Online Appendix of "Green Targeted Lending Operations in the Euro Area"

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A Theoretical Model

This model examines two distinct types of intermediate firms: green and brown. They are subject to financial frictions and reserve requirements. The model is based on the work of Le (2023) and is calibrated to the Euro Area[1]. We introduce wage stickiness and two separate lending rate rules, making this model a further development of the previous work.

A Households

Households maximize the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[log(C_t - \kappa C_{t-1}) - \Psi \frac{H_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

with respect to the budget constraint:

$$C_t + I_t + \frac{D_t}{P_t} = r_t^k K_{t-1} + w_t H_t + R_{t-1} \frac{D_{t-1}}{P_t} + T_t \tag{2}$$

The capital accumulation process is as follows:

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{3}$$

B Labour Union

Households operate under a labour union that creates wage stickiness. Hence, they face quadratic nominal wage adjustment costs à la Rotemberg (1982), κ_w. At time t, union j sets its wage $W_{j,t}$ to maximize the utility of its average worker. To accommodate the nonlinear solving exercise later, we derive the nonlinear wage Phillips curve is defined as follows:

[1] We keep the derivations brief. Viewers can read Le (2023) for the detailed version.

$$\max_{W_{j,t}} \sum_{s=0}^{\infty} E_{t+s} \left[log(C_{t+s} - \kappa C_{t+s-1}) - \Psi \frac{H_{t+s}^{1+\varphi}}{1+\varphi} - \frac{\kappa_w}{2} \left(\frac{W_{j,t+s}}{W_{j,t+s-1}} - 1 \right)^2 \right], \tag{4}$$

$$\text{s.t.} \quad H_{j,t+s} = \left(\frac{W_{j,t+s}}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \tag{5}$$

Imposing symmetry , we have $W_{j,t} = W_t$ and $H_{j,t} = H_t$ which leads to:

$$\Pi_t^w \left(1 + \pi_t^w\right) = \frac{\varepsilon_w}{\kappa_w} H_t \left(\Psi H_t^{\varphi} - \frac{\varepsilon_w - 1}{\varepsilon_w} W_t \lambda_t \right) + \beta E_t \left[\Pi_{t+1}^w \left(1 + \Pi_{t+1}^w\right) \right] \tag{6}$$

where $\lambda_t = \frac{1}{C_t - \kappa C_{t-1}} - \beta \frac{\kappa}{C_{t+1} - \kappa C_t}$ and $\Pi_t^W = \frac{W_t}{W_{t-1}} - 1$.

C Wholesale and Intermediate Goods Sectors

Here, we have green and brown firms, adhering to the framework sketched in Bernanke et al. (1999). $Y_{G,t}$ and $Y_{B,t}$ denote the production of green and brown sectors. The composition of wholesale goods, Γ_t, is described as:

$$\Gamma_t = (\phi Y_{G,t}^{\frac{\sigma-1}{\sigma}} + (1-\phi) Y_{B,t}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{7}$$

Cost-minimization implies:

$$Y_{G,t} = \phi^{\sigma} \left(\frac{p_{G,t}}{p_{wc,t}} \right)^{-\sigma} \Gamma_t, Y_{B,t} = (1-\phi)^{\sigma} \left(\frac{p_{B,t}}{p_{wc,t}} \right)^{-\sigma} \Gamma_t \tag{8}$$

$p_{G,t}$ and $p_{B,t}$ are the prices of goods from green firms and brown firms.

$$p_{wc,t} = (\phi^{\sigma} p_{G,t}^{1-\sigma} + (1-\phi)^{\sigma} p_{B,t}^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{9}$$

Brown firms face an additional cost for carbon price and abatement expenses.

$$Y_{it} = A_{it}^{env} \omega_{it} K_{it}^{1-\alpha} [(H_{it}^e)^{1-\theta} (H_{it})^{\theta}]^{\alpha}, \quad i = G, B \tag{10}$$

