Measurements of Differential Cross Sections of Inclusive η Production in e^+e^- Annihilation at Energies from 2.0000 to 3.6710 GeV: Supplemental Material

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(Dated: April 23, 2024)

I. SUMMARY OF THE OBSERVED YIELDS FOR THE $e^+e^- \to \eta + X$ PROCESS

Table I shows the obtained N_η^{obs} in different momentum ranges at different c.m. energies.

TABLE I. The obtained N_η^{obs} in different momentum ranges at different c.m. energies, where the uncertainties are statistical.

p_η (GeV/c)	$\sqrt{s}$ = 2.0000 GeV	$\sqrt{s}$ = 2.2000 GeV	$\sqrt{s}$ = 2.3960 GeV	$\sqrt{s}$ = 2.6444 GeV	$\sqrt{s}$ = 2.9000 GeV	$\sqrt{s}$ = 3.0500 GeV	$\sqrt{s}$ = 3.5000 GeV	$\sqrt{s}$ = 3.6710 GeV
0.00 - 0.10	41 ± 11	38 ± 12	175 ± 25	94 ± 17	254 ± 30	32 ± 10	5 ± 5	15 ± 6
0.10 - 0.20	276 ± 33	416 ± 34	1607 ± 69	667 ± 47	1786 ± 78	235 ± 28	35 ± 13	80 ± 16
0.20 - 0.30	771 ± 42	1169 ± 51	4324 ± 104	1716 ± 71	4613 ± 118	595 ± 42	136 ± 22	120 ± 23
0.30 - 0.40	1294 ± 51	1716 ± 58	6911 ± 123	2598 ± 80	6813 ± 136	785 ± 48	136 ± 24	148 ± 24
0.40 - 0.50	1241 ± 47	1907 ± 59	7750 ± 123	3045 ± 81	8251 ± 134	1023 ± 51	222 ± 26	210 ± 26
0.50 - 0.60	1079 ± 42	1443 ± 49	6522 ± 108	2852 ± 75	7657 ± 126	888 ± 44	229 ± 24	227 ± 25
0.60 - 0.70	516 ± 29	865 ± 38	4187 ± 83	2196 ± 62	6368 ± 109	840 ± 40	165 ± 21	224 ± 23
0.70 - 0.80	653 ± 29	640 ± 30	2548 ± 63	1761 ± 52	5393 ± 94	665 ± 34	177 ± 18	198 ± 20
0.80 - 0.90	191 ± 15	449 ± 24	1811 ± 51	983 ± 38	3999 ± 77	557 ± 29	122 ± 15	151 ± 17
0.90 - 1.00	–	128 ± 12	1473 ± 43	596 ± 29	2576 ± 4	387 ± 23	112 ± 13	125 ± 15
1.00 - 1.10	–	–	694 ± 28	490 ± 25	1316 ± 43	229 ± 18	76 ± 10	93 ± 12
1.10 - 1.20	–	–	–	364 ± 20	960 ± 35	124 ± 13	61 ± 9	85 ± 11
1.20 - 1.30	–	–	–	–	876 ± 32	101 ± 11	41 ± 7	62 ± 9
1.30 - 1.40	–	–	–	–	–	73 ± 9	25 ± 6	56 ± 8
1.40 - 1.50	–	–	–	–	–	–	19 ± 5	29 ± 6

II. SUMMARY OF THE CORRECTION FACTORS FOR THE $e^+e^- \to \eta + X$ PROCESS

Table II shows the obtained f_η for different momentum ranges at different c.m. energies.

TABLE II. The obtained f_η for different momentum ranges at different c.m. energies, where the uncertainties are statistical.

