon_{\mathrm{DT(ST)}}$ is the DT (ST) efficiency.

The BF of the signal side is determined by

$$\mathcal{B}_{\mathrm{sig}} = \frac{N_{\mathrm{DT}}}{\mathcal{B}_{\pi^0\to\gamma\gamma}\sum_i N^i_{\mathrm{ST}}\varepsilon^i_{\mathrm{DT}}/\varepsilon^i_{\mathrm{ST}}}\,, \tag{32}$$

where the N_{DT} and N^i_{ST} yields are obtained from the data sample, while $\varepsilon^i_{\mathrm{DT}}$ and $\varepsilon^i_{\mathrm{ST}}$ are obtained from the generic MC samples, where i indicates the tag mode. In particular, $\varepsilon^i_{\mathrm{DT}}$ is determined by the amplitude analysis model used in the generic MC samples.

We determine the signal BF $\mathcal{B}_{\mathrm{sig}}$ by

$$\mathcal{B}_{\mathrm{sig}} = \frac{\sum_n N_{n\mathrm{DT}}}{\mathcal{B}_{\pi^0\to\gamma\gamma}\sum_n\sum_i N^i_{n\mathrm{ST}}\varepsilon^i_{n\mathrm{DT}}/\varepsilon^i_{n\mathrm{ST}}}\,, \tag{33}$$

where i denotes the tag mode and n indicates the data sample at 4.178 GeV, 4.189-4.219 GeV or 4.226 GeV. For the numerator, $\sum_n N_{n\mathrm{DT}}$, we fit the combined data sample to obtain the total DT data yield.

A. Event Selection

For the BF measurement, it is necessary to guarantee that the DT sample is a strict subset of the ST sample. Therefore, we select the ST events ahead of selecting the DT candidates. For this measurement, the event selection criteria are relaxed or modified in order to increase the signal yield. Here, the background level does not play as crucial a role as in the amplitude analysis.

In order to reject the soft pions from D^* decays, all the π mesons are required to satisfy $P_\pi > 100$ MeV/c, and the χ^2 of the kinematic fit for the $\pi^0 \to \gamma\gamma$ decay must be less than 20. The new criteria for selecting K_S^0 are $487 < M_{\pi^+\pi^-} < 511$ (MeV/c^2) and that the vertex

