Supplemental Material for "Observation of a vector charmonium-like state at 4.7 GeV/c^2 and search for Z_{cs} states in $e^+e^- \to K^+K^-J/\psi$ "

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$M(\ell^+\ell^-)$ at each center-of-mass energy

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Fig. 1. The $M(\ell^+\ell^-)$ distribution at each c.m. energy. Dots with error bars are data, the red histograms are signal MC and the blue shaded histograms stand for the background from the inclusive MC samples.



Fig. 2. The Dalitz plots of the data samples at each c.m. energy.

CROSS SECTIONS

¹⁸¹ The c.m. energies (\sqrt{s}) , integrated luminosities (\mathcal{L}) , numbers of events in the signal region (N^{obs}) and in the ¹⁸² sideband regions (N^{side}) , signal yields (N^{sig}) , event selection efficiencies (ϵ) , ISR correction factors $((1 + \delta))$, vacuum ¹⁸³ polarization factors $(|1 + \Pi|^2)$, Born cross sections (σ^{Born}) , and Born cross section ratios of $e^+e^- \rightarrow K_S^0 K_S^0 J/\psi$ to ¹⁸⁴ $e^+e^- \rightarrow K^+ K^- J/\psi \left(\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}\right)$ are shown in Table 1.

In the maximum likelihood fit to the dressed cross sections of $e^+e^- \to K^+K^-J/\psi$, assuming the obtained signal events obey Poisson ($N^{\text{sig}} \leq 10$) or asymmetric Gaussian ($N^{\text{sig}} > 10$). The Poisson is defined as

$$P_{\text{Poisson}} = (N^{\text{fit}} + f \cdot N^{\text{side}})^{N^{\text{obs}}} \cdot \frac{e^{-(N^{\text{fit}} + f \cdot N^{\text{side}})}}{N^{\text{obs}}!},$$
(1)

Tab. 1. The c.m. energies (\sqrt{s}) , integrated luminosities (\mathcal{L}) , numbers of events in the signal region (N^{obs}) and in the sideband regions (N^{side}) , signal yields (N^{sig}) , event selection efficiencies (ϵ) , ISR correction factors $((1 + \delta))$, vacuum polarization factors $(\frac{1}{|1-\Pi|^2})$, Born cross sections (σ^{Born}) , and Born cross section ratios $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$. The first uncertainties of σ^{Born} and $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$ are statistical, and the second ones systematic. The uncertainties of N^{sig} are only statistical.

$\sqrt{s} \; (\text{GeV})$	$\mathcal{L} (\mathrm{pb}^{-1})$	$N^{\rm obs}$	$N^{\rm side}$	N^{sig}	ϵ	$(1+\delta)$	$\frac{1}{ 1-\Pi ^2}$	$\sigma^{\rm Born}$ (pb)	$\frac{\sigma^{\rm Born}(K^0_S K^0_S J/\psi)}{\sigma^{\rm Born}(K^+ K^- J/\psi)}$
4.61	103.65	3	1	$2.75^{+2.09}_{-1.45}$	0.322	1.137	1.055	$0.57^{+0.44}_{-0.30}\pm0.04$	$0.379^{+0.674}_{-0.354}\pm0.026$
4.63	521.53	38	7	$36.25_{-5.88}^{+6.54}$	0.306	1.181	1.054	$1.53^{+0.28}_{-0.25}\pm0.10$	$0.116^{+0.162}_{-0.084} \pm 0.008$
4.64	551.65	22	10	$19.50\substack{+5.09\\-4.45}$	0.304	1.200	1.054	$0.77^{+0.20}_{-0.18}\pm0.05$	$0.832^{+0.525}_{-0.306}\pm0.056$
4.66	529.43	26	6	$24.50^{+5.47}_{-4.82}$	0.308	1.158	1.054	$1.03^{+0.23}_{-0.20} \pm 0.07$	$0.654^{+0.351}_{-0.266} \pm 0.044$
4.68	1667.39	94	26	$87.50^{+10.12}_{-9.46}$	0.334	1.043	1.054	$1.20^{+0.14}_{-0.13} \pm 0.08$	$0.772^{+0.172}_{-0.158} \pm 0.052$
4.70	536.54	40	10	$37.50^{+6.71}_{-6.06}$	0.365	0.953	1.055	$1.60^{+0.29}_{-0.26}\pm0.11$	$0.635^{+0.247}_{-0.175} \pm 0.043$
4.74	163.87	18	0	$18.00^{+4.59}_{-3.92}$	0.403	0.886	1.055	$2.44^{+0.62}_{-0.53}\pm0.23$	$0.111^{+0.184}_{-0.099}\pm0.007$
4.75	366.55	37	9	$34.75_{-5.81}^{+6.47}$	0.398	0.892	1.055	$2.12^{+0.39}_{-0.35}\pm0.20$	$0.562^{+0.248}_{-0.184}\pm0.038$
4.78	511.47	60	12	$57.00^{+8.13}_{-7.47}$	0.388	0.931	1.055	$2.45^{+0.35}_{-0.32}\pm0.23$	$0.399^{+0.172}_{-0.133} \pm 0.027$
4.84	525.16	34	7	$32.25_{-5.55}^{+6.21}$	0.358	1.015	1.056	$1.34^{+0.26}_{-0.23}\pm0.12$	$0.474^{+0.240}_{-0.172}\pm0.032$
4.92	207.82	13	4	$12.00^{+3.98}_{-3.33}$	0.328	1.082	1.056	$1.29^{+0.43}_{-0.36}\pm0.12$	$0.384^{+0.348}_{-0.223}\pm0.026$
4.95	159.28	9	4	$8.00^{+3.38}_{-2.73}$	0.312	1.103	1.056	$1.15^{+0.49}_{-0.39}\pm0.11$	$0.180^{+0.332}_{-0.169}\pm0.012$