p_η (GeV/c)	$\sqrt{s}$ = 2.0000 GeV	$\sqrt{s}$ = 2.2000 GeV	$\sqrt{s}$ = 2.3960 GeV	$\sqrt{s}$ = 2.6444 GeV	$\sqrt{s}$ = 2.9000 GeV	$\sqrt{s}$ = 3.0500 GeV	$\sqrt{s}$ = 3.5000 GeV	$\sqrt{s}$ = 3.6710 GeV
0.00 - 0.10	3.280 ± 0.123	3.435 ± 0.128	3.974 ± 0.075	4.352 ± 0.119	4.495 ± 0.090	4.489 ± 0.085	5.119 ± 0.109	4.873 ± 0.104
0.10 - 0.20	3.170 ± 0.063	3.313 ± 0.064	3.369 ± 0.033	3.763 ± 0.056	3.842 ± 0.042	3.864 ± 0.042	4.313 ± 0.053	4.277 ± 0.052
0.20 - 0.30	2.797 ± 0.049	2.926 ± 0.051	3.088 ± 0.028	3.243 ± 0.043	3.403 ± 0.034	3.423 ± 0.034	3.845 ± 0.043	3.829 ± 0.042
0.30 - 0.40	2.623 ± 0.048	2.734 ± 0.048	2.879 ± 0.026	3.016 ± 0.040	3.110 ± 0.031	3.157 ± 0.031	3.470 ± 0.038	3.526 ± 0.039
0.40 - 0.50	2.511 ± 0.049	2.657 ± 0.050	2.679 ± 0.025	2.826 ± 0.038	2.974 ± 0.031	2.937 ± 0.030	3.141 ± 0.035	3.207 ± 0.037
0.50 - 0.60	2.434 ± 0.051	2.481 ± 0.051	2.644 ± 0.027	2.729 ± 0.039	2.818 ± 0.029	2.819 ± 0.030	2.924 ± 0.034	3.077 ± 0.038
0.60 - 0.70	2.254 ± 0.049	2.384 ± 0.054	2.632 ± 0.029	2.648 ± 0.040	2.696 ± 0.029	2.732 ± 0.029	2.718 ± 0.031	2.910 ± 0.035
0.70 - 0.80	2.367 ± 0.048	2.401 ± 0.056	2.558 ± 0.031	2.701 ± 0.044	2.712 ± 0.030	2.746 ± 0.031	2.727 ± 0.033	2.809 ± 0.034
0.80 - 0.90	2.939 ± 0.089	2.368 ± 0.054	2.540 ± 0.033	2.719 ± 0.050	2.673 ± 0.032	2.718 ± 0.032	2.702 ± 0.033	2.799 ± 0.035
0.90 - 1.00	–	2.826 ± 0.089	2.516 ± 0.033	2.653 ± 0.055	2.809 ± 0.037	2.722 ± 0.034	2.713 ± 0.034	2.818 ± 0.036
1.00 - 1.10	–	–	3.057 ± 0.053	2.419 ± 0.051	2.749 ± 0.040	2.794 ± 0.038	2.812 ± 0.037	2.742 ± 0.035
1.10 - 1.20	–	–	–	3.147 ± 0.075	2.787 ± 0.047	2.918 ± 0.046	2.934 ± 0.041	2.889 ± 0.039
1.20 - 1.30	–	–	–	–	3.126 ± 0.056	2.774 ± 0.048	2.917 ± 0.042	2.844 ± 0.040
1.30 - 1.40	–	–	–	–	–	3.351 ± 0.065	2.928 ± 0.044	2.821 ± 0.040
1.40 - 1.50	–	–	–	–	–	–	3.096 ± 0.053	2.999 ± 0.047

III. COMPARISONS ON SOME OBSERVABLES OF η BETWEEN DATA AND LUARLW MC

Figure 1 shows the event level comparison between data and LUARLW MC on the distributions of some reconstructed observables of η including the cosine of the polar angle ($\cos\theta$), the azimuthal angle (ϕ), the multiplicity of η (N_η) and the helicity of η. The good agreement between data and MC shows that the LUARLW model can reasonably reproduce the properties of η mesons.

