

Measurement of $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections at center-of-mass energies from 2.00 to 3.08 GeV

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Using data corresponding to an integrated luminosity of 651 pb^{-1} accumulated at 22 center-of-mass energies from 2.00 to 3.08 GeV by the BESIII experiment, the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ is studied. The cross sections for $e^+e^- \rightarrow \phi\pi^+\pi^-$ are consistent with previous results, but with improved precision. A combine fit to the cross section line shape is performed, which reveals contributions from two structures: the first one has a mass of $M = 2174 \pm 23 \pm 4 \text{ MeV}/c^2$ and a width of $\Gamma = 207 \pm 49 \pm 5 \text{ MeV}$ and the second one has a mass of $M = 2276 \pm 42 \pm 6 \text{ MeV}/c^2$ and a width of $\Gamma = 320 \pm 112 \pm 6 \text{ MeV}$, where the first uncertainties are statistical and the second systematic.

I. INTRODUCTION

The study of the hadron spectrum is important to understand the non-perturbative behavior of quantum chromodynamics (QCD). For the low-energy region, the vector mesons ρ , ω , ϕ and their low-lying excited states are copiously produced in e^+e^- collision experiments. The experimental results for these states have been tabulated by the Particle Data Group (PDG) [1], but the higher lying excitations are not fully identified yet, especially in the region around 2.0 GeV. Further measurements are needed to resolve the situation involving resonances such as the $\rho(2000)$, $\rho(2150)$ and $\phi(2170)$ states.

The $\phi(2170)$ resonance was first observed by the BABAR Collaboration via the initial-state-radiation (ISR) process $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$ [2, 3], and later confirmed by the Belle, BESII, and BESIII experiments [4–8]. This observation stimulated speculation that the $\phi(2170)$ resonance might be a strangeonium counterpart of the charmonium resonance $\psi(4260)$ due to similarities in their

production and decay pattern [9]. Considerable efforts have been made theoretically to understand the nature of the $\phi(2170)$ resonance and abundant interpretations have been proposed, including a traditional $s\bar{s}$ state [10–15], an $s\bar{s}g$ hybrid [9, 16], an $ss\bar{s}\bar{s}$ tetra-quark state [17–24], a $\Lambda\bar{\Lambda}$ bound state [25–29] and an ordinary $\phi K\bar{K}$ or $\phi f_0(980)$ resonance produced by interactions between the final state particles [30, 31]. The model predictions differ in both mass and width of the resonance. Further experimental studies are therefore crucial to clarify its nature.

Though many experiments have been carried out to study the $\phi(2170)$ resonance [2–8, 32], the results of the measurements vary substantially. For example, the mass and width of the $\phi(2170)$ resonance obtained from the process $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi\pi^+\pi^-$ [5] shows smaller values than other experimental measurements. Recently, more studies related to the $\phi(2170)$ resonance have been carried out by the BESIII experiment. A partial-wave analysis of the $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ process [33] found that

the partial widths of the $\phi(2170)$ resonance are sizable for the $K(1460)^+K^-$, $K_1(1400)^+K^-$, and $K_1(1270)^+K^-$ decay channels, but much smaller for $K^*(892)^+K^*(892)^-$ and $K^*(1410)^+K^-$. Several theoretical expectations are challenged by the results according to Ref. [10]. Attempts have also been made to study channels with simpler topologies, including the processes $e^+e^- \rightarrow K^+K^-$, where a resonance with a mass of $(2239.2 \pm 7.1 \pm 11.3)$ MeV/ c^2 and a width of $(139.8 \pm 12.3 \pm 20.6)$ MeV is seen [34]; $e^+e^- \rightarrow \phi K^+K^-$ [35], where a sharp enhancement is observed in the Born cross section line-shape at a center-of-mass (c.m.) energy of $\sqrt{s} = 2.2324$ GeV; $e^+e^- \rightarrow \phi\eta'$ [36], where a resonance with a mass of $(2177.5 \pm 5.1 \pm 18.6)$ MeV/ c^2 and a width of $(149.0 \pm 15.6 \pm 8.9)$ MeV is seen; $e^+e^- \rightarrow \omega\eta$ [37], a resonance with a mass of $(2179 \pm 21 \pm 3)$ MeV/ c^2 and a width of $(89 \pm 28 \pm 5)$ MeV is observed with a significance of 6.1σ ; $e^+e^- \rightarrow \phi\eta$ [38], a resonant structure is observed with parameters determined to be $M = (2163.5 \pm 6.2 \pm 3.0)$ MeV/ c^2 and $\Gamma = (31.1_{-11.6}^{+21.1} \pm 1.1)$ MeV; and $e^+e^- \rightarrow K_S^0 K_L^0$ [39], a resonant structure around 2.2 GeV is observed, with a mass and width of $2273.7 \pm 5.7 \pm 19.3$ MeV/ c^2 and $86 \pm 44 \pm 51$ MeV respectively. The Breit-Wigner parameters of $\phi(2170)$ are not consistent between the different studies, especially concerning the width.

In addition, a resonance-like structure which we called R(2400) might exist around 2.4 GeV in the $\phi\pi^+\pi^-$ cross section line-shape. The R(2400) was first studied by the Belle [5] experiment. Later, Shen and Yuan [40] performed a fit to the R(2400) structure using the combined data of the Belle and *BABAR* experiments. The mass and the width are determined to be (2436 ± 26) MeV/ c^2 and (121 ± 35) MeV, respectively. However, its statistical significance is less than 3σ . An interpretation is proposed for R(2400) as a partner state of the $\phi(2170)$ resonance [41]. Therefore, a precise measurement of $e^+e^- \rightarrow \phi\pi^+\pi^-$ is desirable to establish the mass and width of the $\phi(2170)$ resonance and to search for the possible structure near 2.4 GeV.