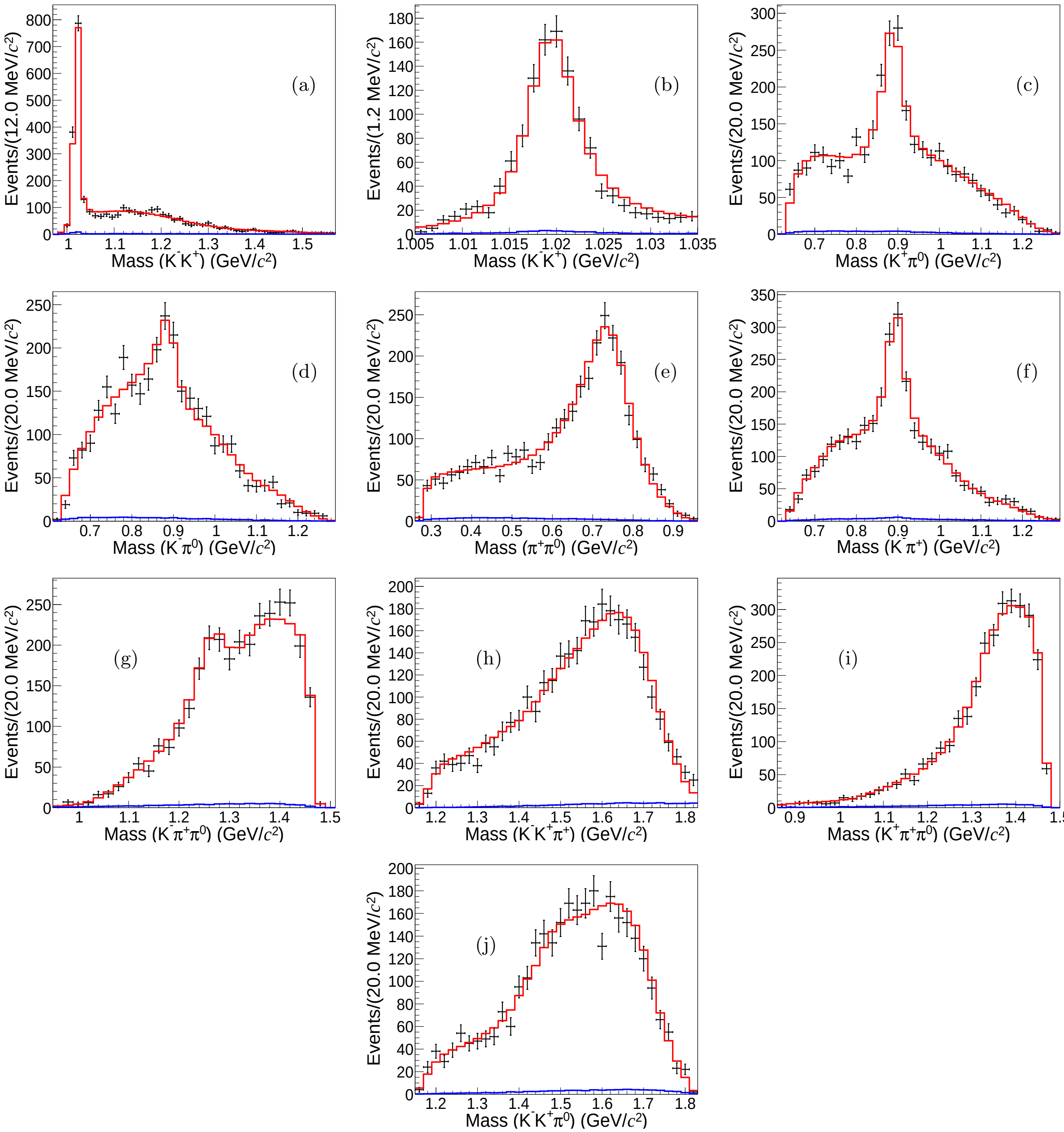

Figure 3. Invariant mass distributions of the total data sample: data (points with error bars - black), the fit histograms (red) and the backgrounds (blue). Plot (b) shows the ϕ mass region with an expanded scale.

fit χ^2 must be less than 100.

For the ST selection, if there are multiple candidates for a tag mode, we retain the one with $M_{\rm rec}$ closest to the nominal $M_{D_s^{*\pm}}$ [14]. The $M_{\rm rec}$ windows are given in Table II. If the D_s^+ meson and D_s^- meson can be simultaneously reconstructed as ST in an event, both of them are accepted. After the ST selection, if multiple signal candidates are obtained, the one with average mass $\bar{M}$ $(=(M_{D_s^+}+M_{D_s^-})/2)$ closest to the nominal $M_{D_s^\pm}$ is chosen. $\bar{M}_{D_s^\pm}$ of every candidate must lie in the interval [1.87, 2.06] GeV/c^2, and events with both $M_{\rm rec}$ for the D_s^+ meson and $M_{\rm rec}$ for the D_s^- meson smaller than 2.1 GeV are rejected.

Table X. The phase systematic uncertainty sources (in units of statistical standard deviations) are (1) mass and width, (2) shape of the ρ^+ meson, (3) parameters of the $a_0^0(980)$ meson, (4) R value, (5) experimental effects, (6) background, and (7) fit bias.

Phase (ϕ)	1	2	3	4	5	6	7	Total
$D_s^+[S] \to \phi\rho^+$	0 (fixed)							
$D_s^+[P] \to \phi\rho^+$	0.12	0.02	0.01	0.25	0.00	0.08	0.06	0.30
$D_s^+[D] \to \phi\rho^+$	0.05	0.10	0.00	0.21	0.11	0.02	0.10	0.28
$D_s^+[S] \to \bar{K}^{*0}K^{*+}$	0.41	0.02	0.01	0.37	0.03	0.15	0.06	0.57
$D_s^+[P] \to \bar{K}^{*0}K^{*+}$	0.31	0.05	0.01	0.20	0.03	0.25	0.05	0.45
$D_s^+[D] \to \bar{K}^{*0}K^{*+}$	0.24	0.01	0.01	0.37	0.02	0.09	0.05	0.45
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270) \to K^-\rho^+$	1.56	0.06	0.01	0.75	0.00	0.08	0.09	1.74
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^*\pi$	0.76	0.03	0.01	0.29	0.01	0.07	0.11	0.83
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^*\pi$	0.77	0.02	0.01	0.20	0.02	0.13	0.07	0.81
$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to K^*\pi$	0.57	0.12	0.01	0.19	0.04	0.04	0.08	0.62
$D_s^+ \to a_0^0(980)\rho^+$	0.50	0.04	0.06	0.68	0.04	0.04	0.06	0.86
$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*\mp}K^\pm$	0.26	0.03	0.00	0.38	0.03	0.10	0.05	0.48
$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to a_0^0(980)\pi^0$	0.26	0.01	0.04	0.20	0.10	0.28	0.19	0.48
$D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to a_0^0(980)\pi^0$	0.65	0.02	0.09	0.32	0.02	0.15	0.06	0.75

B. Data Yields, Efficiencies and BFs

The ST yields are determined from fits to the $M_{D_s^-}$ distributions of data, as shown in Fig. 4. In the fits, we use an MC-simulated shape convolved with a Gaussian function to describe the signal shape of $M_{D_s^-}$ and a 2$^{\rm nd}$-order polynomial function to describe the combinatorial background. For the tag mode $D_s^- \to K_S^0K^-$, there is some peaking background coming from $D^- \to K_S^0\pi^-$. We take the shape of this background from the generic MC samples and add it to the fit leaving its yield floating. For the tag mode $D_s^- \to \pi^-\eta'$, there is peaking background coming from $D_s^- \to \eta\pi^+\pi^-\pi^-$. We take the shape and the yield of this background from the generic MC samples and add it to the fit. The DT yields are obtained from an unbinned fit to the signal D_s^+ mass spectrum of the combined data sample, which is shown in Fig. 5. We determine the number of $D_s^+ \to K^-K^+\pi^+\pi^0$ decays to be $\sum_n N_{n\rm DT} = 4365 \pm 83$. Tables XII-XIV summarize the ST efficiencies, DT efficiencies, and ST yields in data samples at 4.178-4.226 GeV.