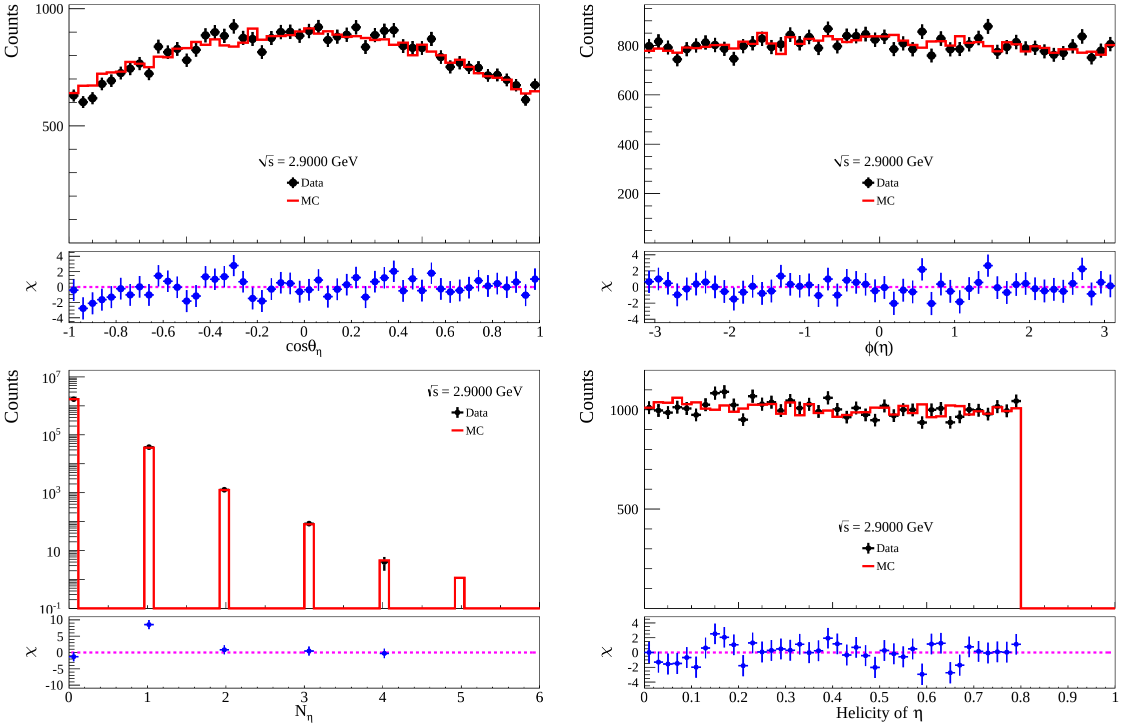

FIG. 1. Event level comparisons between LUARLW MC and data at $\sqrt{s} = 2.9000$ GeV in terms of: the cosine of the polar angle (θ) distribution of η, the azimuth angle (ϕ) of η, the number of reconstructed η, and the helicity distributions in each event, respectively. The pull variable χ, defined as the residual between data and MC, normalized by the uncertainty of the data, is shown on the bottom of each figure.

IV. NORMALIZED DIFFERENTIAL CROSS SECTIONS OF THE $e^+e^- \to \eta + X$ PROCESS

Table III and IV shows the normalized differential cross sections of the $e^+e^- \to \eta+X$ process at different momentum ranges at different c.m. energy points. The cross sections from the first four c.m. energy points are listed in Table III, and the last four c.m. energy points are listed in Table IV.

TABLE III. The normalized differential cross section of the $e^+e^- \to \eta + X$ process at different momentum ranges at the first four c.m. energy points, where the first uncertainties are statistical and the second are systematic. Systematic uncertainties are regarded as uncorrelated between different momentum ranges except 2% of that due to the reconstruction of the photons.

