First Measurement of the Decay Asymmetry of pure W-exchange Decay $\Lambda_c^+ \to \Xi^0 K^+$

First Maseurement of the Decay Asymmetry of pure W-exchange Decay A⁺_n → EⁿA⁺
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Based on 4.4 fb⁻¹ of e^+e^- annihilation data collected at the center-of-mass energies between 4.60 and 4.70 GeV with the BESIII detector at the BEPCII collider, the pure W-exchange decay $\Lambda_c^+ \rightarrow$ $\Xi^0 K^+$ is studied with a full angular analysis. The corresponding decay asymmetry is measured for the first time to be $\alpha_{\equiv 0K^+} = 0.01 \pm 0.16$ (stat.) ± 0.03 (syst.). This result reflects the interference between the S- and P-wave amplitudes. The phase shift between S- and P-wave amplitudes is $\delta_n - \delta_s = -1.55 \pm 0.25 (\text{stat.}) \pm 0.05 (\text{syst.})$ rad.

in the Standard Model (SM) of particle physics. The ground state of the singly-charmed baryons, Λ_c^+ , was discovered in 1979 [1]. Many studies have since been made of the properties of charmed baryons, such as the decay branching fractions (BFs) and decay asymmetries. But experimental results of the decay asymmetries, which are sensitive to the different amplitudes in the decay dynamics, have been limited. Since 2014, there has been some progress on the weak hadronic decays of Λ_c^+ , $\Xi_c^{+,0}$, and Ω_c^0 , both experimentally and theoretically [3, 4]. This provides crucial information about the properties of all the singly-charmed baryons and the searches for doubly-charmed baryons $(\Xi_{cc} \text{ and } \Omega_{cc})$ [5]. Nonetheless, the understanding of the decay dynamics of charmed baryons is still limited, due to the lack of precision experimental measurements and the difficulties in the theoretical treatment of strong interaction effects.

Compared to heavy meson decays, charmed baryon decays have a significant dependence on nonfactorizable contributions from W-exchange diagrams. However, these contributions cannot currently be calculated using theoretical approaches. Additionally, no experimental measurements exist for the decay asymmetries of Wexchange hadronic decays. An example of such a process is the decay $\Lambda_c^+ \to \Xi^0 K^+$, which can only be produced via a W-exchange process as depicted in Fig. 1. Experimental measurements of the asymmetry parameters of the decay $\Lambda_c^+ \to \Xi^0 K^+$ can aid understanding of the internal dynamics and can also explore charge-parity (*CP*) violation in baryons [5].



FIG. 1. Feynman diagrams for $\Lambda_c^+ \to \Xi^0 K^+$.

Table I lists the theoretical calculations of the BF and asymmetry parameters of $\Lambda_c^+ \to \Xi^0 K^+$, as well as the experimental measurements of the BF. Various predictions of the BFs based on the covariant confined quark model (CCQM), the pole model, and current algebra (CA) in the 1990s are all smaller than the experimental results [3]. This is explained as a strong cancellation in the S- and P-wave amplitudes. Moreover, the decay asymmetry parameter was predicted to be zero in these models owing to the vanishing Swave amplitude [7–11]. This long-standing puzzle has recently experienced a renewed interest in the theoretical community [5, 12, 13], especially after the report of new BF measured by BESIII in 2018 [14]. To reproduce the relatively large experimental branching fraction of this decay, Ref. [5] adopted a variant of the CA approach and obtained a larger $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+) \simeq 0.71\%$. This modification introduces a large positive decay asymmetry of 0.90, which is quite close to the calculations based on SU(3) symmetry [12, 13]. However, regarding the significant enhancement of $\alpha_{\Xi^0 K^+}$ from 0 to about 0.9, the authors of Ref. [15] pointed out that the particular construction of the *S*-wave amplitude in Ref. [5] is not well-justified. So experimental measurement of the asymmetry parameter of the decay $\Lambda_c^+ \to \Xi^0 K^+$ will be crucial to test these calculations and confirm the vanishing *S*-wave contribution [15].

In the SM, the amplitude for a spin-1/2 baryon decaying into a spin-1/2 baryon and a spin-0 meson can be written as $M = i\bar{u}_f(A - B\gamma_5)u_i$, where A and B are constants, u_i and \bar{u}_f are spinors describing the initial and final baryons [8]. For the decay $\Lambda_c^+ \to \Xi^0 K^+$, the decay asymmetry is defined by $\alpha_{\Xi^0 K^+} = 2\text{Re}(s^*p)/(|s|^2 + |p|^2)$, where s = A and $p = |\vec{p}_{\Xi^0}|B/(E_{\Xi^0} + m_{\Xi^0})$; here E_{Ξ^0} and \vec{p}_{Ξ^0} are the energy and momentum of the Ξ^0 in the Λ_c^+ rest frame [3]. The effect of the S- and P-wave phase shift difference, $\delta_p - \delta_s$, is not well accounted for in the theoretical calculations of decay asymmetries. It can be extracted from experiments combined with the BF cited from the Particle Data Group (PDG) [3] and provides an important experimental parameter for the theoretical prediction of CP violation [16].

In this Letter, we present the first measurement of the decay asymmetry of $\Lambda_c^+ \to \Xi^0 K^+$ and its decay dynamic parameters $(|A|, |B|, \text{ and } \delta_p - \delta_s)$. A multidimensional angular analysis of the cascade-decay $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, $\Lambda_c^+ \to \Xi^0 K^+, \, \Xi^0 \to \Lambda \pi^0, \text{ and } \Lambda \to p \pi^- \text{ is performed}$ using a technique similar to that used to measure the asymmetry parameters of $\Lambda_c^+ \to p K_S^0$, $\Lambda \pi^+$, $\Sigma^+ \pi^0$, and $\Sigma^0 \pi^+$ [17]. The data samples used in this analysis, with an integrated luminosity of 4.4 fb^{-1} [18], were collected at center-of-mass (CM) energies of 4.60, 4.63, 4.64, 4.66, 4.68, and 4.70 GeV with the BESIII detector at the BEPCII collider. Details about BEPCII as well as BESIII and its sub-detectors - a helium-based multilaver drift chamber (MDC), a plastic scintillator time-offlight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC) – can be found in Refs. [19-23]. Large data samples taken around the $\Lambda_c^+ \bar{\Lambda}_c^-$ production threshold allow to measure decay asymmetry of low BF decays with the single-tag technique. The lowbackground environment is more favorable to measure $\alpha_{\Xi^0K^+}$ accurately. Charge-conjugate modes are always implied unless explicitly stated otherwise.