In this paper, the measurement of cross sections for the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ at 22 c.m. energies (\sqrt{s}) is reported from 2.00 to 3.08 GeV.

II. DETECTOR AND DATA SAMPLES

The BESIII detector [42] records symmetric e^+e^- collisions provided by the BEPCII storage ring [43], which operates with a peak luminosity of 1×10^{33} cm $^{-2}$ s $^{-1}$ in the c.m. energy range between 2.0000 and 4.9000 GeV. BESIII has collected large data samples in this energy region [44]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in

a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/ c is 0.5%, and the dE/dx resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end-cap region is 110 ps. The end-cap TOF system was upgraded in 2015 using multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [45–47].

Simulated Monte Carlo (MC) samples of signal and background processes are produced to optimize the event selection criteria, determine the detection efficiency and estimate the background contamination. The response of the detector is reproduced using a GEANT4-based [48] MC simulation software package, which includes the geometric and material description of the BESIII detector, the detector response and digitization models.

Background samples of QED processes are produced with the BABAYAGA [49] generator and inclusive hadronic processes are generated with the LUARLW [50] generator. Signal MC samples of the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ with one million events are generated according to the result of a Partial Wave Analysis (PWA) at each energy point (detailed in Sec. V). The signal MC samples are used to determine the reconstruction efficiency, and the correction factors for ISR and vacuum polarization (VP).

III. EVENT SELECTION AND BACKGROUND ANALYSIS

Signal events of the $e^+e^- \rightarrow \phi\pi^+\pi^-$ process are reconstructed via the $\phi \rightarrow K^+K^-$ decay. Charged track candidates are reconstructed from hits in the MDC and need to satisfy $|\cos\theta| < 0.93$, where θ is the polar angle with respect to the symmetry axis of the MDC. The closest approach to the interaction point is required to be less than 10 cm along the symmetry axis and less than 1 cm in the perpendicular plane. Combined TOF and dE/dx information is used to perform the particle identification (PID), obtaining probabilities for the π , K and p hypotheses. The particle type with the largest probability is assigned to each track. Since the tracking efficiency decreases sharply in the low momentum region below 0.5 GeV/ c , and most kaon candidates are expected to have a low momentum, one kaon is allowed to be missing in this study to increase the selection efficiency. Including events with one missing kaon increases number of signal events by factor of 3 at 2.00 GeV and 30% at 3.08 GeV. A candidate event is, therefore, expected to have two pions and at least one kaon reconstructed.

A vertex fit to the $\pi^+\pi^-K^\pm$ combination is then ap-

plied and required to have converged for an event to be kept for further analysis. For events with four charged tracks, both $\pi^+\pi^-K^+$ and $\pi^+\pi^-K^-$ combinations are tested. Under the hypothesis that one kaon is missing, a one-constraint (1C) kinematic fit is performed to the combinations that are kept after the vertex fit. For each event, the $\pi^+\pi^-K^\pm$ combination with the smallest χ^2 of the 1C kinematic fit ($\chi_{1C}^2(\pi^+\pi^-KK_{miss})$) is retained. Finally, events with $\chi_{1C}^2 \geq 10$ are rejected. After applying the selection criteria, we use the momenta of the particles obtained from the kinematic fit in the further analysis.

Events passing the selection criteria described above are shown in Fig. 1 for the data at $\sqrt{s} = 2.1250$ GeV. The invariant mass of the K^+K^- pairs shows a clear signal band around the ϕ mass. The enhancement around 0.98 GeV/c^2 in the $\pi^+\pi^-$ invariant mass indicates a correlation between $f_0(980)$ and ϕ production due to the process $e^+e^- \rightarrow \phi f_0(980)$.

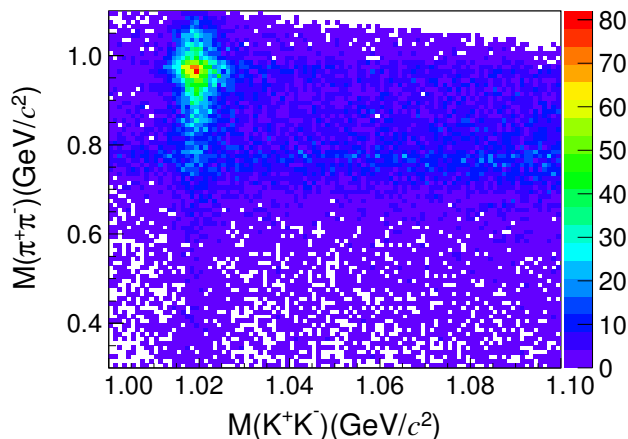


FIG. 1: Distribution of $M_{\pi^+\pi^-}$ versus $M_{K^+K^-}$ for the data at $\sqrt{s} = 2.1250$ GeV.

The distribution of the K^+K^- invariant mass is shown in Fig. 2. The range of $|M_{K^+K^-} - m_\phi| < 0.01$ GeV/c^2 is regarded as the signal region in the following study, where $m_\phi = 1019.461$ MeV/c^2 is the world average ϕ mass from the PDG [1]. The sideband regions, defined as $[0.995, 1.005]$ and $[1.035, 1.045]$ GeV/c^2 , are used to study non- ϕ background contributions.