p_η (GeV/c)	$\sqrt{s} = 2.0000$ GeV	$\sqrt{s} = 2.2000$ GeV	$\sqrt{s} = 2.3960$ GeV	$\sqrt{s} = 2.6444$ GeV
0.00-0.10	0.010±0.003±0.002	0.008±0.003±0.001	0.010±0.001±0.002	0.013±0.002±0.002
0.10-0.20	0.065±0.008±0.006	0.081±0.007±0.006	0.075±0.003±0.006	0.080±0.006±0.010
0.20-0.30	0.160±0.009±0.008	0.200±0.009±0.025	0.186±0.005±0.006	0.177±0.008±0.011
0.30-0.40	0.252±0.011±0.013	0.274±0.010±0.055	0.277±0.006±0.023	0.250±0.008±0.010
0.40-0.50	0.232±0.010±0.019	0.296±0.011±0.046	0.289±0.005±0.036	0.274±0.008±0.013
0.50-0.60	0.195±0.009±0.012	0.209±0.008±0.016	0.240±0.005±0.022	0.248±0.007±0.013
0.60-0.70	0.086±0.005±0.032	0.121±0.006±0.010	0.153±0.003±0.008	0.185±0.006±0.015
0.70-0.80	0.115±0.006±0.020	0.090±0.005±0.021	0.091±0.003±0.008	0.152±0.005±0.009
0.80-0.90	0.042±0.004±0.012	0.062±0.004±0.006	0.064±0.002±0.012	0.085±0.004±0.004
0.90-1.00	–	0.021±0.002±0.005	0.052±0.002±0.006	0.050±0.003±0.008
1.00-1.10	–	–	0.030±0.001±0.009	0.038±0.002±0.015
1.10-1.20	–	–	–	0.036±0.002±0.007

TABLE IV. The normalized differential cross section of the $e^+e^- \to \eta + X$ process at different momentum ranges at the last four c.m. energy points, where the first uncertainties are statistical and the second are systematic. Systematic uncertainties are regarded as uncorrelated between different momentum ranges except 2% of that due to the reconstruction of the photons.

p_η (GeV/c)	$\sqrt{s} = 2.9000$ GeV	$\sqrt{s} = 3.0500$ GeV	$\sqrt{s} = 3.5000$ GeV	$\sqrt{s} = 3.6710$ GeV
0.00-0.10	0.014±0.002±0.003	0.013±0.004±0.027	0.011±0.011±0.002	0.026±0.011±0.008
0.10-0.20	0.081±0.004±0.013	0.083±0.010±0.010	0.063±0.024±0.008	0.126±0.025±0.009
0.20-0.30	0.186±0.005±0.018	0.187±0.013±0.010	0.218±0.034±0.023	0.170±0.033±0.018
0.30-0.40	0.251±0.006±0.016	0.228±0.014±0.008	0.196±0.034±0.037	0.192±0.032±0.027
0.40-0.50	0.291±0.006±0.017	0.276±0.014±0.013	0.290±0.034±0.058	0.249±0.031±0.044
0.50-0.60	0.256±0.005±0.009	0.230±0.012±0.008	0.279±0.029±0.056	0.258±0.029±0.032
0.60-0.70	0.204±0.004±0.014	0.211±0.010±0.008	0.187±0.024±0.031	0.240±0.025±0.025
0.70-0.80	0.173±0.004±0.014	0.168±0.009±0.006	0.201±0.021±0.018	0.205±0.021±0.021
0.80-0.90	0.127±0.003±0.014	0.139±0.007±0.007	0.137±0.017±0.010	0.156±0.017±0.009
0.90-1.00	0.086±0.001±0.010	0.097±0.006±0.011	0.126±0.015±0.009	0.130±0.015±0.005
1.00-1.10	0.043±0.002±0.008	0.059±0.005±0.007	0.088±0.012±0.004	0.094±0.012±0.007
1.10-1.20	0.032±0.001±0.003	0.033±0.003±0.003	0.075±0.011±0.004	0.090±0.012±0.004
1.20-1.30	0.032±0.001±0.010	0.026±0.003±0.004	0.049±0.008±0.005	0.065±0.010±0.003
1.30-1.40	–	0.023±0.003±0.004	0.030±0.007±0.003	0.059±0.009±0.008
1.40-1.50	–	–	0.025±0.006±0.002	0.033±0.007±0.004

V. NORMALIZED DIFFERENTIAL CROSS SECTIONS AS FUNCTION OF z

In the main text, the normalized differential cross sections of the $e^+e^- \to \eta + X$ process are given as function of hadron momentum. According to the relation $z \equiv 2\sqrt{{p_\eta}^2c^2 + {M_\eta}^2c^4}/\sqrt{s}$, these cross sections could be converted to be z-dependent by

$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})}\frac{\mathrm{d}\sigma(e^+e^- \to \eta + X)}{\mathrm{d}z_\eta} = \frac{\sqrt{s}}{2}\sqrt{1 + \frac{{M_\eta}^2c^2}{{p_\eta}^2}}\frac{1}{\sigma(e^+e^- \to \text{hadrons})}\frac{\mathrm{d}\sigma(e^+e^- \to \eta + X)}{\mathrm{d}p_\eta}. \tag{1}$$

The z-dependent cross sections for the $e^+e^- \to \eta + X$ process are shown in Fig. 2.