Inserting the values of the ST and DT data yields and the ST and DT efficiencies into Eq. 33, we determine the BF of the $D_s^+ \to K^-K^+\pi^+\pi^0$ decay to be

$$\mathcal{B}_{\rm sig} = (5.42 \pm 0.10_{\rm stat.})\% \,. \quad (34)$$

C. Systematic Uncertainties in the BF

The sources of the systematic uncertainties in the BF measurement are considered as follows.

- *$K^\pm$ meson and $\pi^\pm$ meson tracking/PID efficiencies*
  The ratios between data and MC efficiencies are weighted by the corresponding momentum spectra of signal MC events. The systematic uncertainty from tracking efficiency of each charged particle is estimated to be 0.5%, and that from PID efficiency is also 0.5%. The tracking efficiency systematic uncertainties are added linearly for the three charged tracks, as are the PID efficiency systematic uncertainties.

- *π^0 meson reconstruction efficiency*
  We assign 2.0% as the systematic uncertainty in the π^0 reconstruction according to the studies in Ref. [49].

- *The numbers of ST D_s^- candidates*
  The BF measurement is not sensitive to systematic uncertainties coming from modifying the polynomial function order, the fit ranges or the bin sizes. An uncertainty of 0.56% was estimated from alternative fits with different signal shapes. According to Tables XII-XIV, the total ST yield of the eight tag modes is 441684 ± 1766, corresponding to the relative statistical uncertainty of 0.40%. The sum of these terms in quadrature is 0.69%.

- *MC statistics*
  The uncertainties of the ST and DT efficiencies are considered, but the DT uncertainties dominate. The uncertainty of the MC statistics is given by $\sqrt{\sum_i f_i(\frac{\delta\epsilon_i}{\epsilon_i})^2}$, where f_i is the tag yield fraction and ϵ_i is the average DT efficiency of tag mode i. We obtain an uncertainty of 0.34% for this term.

- *The shape of the signal D_s^+ mass*
  The systematic uncertainty due to the shape of the

Table XI. The FF systematic uncertainty sources (in units of statistical standard deviations) are (1) mass and width, (2) shape of ρ^+ meson, (3) parameters of $a_0^0(980)$ meson, (4) R value, (5) experimental effects, (6) background, and (7) fit bias. The last row is the systematic uncertainty of the ratio $\frac{\mathcal{B}(D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^{*-}\pi^+)}{\mathcal{B}(D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^-\rho^+)}$.