Simulated event samples produced with a GEANT4based [24] Monte Carlo (MC) package, which provides the geometric description of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The simulation

TABLE I. Theoretical calculations and experimental measurements of the BF, $\alpha_{\Xi^0 K^+}$, |A|, |B| and $\delta_p - \delta_s$ of $\Lambda_c^+ \to \Xi^0 K^+$. G_F is the Fermi constant. The superscript *a* denotes a model with SU(3) symmetry, while model *b* includes SU(3) symmetrybreaking effects. The PDG Fit BF also includes a CLEO result on $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)/\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ [3].

Theory or experiment	$\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$	$\alpha_{\Xi^0K^+}$	A	B	$\delta_p - \delta_s$
	$(\times 10^{-3})$		$(\times 10^{-2}G_F \text{ GeV}^2)$	$(\times 10^{-2}G_F \ {\rm GeV}^2)$	(rad)
Körner (1992), CCQM [7]	2.6	0	-	-	-
Xu (1992), Pole [8]	1.0	0	0	7.94	-
Źencaykowski (1994), Pole [9]	3.6	0	-	-	-
Ivanov (1998), CCQM $[10]$	3.1	0	-	-	-
Sharma (1999), CA [11]	1.3	0	-	-	-
Geng (2019) , SU (3) [12]	5.7 ± 0.9	$0.94^{+0.06}_{-0.11}$	2.7 ± 0.6	16.1 ± 2.6	-
Zou (2020), CA [5]	7.1	0.90	4.48	12.10	-
Zhong (2022), $SU(3)^a$ [13]	$3.8^{+0.4}_{-0.5}$	$0.91\substack{+0.03 \\ -0.04}$	3.2 ± 0.2	$8.7\substack{+0.6\\-0.8}$	-
Zhong (2022), $SU(3)^b$ [13]	$5.0^{+0.6}_{-0.9}$	0.99 ± 0.01	$3.3^{+0.5}_{-0.7}$	$12.3^{+1.2}_{-1.8}$	-
BESIII (2018) [14]	$5.90 \pm 0.86 \pm 0.39$	-	-	-	-
PDG Fit (2022) [3]	5.5 ± 0.7	-	-	-	-

models the beam energy spread and initial state radiation (ISR) in the e^+e^- annihilations with the generator KKMC [26, 27]. The inclusive MC sample includes the production of open charm processes, the ISR production of vector charmonium(-like) states, and the continuum processes incorporated in KKMC. All particle decays are modelled with EVTGEN [28, 29] using BFs either taken from the PDG [3], when available, or otherwise estimated with LUNDCHARM [30, 31]. Final state radiation from charged final state particles is incorporated using the PHOTOS package [32]. The phase space MC sample is generated uniformly over phase space, which is $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ followed by $\Lambda_c^+ \rightarrow \Xi^0 K^+$, $\Xi^0 \rightarrow \Xi^0 K^ \Lambda(\to p\pi^-)\pi^0(\to \gamma\gamma)$ and $\bar{\Lambda}_c^-$ decaying inclusively. For the signal MC sample, the signal process is generated by the helicity formalism using the decay asymmetry parameters measured in this work or cited from the PDG [3].

The tracks of K^+ candidates detected in the MDC are required to be within a polar angle (θ) range of $|\cos\theta| < 0.93$, where θ is defined with respect to the z axis, which is the symmetry axis of the MDC. Besides, the distance of the closest approach to the interaction point (IP) must be less than 10 cm along the z axis, $|V_z|$, and less than 1 cm in the transverse plane, $|V_{xy}|$. The particle identification (PID) combining the dE/dx information from the MDC and the flight time from the TOF is implemented to evaluate the likelihoods $\mathcal{L}(h)$ ($h = \pi, K, p$) for different hadron hypotheses. Charged tracks satisfying $\mathcal{L}(K) > \mathcal{L}(\pi)$, and $\mathcal{L}(K) > 0$ are assigned as kaon candidates.

The π^0 candidates are formed by photon pairs with an invariant mass in the interval 0.115 $< M_{\gamma\gamma} <$ 0.150 GeV/ c^2 . The γ candidates are reconstructed from showers in the EMC that are not associated with any charged track. The deposited energy must be larger than 25 MeV in the barrel region $(|\cos\theta| < 0.80)$ or 50 MeV in the end cap region $(0.86 < |\cos\theta| < 0.92)$. To suppress electronic noise and showers unrelated to the event, the difference between the EMC time and the event start time is required to be within [0, 700] ns. A massconstrained fit to the known π^0 mass [3] is applied to the photon pair and the resulting energy and momentum of the π^0 is used for further analysis.