Tab. 2. The four solutions of $(\Gamma_{ee}\mathcal{B})$ and ϕ for the third Breit-Wigner function, which represents the resonance Y(4710). The uncertainties are statistical only.

$(\Gamma_{ee}\mathcal{B})_3 \text{ (eV)}$	0.16 ± 0.04	0.20 ± 0.06	1.29 ± 0.23	1.61 ± 0.36
$\phi_3 \text{ (rad)}$	0.25 ± 0.43	-1.60 ± 0.45	-0.92 ± 0.19	-2.76 ± 0.16

¹⁸⁷ while the asymmetric Gaussian is defined as

$$P_{\text{Gaussian}} = \begin{cases} \frac{1}{\sqrt{2\pi} \cdot (\sigma_l + \sigma_h)} \cdot e^{-\frac{(N^{\text{fit}} - N^{\text{sig}})^2}{2\sigma_l^2}}, & N^{\text{fit}} < N^{\text{sig}} \\ \frac{1}{\sqrt{2\pi} \cdot (\sigma_l + \sigma_h)} \cdot e^{-\frac{(N^{\text{fit}} - N^{\text{sig}})^2}{2\sigma_h^2}}, & N^{\text{fit}} \ge N^{\text{sig}} \end{cases}$$
(2)

where f is the normalization factor of signal to sideband region (0.5), N^{fit} is the expected number of signal events, and σ_l and σ_h are the low and high uncertainties of the number of signal events. The likelihood is structured as

$$L = \prod_{i} P_i,\tag{3}$$

where P_i is P_{Poisson} if $N^{\text{sig}} \leq 10$ or P_{Gaussian} if $N^{\text{sig}} > 10$.

Figure 3 shows the four solutions of the fits to the dressed cross sections of $e^+e^- \rightarrow K^+K^-J/\psi$ with the coherent sum of three Breit-Wigner functions, and Table 2 shows the quantities of the four solutions. The systematic uncertainties of the fit parameters are listed in Table 3.

The Born cross section ratios $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$, shown in Table 1 and Fig. 4, are determined by the ratio likelihood simulations, where the Born cross sections of $e^+e^- \to K_S^0 K_S^0 J/\psi$ ($\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)$) are obtained from Ref. [2]. The systematic uncertainties of $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$ are estimated by taking into account correlations among uncertainties (the kinematic fit, ISR correction, K_S^0 reconstruction, and MUC depth are unrelated).

(the kinematic fit, ISR correction, K_S^0 reconstruction, and MUC depth are unrelated). The average Born cross ratio $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$ over $\sqrt{s} = 4.61 - 4.95$ GeV is determined to be $0.512^{+0.074}_{-0.060} \pm 0.035$ based on combined ratio likelihood simulations, where the first uncertainties are statistical, while the second one systematic. The common items of the systematic uncertainties have been canceled. The P-value for the ratio being greater then 0.5 is 0.621 which indicates a 0.31σ significance isospin-violation effect in $e^+e^- \rightarrow K\bar{K}J/\psi$.