An accumulation of events exists around the mass of the ρ meson in Fig 1. This indicates a non-negligible background contribution from the $e^+e^- \rightarrow \rho K^+K^-$ process. Based on a study of the ϕ sideband and an analysis of the inclusive MC sample, the $e^+e^- \rightarrow K^*(892)K^\pm\pi^\pm$ process is found to be the dominant background source. Peaking background in the ϕ signal region is negligible.

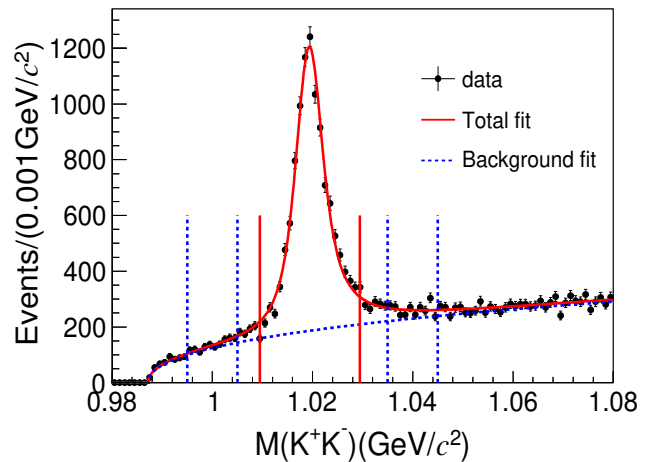


FIG. 2: Fit to the $M_{K^+K^-}$ distribution for the data at $\sqrt{s} = 2.1250$ GeV: the signal is described by a P-wave Breit-Wigner (BW) function convolved with a Gaussian function, and the background is described by a reversed ARGUS function. The range between the two red vertical solid lines is regarded as signal region, and the ranges between the two blue vertical dashed lines on each side of the signal peak are regarded as the sideband regions.

IV. SIGNAL YIELDS

The $e^+e^- \rightarrow \phi\pi^+\pi^-$ signal yields are obtained from unbinned maximum likelihood fits to the K^+K^- invariant mass in the region $[2 \cdot m_{K^\pm}, 1.08]$ GeV/c^2 . In the fit, the ϕ peak is modeled as a P-wave BW function convolved with a Gaussian function to account for a difference in detector resolution and an offset in calibration between data and the MC simulation [35]. The P-wave BW function is defined in the form

$$f(m) = |A(m)|^2 \cdot p^{2l+1}, \quad (1)$$

$$A(m) = \frac{1}{m^2 - m_\phi^2 + im\Gamma(m)} \frac{B(p)}{B(p')}, \quad (2)$$

$$\Gamma(m) = \left(\frac{p}{p'}\right)^{2l+1} \left(\frac{m_\phi}{m}\right) \Gamma_\phi \left[\frac{B(p)}{B(p')}\right], \quad (3)$$

$$B(p) = \frac{1}{\sqrt{1 + (Rp)^2}}, \quad (4)$$

where p is the momentum of the kaon in the rest frame of the K^+K^- system, p' is the momentum of the kaon at the ϕ peak mass, and Γ_ϕ is the width of the ϕ resonance [1]. The angular momentum (l) is equal to one, $B(p)$ is the

Blatt-Weisskopf form factor, and $R = 3 \text{ GeV}^{-1}$ is the radius of the centrifugal barrier [51].

Since no peaking background is expected in the signal area, the background is parameterized with a reversed ARGUS function [52]. The parameters of the Gaussian function and the reversed ARGUS function are determined in a fit to the data. The fit result for the c.m. energy $\sqrt{s} = 2.1250 \text{ GeV}$ is shown in Fig. 2. We obtain a similar fit quality for all c.m. energies.

V. CROSS SECTION CALCULATION

The dressed cross section of the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ is calculated by:

$$\sigma^{\text{D}} = \frac{N^{\text{obs}}}{\mathcal{L} \cdot (1 + \delta\gamma) \cdot \epsilon \cdot \mathcal{B}}, \quad (5)$$

where N^{obs} is the signal yield; \mathcal{L} is the integrated luminosity; $(1 + \delta\gamma)$ is the initial state radiation (ISR) correction factor; ϵ is the detection efficiency and \mathcal{B} is the branching fraction of the decay $\phi \rightarrow K^+K^-$. The ISR correction factor is handled by generator CONEXC [53], depending on the input cross sections. The lowest-order Born cross section is defined as $\sigma^{\text{B}} = \sigma^{\text{D}}/(1/|1 - \Pi|^2)$, where $1/|1 - \Pi|^2$ is the vacuum polarization (VP) correction factor, available from F. Jegerlehner group [54].

To adequately describe the data in our MC simulation, the PWA MC sample is generated at each energy point. For a few low-statistic points, the PWA parameters obtained at the most adjacent high-statistic energies are applied. The quasi-two-body decay amplitudes in the sequential decays are constructed using covariant tensor amplitudes [55]. The $e^+e^- \rightarrow \phi\pi^+\pi^-$ process is found to be well described by four subprocesses: $e^+e^- \rightarrow \phi f_0(980)$, $\phi\sigma$, $\phi f_0(1370)$ and $\phi f_2(1270)$. The intermediate states are parametrized with relativistic BW functions, except for the σ and $f_0(980)$, which are described with using the model described in [56] and by a Flatté formula [57], respectively. The resonance parameters of the $f_0(980)$ and the wide resonance σ in the fit are fixed to those in Ref. [57] and Refs. [56, 57], respectively, and those of other intermediate states are fixed to the PDG values. The relative magnitudes and phases of the individual intermediate processes are determined by performing an unbinned maximum likelihood fit using MINUIT [58]. To describe the background below the ϕ peak, sideband events are added to the likelihood with negative weights.