The Λ candidates are obtained from two oppositely charged tracks in the final states $p\pi^{-}$. Their distance of closest approaches to the IP must be within ± 20 cm along the beam direction. In order to improve signal purity, the proton PID is required to satisfy $\mathcal{L}(p) >$ $\mathcal{L}(K), \ \mathcal{L}(p) > \mathcal{L}(\pi) \text{ and } \mathcal{L}(p) > 0.$ The two daughter tracks are constrained to originate from a common decay vertex by requiring the vertex fit χ^2 to be less than 20. The momentum of the Λ corrected by the vertex fit is used in subsequent analysis. Furthermore, the distance of the Λ decay vertex from the IP is required to be greater than twice the fitted vertex resolution. The $p\pi^{-}$ invariant mass is required to lie in the Λ mass region (1.111, 1.121) GeV/ c^2 . The Ξ^0 candidates are formed by $\Lambda \pi^0$ combinations and the invariant mass $M_{\Lambda \pi^0}$ must be within the mass region (1.30, 1.33) GeV/ c^2 . These mass regions are selected to be about three times the resolution.

Two kinematic variables, the energy difference $\Delta E \equiv E_{\Lambda_c^+} - E_{\text{beam}}$ and the beam-constrained mass $M_{\text{BC}} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_{\Lambda_c^+}|^2/c^2}$, are defined to identify $\Lambda_c^+ \rightarrow \Xi^0 K^+$ candidates. Here, $E_{\Lambda_c^+}$ and $\vec{p}_{\Lambda_c^+}$ are the reconstructed energy and momentum of the Λ_c^+ candidates calculated in the e^+e^- rest frame, and E_{beam} is the average energy of the e^+ and e^- beams. All candidates are required to satisfy $-0.05 < \Delta E < 0.05$ GeV and

2.25 GeV/ $c^2 < M_{\rm BC} < E_{\rm beam}$. If more than one candidate satisfies the above requirements, the one with minimal $|\Delta E|$ is kept. After applying these conditions, the $M_{\rm BC}$ distribution in data collected at 4.60 GeV is shown in the Fig. 2. In the fit to this data, the correctly and mis-reconstructed signal shapes are modeled with the MC-simulated shape convolved with a Gaussian function representing the resolution difference between data and MC simulation, and the background shape is described by an ARGUS function [33]. Finally, 378 ± 21 signal events are obtained by combining the six energy points.



FIG. 2. Distribution of the $M_{\rm BC}$ fitting result at 4.60 GeV. Black points with error bars are data; the blue shaded region indicates the combinatorial background events and pink shaded region is the mis-reconstructed signal events.

The decay asymmetry parameters are determined by analyzing the multi-dimensional angular distributions, where the full cascade-decay chain is considered. The joint angular formula is obtained using helicity basis [34]. Figure 3 illustrates the definitions of the helicity angles for the three-level cascade decay $\Lambda_c^+ \rightarrow \Xi^0 K^+, \ \Xi^0 \rightarrow$ $\Lambda \pi^0$, and $\Lambda \to p\pi^-$ following the process of $e^+e^- \to$ $\gamma^* \to \Lambda_c^+ \bar{\Lambda}_c^-$. In the helicity frame of the $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$, θ_0 is the polar angle of the Λ_c^+ with respect to the e^+ beam axis in the e^+e^- CM system. For the $\Lambda_c^+ \to \Xi^0 K^+$ decay, ϕ_1 is the angle between the $e^+\Lambda_c^+$ and Ξ^0K^+ planes, and θ_1 is the polar angle of the Ξ^0 momentum in the Λ_c^+ rest frame with respect to the Λ_c^+ momentum in the CM frame. For the $\Xi^0 \to \Lambda \pi^0$ decay, ϕ_2 is the angle between the $\Xi^0 K^+$ and $\Lambda \pi^0$ planes, and θ_2 is the polar angle of the Λ momentum with respect to the Ξ^0 momentum in the rest frame of Λ_c^+ . For the helicity angles describing the $\Lambda \to p\pi^-$ decay, ϕ_3 is the angle between the $\Lambda \pi^{\bar{0}}$ and $p\pi^-$ planes, and θ_3 is the polar angle of the proton with respect to the Λ momentum in the Ξ^0 rest frame.

In Ref. [34], Δ_0 is defined as the phase shift between two individual helicity amplitudes, $\mathcal{H}_{\lambda_1,\lambda_2}$, for the Λ_c^+ production process $\gamma^* \to \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$ with total helicities $|\lambda_1 - \lambda_2| = 0$ and 1, respectively. In the case where one-photon exchange dominates the production process, Δ_0 is also the phase between the



FIG. 3. Definitions of the helicity frames and related angles for $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-, \Lambda_c^+ \to \Xi^0 K^+, \Xi^0 \to \Lambda \pi^0$, and $\Lambda \to p\pi^-$.

electric and magnetic form factors of Λ_c^+ [35, 36], and α_0 is the angular distribution parameter of Λ_c^+ defined by the helicity amplitude $\alpha_0 = (|\mathcal{H}_{1/2,-1/2}|^2 - 2|\mathcal{H}_{1/2,1/2}|^2)/(|\mathcal{H}_{1/2,-1/2}|^2+2|\mathcal{H}_{1/2,1/2}|^2)$. Similarly, the $\Lambda_c^+ \to \Xi^0 K^+$ decay is described by two parameters, $\alpha_{\Xi^0 K^+}$ and $\Delta_{\Xi^0 K^+}$, where the latter one is the phase shift between the two helicity amplitudes. The Lee-Yang parameters [1, 34] can be obtained with the relations

$$\beta_{\Xi^0 K^+} = \sqrt{1 - (\alpha_{\Xi^0 K^+})^2} \sin \Delta_{\Xi^0 K^+},$$

$$\gamma_{\Xi^0 K^+} = \sqrt{1 - (\alpha_{\Xi^0 K^+})^2} \cos \Delta_{\Xi^0 K^+}.$$
(1)

In this analysis, the common free parameters $(\alpha_{\Xi^0 K^+})$ and $\Delta_{\Xi^0 K^+}$) describing the angular distributions for the six data sets are determined by a simultaneous unbinned maximum likelihood fit. The likelihood function is constructed from the joint probability density function (PDF) by

$$\mathcal{L}_{\text{total}} = \sum_{i=1}^{\text{energy}} \mathcal{L}_{\text{data}} = \prod_{i=1}^{N_{\text{data}}} f_s(\vec{\xi_i}).$$
(2)