ω_{it} signifies an idiosyncratic productivity shock in sector i, drawn from $F(\cdot)$ which is a non-negative support distribution.

$$A_t^{env} = \left(1 - \left(d_0 + d_1\, X_t + d_2\, X_t^2\right)\right) A_t \tag{11}$$

$$ln(A_t) = (1-\rho_a)\, ln(A) + \rho_a\, ln(A_{t-1}) + \epsilon_t^A, \quad \epsilon_t^A \sim \mathcal{N}(0,\, \sigma_a^2) \tag{12}$$

Equation 12 is the AR(1) TFP shock process.

The environmental part includes the emission, e_t, the pollution, X_t, and the abatement, z_t.

$$X_t = \eta\, X_{t-1} + e_t + e_t^{row} \tag{13}$$

$$e_t = \gamma_1\, (1-\mu_t)\, Y_{B,t} \tag{14}$$

$$z_t = \theta_1\, \mu_t^{\theta_2}\, Y_{B,t} \tag{15}$$

Green firms must cover wages and capital costs for their operations within their capital constraints.

$$N_{G,t-1} + B_{G,t} = w_t H_{G,t} + w_{G,t}^e H_{G,t}^e + r_t^k K_{G,t} \tag{16}$$

The brown firms's working capital constraints take the form:

$$N_{B,t-1} + B_{B,t} = w_t H_{B,t} + w_{B,t}^e H_{B,t}^e + r_t^k K_{B,t} + \underbrace{\theta_1 \mu_t^{\theta_2} Y_{B,t}}_{z_t} + \tau_t^e \underbrace{(1-\mu_t)\gamma_1 Y_{B,t}}_{e_t} \tag{17}$$

where $N_{i,t-1}$ represents the start-up equity and $B_{i,t}$ designates external loans.

D Green and Brown Banks

In this section, we provide a detailed breakdown of the banking sector and the introduction of dual rates. In this paper, we maintain the reserve requirement ratio at a steady state level, which is the primary distinction from Le (2023).

Following the BGG model, firms will choose bankruptcy if their productivity falls too low. As a result, some firms go bankrupt each period, which elevates the default risk associated with bank loans.

$$(r_{G,t} - 1)(1 - \tau_t) = R_t^{env} - 1 \tag{18}$$

$$(r_{B,t} - 1)(1 - \tau_t) = R_t - 1 \tag{19}$$

Following the suggestion of green targeted lending operations from the ECB, this paper introduces the second operational rate for green lending banks, R_t^{env}. Instead of one Taylor rule, we introduce a Taylor rule type for R_t^{env} also.

$$\ln\left(\frac{R_t^{env}}{R_{ss}}\right) = \rho_r ln\left(\frac{R_{t-1}}{R_{ss}}\right) + (1-\rho_r)\left(\rho_y ln\left(\frac{Y_t}{Y_{t-1}}\right) + \rho_\pi ln\left(\frac{\Pi_t}{\Pi_{ss}}\right) + \rho_e ln\left(\frac{e_t}{e_{ss}}\right)\right) \tag{20}$$

$$\ln\left(\frac{R_t}{R_{ss}}\right) = \rho_r ln\left(\frac{R_{t-1}}{R_{ss}}\right) + (1-\rho_r)\left(\rho_y ln\left(\frac{Y_t}{Y_{t-1}}\right) + \rho_\pi ln\left(\frac{\Pi_t}{\Pi_{ss}}\right)\right) \tag{21}$$

The remaining financial sectors are taken from Le (2023) and we use the same notation for convenience. For simplicity, we represent the total emission costs:

$$cost_t^E = \frac{\theta_1(\mu_t)^{\theta_2} y_t^B + \tau_t^e(1-\mu_t)\gamma_1 y_t^B}{\tilde{A}_{B,t}(N_{B,t-1} + B_{B,t})} \tag{22}$$

where $\bar{\omega}_{B,t}$ is defined as:

$$\bar{\omega}_{B,t} = \frac{Z_{B,t}B_{B,t}}{\tilde{A}_{B,t}(N_{B,t-1} + B_{B,t})(1 - cost_t^E)} \tag{23}$$