The signal MC simulation based on the PWA results has reasonable agreement with the experimental data at all c.m. energies. The comparison of the MC simulation and experimental data in the signal region for the

$M(\pi^+\pi^-)$ distribution at $\sqrt{s} = 2.1250 \text{ GeV}$ is shown in Fig. 3.

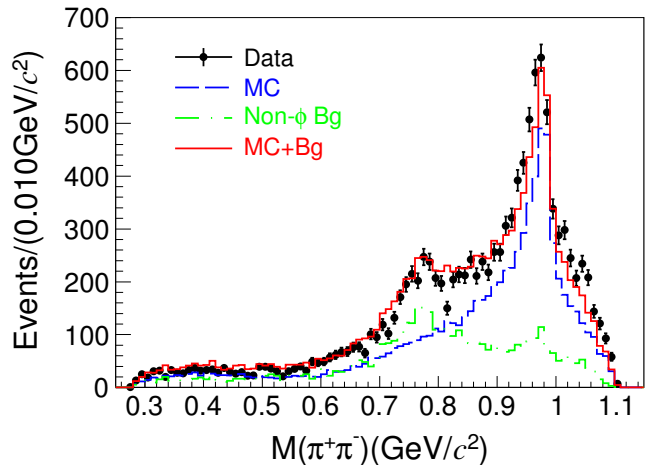


FIG. 3: Invariant mass distribution of the $\pi^+\pi^-$ candidates for the data at $\sqrt{s} = 2.1250 \text{ GeV}$. The black dots with uncertainties are experimental data, the blue dashed line is the signal MC distribution following the PWA solution, the green dash-dotted line is the non- ϕ background estimated from the ϕ sideband region and the red solid line is the sum of the former two distributions.

The efficiency ϵ and the ISR correction factor $(1 + \delta\gamma)$ depend on the input cross section line-shape and need to be determined using an iterative procedure. The BABAR result [6] is used as the initial input cross section and the updated cross section is obtained through the resulting MC simulation. This procedure is repeated until the measured cross section converges. Dressed cross sections for $e^+e^- \rightarrow \phi\pi^+\pi^-$ at each energy point are listed in Table V, together with Born cross sections and vacuum polarization correction factors. Dressed cross sections are shown in Fig. 4.

VI. SYSTEMATIC UNCERTAINTY

Systematic uncertainties in the cross section measurement come from the luminosity measurement, tracking efficiency, PID efficiency, kinematic fit, signal and background shape, fitting range, radiative correction, MC sample size and the branching fraction of the decay $\phi \rightarrow K^+K^-$.

1. The integrated luminosity is measured using large angle Bhabha events, with a signal uncertainty of 1.0% [59].
2. The tracking efficiency uncertainty is estimated to be 1.0% for each track [35]. Thus, 3.0% is taken as the systematic uncertainty for the two pion and one kaon tracks.

TABLE I: Cross sections of the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ at different c.m. energies. The \sqrt{s} is the c.m. energy; \mathcal{L} is the integrated luminosity; N^{obs} is the yield of signal events; $1 + \delta^\gamma$ is the ISR correction factor; $1/[1 - \Pi]^2$ is the VP correction factor; ϵ is the detection efficiency; σ^{B} is the Born cross section and σ^{D} is the dressed cross section. The first uncertainty for the cross section is statistical and the second systematic, where the systematic uncertainty is obtained as a sum in quadrature of all correlated and uncorrelated parts.