Here, $f_s(\vec{\xi_i})$ is the PDF of the signal process, N_{data} is the number of events in data and *i* is the event index. The

signal PDF $f_s(\vec{\xi_i})$ is formulated as

$$f_s(\vec{\xi_i}) = \frac{\epsilon(\vec{\xi_i})|M(\vec{\xi_i};\vec{\eta})|^2}{\int \epsilon(\vec{\xi_i})|M(\vec{\xi_i};\vec{\eta})|^2 \mathrm{d}\vec{\xi_i}} , \qquad (3)$$

where $\vec{\xi_i}$ denotes the kinematic angular observables $(\theta_{0,1,2,3} \text{ and } \phi_{1,2,3})$ and $\vec{\eta}$ denotes the free parameters $(\alpha_{\Xi^0K^+} \text{ and } \Delta_{\Xi^0K^+})$ to be determined. The $M(\vec{\xi_i};\vec{\eta})$ is the total decay amplitude [34] and $\epsilon(\vec{\xi_i})$ is the detection efficiency parameterized in terms of the kinematic variables $\vec{\xi_i}$. The background contribution to the joint likelihood is subtracted according to the calculated likelihoods for the combinatorial background based on the inclusive MC simulation and for the misreconstructed signal events based on the signal MC simulation. The integration of the normalization factor is calculated with a large phase space MC sample as

$$\int \epsilon(\vec{\xi_i}) |M(\vec{\xi_i}; \vec{\eta})|^2 \mathrm{d}\vec{\xi_i} = \frac{1}{N_{\text{gen}}} \sum_{k_{\text{MC}}}^{N_{\text{MC}}} |M(\vec{\xi_{k_{\text{MC}}}}; \vec{\eta})|^2 , \quad (4)$$

where N_{gen} is the total number of the simulated phase space MC events, N_{MC} is the number of the phase space MC events surviving all selection criteria and k_{MC} is the event index.

Using the MINUIT package [38], we minimize the negative logarithmic likelihood with background subtraction over the six data samples. Here, α_0 , Δ_0 , $\alpha_{\Lambda\pi^0}$, $\alpha_{\bar{\Lambda}\pi^0}$, $\Delta_{\Lambda\pi^0}$, and $\Delta_{\bar{\Lambda}\pi^0}$ are fixed to individual values measured by BESIII [39, 40], $\alpha_{p\pi^-}$ and $\alpha_{\bar{p}\pi^+}$ are fixed to the values from the PDG [3]. In the fit, $\alpha_{\Xi^0K^+}$ and $\Delta_{\Xi^0K^+}$ are free parameters, and $\alpha_{\Xi^0K^+}$ $-\alpha_{\bar{\Xi}^0 K^-}$ and $\Delta_{\Xi^0 K^+} = -\Delta_{\bar{\Xi}^0 K^-}$ as required under the CP invariance assumption. The projections of the best fit onto several variables are shown in Fig. 4. The data are compared with the MC events weighted by the nominal fitting result. From this fit, we obtain $\alpha_{\Xi^0 K^+} = 0.01 \pm 0.16$ and $\Delta_{\Xi^0 K^+} = 3.84 \pm 0.90$ rad. Hence, the other two Lee-Yang parameters are calculated to be $\beta_{\Xi^0 K^+} = -0.64 \pm 0.69$ and $\gamma_{\Xi^0 K^+} = -0.77 \pm 0.58$, where the uncertainties are statistical only.

The systematic uncertainties arise mainly from the reconstruction of final state particles, which is studied with $J/\psi \rightarrow K_s^0 K^{\pm} \pi^{\mp}$ for kaon, $\Lambda_c^+ \rightarrow \Lambda X$ for Λ , $\psi(3686) \rightarrow J/\psi \pi^0 \pi^0$ and $e^+e^- \rightarrow \omega \pi^0$ for π^0 . For ΔE requirement and $M_{\rm BC}$ signal regions, which are estimated by smearing the phase space MC samples using resolution parameters. For the background subtraction, which is taking into account the background shape and size. The uncertainties from the quoted values of α_0 , Δ_0 , $\alpha_{\Lambda\pi^0}$, $\alpha_{\bar{\Lambda}\pi^0}$, $\Delta_{\Lambda\pi^0}$, $\Delta_{\bar{\Lambda}\pi^0}$, $\alpha_{p\pi^-}$ and $\alpha_{\bar{p}\pi^+}$ are estimated by Gaussian sampling considered their uncertainties and refit the angular distribution, and take the values in one time uncertainty of Gaussian fit as the uncertainties of this part. The fit bias, which is

the difference before and after the correction from pull distribution check. Systematic uncertainties from all sources are combined in quadrature to calculate the total systematic uncertainties. All systematic uncertainties are summarized in Table II and the relevant details can be found in Ref. [34].



FIG. 4. Projections of the best fit onto various variables. Black points with error bars are data; red solid lines are phase space MC events re-weighted by angular distribution formula, and represent the fitting result; the blue shaded region denotes the combinatorial background events and the pink shaded region is the mis-reconstructed signal events.

TABLE II. Systematic uncertainties in $\alpha_{\Xi^0 K^+}$ and $\Delta_{\Xi^0 K^+}$.

Source	$\alpha_{\Xi^0K^+}$	$\Delta_{\Xi^0 K^+}[\mathrm{rad}]$
Tracking and PID efficiencies	negligible	negligible
π^0 reconstruction	negligible	negligible
Λ reconstruction	0.01	0.01
ΔE and $M_{\rm BC}$ signal regions	negligible	negligible
Background subtraction	0.03	0.08
Input parameters	0.01	0.14
Fit bias	negligible	0.05
Total	0.03	0.17

The $\delta_p - \delta_s$ can be expressed by $\delta_p - \delta_s =$



FIG. 5. The comparison between this work and theoretical predictions, where the branching fraction is taken from PDG (2022) [3]. The blue symbols are theoretical predictions and the red star is result from this work. The definitions of the superscripts a and b can be found in Table I and the theory acronyms are explained in the text.