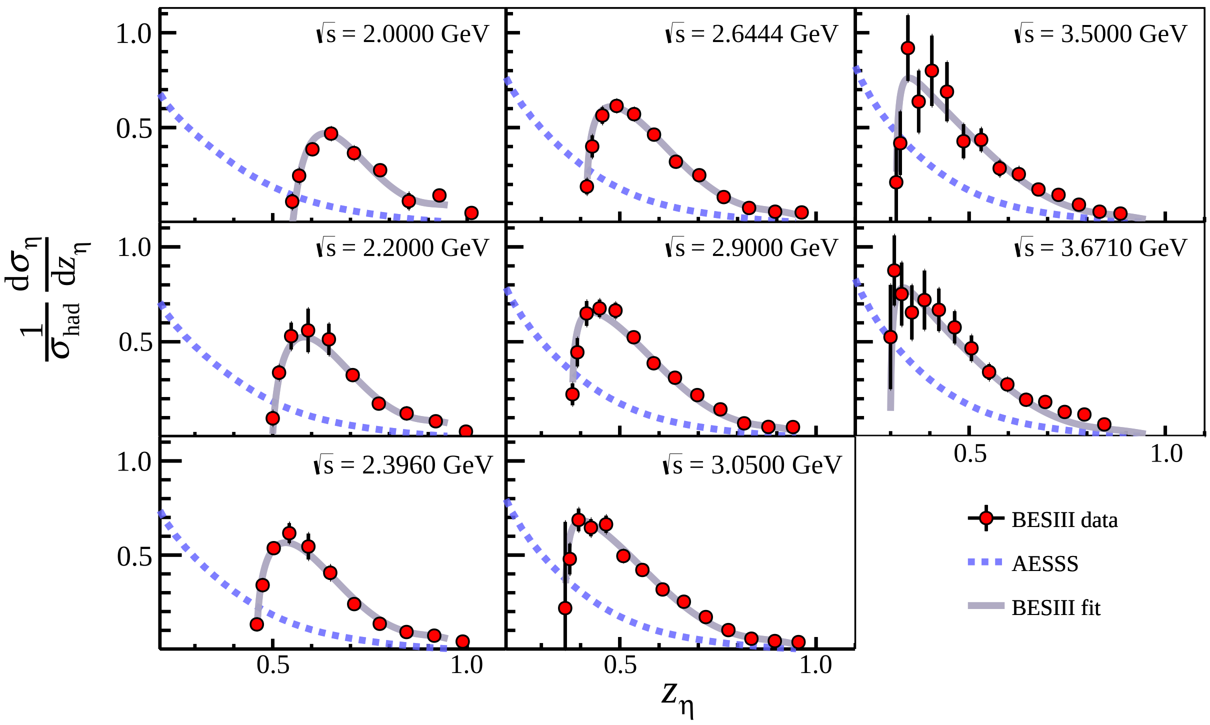

FIG. 2. Normalized differential cross sections for $e^+e^- \to \eta + X$ process as function of z. The points with error bars are the measured values. The blue dotted curves denote the predictions by using the AESSS FFs, while the curves in gray denote the calculations by using the newly extracted FFs from our fit.

VI. A GLOBAL ANALYSIS OF η FRAGMENTATION FUNCTIONS WITH BESIII DATA

As new BESIII experimental data become available, especially for its low energy scales, they can provide additional insights into the hadronization process. Up to now, the AESSS [1] FFs at NLO accuracy stand as the primary and sole resource for η. The AESSS FFs are extracted from data of η production in e^+e^- annihilation with $\sqrt{s}$ approximately at 10, 30, and 90 GeV, and in proton-proton collisions at a $\sqrt{s}$ of about 200 GeV. However, the predictions from AESSS FFs significantly deviate from experimental results within the low-energy range of the BESIII experiment.