\sqrt{s} (GeV)	\mathcal{L} (pb $^{-1}$)	N^{obs}	$(1 + \delta^\gamma)$	$1/[1 - \Pi]^2$	ϵ	σ^{B} (pb)	σ^{D} (pb)
2.0000	10.1 \pm 0.1	577.0 \pm 46.8	0.98	1.04	0.34	342.1 \pm 27.7 \pm 24.7	354.7 \pm 28.8 \pm 25.6
2.0500	3.34 \pm 0.03	191.5 \pm 24.6	0.96	1.04	0.38	309.4 \pm 39.7 \pm 20.4	321.2 \pm 41.3 \pm 21.1
2.1000	12.2 \pm 0.1	1100.7 \pm 51.1	0.95	1.04	0.40	470.0 \pm 21.8 \pm 24.1	488.3 \pm 22.7 \pm 25.0
2.1250	108 \pm 1	9372.1 \pm 144.1	0.95	1.04	0.42	427.9 \pm 6.6 \pm 23.0	444.6 \pm 6.8 \pm 23.9
2.1500	2.84 \pm 0.02	220.0 \pm 20.0	0.96	1.04	0.37	419.9 \pm 38.2 \pm 31.7	436.7 \pm 39.7 \pm 33.0
2.1750	10.6 \pm 0.1	760.8 \pm 39.1	1.00	1.04	0.37	372.3 \pm 19.1 \pm 23.9	387.2 \pm 19.9 \pm 24.9
2.2000	13.7 \pm 0.1	706.1 \pm 38.3	1.08	1.04	0.37	257.0 \pm 13.9 \pm 19.6	267.2 \pm 14.5 \pm 20.4
2.2324	11.9 \pm 0.1	435.2 \pm 29.5	1.19	1.04	0.33	180.6 \pm 12.2 \pm 14.8	188.0 \pm 12.7 \pm 15.4
2.3094	21.1 \pm 0.1	587.3 \pm 37.7	1.19	1.04	0.36	127.5 \pm 8.2 \pm 6.6	132.7 \pm 8.5 \pm 6.8
2.3864	22.5 \pm 0.2	697.3 \pm 37.0	1.13	1.04	0.39	137.6 \pm 7.3 \pm 8.2	143.3 \pm 7.6 \pm 8.5
2.3960	66.9 \pm 0.5	1977.7 \pm 65.5	1.13	1.04	0.39	131.1 \pm 4.3 \pm 7.8	136.5 \pm 4.5 \pm 8.1
2.5000	1.10 \pm 0.01	18.7 \pm 5.2	1.21	1.04	0.38	72.8 \pm 20.2 \pm 4.8	75.7 \pm 21.1 \pm 5.0
2.6444	33.7 \pm 0.2	501.0 \pm 33.2	1.31	1.04	0.34	65.9 \pm 4.4 \pm 4.8	68.5 \pm 4.5 \pm 5.0
2.6464	34.0 \pm 0.3	423.4 \pm 29.8	1.31	1.04	0.34	55.3 \pm 3.9 \pm 4.0	57.4 \pm 4.0 \pm 4.1
2.7000	1.03 \pm 0.01	22.0 \pm 6.1	1.64	1.04	0.33	77.4 \pm 21.5 \pm 5.5	80.4 \pm 22.3 \pm 5.7
2.8000	1.01 \pm 0.01	11.6 \pm 4.1	1.45	1.04	0.31	49.8 \pm 17.6 \pm 3.0	51.6 \pm 18.2 \pm 3.1
2.9000	105 \pm 1	687.0 \pm 37.7	1.45	1.03	0.30	29.3 \pm 1.6 \pm 1.8	30.3 \pm 1.7 \pm 1.8
2.9500	15.9 \pm 0.1	114.4 \pm 14.5	1.47	1.03	0.30	32.7 \pm 4.1 \pm 2.5	33.6 \pm 4.3 \pm 2.5
2.9810	16.1 \pm 0.1	72.4 \pm 15.2	1.48	1.02	0.30	20.4 \pm 4.3 \pm 1.5	20.9 \pm 4.4 \pm 1.5
3.0000	15.9 \pm 0.1	74.6 \pm 13.4	1.49	1.02	0.29	21.8 \pm 3.9 \pm 1.9	22.3 \pm 4.0 \pm 1.9
3.0200	17.3 \pm 0.1	78.2 \pm 12.2	1.49	1.01	0.29	21.1 \pm 3.3 \pm 1.6	21.4 \pm 3.3 \pm 1.6
3.0800	126 \pm 1	576.9 \pm 34.8	1.59	0.92	0.27	23.5 \pm 1.4 \pm 1.5	21.5 \pm 1.3 \pm 1.3

- The PID efficiency uncertainty is estimated to be 1.0% per π^\pm and 1.0% per K^\pm [35]. So 3.0% is taken as the systematic uncertainty on the PID efficiency.
- The uncertainty in $\mathcal{B}(\phi \rightarrow K^+K^-)$ is taken from the PDG [1].
- The uncertainty from the kinematic fit comes from the inconsistency between the data and MC simulation of the helix parameters. Following the procedure described in Ref. [60], the helix parameters for the charged tracks of MC samples are corrected to eliminate the inconsistency during uncertainty study. The agreement of χ^2 distributions between data and the MC simulation is significantly improved. Half of the difference between the selection efficiencies with and without the helix parameter correction is taken as the systematic uncertainty.
- Uncertainties due to the choice of signal shape, background shape and fitting range are estimated by introducing the changes below. The ϕ signal is described by a P-wave BW function convolved with a Gaussian function. To estimate the signal shape uncertainty, the signal shape is changed to the shape from the signal MC simulation convolved with a Gaussian function and the resulting difference is taken as the uncertainty from the

signal model. To estimate the background model uncertainty the background function is modified from a reversed ARGUS function to the function of $f(M) = (M - M_a)^c (M_b - M)^d$, where M_a and M_b are the lower and upper edges of the mass distribution while c and d are the parameters which were determined in the fit. The fit range is extended from [0.98, 1.08] GeV/ c^2 to [0.98, 1.10] GeV/ c^2 to estimate the fit-range uncertainty. The differences between the number of signal events before and after the changes are taken as the systematic uncertainties.

- Uncertainties in the possible distortions of the cross section line-shape introduce systematic uncertainties in the radiative correction factor and the efficiency. These are estimated by using the cross section line-shape function $\sigma = \sigma(\sqrt{s}; p_1, p_2, \dots)$ obtained from the iteration described in Sec. V, where p_i ($i = 1, 2, \dots$) are the parameters which are determined in the fit. All parameters are randomly varied within their uncertainties and the resulting parametrization of the line-shape is used to recalculate $(1 + \delta)$, ϵ and the corresponding cross sections. This procedure is repeated 1000 times and the standard deviation of the resulting cross sections is taken as a systematic uncertainty.

8. The uncertainty from the MC sample size is estimated by the number of generated events.

The first four sources of uncertainty are correlated between different energies, while others are uncorrelated. In the fit of cross section lineshapes, the minimized χ^2 is constructed by incorporating both statistical and systematic uncertainties considering the correlated and uncorrelated terms as formula

$$\chi^2 = \Delta X^T M^{-1} \Delta X, \quad (6)$$

where ΔX is the difference between the measured cross section and the expected value calculated by function at each c.m. energy. The M is the covariance matrix of elements

$$M_{ij} = \begin{cases} (\Delta_i^{\text{sys}})^2 + (\Delta_i^{\text{sta}})^2, & i = j \\ (\sigma_i \cdot \varepsilon_s) \cdot (\sigma_j \cdot \varepsilon_s), & i \neq j \end{cases}, \quad (7)$$

where the index $i(j)$ represents the $i(j)$ -th data set; Δ_i^{sta} is the asymmetrically statistical uncertainty for i -th data set; Δ_i^{sys} is the total systematic uncertainty and ε_s is the relative correlated systematic uncertainty.