FF	1	2	3	4	5	6	7	Total
$D_s^+[S]\to\phi\rho^+$	0.28	0.03	0.07	0.38	0.33	0.07	0.08	0.59
$D_s^+[P]\to\phi\rho^+$	0.38	0.08	0.13	0.22	0.11	0.23	0.09	0.54
$D_s^+[D]\to\phi\rho^+$	0.08	0.15	0.01	0.56	0.09	0.05	0.06	0.60
$D_s^+\to\phi\rho^+$	0.33	0.06	0.01	0.99	0.50	0.16	0.08	1.18
$D_s^+[S]\to\bar{K}^{*0}K^{*+}$	0.28	0.14	0.08	0.16	0.11	0.18	0.17	0.45
$D_s^+[P]\to\bar{K}^{*0}K^{*+}$	0.04	0.00	0.00	0.34	0.14	0.02	0.07	0.37
$D_s^+[D]\to\bar{K}^{*0}K^{*+}$	0.16	0.06	0.02	0.60	0.16	0.31	0.06	0.72
$D_s^+\to\bar{K}^{*0}K^{*+}$	0.18	0.08	0.04	0.12	0.26	0.14	0.10	0.39
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^-\rho^+$	0.22	0.19	0.05	0.46	0.16	0.04	0.06	0.57
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[S]\to\bar{K}^{*0}\pi^0$	0.79	0.08	0.01	0.46	0.12	0.06	0.08	0.93
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[S]\to K^{*-}\pi^+$	0.78	0.08	0.01	0.46	0.12	0.06	0.08	0.93
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[S]\to K^*\pi$	0.79	0.08	0.01	0.47	0.12	0.06	0.08	0.93
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[D]\to\bar{K}^{*0}\pi^0$	0.11	0.15	0.01	0.46	0.04	0.01	0.09	0.51
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[D]\to K^{*-}\pi^+$	0.10	0.15	0.01	0.46	0.04	0.01	0.09	0.50
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)[D]\to K^*\pi$	0.09	0.15	0.01	0.46	0.04	0.01	0.09	0.50
$D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^*\pi$	0.76	0.01	0.01	0.64	0.13	0.07	0.14	1.01
$D_s^+\to\bar{K}_1^0(1400)K^+,\ \bar{K}_1^0(1400)[S]\to\bar{K}^{*0}\pi^0$	0.25	0.07	0.01	0.27	0.09	0.25	0.07	0.47
$D_s^+\to\bar{K}_1^0(1400)K^+,\ \bar{K}_1^0(1400)[S]\to K^{*-}\pi^+$	0.23	0.07	0.01	0.26	0.09	0.25	0.07	0.45
$D_s^+\to\bar{K}_1^0(1400)K^+,\ \bar{K}_1^0(1400)[S]\to K^*\pi$	0.23	0.07	0.01	0.24	0.09	0.25	0.07	0.43
$D_s^+\to a_0^0(980)\rho^+$	0.23	0.04	0.01	0.65	0.08	0.12	0.08	0.71
$D_s^+\to f_1(1420)\pi^+,\ f_1(1420)\to K^{*-}K^+$	0.27	0.08	0.07	0.28	0.04	0.10	0.08	0.42
$D_s^+\to f_1(1420)\pi^+,\ f_1(1420)\to K^{*+}K^-$	0.25	0.08	0.07	0.28	0.04	0.10	0.08	0.41
$D_s^+\to f_1(1420)\pi^+,\ f_1(1420)\to K^{*\mp}K^\pm$	0.24	0.08	0.07	0.24	0.04	0.10	0.08	0.38
$D_s^+\to f_1(1420)\pi^+,\ f_1(1420)\to a_0^0(980)\pi^0$	0.27	0.08	0.10	0.19	0.19	0.14	0.27	0.50
$D_s^+\to\eta(1475)\pi^+,\ \eta(1475)\to a_0^0(980)\pi^0$	0.49	0.18	0.11	0.16	0.21	0.20	0.16	0.65
$\frac{\mathcal{B}(D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^{*-}\pi^+)}{\mathcal{B}(D_s^+\to\bar{K}_1^0(1270)K^+,\ \bar{K}_1^0(1270)\to K^-\rho^+)}$	0.68	0.03	0.02	0.44	0.09	0.04	0.12	0.82

Table XII. The efficiencies and ST yields at $E_{\rm cm}$ = 4.178 GeV.

Tag mode	Mass window (GeV/c^2)	$N_{\rm ST}$	$\varepsilon_{\rm ST}(\%)$	$\varepsilon_{\rm DT}(\%)$
$D_s^-\to K_S^0K^-$	[1.948, 1.991]	31668 ± 315	46.95 ± 0.07	8.75 ± 0.09
$D_s^-\to K^+K^-\pi^-$	[1.950, 1.986]	135867 ± 610	39.00 ± 0.03	7.09 ± 0.03
$D_s^-\to K_S^0K^-\pi^0$	[1.946, 1.987]	11284 ± 512	15.32 ± 0.11	2.92 ± 0.05
$D_s^-\to K_S^0K^-\pi^+\pi^-$	[1.958, 1.980]	8032 ± 273	20.29 ± 0.12	3.36 ± 0.07
$D_s^-\to K_S^0K^+\pi^-\pi^-$	[1.953, 1.983]	15645 ± 289	21.70 ± 0.06	3.76 ± 0.05
$D_s^-\to\pi^-\eta_{\gamma\gamma}$	[1.930, 2.000]	18071 ± 560	43.07 ± 0.15	7.92 ± 0.10
$D_s^-\to\pi^-\eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$	[1.940, 1.996]	7629 ± 147	18.72 ± 0.06	3.19 ± 0.06
$D_s^-\to K^-\pi^+\pi^-$	[1.953, 1.983]	16942 ± 548	45.80 ± 0.22	8.39 ± 0.10

signal is studied by fitting without the convolved Gaussian function. The difference of the DT yield is taken as the systematic uncertainty and is determined to be 0.5%.

- *Background shape of the signal D_s^+ meson*
  For the background shape of the signal D_s^+, the MC-simulated shape is used to replace the nominal one, and an uncertainty of 0.75% is obtained.

- *Bias of the measurement method*
  Ten updated inclusive generic MC samples are used as fake data to estimate the possible fit bias. The BF for each sample is determined, and the relative difference between the average of BFs and the MC truth value is 0.16%, which is considered negligible.