In [2], we conduct a comprehensive global QCD analysis of single inclusive production of η at NNLO accuracy. We incorporate existing SIA world data along with our low-Q scale BESIII data, aiming to probe the effective region of leading twist QCD collinear factorization. Furthermore, we adopt a parameterized functional approach to investigate the contributions from higher twist. This methodology provides a testing framework for analyzing experimental results over a wide range of energy scales, from low to high, extending traditional theoretical descriptions to encompass the BESIII regime. In the following, we have summarized the essential information for the QCD based analysis for the η inclusive production.

A. Physical Observable

The cross section for the single-inclusive hadron production in electron-positron annihilation ($e^+ + e^- \to h + X$), normalized to the total cross section σ_{tot}, can be schematically written as

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^h}{dz} = \frac{1}{\sigma_{tot}}\sum_{k=T,L}\frac{d\sigma^h_k}{dz}, \tag{2}$$

where $z = 2P_h \cdot q/Q^2$ is the scaling variable ($z = 2E_h/\sqrt{s}$ in electron-positron center of mass frame), P_h and q are the four momenta of the observed hadron and time-like γ/Z boson, respectively, and $Q^2 = q^2 = s$ with $\sqrt{s}$ being the center of mass energy.

More specifically, the transverse (T) and longitudinal (L) part in Eq. (2) can be factorised as a convolution of perturbative coefficient functions C_i^k with $k = T,\ L$ and non-perturbative fragmentation functions (FFs) D_i^h,

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^h_k}{dz} = \frac{1}{\sigma_{tot}}\sum_i C_i^k(z, \frac{Q^2}{\mu^2}) \otimes D_i^h(z, \mu^2). \tag{3}$$

In our calculation, both factorization and renormalization scales are set equal to the center of mass energy of the collision, $\mu_R = \mu_F = \sqrt{s} = Q$. The symbol $\otimes$ denotes the standard convolution integral defined as

$$[f \otimes g](z) = \int_0^1 dx \int_0^1 dy f(x) g(y) \delta(z - xy). \tag{4}$$

In pQCD, the coefficient functions can be calculated as a perturbative series in $a_s = \alpha_s/4\pi$ with α_s the QCD running coupling,

$$C_i^k = C_i^{k,(0)} + a_s C_i^{k,(1)} + a_s^2 C_i^{k,(2)} + \dots. \tag{5}$$

To minimize corrections from higher-order effects, we employ the most precise results available for all perturbative coefficients up to NNLO. The expressions for σ_{tot} and the coefficient C are detailed in Refs. [3] and [4–7], respectively.

In our study, we derive fragmentation functions for η using cross-sections from electron-positron single inclusive annihilation (SIA). This analysis integrates our BESIII datasets and comprehensive world datasets [8–19]. The latter cover a broad spectrum of center-of-mass energies from 9.46 GeV at ARGUS to 91.2 GeV at ALEPH and OPAL. The datasets used in this analysis are detailed in Section VI C. For each dataset, the experiment name, associated references, and the number of data points included in the fit are specified. In the small-z region, soft gluon effects render the DGLAP evolution equations unstable, resulting in discrepancies between theoretical models and experimental data. Consequently, all theoretical models restrict their analyses to data points where $z \geq z_{min}$, with z_{min} defining the lower bound of z. In our analysis, we restrict the data to $z \geq 0.05$ and impose an upper limit at $z < 0.95$ to mitigate the significant enhancement from threshold logarithms $\propto \log(1-z)$.

B. Fitting framework

The fitting framework we employ is grounded in the methodology outlined in Ref. [20], where the fitting process is performed in Mellin space. Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [21–24] evaluate the FFs with the energy scale Q^2 as:

$$\frac{\partial}{\partial lnQ^2} D_i^h(z,Q) = \sum_j [P_{ji} \otimes D_j^h](z,Q), \tag{6}$$

where P_{ji} are the time-like splitting functions, and z is the fraction of the four-momentum of the parton taken by the η meson.