VII. LINE-SHAPE FITTING

The measured cross sections are shown in Fig. 4 together with results from the *BABAR* and Belle experiments. Clear structures can be seen around 2.1 and 2.4 GeV, where the first is identified as the $\phi(2170)$ resonance and the second is denoted as $R(2400)$. By using two BWs, a fit to $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections, with previous results and results from this research as combined data, is performed to parameterize $\phi(2170)$. The form of BW is taken as:

$$\text{BW}(\sqrt{s}) = \frac{M_r}{\sqrt{s}} \frac{\sqrt{12\pi}\Gamma_{ee}Br\Gamma_r}{s - M^2 + iM_r\Gamma(\sqrt{s})} \sqrt{\frac{\Phi(\sqrt{s})}{\Phi(M_r)}}, \quad (8)$$

where M_r and Γ_r are the mass and width of the resonant structure; $\Gamma_{ee}Br$ is the electric partial width times the branching fraction of the resonance decaying to corresponding intermediate states; Φ is the phase space factor calculated from MC integral using PWA amplitude and $\Gamma(\sqrt{s}) = \Gamma_r \cdot (\Phi(\sqrt{s})/\Phi(M_r))$ is energy-dependent width. The total cross section is taken as:

$$\sigma(\sqrt{s}) = \left| \sum \text{BW}(\sqrt{s})\phi_P \right|^2, \quad (9)$$

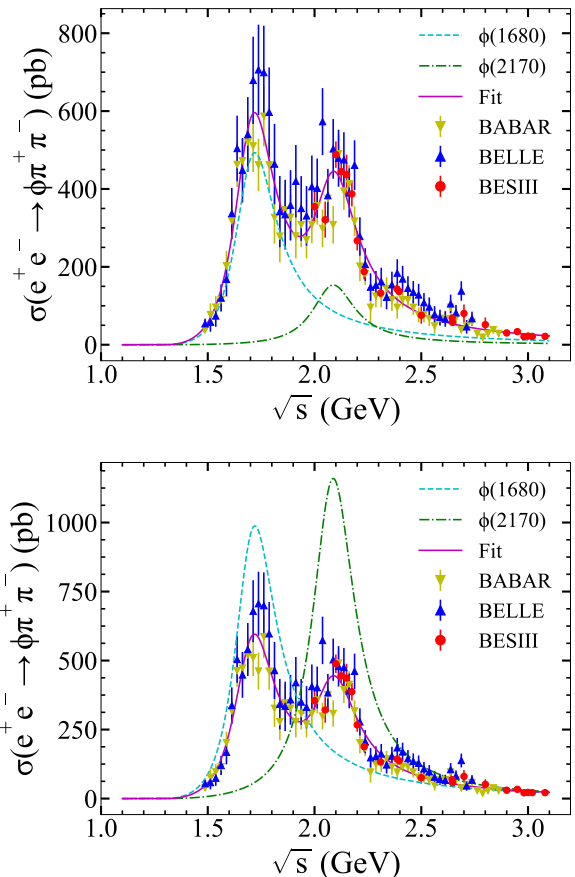


FIG. 4: Dressed cross sections of the process $e^+e^- \rightarrow \phi\pi^+\pi^-$. Red dots with error bars represent results from BESIII. Blue upside and yellow downside triangles with error bars represent results from Belle and *BABAR*. Purple solid line represents total fit function. Cyan and green dashed line represent contributions from $\phi(1680)$ and $\phi(2170)$. The first figure shows the construction interference of two multi-solutions, while second figure shows the destruction one.

TABLE II: Fit result with two BWs to fit the $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections, where mass is reported in MeV/c^2 ; width is reported in MeV; $Br\Gamma_{e^+e^-}$ is reported in eV and ϕ_P is phase angle between two BWs.

Parameters	Solution I	Solution II
$M_r(\phi(1680))$		1694 ± 8
$\Gamma_r(\phi(1680))$		227 ± 32
$Br\Gamma_{ee}(\phi(1680))$	21.8 ± 1.3	43.6 ± 1.5
$M_r(\phi(2170))$		2076 ± 10
$\Gamma_r(\phi(2170))$		243 ± 21
$Br\Gamma_{ee}(\phi(2170))$	10.8 ± 1.0	81.9 ± 5.5
$\phi_P(\phi(2170)/\phi(1680))$	0.75 ± 0.14	-2.07 ± 0.05

where ϕ_P is the phase angle between BWs. The fit result is shown in Fig. 4 and Table II.

Two multi-solutions are found with equal fit quality of $\chi^2/\text{ndf} = 193/122$, where ndf is the number of degrees of freedom. The statistical significance of $\phi(1680)$ and $\phi(2170)$ are both larger than 10σ . To further parameterize R(2400), a fit with three BWs is carried out with a BW for R(2400). The fit result is shown in Fig. 5 and Table III.