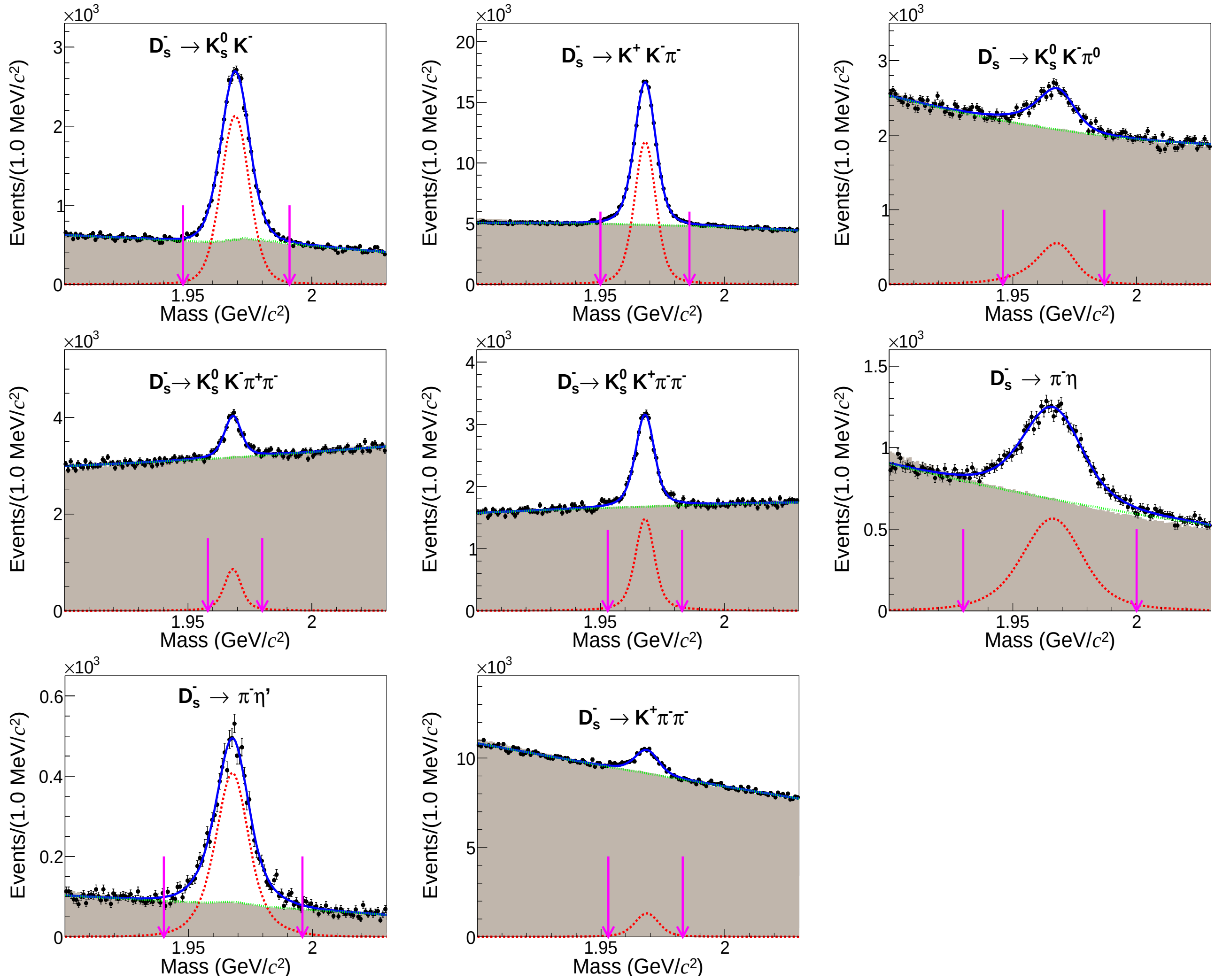

Figure 4. Fits to the $M_{D_s^-}$ distributions of ST candidates selected from the 4.178 GeV data sample, where the dots with error bars are data, the solid blue curve shows the best fit, the red dotted curve shows the signal shape, the green dashed line shows the shape of the combinatorial backgrounds, the brown area shows the background estimated by the generic MC samples, and the pairs of pink arrows are the mass windows. In the plots for $D_s^- \to K_S^0 K^-$ and $D_s^- \to \pi^- \eta'$ decays, the green dashed lines include contributions from $D^- \to K_S^0 \pi^-$ and $D_s^- \to \eta \pi^+ \pi^- \pi^-$ backgrounds, respectively.

- *MC model*
  To determine the uncertainty from the amplitude model, we randomly perturb the parameters (magnitudes and phases) of the amplitude model 400 times within their statistical uncertainties according to the covariant matrix of the nominal fit to obtain the DT efficiency distribution. Then we fit the DT efficiency distribution with a Gaussian function for which the relative width σ/μ is 0.4%.