The contract optimization process takes the following form:

$$\max \tilde{A}_{B,t}(N_{B,t-1} + B_{B,t})(1 - cost_t^E) f(\bar{\omega}_{B,t}) \tag{24}$$

$$\tilde{A}_{B,t}(N_{B,t-1} + B_{B,t})(1 - cost_t^E) g(\bar{\omega}_{B,t}) \geq r_{B,t} B_{B,t} \tag{25}$$

The contract optimization process gives us:

$$\frac{N_{B,t-1}}{(N_{B,t-1}+B_{B,t})(1-cost_t^E)} = -\frac{g'(\bar{\omega}_{B,t})}{f'(\bar{\omega}_{B,t})}\frac{\tilde{A}_{B,t}f(\bar{\omega}_{B,t})}{r_{B,t}} \tag{26}$$

At the end of period t, the total net worth includes the profits of firms that have survived and the income of the entrepreneur in that sector. The variable δ_B represents the manager's survival rate.

$$N_{B,t} = w^e_{B,t}H^e_{B,t} + \delta_B \tilde{A}_{Bt}(N_{B,t-1}+B_{B,t})(1-cost_t^E)f(\bar{\omega}_{B,t}) \tag{27}$$

The problem of the green sector is identical with zero environmental policy cost, $cost_t^E = 0$.

E Monetary Authority and Market Clearing Conditions

The monetary authority manages the standard Taylor rule, the green lending rule, and the macroprudential rate. However, for this paper, we maintain a fixed response in the macroprudential rate.

$$\ln\left(\frac{R_t^{env}}{R_{ss}}\right) = \rho_r ln\left(\frac{R_{t-1}}{R_{ss}}\right) + (1-\rho_r)\left(\rho_y ln\left(\frac{Y_t}{Y_{t-1}}\right) + \rho_\pi ln\left(\frac{\Pi_t}{\Pi_{ss}}\right) + \rho_e ln\left(\frac{e_t}{e_{ss}}\right)\right) \tag{28}$$

$$\ln\left(\frac{R_t}{R_{ss}}\right) = \rho_r ln\left(\frac{R_{t-1}}{R_{ss}}\right) + (1-\rho_r)\left(\rho_y ln\left(\frac{Y_t}{Y_{t-1}}\right) + \rho_\pi ln\left(\frac{\Pi_t}{\Pi_{ss}}\right)\right) \tag{29}$$

$$\tau_t^i = \tau_{ss} \tag{30}$$

$$\begin{aligned} Y_t = C_t + I_t + G_t + \theta_1 \mu_t^{\theta_2} y_{B,t} + \frac{\kappa_P}{2}(\pi_t - \bar{\pi})^2 Y_t + \tilde{A}_{G,t}(\frac{n_{G,t-1}}{\pi_t} + b_{G,t})m_g \int_0^{\omega_{\bar{G},t}} \omega\, dF(\omega) \\ + \tilde{A}_{B,t}(\frac{n_{B,t-1}}{\pi_t} + b_{B,t})(1-cost_t^E)m_b \int_0^{\omega_{\bar{B},t}} \omega\, dF(\omega) \end{aligned} \tag{31}$$

G_t denotes autonomous government spending. Capital and labour market equilibrium necessitate:

$$K_{t-1} = K_{G,t} + K_{B,t}, \tag{32}$$

$$H_t = H_{G,t} + H_{B,t} \tag{33}$$

The loans market clearing implies:

$$\frac{B_{G,t}}{(1-\tau_t)} + \frac{B_{B,t}}{(1-\tau_t)} = D_t \tag{34}$$

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