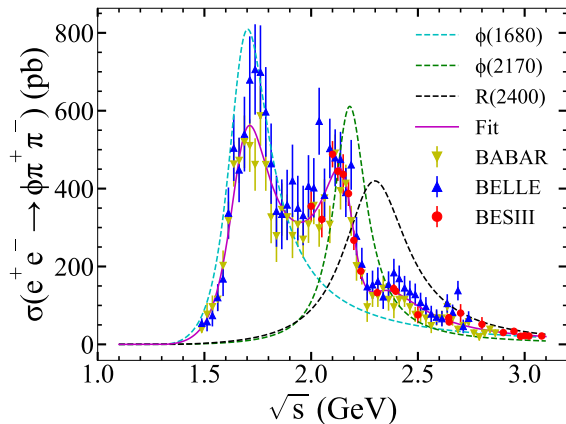


FIG. 5: The fit to cross sections of the process $e^+e^- \rightarrow \phi\pi^+\pi^-$ with three BWs. Red dots with error bars represent results from BESIII. Blue upside and yellow downside triangles with error bars represent results from *BABAR* and Belle. Purple solid line represents total fit result. Cyan, green and black dashed line represent contributions from $\phi(1680)$, $\phi(2170)$ and R(2400). Only one of four parallel multi-solutions are shown.

Four multi-solutions are found with equal fit quality of $\chi^2/\text{ndf} = 118/118$. The statistical significance of R(2400) is found to be 8.0σ . The systematic uncertainties of the resonance parameters mainly come from the cross section fitting. The uncertainties associated with the fit procedure include effects from the choice of $\phi(1680)$ model. To estimate the uncertainty from the choice of $\phi(1680)$ model, an alternative fit is carried out by fixing the mass and width of $\phi(1680)$ to the PDG average parameters [1]. The difference between the nominal and alternative fit result is considered as the systematic uncertainty. The systematic uncertainties of the $\phi(2170)$ resonance mass and width are obtained to be 4 MeV/ c^2 and 5 MeV, while for R(2400) mass and width we find 6 MeV/ c^2 and 6 MeV, respectively.

VIII. CONCLUSION

In summary, the cross sections of the $e^+e^- \rightarrow \phi\pi^+\pi^-$ process is measured using data samples collected with the BESIII detector at 22 c.m. energies from 2.00

GeV to 3.08 GeV. The measured cross section is consistent with previous results from the *BABAR* [6], Belle [5] and BESIII [61] experiments, but with improved precision.

For the cross section line shape, a least χ^2 fit is performed to study possible resonant contributions. The $\phi(2170)$ resonance is observed with a mass of $M = 2174 \pm 23 \pm 4$ MeV/ c^2 and a width of $\Gamma = 207 \pm 49 \pm 5$ MeV, where these first uncertainties are statistical and the second systematic. The central value of the $\phi(2170)$ width obtained in this work is consistent with existing results [4–8].

In addition, another structure R(2400) is observed at around 2.4 GeV. Comparing our fit with and without a Breit-Wigner to account for the R(2400), a statistical significance of 8.0σ is found in favor of including the third Breit-Wigner resonance. It is parameterized with a mass of $M = 2276 \pm 42 \pm 6$ MeV/ c^2 and a width of $\Gamma = 320 \pm 112 \pm 6$ MeV, where the first uncertainties are statistical and the second systematic. The uncertainties of the R(2400) parameters are large in this work. This is a result of the lack of energy points in the 2.4 GeV region. Collecting additional data samples in this region will help to reduce these uncertainties. Moreover, since the structure at 2.4 GeV has also been seen in the $K^+K^-f_0(980)$ mode with $f_0(980) \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ [2, 3], a future study of this channel with a more robust amplitude analysis will be helpful to improve knowledge of the R(2400) state.

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TABLE III: Fit result with three BWs to fit the $e^+e^- \rightarrow \phi\pi^+\pi^-$ cross sections, where mass is reported in MeV/c^2 ; width is reported in MeV ; $Br\Gamma_{e^+e^-}$ is reported in eV and ϕ_P is phase angle between two BWs.

Parameters	Solution I	Solution II	Solution III	Solution IV
$M_r(\phi(1680))$			1675 ± 7	
$\Gamma_r(\phi(1680))$			238 ± 29	
$Br\Gamma_{ee}(\phi(1680))$	35.3 ± 1.5	39.0 ± 1.3	20.7 ± 1.3	18.8 ± 0.9
$M_r(\phi(2170))$			2174 ± 23	
$\Gamma_r(\phi(2170))$			207 ± 49	
$Br\Gamma_{ee}(\phi(2170))$	60.2 ± 34.5	329 ± 211	123 ± 83	22.9 ± 12.6
$\phi_P(\phi(2170)/\phi(1680))$	0.75 ± 0.14	-0.99 ± 0.60	2.90 ± 0.87	2.46 ± 0.68
$M_r(R(2400))$			2276 ± 42	
$\Gamma_r(R(2400))$			320 ± 112	
$Br\Gamma_{ee}(R(2400))$	50.7 ± 35.7	350 ± 247	112 ± 82	15.6 ± 9.0
$\phi_P(R(2400)/\phi(1680))$	-2.92 ± 0.65	2.91 ± 0.52	0.03 ± 0.27	0.56 ± 0.84