The systematic uncertainties in the BF are summarized in Table XV. The total systematic uncertainty is obtained by adding them in quadrature. Finally, we obtain the BF of the $D_s^+ \to K^- K^+ \pi^+ \pi^0$ decay to be

$$\mathcal{B}_{\rm sig} = (5.42 \pm 0.10_{\rm stat.} \pm 0.17_{\rm syst.})\%. \tag{35}$$

VI. CONCLUSION

This paper presents the first amplitude analysis of the decay $D_s^+ \to K^- K^+ \pi^+ \pi^0$. We obtain the BF $\mathcal{B}(D_s^+ \to K^- K^+ \pi^+ \pi^0)$ to be $(5.42 \pm 0.10_{\rm stat.} \pm 0.17_{\rm syst.})\%$. Using the FFs listed in Table IX and Table XI, the BFs for the intermediate processes are calculated and listed in Table XVI. The $D_s^+ \to \phi \rho^+$ and $D_s^+ \to \bar{K}^{*0} K^{*+}$ decays are found to be dominant, and the decays involving $K_1(1270), K_1(1400), \eta(1475), f_1(1420)$, and $a_0^0(980)$ mesons are also observed with significances larger than 5σ. Compared to the PDG [14] values of $\mathcal{B}(D_s^+ \to K^- K^+ \pi^+ \pi^0) = (6.3 \pm 0.6)\%$, $\mathcal{B}(D_s^+ \to \phi \rho^+) = (8.4^{+1.9}_{-2.3})\%$, and $\mathcal{B}(D_s^+ \to \bar{K}^{*0} K^{*+}) = (7.2 \pm 2.6)\%$, which were previously measured by the CLEO and ARGUS experiments, respectively, the BFs $((5.42 \pm 0.10_{\rm stat.} \pm$

Table XIII. The efficiencies and ST yields at $E_{\rm cm}$ = 4.189-4.219 GeV.

Tag mode	Mass window (GeV/c^2)	$N_{\rm ST}$	$\varepsilon_{\rm ST}(\%)$	$\varepsilon_{\rm DT}(\%)$
$D_s^- \to K_S^0 K^-$	[1.948, 1.991]	18304 ± 260	46.87 ± 0.09	9.08 ± 0.11
$D_s^- \to K^+ K^- \pi^-$	[1.950, 1.986]	80417 ± 508	38.82 ± 0.04	7.28 ± 0.04
$D_s^- \to K_S^0 K^- \pi^0$	[1.946, 1.987]	6730 ± 462	14.88 ± 0.15	3.11 ± 0.07
$D_s^- \to K_S^0 K^- \pi^+ \pi^-$	[1.958, 1.980]	5252 ± 285	20.07 ± 0.16	3.32 ± 0.08
$D_s^- \to K_S^0 K^+ \pi^- \pi^-$	[1.953, 1.983]	8923 ± 230	21.53 ± 0.08	3.86 ± 0.07
$D_s^- \to \pi^- \eta_{\gamma\gamma}$	[1.930, 2.000]	10034 ± 355	42.37 ± 0.21	8.15 ± 0.13
$D_s^- \to \pi^- \eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$	[1.940, 1.996]	4382 ± 112	18.66 ± 0.07	3.45 ± 0.09
$D_s^- \to K^- \pi^+ \pi^-$	[1.953, 1.983]	10051 ± 529	45.38 ± 0.30	8.41 ± 0.13

Table XIV. The efficiencies and ST yields at $E_{\rm cm}$ = 4.226 GeV.

Tag mode	Mass window (GeV/c^2)	$N_{\rm ST}$	$\varepsilon_{\rm ST}(\%)$	$\varepsilon_{\rm DT}(\%)$
$D_s^- \to K_S^0 K^-$	[1.948, 1.991]	6550 ± 159	46.42 ± 0.18	8.81 ± 0.18
$D_s^- \to K^+ K^- \pi^-$	[1.950, 1.986]	28290 ± 328	38.27 ± 0.07	7.30 ± 0.07
$D_s^- \to K_S^0 K^- \pi^0$	[1.946, 1.987]	2145 ± 219	15.22 ± 0.28	2.97 ± 0.11
$D_s^- \to K_S^0 K^- \pi^+ \pi^-$	[1.958, 1.980]	1708 ± 217	19.45 ± 0.30	3.38 ± 0.14
$D_s^- \to K_S^0 K^+ \pi^- \pi^-$	[1.953, 1.983]	3242 ± 170	21.31 ± 0.15	3.90 ± 0.12
$D_s^- \to \pi^- \eta_{\gamma\gamma}$	[1.930, 2.000]	3699 ± 244	41.94 ± 0.40	8.12 ± 0.22
$D_s^- \to \pi^- \eta'_{\pi^+\pi^-\eta_{\gamma\gamma}}$	[1.940, 1.996]	1646 ± 75	18.45 ± 0.13	3.37 ± 0.14
$D_s^- \to K^- \pi^+ \pi^-$	[1.953, 1.983]	4915 ± 423	44.75 ± 0.57	8.41 ± 0.22

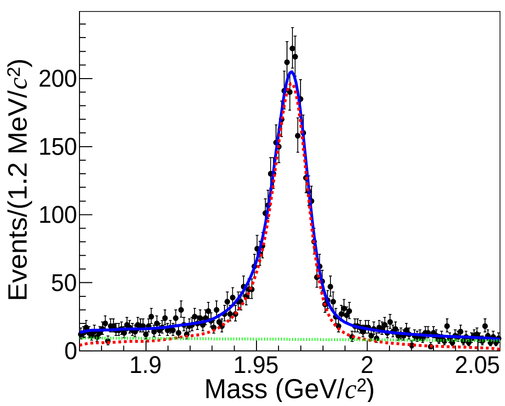

Figure 5. Invariant mass distribution of the DT $D_s^+ \to K^- K^+ \pi^+ \pi^0$ events. The black dots with error bars are data. The red dashed line represents the MC-simulated shape convolved with a Gaussian function. The green dashed line represents the MC background shape, which is fitted by a 1$^{\rm st}$-order Chebychev polynomial. The blue solid line represents the total fitted shape.