Fig. 3. Four solutions of the fits to the dressed cross sections of $e^+e^- \rightarrow K^+K^-J/\psi$ with the coherent sum of three Breit-Wigner functions (solid curve). The dash (dash-dot-dot or dash-dot) curve shows the contribution from the three structures Y(4710) (Y(4230) or Y(4500)). The solid dots with error bars are the cross sections from this study, and the dash dots with error bars are the cross sections from Ref. [1]. The error bars are statistical uncertainty only.

Tab. 3. The systematic uncertainties in the measurement of resonance parameters, including that due to the c.m. energy (\sqrt{s}) , the parameterization of the fit function (Fitting), the c.m. energy spread (ES), and the uncommon $(\sigma_1^{\text{Dress}})$ and common $(\sigma_2^{\text{Dress}})$ systematic uncertainties from the cross section measurement. The symbol "–" represents the uncertainty, which can be neglected.

Parameter	\sqrt{s}	Fitting	ES	$\sigma_1^{\mathrm{Dress}}$	$\sigma_2^{\mathrm{Dress}}$	Sum
$M_3 \; ({\rm MeV}/c^2)$	0.80	20.91	0.11	3.03	_	21.1
$\Gamma_3 \ ({\rm MeV})$	_	29.67	0.01	1.09	_	29.7
	_	0.09	_	_	0.01	0.09
$(\Gamma_{aa}\mathcal{B})_{2}$ (eV)	_	0.01	_	_	0.01	0.01
(1 ee~)3 (01)		0.26		0.03	0.04	0.26
	_	0.06	_	0.03	0.05	0.08
	_	0.40	_	0.08	_	0.41
ϕ_2 (rad)	_	0.69	_	0.08	_	0.69
\$3 (1000)		0.42	0.01	0.02	_	0.42
	_	0.70	_	0.02	_	0.70



Fig. 4. Ratio of Born cross section $\sigma^{\text{Born}}(e^+e^- \to K^0_S K^0_S J/\psi)$ to $\sigma^{\text{Born}}(e^+e^- \to K^+ K^- J/\psi)$, where the error bars are statistical only.

Systematic uncertainties for the Born cross sections of Z_{cs} states include the detection efficiencies, vacuum polarization and ISR factor, which are estimated in the cross section measurement of $e^+e^- \rightarrow K^+K^-J/\psi$. These uncertainties are multiplicative.

Tab. 4. Systematic uncertainties on the Z_{cs} Born cross sections. The first five uncertainties are additive systematic uncertainties on the yields of Z_{cs} while the last two uncertainties are multiplicative.

Source	Systematic uncertainty on <i>yields</i> or efficiencies	
Detector resolution	0.1	
Efficiency curves	Negligible	
Signal model	0.2	
Backgrounds	0.1	
f states	0.3	
$\overline{Z_{cs}}$ resonance parameters	See the main texts	
Detection efficiency	15%	

In addition, we take into account the systematic uncertainties from the detector resolution, efficiency curves, 206 signal models of Z_{cs} , backgrounds and f states, Z_{cs} resonance parameters, which are summarized in Table 4. The 207 difference of detector resolution between data and MC is estimated to be $3.2 \text{ MeV}/c^2$ by studying the control sample 208 of $e^+e^- \to K^+D^{*0}D_s^{*-}$. We smear the resolution function and redo the $M_{\rm max}(K^{\pm}J/\psi)$ fit and the change is taken 209 as the systematic uncertainty. Through the similar procedure to estimate systematic uncertainties, we vary the 210 efficiency curves within $\pm 1\sigma$ uncertainties, change the signal model under different J^P assumptions, vary the kernel 211 width parameter of the background shapes and add shapes of f state at $\sqrt{s} > 4.70$ GeV. We also generate signal MC 212 samples for each signal model, and we take the largest difference between the resultant efficiencies and the nominal 213 efficiencies as the systematic uncertainty. We vary the resonance parameters within $\pm 1\sigma$ regions and take the largest 214 changes of the yields as the uncertainties. 215

These additive systematic uncertainties on the fitted yields are then converted into Born cross sections. Then the the root of quadratic sum of converted uncertainties and multiplicative uncertainties, which will be discussed later, is assigned as the σ of a Gaussian function. Then the Gaussian function will be used to convolve with the distributions of the nominal $-\ln L$ values.