 $\arctan(\sqrt{1-\alpha_{\Xi^0K^+}^2}\sin\Delta_{\Xi^0K^+}/\alpha_{\Xi^0K^+})$ and the derivation of |A| and |B| can be found in Ref. [34]. The study has uncovered two distinct physical solutions, with one group characterized by $|A| = 1.6^{+1.9}_{-1.6}(\text{stat.}) \pm 0.4(\text{syst.})$ and $|B| = 18.3 \pm 2.8(\text{stat.}) \pm 0.7(\text{syst.})$, and the other by $|A| = 4.3 \pm 0.7(\text{stat.}) \pm 0.2(\text{syst.})$ and $|B| = 6.7^{+8.3}_{-6.7}(\text{stat.}) \pm 1.6(\text{syst.})$.

In summary, by analyzing 4.4 fb⁻¹ of e^+e^- annihilation data collected at the CM energies between 4.60 and 4.70 GeV with the BESIII detector, the pure W-exchange decay $\Lambda_c^+ \to \Xi^0 K^+$ from the $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$ production has been studied. The decay asymmetry parameters are measured for the first time as $\alpha_{\Xi^0 K^+} = 0.01 \pm$ $0.16(\text{stat.}) \pm 0.03(\text{syst.})$ and $\Delta_{\Xi^0 K^+} = 3.84 \pm 0.90(\text{stat.}) \pm$ 0.17(syst.) rad. The other two Lee-Yang parameters are calculated to be $\beta_{\Xi^0 K^+} = -0.64 \pm 0.69 (\text{stat.}) \pm 0.13 (\text{syst.})$ and $\gamma_{\Xi^0 K^+} = -0.77 \pm 0.58 \text{(stat.)} \pm 0.11 \text{(syst.)}$. The comparison between this work and theoretical predictions is shown in Fig. 5. Our measurement of $\alpha_{\Xi^0 K^+}$ is in good agreement with zero, which is consistent with the theoretical predictions from the 1990s. The decay dynamics parameters |A|, |B|, and $\delta_p - \delta_s$ are derived. The value of $\delta_p - \delta_s$ is observed to be -1.55 ± 0.25 (stat.) \pm This is of great significance for decay 0.05(syst.).asymmetries, as $\cos(\delta_p - \delta_s)$ measured in this study is close to zero, an effect that had not been anticipated in previous literature. This measurement fills the longstanding puzzle and deepens our understanding of the strong dynamics in charmed baryon sector.

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Supplemental Material for "First Measurement of the Decay Asymmetry of pure W-exchange Decay $\Lambda_c^+\to \Xi^0 K^+$ "

i. DECAY ASYMMETRY PARAMETERS

For the process $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-, \Lambda_c^+ \to BP$ and $\bar{\Lambda}_c^-$ decaying to anything, where *B* and *P* denote a $J^P = \frac{1}{2}^+$ baryon and a $J^P = 0^-$ pseudoscalar meson, respectively, the amplitude can be constructed using the helicity basis. For the weak non-leptonic decay $\Lambda_c^+ \to BP$, the Lee-Yang variables [1] α_{BP} , β_{BP} and γ_{BP} are defined with respect to the *s*-wave and *p*-wave amplitudes, such as

$$\alpha_{BP} = \frac{2\text{Re}(s^*p)}{|s|^2 + |p|^2}, \quad \beta_{BP} = \frac{2\text{Im}(s^*p)}{|s|^2 + |p|^2}, \quad \gamma_{BP} = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2}, \quad (i.1)$$

where s and p are the parity-odd and parity-even decay amplitudes. In a non-relativistic picture, they correspond to the L = 0 (S-wave) and L = 1 (P-wave) orbital angular momenta of the baryon-meson system, respectively.

The parameters α_{BP} , β_{BP} , and γ_{BP} satisfy

$$\alpha_{BP}^2 + \beta_{BP}^2 + \gamma_{BP}^2 = 1. \tag{i.2}$$

We work with helicity amplitudes. For $\Lambda_c^+ \to B(\frac{1}{2}^+) P(0^-)$ decays, we have two helicity amplitudes, $\mathcal{H}_{\frac{1}{2}}$ and $\mathcal{H}_{-\frac{1}{2}}$. Using the relations $s = \frac{1}{\sqrt{2}} (\mathcal{H}_{\frac{1}{2}} + \mathcal{H}_{-\frac{1}{2}})$, $p = \frac{1}{\sqrt{2}} (\mathcal{H}_{\frac{1}{2}} - \mathcal{H}_{-\frac{1}{2}})$, the asymmetry parameters defined with helicity amplitudes are

$$\alpha_{BP} = |\mathcal{H}_{\frac{1}{2}}|^2 - |\mathcal{H}_{-\frac{1}{2}}|^2,$$

$$\beta_{BP} = \sqrt{1 - \alpha_{BP}^2} \sin\Delta_{BP},$$

$$\gamma_{BP} = \sqrt{1 - \alpha_{BP}^2} \cos\Delta_{BP},$$

(i.3)

where we take the normalization $|\mathcal{H}_{\frac{1}{2}}|^2 + |\mathcal{H}_{-\frac{1}{2}}|^2 = 1$, and Δ_{BP} is the phase angle difference between two helicity amplitudes $\mathcal{H}_{\frac{1}{2}}$ and $\mathcal{H}_{-\frac{1}{2}}$.