$0.17_{\rm syst.}$)%, ($6.22 \pm 0.17_{\rm stat.} \pm 0.24_{\rm syst.}$)% and ($5.46 \pm 0.23_{\rm stat.} \pm 0.18_{\rm syst.}$)%) obtained in this work have a much better precision. The measurement of $\mathcal{B}(D_s^+ \to \phi\rho^+)$ is consistent with the theory prediction [12] ($\mathcal{B}(D_s^+ \to \phi\rho^+) = 5.70\%$). The ratio $R_{K_1(1270)} \equiv \frac{\mathcal{B}(K_1^0(1270)\to K^*\pi)}{\mathcal{B}(K_1^0(1270)\to K\rho)}$ mentioned in Table I is determined to be $0.51 \pm 0.12_{\rm stat.} \pm 0.09_{\rm syst.}$ in this analysis. Our result is consistent with the results using CLEO data [24] and Belle data (Fit 1) [22] within uncertainties.

Table XV. The systematic uncertainties for the branching fraction measurement.

Source	Uncertainty (%)
Tracking efficiency	1.5
PID efficiency	1.5
π^0 reconstruction efficiency	2.0
Number of D_s^-	0.7
MC statistics	0.3
Signal shape	0.5
Background shape	0.8
MC model	0.4
Total	3.2

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Table XVI. The BFs of intermediate processes with final states $K^-K^+\pi^+\pi^0$. $K^*\pi$ indicates $\bar{K}^{*0}\pi^0$ and $K^{*-}\pi^+$. For decays with $a_0^0(980)$ in the final state, the quoted BFs include $\mathcal{B}(a_0^0(980) \to K^+K^-)$. The first and second uncertainties are statistical and systematic, respectively.

Process	BF (%)
$D_s^+[S] \to \phi\rho^+$	$2.31 \pm 0.08 \pm 0.08$
$D_s^+[P] \to \phi\rho^+$	$0.45 \pm 0.04 \pm 0.02$
$D_s^+[D] \to \phi\rho^+$	$0.26 \pm 0.04 \pm 0.03$
$D_s^+ \to \phi\rho^+$	$3.06 \pm 0.08 \pm 0.12$
$D_s^+[S] \to \bar{K}^{*0}K^{*+}$	$0.84 \pm 0.05 \pm 0.03$
$D_s^+[P] \to \bar{K}^{*0}K^{*+}$	$0.33 \pm 0.03 \pm 0.01$
$D_s^+[D] \to \bar{K}^{*0}K^{*+}$	$0.21 \pm 0.03 \pm 0.02$
$D_s^+ \to \bar{K}^{*0}K^{*+}$	$1.21 \pm 0.05 \pm 0.04$
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270) \to K^-\rho^+$	$0.50 \pm 0.04 \pm 0.03$
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^*\pi$	$0.10 \pm 0.02 \pm 0.02$
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^*\pi$	$0.04 \pm 0.01 \pm 0.01$
$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270) \to K^*\pi$	$0.14 \pm 0.02 \pm 0.02$
$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400) \to K^*\pi$	$0.39 \pm 0.05 \pm 0.03$
$D_s^+ \to a_0^0(980)\rho^+$	$0.07 \pm 0.02 \pm 0.01$
$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*\mp}K^\pm$	$0.07 \pm 0.02 \pm 0.01$
$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to a_0^0(980)\pi^0$	$0.04 \pm 0.01 \pm 0.01$
$D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to a_0^0(980)\pi^0$	$0.17 \pm 0.02 \pm 0.01$

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Appendix A: Fixed Relations of some Amplitudes

The amplitudes that are fixed by Clebsch Gordan coefficients and charge conjugation relations in this analysis are listed in Table XVII. The amplitudes with fixed relation share the same magnitude (ρ) and phase (ϕ).

Appendix B: Amplitudes Tested

Other tested amplitudes which are found to have a significance smaller than 3σ based on the nominal fit model are listed below.