Multiplicative uncertainties are related to the detection efficiencies, vacuum polarization and ISR factor, which part of which has been estimated in the measurement of the line shape. Except for the uncertainty of MC model which is irrelevant to detection efficiencies of Z_{cs} , all uncertainties in the measurement of line shape will be considered in the convolution of systematic uncertainties. Different spin-parity assumptions will affect the angular distribution of final states thus affect the detection efficiencies. We take the largest relative change among all energies to the nominal uncertainty in different spin-parity assumptions, 15.0%, as the systematic uncertainty.

The convolved $-\ln L$ distributions are shown in red curves in Figs. 5 and 6, which are used to determine the upper limits at the 90% confidence level. The upper limits are shown in Table 5.

Tab. 5. The detection efficiencies ϵ , and upper limits $\sigma^{\text{Born}}(e^+e^- \to K^-Z_{cs}^+ + c.c.) \cdot \mathcal{B}(Z_{cs}^+ \to K^+J/\psi)$ of Born cross sections for Z_{cs} states at the 90% confidence level, where the systematic uncertainties are incorporated. The VP and ISR factors are taken from Table 1.

\sqrt{s} (GeV)	$\epsilon(Z_{cs}(3985))$	$\epsilon(Z_{cs}(4000))$	$\sigma^{\text{Born}} \cdot \mathcal{B}[Z_{cs}(3985)]^{\text{UL}} \text{ (pb)}$	$\sigma^{\text{Born}} \cdot \mathcal{B}[Z_{cs}(4000)]^{\text{UL}} \text{ (pb)}$
4.63	0.408	0.409	0.2	0.9
4.64	0.402	0.399	0.2	0.7
4.66	0.404	0.396	0.2	0.7
4.68	0.415	0.414	0.1	0.8
4.70	0.428	0.429	0.2	3.3
4.74	0.456	0.449	0.6	1.9
4.75	0.444	0.445	0.3	1.5
4.78	0.426	0.428	0.3	0.8
4.84	0.397	0.401	0.3	1.4
4.92	0.376	0.375	0.6	1.3



Fig. 5. The scans for the upper limits of Born cross sections of $e^+e^- \rightarrow K^- Z_{cs}(3985)^+ + c.c.$. The blue arrows indicate the upper limits at the 90% confidence level by integrating the red regions consisting of smeared likelihood values with systematic uncertainties considered.



Fig. 6. The scans for the upper limits of Born cross sections of $e^+e^- \rightarrow K^- Z_{cs}(4000)^+ + c.c.$. The blue arrows indicate the upper limits at the 90% confidence level by integrating the red regions consisting of smeared likelihood values with systematic uncertainties considered.

PARTIAL WAVE ANALYSIS

Partial wave analysis (PWA) using helicity formalism is performed on each c.m. energy to study the intermediate 229 states. However, no significant Z_{cs} signal is detected because of the limited statistics. The $f_0(0^{++})$ and $f_2(2^{++})$ 230 components can be not well distinguished either. The PWA results with different $f_{0,2}$ combinations are used to 231 generate alternative signal MC samples to do efficiency uncertainty study for $e^+e^- \rightarrow K^+K^-J/\psi$ cross section 232 measurement. Figures 7 and 8 show the comparison of distributions between data and one of the PWA results at four 233 c.m. energies with higher statistics ($\sqrt{s} = 4.68, 4.70, 4.78, 4.84$ GeV). For the data sets with $\sqrt{s} \le 4.70$ GeV, the PWA 234 results are based on the $f_0(980) + f_0(1500)$ assumption. For the data sets with $\sqrt{s} > 4.70$ GeV, the PWA results are 235 based on a single $f_0(x)$ with mass and width free in the fit. 236



Fig. 7. The comparisons of $M(K^{\pm}J/\psi)$ and $M(K^{+}K^{-})$ between real data and the PWA result. The black dots with error bars are real data, the red line is the sum of the fit result and background from J/ψ sideband. The PWA results at (a, b) $\sqrt{s} = 4.68$ GeV and (c, d) $\sqrt{s} = 4.70$ GeV are based on the $f_0(980) + f_0(1500)$ assumption.



Fig. 8. The comparisons of $M(K^{\pm}J/\psi)$ and $M(K^{+}K^{-})$ between real data and the PWA result. The black dots with error bars are real data, the red line is the sum of the fit result and background from J/ψ sideband. The PWA results at (a, b) $\sqrt{s} = 4.78$ GeV and (c, d) $\sqrt{s} = 4.84$ GeV are based on a single $f_0(x)$ with mass and width free in the fit.

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