If the Λ_c^+ and $\bar{\Lambda}_c^-$ decays respect CP symmetry, we have relations between the Λ_c^+ and $\bar{\Lambda}_c^-$ asymmetry parameters

$$\alpha_{\bar{B}\bar{P}} = -\alpha_{BP}, \ \beta_{\bar{B}\bar{P}} = -\beta_{BP}, \ \gamma_{\bar{B}\bar{P}} = \gamma_{BP}. \tag{i.4}$$

ii. JOINT ANGULAR DISTRIBUTION FORMULA FOR THE DECAY $\Lambda_c^+ \to \Xi^0 K^+$

In the helicity frame of the $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$ system, θ_0 is the polar angle of the Λ_c^+ with respect to the e^+ beam axis in the e^+e^- CM system. For the $\Lambda_c^+ \to \Xi^0 K^+$ decay, ϕ_1 is the angle between the $e^+\Lambda_c^+$ and $\Xi^0 K^+$ planes and θ_1 is the polar angle of the Ξ^0 momentum in the Λ_c^+ rest frame with respect to the Λ_c^+ momentum in the CM frame. For the helicity system describing $\Xi^0 \to \Lambda \pi^0$ decay, ϕ_2 is the angle between the $\Xi^0 K^+$ and $\Lambda \pi^0$ planes and θ_2 is the polar angle of the Λ momentum with respect to the Ξ^0 momentum in the rest frame of Λ_c^+ . For the helicity angles describing the $\Lambda \to p\pi^-$ decay, ϕ_3 is the angle between the $\Lambda \pi^0$ and $p\pi^-$ planes and θ_3 is the polar angle of the proton with respect to the Λ momentum in the Ξ^0 rest frame. In the $\Lambda_c^+ \to \Xi^0 K^+$ process, as shown in Table I, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 indicate the helicity of $\Lambda_c^+, \bar{\Lambda}_c^-, \Xi^0, \Lambda$ and p. $A_{\lambda_1,\lambda_2}, B_{\lambda_3}, C_{\lambda_4}$ and D_{λ_5} are the helicity amplitudes.

TABLE I. Definition of decays, helicity angles and amplitudes, where λ_i indicates the helicity values for the corresponding hadron.

Level	Decay	Helicity angle	Helicity amplitude
0	$e^+e^- \to \Lambda_c^+(\lambda_1) \bar{\Lambda}_c^-(\lambda_2)$	$(heta_0)$	A_{λ_1,λ_2}
1	$\Lambda_c^+ \to \Xi^0(\lambda_3) K^+$	$_{(heta_1,\phi_1)}$	B_{λ_3}
2	$\Xi^0 o \Lambda(\lambda_4) \pi^0$	$(heta_2,\phi_2)$	C_{λ_4}
3	$\Lambda \to p(\lambda_5) \pi^-$	$(heta_3,\phi_3)$	D_{λ_5}

The joint angular distribution is defined as

$$\frac{d\Gamma}{d\cos\theta_{0}\,d\cos\theta_{1}\,d\cos\theta_{2}\,d\cos\theta_{3}\,d\phi_{1}\,d\phi_{2}\,d\phi_{3}} \propto \sum_{\lambda_{1},\lambda_{2},\lambda_{3},\lambda_{4},\lambda_{5}} \rho^{\lambda_{1}-\lambda_{2},\lambda_{1}'-\lambda_{2}}(\theta_{0})\,D_{\lambda_{1},\lambda_{3}}^{\frac{1}{2}*}(\theta_{1},\phi_{1},0)\,D_{\lambda_{1}',\lambda_{3}'}^{\frac{1}{2}}(\theta_{1},\phi_{1},0)\,B_{\lambda_{3}}\,B_{\lambda_{3}'}^{*}$$
(ii.1)
$$D_{\lambda_{3},\lambda_{4}}^{\frac{1}{2}*}(\theta_{2},\phi_{2},0)\,D_{\lambda_{3}',\lambda_{4}'}^{\frac{1}{2}}(\theta_{2},\phi_{2},0)\,C_{\lambda_{4}}\,C_{\lambda_{4}'}^{*}\,D_{\lambda_{4},\lambda_{5}}^{\frac{1}{2}*}(\theta_{3},\phi_{3},0)\,D_{\lambda_{4}',\lambda_{5}}^{\frac{1}{2}}(\theta_{3},\phi_{3},0)\,|D_{\lambda_{5}}|^{2},$$