- **Cascade amplitudes**
  - $D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^-\rho^+$
  - $D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[D] \to K^*\pi$
  - $D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[P] \to \bar{K}_0^*(1430)\pi$
  - $D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S,D] \to K^-\rho^+$
  - $D_s^+[P] \to \phi(1680)\pi^+$, $\phi(1680)[P] \to K^{*\mp}K^\pm$
  - $D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to K^{*\mp}K^\pm$
  - $D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to K^{*\mp}K^\pm$
  - $D_s^+ \to \eta(1295)\pi^+$, $\eta(1295) \to a_0^0(980)\pi^0$
  - $D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to a_0^0(980)\pi^0$
  - $D_s^+ \to f_1(1285)\pi^+$, $f_1(1285) \to a_0^0(980)\pi^0$
  - $D_s^+ \to f_1(1285)\pi^+$, $f_1(1285) \to K^{*\mp}K^\pm$
  - $D_s^+ \to f_1(1510)\pi^+$, $f_1(1510) \to K^{*\mp}K^\pm$
- **Three-body amplitudes**
  - $D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[P] \to (K\pi)_{\text{S-wave}}\pi$
  - $D_s^+[S,P,D] \to (K^-\pi^+)_V K^{*+}$
  - $D_s^+[S,P,D] \to \bar{K}^{*0}(K^+\pi^0)_V$
  - $D_s^+[S,P,D] \to (K^-K^+)_V\rho^+$
  - $D_s^+[S,P,D] \to \phi(\pi^+\pi^0)_V$
  - $D_s^+[S,P,D] \to \phi(1680)(\pi^+\pi^0)_V$
  - $D_s^+ \to (K^-\rho^+)_A[S,D]K^+$
  - $D_s^+ \to (K^*\pi)_A[S,D]K^+$
  - $D_s^+ \to (K^-\rho^+)_P K^+$
  - $D_s^+ \to (K^-\rho^+)_V K^+$
  - $D_s^+ \to (K^{*\mp}K^\pm)_P\pi^+$
  - $D_s^+ \to (K^{*\mp}K^\pm)_V\pi^+$
  - $D_s^+[P] \to (K^-K^+)_S\rho^+$
  - $D_s^+[P] \to \phi(\pi^+\pi^0)_S$
  - $D_s^+[P] \to (K^-\pi^+)_S K^{*+}$
  - $D_s^+[P] \to \bar{K}^{*0}(K^+\pi^0)_S$
  - $D_s^+[P] \to (K^-\pi^+)_{\text{S-wave}}K^{*+}$

Table XVII. The fixed relations of some amplitudes.

Index	Amplitude	Relation
A_1	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to \bar{K}^{*0}\pi^0$	
A_2	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^{*-}\pi^+$	
A	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[S] \to K^*\pi$	$A_1 - \sqrt{2} * A_2$
A_1	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to \bar{K}^{*0}\pi^0$	
A_2	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^{*-}\pi^+$	
A	$D_s^+ \to \bar{K}_1^0(1270)K^+$, $\bar{K}_1^0(1270)[D] \to K^*\pi$	$A_1 - \sqrt{2} * A_2$
A_1	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to \bar{K}^{*0}\pi^0$	
A_2	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to K^{*-}\pi^+$	
A	$D_s^+ \to \bar{K}_1^0(1400)K^+$, $\bar{K}_1^0(1400)[S] \to K^*\pi$	$A_1 - \sqrt{2} * A_2$
A_1	$D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to K^{*-}K^+$	
A_2	$D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to K^{*+}K^-$	
A	$D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to K^{*\mp}K^\pm$	$A_1 - A_2$
A_1	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*-}K^+$	
A_2	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*+}K^-$	
A	$D_s^+ \to f_1(1420)\pi^+$, $f_1(1420) \to K^{*\mp}K^\pm$	$A_1 - A_2$ [50]

- $D_s^+[P] \to \bar{K}^{*0}(K^+\pi^0)_{\text{S-wave}}$
- $D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to (K^\mp\pi^0)_V K^\pm$
- $D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to (K^\mp\pi^0)_V K^\pm$
- $D_s^+ \to \eta(1405)\pi^+$, $\eta(1405) \to (K^\mp\pi^0)_{\text{S-wave}} K^\pm$
- $D_s^+ \to \eta(1475)\pi^+$, $\eta(1475) \to (K^\mp\pi^0)_{\text{S-wave}} K^\pm$

- **Four-body non-resonance amplitudes**

- $D_s^+ \to ((K\pi)_{\text{S-wave}}\pi)_A K^+$
- $(K^-(\pi^+\pi^0)_V)_P K^+$
- $(K^-(\pi^+\pi^0)_V)_V K^+$
- $D_s^+ \to ((K^\mp\pi^0)_V K^\pm)_P \pi^+$
- $D_s^+ \to ((K^\mp\pi^0)_V K^\pm)_V \pi^+$
- $((K\pi)_V\pi)_A[S,D]K^+$
- $D_s^+ \to ((\pi^+\pi^0)_V K^-)_A[S,D]K^+$
- $D_s^+[S,P,D] \to (K^-K^+)_V(\pi^+\pi^0)_V$
- $D_s^+[S,P,D] \to (K^-\pi^+)_V(K^+\pi^0)_V$
- $D_s^+ \to (K^-\pi^+)_S(K^+\pi^0)_S$
- $D_s^+ \to (K^-K^+)_S(\pi^+\pi^0)_S$

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