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- [1] R. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [2] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **74**, 091103 (2006).
- [3] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **76**, 012008 (2007).
- [4] M. Ablikim *et al.* (BES Collaboration), *Phys. Rev. Lett.* **100**, 102003 (2008).
- [5] C. P. Shen *et al.* (Belle Collaboration), *Phys. Rev. D* **80**, 031101 (2009).
- [6] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **86**, 012008 (2012).
- [7] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **91**, 052017 (2015).
- [8] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **99**, 012014 (2019).
- [9] G. J. Ding and M. L. Yan, *Phys. Lett. B* **650**, 390 (2007).
- [10] G. J. Ding and M. L. Yan, *Phys. Lett. B* **657**, 49 (2007).
- [11] X. Wang *et al.*, *Phys. Rev. D* **85**, 074024 (2012).
- [12] S. S. Afonin and I. V. Pusenkov, *Phys. Rev. D* **90**, 094020 (2014).
- [13] C. Q. Pang, *Phys. Rev. D* **99**, 074015 (2019).
- [14] C. G. Zhao *et al.*, *Phys. Rev. D* **99**, 114014 (2019).
- [15] Q. Li *et al.*, *Chin. Phys. C* **45**, 023116 (2021).
- [16] J. Ho *et al.*, *Phys. Rev. D* **100**, 034012 (2019).
- [17] Z. G. Wang, *Nucl. Phys. A* **791**, 106 (2007).
- [18] H. X. Chen *et al.*, *Phys. Rev. D* **78**, 034012 (2008).
- [19] N. V. Drenska *et al.*, *Phys. Lett. B* **669**, 160 (2008).
- [20] C. R. Deng *et al.*, *Phys. Rev. D* **82**, 074001 (2010).
- [21] H. W. Ke and X. Q. Li, *Phys. Rev. D* **99**, 036014 (2019).
- [22] S. S. Agaev *et al.*, *Phys. Rev. D* **101**, 074012 (2020).
- [23] R. R. Dong *et al.*, *Eur. Phys. J. C* **749** (2020).
- [24] F. X. Liu *et al.*, *Phys. Rev. D* **103** (2021), 10.1103/physrevd.103.016016.
- [25] E. Klempt and A. Zaitsev, *Phys. Rep.* **454**, 1 (2007).
- [26] L. Zhao *et al.*, *Phys. Rev. D* **87**, 054034 (2013).
- [27] C. Deng *et al.*, *Phys. Rev. D* **88**, 074007 (2013).
- [28] Y. B. Dong *et al.*, *Phys. Rev. D* **96**, 074027 (2017).
- [29] D. Y. C. Y. L. Yang and Z. Lu, *Phys. Rev. D* **100**, 073007 (2019).
- [30] A. M. Torres *et al.*, *Phys. Rev. D* **78**, 074031 (2008).
- [31] S. Gomez-Avila *et al.*, *Phys. Rev. D* **79**, 034018 (2009).
- [32] B. Aubert *et al.*, *Phys. Rev. D* **77**, 092002 (2008).
- [33] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **124**, 112001 (2020).
- [34] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **99**, 032001 (2019).
- [35] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **100**, 032009 (2019).
- [36] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **102**, 012008 (2020).
- [37] M. Ablikim *et al.*, *Phys. Lett. B* **813**, 136059 (2021).
- [38] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **104**, 032007 (2021).
- [39] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **104**, 092014 (2021).
- [40] C. P. Shen and C. Z. Yuan, *Chin. Phys. C* **34**, 1045 (2010).
- [41] C. P. S. H. X. Chen and S. L. Zhu, *Phys. Rev. D* **98**, 014011 (2018).
- [42] M. Ablikim *et al.* (BESIII Collaboration), *Nucl. Instrum. Meth. A* **614**, 345 (2010).
- [43] C. H. Yu *et al.*, *Proceedings of IPAC2016, Busan, Korea (2016)*, 10.18429/JACoW-IPAC2016-TUYA01.
- [44] M. Ablikim *et al.* (BESIII Collaboration), *Chin. Phys. C* **44**, 040001 (2020).
- [45] X. Li *et al.*, *Rad. Det. Tech. Meth.* **1**, 13 (2017).
- [46] Y. X. Guo *et al.*, *Radiat. Detect. Technol. Methods* **1**, 15 (2017).
- [47] P. Cao *et al.*, *Nucl. Instrum. Meth. A* **953**, 163053 (2020).

- [48] S. Agostinelli *et al.* (GEANT4 Collaboration), *Nucl. Instrum. Meth. A* **506**, 250 (2003).
- [49] G. M. G. Balossini, C. M. C. Calame and O. N. F. Phiccinini, *Nucl. Phys. B* **758**, 227 (2006).
- [50] B. Andersson and H. M. Hu, (1999), [arXiv:hep-ph/9910285](https://arxiv.org/abs/hep-ph/9910285).
- [51] F. V. Hippel and C. Quigg, *Phys. Rev. D* **5**, 624 (1972).
- [52] H. Albrecht *et al.* (ARGUS Collaboration), *Physics Letters B* **340**, 217 (1994).
- [53] R. G. Ping *et al.*, *Chin. Phys. C* **40**, 113002 (2016).
- [54] F. Jegerlehner, *Z. Phys. C* **32**, 195 (1986).
- [55] B. S. Zou and D. V. Bugg, *Phys. J. A* **16**, 537 (2003).
- [56] M. Ablikim *et al.* (BES Collaboration), *Phys. Lett. B* **645**, 19 (2007).
- [57] M. Ablikim *et al.* (BES Collaboration), *Phys. Lett. B* **598**, 149 (2004).
- [58] F. James and M. Roos, *Comput. Phys. Commun.* **10**, 343 (1975).
- [59] M. Ablikim *et al.* (BESIII Collaboration), *Chin. Phys. C* **41**, 063001 (2017).
- [60] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **87**, 012002 (2013).
- [61] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **99**, 011101 (2019).