We take the parameterization form of FFs at initial scale μ_0 following the approach in the series of DSS global QCD analyses [25–28]:

$$D_i^h(z,\mu_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i,\beta_i+1]+\gamma_i B[2+\alpha_i,\beta_i+\delta_i+1]}, \tag{7}$$

$B[a,b]$ denotes the Euler Beta function, with a and b chosen such that N_i is normalized to the second moment $\int_0^1 zD_i^h(z,\mu_0)dz$ of the FFs. Given the limitation of using SIA data, which precludes differentiation between quark and antiquark fragmentation functions, our analysis adopts the quark combination $q^+ = q + \bar{q}$ for the parametrization. In Equation (7), the index i represents the quark combinations u^+, d^+, s^+, c^+, b^+, and the gluon g.

TABLE V. Parameters describing the NNLO FFs ($i^+ = i + \bar{i}$) for η. Inputs for light and gluon FFs are set at the initial scale $\mu_0 = 1.0$ GeV. Inputs for the charm and bottom FFs refer to $\mu = m_c$ and $\mu = m_b$, respectively.

Parameter	N	α	β	γ	δ
$u^+ = d^+ = s^+$	N_{u^+}	α_{u^+}	β_{u^+}	γ_{u^+}	δ_{u^+}
g	N_g	α_g	β_g	-	-
c^+	N_{c^+}	α_{c^+}	β_{c^+}	-	-
b^+	N_{b^+}	α_{b^+}	β_{b^+}	-	-

Utilizing the isospin symmetry, we employ the AESSS scheme [1], which posits $D^\eta_{u^+} = D^\eta_{d^+} = D^\eta_{s^+}$. Considering the number of experimental data points from e^+e^- annihilation to determine the η FFs is rather limited, we fix $\gamma_{c^+,b^+,g}$ and $\delta_{c^+,b^+,g}$ to zero. During the scale evolution of the fragmentation functions, charm and bottom quarks are included in the scale evolution above $\mu = m_c$ and $\mu = m_b$, with $m_c = 1.43$ GeV and $m_b = 4.3$ GeV denoting the masses of the charm and bottom quarks, respectively. Following these considerations, the free parameters for η are summarized in Table V.

Parameters defining the fragmentation functions (FFs) for quarks and gluons into eta, delineated in Equation (7), are ascertained by standard χ^2 minimization. The minimized χ^2 value is delineated by the following equation:

$$\chi^2 = \sum_{j=1}^{N} \frac{(T_j - E_j)^2}{\Delta E_j^2}, \tag{8}$$

where E_j represents the experimentally measured value, ΔE_j signifies the associated uncertainty, and T_j corresponds to the theoretical prediction for the parameters in Equation (7). For the experimental uncertainties ΔE_j, we currently account for both statistical and systematic errors in quadrature.

Fixed-order NNLO calculations in SIA are accessible [4–7] and requisite for accurate predictions, allowing the FFs fits to be conducted according to this benchmark.

Given the relatively low energy scale at BESIII, the consideration of hadron mass corrections [29–31] becomes pertinent. Our methodology for hadron mass corrections adheres to the approach outlined in these references. With hadron mass (m_h) effects considered, the scaling variable transitions from $z = 2E_h/\sqrt{s}$ to the designated light-cone scaling variable ξ. It is given by,

$$\xi = \frac{z}{2}\left(1 + \sqrt{1 - \frac{4m_h^2}{Q^2 z^2}}\right). \tag{9}$$

Consequently, the differential cross section in the presence of hadron mass effects for a SIA process need to be modified as

$$\frac{d\sigma^h}{dz} = \frac{1}{1 - \frac{m_h^2}{Q^2\xi^2}} \frac{d\sigma^h}{d\xi}. \tag{10}$$