where $\rho^{\lambda_1 - \lambda_2, \lambda_1' - \lambda_2}(\theta_0) = \sum_{M=\pm 1} d^1_{M,\lambda_1 - \lambda_2}(\theta_0) d^1_{M,\lambda_1' - \lambda_2}(\theta_0) A_{\lambda_1,\lambda_2} A^*_{\lambda_1',\lambda_2}$ corresponds to the Λ_c^+ spin density matrix and $D^J_{m,n}(\phi,\theta,\gamma) = e^{-im\phi} d^J_{m,n}(\theta) e^{-in\gamma}$ is Wigner-D function [2]. The helicity amplitudes A_{λ_1,λ_2} are related to the asymmetry parameters $\alpha_0 = \frac{\left|A_{\frac{1}{2},-\frac{1}{2}}\right|^2 - 2\left|A_{\frac{1}{2},\frac{1}{2}}\right|^2}{\left|A_{\frac{1}{2},-\frac{1}{2}}\right|^2 + 2\left|A_{\frac{1}{2},\frac{1}{2}}\right|^2}$, and helicity B_{λ_3} is related to the asymmetry parameter $\alpha_{\Xi^0K^+} = 0$ $\frac{\left|B_{\frac{1}{2}}\right|^{2} - \left|B_{-\frac{1}{2}}\right|^{2}}{\left|B_{\frac{1}{2}}\right|^{2} + \left|B_{-\frac{1}{2}}\right|^{2}}, \text{ helicity } C_{\lambda_{4}} \text{ is related to the asymmetry parameter } \alpha_{\Lambda\pi^{0}} = \frac{\left|C_{\frac{1}{2}}\right|^{2} - \left|C_{-\frac{1}{2}}\right|^{2}}{\left|C_{\frac{1}{2}}\right|^{2} + \left|C_{-\frac{1}{2}}\right|^{2}} \text{ and helicity } D_{\lambda_{5}} \text{ is related } C_{\lambda_{5}} \text{ is related$ to the asymmetry parameter $\alpha_{p\pi^-} = \frac{\left|D_{\frac{1}{2}}\right|^2 - \left|D_{-\frac{1}{2}}\right|^2}{\left|D_{\frac{1}{2}}\right|^2 + \left|D_{-1}\right|^2}$. Then the joint angular distribution becomes $\frac{1}{d\cos\theta_0} \ d\cos\theta_1 \ d\cos\theta_2 \ d\cos\theta_3 \ d\phi_1 \ d\phi_2 \ d\phi_3$ $\propto 1 + \alpha_0 \cos^2 \theta_0$ + $(1 + \alpha_0 \cos^2 \theta_0) \alpha_{\Xi^0 K^+} \alpha_{\Lambda \pi^0} \cos \theta_2$ + $(1 + \alpha_0 \cos^2 \theta_0) \alpha_{=0 K^+} \alpha_{n\pi^-} \cos \theta_2 \cos \theta_3$ + $(1 + \alpha_0 \cos^2 \theta_0) \alpha_{\Lambda \pi^0} \alpha_{p \pi^-} \cos \theta_3$ $-(1+\alpha_0\cos^2\theta_0) \alpha_{\Xi^0K^+} \sqrt{1-\alpha_{\Lambda\pi^0}^2} \alpha_{p\pi^-}\sin\theta_2\sin\theta_3\cos(\Delta_{\Lambda\pi^0}+\phi_3)$ + $\sqrt{1-\alpha_0^2} \sin \Delta_0 \sin \theta_0 \cos \theta_0 \alpha_{\Xi^0 K^+} \sin \theta_1 \sin \phi_1$ + $\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\alpha_{\Lambda\pi^0}\sin\theta_1\sin\phi_1\cos\theta_2$ + $\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\alpha_{\Xi^0K^+}\alpha_{\Lambda\pi^0}\alpha_{n\pi^-}\sin\theta_1\sin\phi_1\cos\theta_3$ + $\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\alpha_{n\pi} - \sin\theta_1\sin\phi_1\cos\theta_2\cos\theta_3$ (ii.2) $-\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Lambda\pi^0}^2}\ \alpha_{p\pi^-}\sin\theta_1\sin\phi_1\sin\theta_2\sin\theta_3\cos(\Delta_{\Lambda\pi^0}+\phi_3)$ $+\sqrt{1-\alpha_0^2}\,\sin\!\Delta_0\!\sin\!\theta_0\!\cos\!\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\,\,\alpha_{\Lambda\pi^0}\!\cos\!\phi_1\!\sin\!\theta_2\!\sin(\Delta_{\Xi^0K^+}+\phi_2)$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\alpha_{\Lambda\pi^0}\cos\theta_1\sin\phi_1\sin\theta_2\cos(\Delta_{\Xi^0K^+}+\phi_2)$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\alpha_{p\pi^-}\cos\theta_1\sin\phi_1\sin\theta_2\cos(\Delta_{\Xi^0K^+}+\phi_2)\cos\theta_3$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\alpha_{p\pi^-}\cos\phi_1\sin\theta_2\sin(\Delta_{\Xi^0K^+}+\phi_2)\cos\theta_3$ $-\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\sqrt{1-\alpha_{\Lambda\pi^0}^2}\ \alpha_{p\pi^-}\cos\theta_1\sin\phi_1\sin(\Delta_{\Xi^0K^+}+\phi_2)\sin\theta_3\sin(\Delta_{\Lambda\pi^0}+\phi_3)$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\sqrt{1-\alpha_{\Lambda\pi^0}^2}\alpha_{p\pi^-}\cos\theta_1\sin\phi_1\cos\theta_2\cos(\Delta_{\Xi^0K^+}+\phi_2)\sin\theta_3\cos(\Delta_{\Lambda\pi^0}+\phi_3)$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\Xi^0K^+}^2}\sqrt{1-\alpha_{\Lambda\pi^0}^2}\alpha_{p\pi^-}\cos\phi_1\cos(\Delta_{\Xi^0K^+}+\phi_2)\sin\theta_3\sin(\Delta_{\Lambda\pi^0}+\phi_3)$ $+\sqrt{1-\alpha_0^2}\sin\Delta_0\sin\theta_0\cos\theta_0\sqrt{1-\alpha_{\mp 0\ \kappa^+}^2}\sqrt{1-\alpha_{\Lambda\pi^0}^2}\ \alpha_{p\pi^-}\cos\phi_1\cos\theta_2\sin(\Delta_{\pm 0\ \kappa^+}+\phi_2)\sin\theta_3\cos(\Delta_{\Lambda\pi^0}+\phi_3)$

where the α_0 is the angular distribution parameter of $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$ and Δ_0 is the transverse polarization for Λ_c^+ . The decay asymmetry parameters $\alpha_{\Lambda\pi^0}$ and $\alpha_{p\pi^-}$ are taken from PDG [3]. The $\Delta_{\Xi^0K^+} = \delta_{\frac{1}{2}}^B - \delta_{-\frac{1}{2}}^B$ and $\Delta_{\Lambda\pi^0} = \delta_{\frac{1}{2}}^C - \delta_{-\frac{1}{2}}^C$ are the difference of the phase of the helicity amplitude *B* and *C*, respectively. For the charge conjugate mode, $\bar{\Lambda}_c^- \to \bar{\Xi}^0 K^-$, the formula of angular distribution is same.

iii. SYSTEMATIC UNCERTAINTY

The systematic uncertainties arise mainly from the reconstruction of final states, ΔE requirement, $M_{\rm BC}$ signal selection, the background subtraction, the uncertainties from the quoted values of α_0 , Δ_0 , $\alpha_{\Lambda\pi^0}$, $\alpha_{\bar{\Lambda}\pi^0}$, $\Delta_{\Lambda\pi^0}$, $\Delta_{\bar{\Lambda}\pi^0}$, $\Delta_{\bar{\Lambda}\pi^0}$, $\alpha_{\bar{\mu}\pi^-}$, and $\alpha_{\bar{p}\pi^+}$, and the fit bias. Systematic uncertainties from various sources are combined in quadrature to calculate the total systematic uncertainties.

The reconstruction efficiency of charged kaon is studied with the control sample of $J/\psi \to K_S^0 K^{\pm} \pi^{\mp}$ events, that for π^0 with $\psi(3686) \to \pi^0 \pi^0 J/\psi$ and $e^+e^- \to \omega \pi^0$, and that for Λ with $J/\psi \to \bar{p}K^+\Lambda$ and $J/\psi \to \Lambda\bar{\Lambda}$ [4]. The signal MC samples are re-weighted based on the data-MC differences in various momentum ranges resulting in new MC integration and new fitting parameters. The uncertainties related to the ΔE and $M_{\rm BC}$ requirement are evaluated by smearing the signal MC sample with a Gaussian resolution function. The changes of the fit results based on new accepted signal MC events are taken as the systematic uncertainties. All effects mentioned above are negligible except for Λ reconstruction.

For the background subtraction, we consider both background size and the background modelling. The background size including combinational background and mis-reconstructed component is obtained from the fit to the $M_{\rm BC}$ spectrum. The relevant systematic uncertainties are examined by repeating the fits with a alternative background size obtained from the Gaussian sampling of the fitted parameters. The ensemble of fitted parameters obtained will be fit to a Gaussian and the sum of the fitted Gaussian resolution and the difference between the fitted Gaussian mean and the nominal result are assigned as the systematic uncertainty for background size. The uncertainty of the background modelling also considers both combinational background and the mis-reconstructed component. The mis-reconstructed model is examined by an alternative signal MC sample produced with the new input parameters $\alpha_{\Xi^0K^+}$ which are changed within $\pm 1\sigma$. The uncertainty due to the combinational background model is estimated by varying the relative weights between $\Lambda_c^+ \bar{\Lambda}_c^-$ pairs and other hadronic events based on the uncertainties of their cross section ratio.

The systematic uncertainty due to the input parameters is evaluated by varying these parameters within $\pm 1\sigma$ using a Gaussian sampling method. For each parameter, the obtained results are fit to a Gaussian function and the sum of the fitted Gaussian resolution and the difference between the fitted Gaussian mean and the nominal result is taken as the systematic uncertainty. The systematic uncertainties arising from fit bias are investigated via pull distribution checks. The mean values of the pull distributions are used to correct the nominal results. The differences between the uncorrected and corrected results are assigned as systematic uncertainties.

All systematic uncertainties discussed above are summarized in Table II.

Source	$\alpha_{\Xi^0K^+}$	$\Delta_{\Xi^0 K^+}$ [rad]
Tracking and PID efficiencies	negligible	negligible
π^0 reconstruction	negligible	negligible
Λ reconstruction	0.01	0.01
ΔE and $M_{\rm BC}$ signal regions	negligible	negligible
Background subtraction	0.03	0.08
Input parameters	0.01	0.14
Fit bias	negligible	0.05
Total	0.03	0.17

TABLE II. Systematic uncertainties in $\alpha_{\Xi^0 K^+}$ and $\Delta_{\Xi^0 K^+}$.

iv. RELATION WITH WEAK DECAY AMPLITUDE

In the Standard Model, the amplitude for the two-body weak decay $\Lambda_c^+ \to \Xi^0 K^+$ can be parameterized as $\mathcal{M} = i\bar{u}_{\Xi^0}(A - B\gamma_5)u_{\Lambda_c^+}$, based on the S-wave and P-wave amplitudes, s = A and $p = \kappa B$ [3]. Combination the Eq. (i.1) and the relationship between the $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$ and the amplitude, the decay width and decay asymmetry can be

obtained as follows:

$$\Gamma = \frac{\mathcal{B}(\Lambda_{c}^{+} \to \Xi^{0}K^{+})}{\tau_{\Lambda_{c}^{+}}} = \frac{|\vec{p}_{c}|}{8\pi} \Big[\frac{(m_{\Lambda_{c}^{+}} + m_{\Xi^{0}})^{2} - m_{K^{+}}^{2}}{m_{\Lambda_{c}^{+}}^{2}} |A|^{2} + \frac{(m_{\Lambda_{c}^{+}} - m_{\Xi^{0}})^{2} - m_{K^{+}}^{2}}{m_{\Lambda_{c}^{+}}^{2}} |B|^{2} \Big],$$

$$\alpha_{\Xi^{0}K^{+}} = \frac{2\kappa |A| |B| \cos(\delta_{p} - \delta_{s})}{|A|^{2} + \kappa^{2} |B|^{2}},$$

$$\Delta_{\Xi^{0}K^{+}} = \arctan \frac{2\kappa |A| |B| \sin(\delta_{p} - \delta_{s})}{|A|^{2} - \kappa^{2} |B|^{2}},$$
(iv.1)

with $\kappa = |\vec{p}_c|/(E_{\Xi^0} + m_{\Xi^0}) = \sqrt{(E_{\Xi^0} - m_{\Xi^0})/(E_{\Xi^0} + m_{\Xi^0})} \approx 0.234582$ and \vec{p}_c is the momentum of the Ξ^0 baryon in the rest frame of Λ_c^+ particle [5].

Upon solving the system of equations presented in Eq. (iv.1), an exquisite solution can be obtained. The complexity of the solution is such that its exposition is not suitable here.

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