To assess the higher twist contributions within the global analysis framework, we utilize the approach developed by Accardi et al. [32], and extend it to include higher twist effects with a phenomenological function dependent on z:

$$\frac{d\sigma_k^h}{dz} = \frac{d\sigma_k^{h,LT}}{dz}(z,Q^2)\left[1 + \frac{C_{T4}(z)}{Q^2} + \frac{C_{T6}(z)}{Q^4} + \dots\right], \tag{11}$$

where $d\sigma_k^{h,LT}$ represents the leading twist contribution as delineated in Eq.(2). The subsequent terms, specifically the second and third, correspond to the twist-4 and twist-6 corrections to the leading twist, attenuated by $\frac{1}{Q^2}$ and $\frac{1}{Q^4}$ respectively. The higher twist coefficient functions are parameterized by a polynomial function as

$$C_{T4}(x) = h_0 x^{h_1}(1 + h_2 x), C_{T6}(x) = h_3 x^{h_4}(1 + h_5 x), \tag{12}$$

with h_0, h_1, h_2, h_3, h_4 and h_5 as free parameters. In this analysis, we employ C_{T4} and C_{T6} to effectively characterize the data points. More complex parametrizations necessitate extensive data analysis, which lies beyond the scope of the current study.

C. Results

Initially, we apply a global analysis to the SIA data for η employing fixed-order pQCD calculations at NNLO, without hadron mass corrections and higher twist effects. It turns out that we can reproduce the similar χ^2 results in AESSS [1] without the BESIII low energy data, and a satisfactory fit is obtained for η. However, this satisfactory convergence is compromised when incorporating BESIII low energy data, despite the inclusion of hadron mass corrections. This finding compellingly advocates the incorporation of both mass corrections and higher twist effects, which yield χ^2/N_{dp} values of 1.52 for η, indicating of a satisfactory fit quality.

In the following, we discuss the quality of the fits and compare our predictions to the included datasets. The overall statistical quality of our fit, as quantified by the χ^2 per data point (χ^2/N_{dp}), for both individual and combined datasets, is summarized in Tables VI.

The optimal fit parameters for π^0, K_S^0, and η fragmentation functions (FFs), along with the corresponding higher twist effects, are detailed in Tables VII, and VIII.

TABLE VI. The list of input datasets in analyses of η. For each dataset, we indicate the corresponding published reference, the name of the experiments, the center-of-mass energy $\sqrt{s}$ and the value of χ^2 per data point for the individual dataset in our best-fit. The total values of χ^2/N_{dp} have been presented as well.

Exp(η)	$\sqrt{s}$ [GeV]	N_{dp}	χ^2/N_{dp}
ARGUS [8]	9.46	6	5.69
HRS [9]	29.0	13	3.10
MARK II [10]	29.0	7	0.56
JADE [11]	34.4	2	3.77
JADE [12]	35.0	4	0.44
CELLO [13]	35.0	4	0.18
ALEPH [14]	91.2	8	0.59
ALEPH [15]	91.2	18	1.07
ALEPH [16]	91.2	5	11.18
L3 [17]	91.2	5	1.11
L3 [18]	91.2	11	1.19
OPAL [19]	91.2	11	0.90
BESIII	2.00	8	2.38
BESIII	2.20	9	0.67
BESIII	2.39	10	1.25
BESIII	2.64	11	0.36
BESIII	2.90	13	0.67
BESIII	3.05	13	0.55
BESIII	3.50	15	0.72
BESIII	3.67	15	1.42
TOTAL		188	1.52

TABLE VII. Parameters describing the NNLO FFs ($i^+ = i + \bar{i}$) for η with one standard deviation uncertainties. Inputs for light and gluon FFs are set at the initial scale $\mu_0 = 1.0$ GeV. Inputs for the charm and bottom FFs refer to $\mu = m_c$ and $\mu = m_b$, respectively.

Parameter	N	α	β	γ	δ
$u^+ = d^+ = s^+$	0.030 ± 0.003	2.4 ± 0.9	0.4 ± 0.7	341 ± 339	4.0 ± 0.5
g	0.123 ± 0.005	-0.1 ± 0.4	0.5 ± 0.3	-	-
c^+	0.0008 ± 0.0010	46.8 ± 41.7	7.9 ± 8.3	-	-
b^+	0.007 ± 0.001	50.0 ± 43.1	18.6 ± 1.1	-	-

TABLE VIII. Parameters of higher twist effect for η with one standard deviation uncertainties.

	h_0	h_1	h_2	h_3	h_4	h_5
HT	-4.5 ± 0.9	-1.5 ± 0.2	-5.5 ± 0.5	3.0 ± 0.8	-2.8 ± 0.5	-6.8 ± 1.7

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