## Supplemental Material for "First Measurement of the Decay Asymmetry in the Pure W-Boson-Exchange Decay $\Lambda_c^+ \to \Xi^0 K^+$ "

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### I. DECAY ASYMMETRY PARAMETERS

For the process  $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-, \Lambda_c^+ \to BP$  and  $\bar{\Lambda}_c^-$  decaying to anything, where B and P denote a  $J^P = \frac{1}{2}^+$  baryon and a  $J^P = 0^-$  pseudoscalar meson, respectively, the amplitude can be constructed using the helicity basis. For the

weak non-leptonic decay  $\Lambda_c^+ \to BP$ , the Lee-Yang variables [1]  $\alpha_{BP}$ ,  $\beta_{BP}$  and  $\gamma_{BP}$  are defined with respect to the s-wave and p-wave amplitudes, such as

$$\alpha_{BP} = \frac{2\text{Re}(s^*p)}{|s|^2 + |p|^2}, \quad \beta_{BP} = \frac{2\text{Im}(s^*p)}{|s|^2 + |p|^2}, \quad \gamma_{BP} = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2}, \tag{1}$$

where s and p are the parity-odd and parity-even decay amplitudes. In a non-relativistic picture, they correspond to the L = 0 (S-wave) and L = 1 (P-wave) orbital angular momenta of the baryon-meson system, respectively.

The parameters  $\alpha_{BP}$ ,  $\beta_{BP}$ , and  $\gamma_{BP}$  satisfy

$$\alpha_{BP}^2 + \beta_{BP}^2 + \gamma_{BP}^2 = 1.$$
 (2)

We work with helicity amplitudes. For  $\Lambda_c^+ \to B(\frac{1}{2}^+) P(0^-)$  decays, we have two helicity amplitudes,  $\mathcal{H}_{\frac{1}{2}}$  and  $\mathcal{H}_{-\frac{1}{2}}$ . Using the relations  $s = \frac{1}{\sqrt{2}}(\mathcal{H}_{\frac{1}{2}} + \mathcal{H}_{-\frac{1}{2}})$ ,  $p = \frac{1}{\sqrt{2}}(\mathcal{H}_{\frac{1}{2}} - \mathcal{H}_{-\frac{1}{2}})$ , the asymmetry parameters defined with helicity amplitudes are

$$\alpha_{BP} = |\mathcal{H}_{\frac{1}{2}}|^2 - |\mathcal{H}_{-\frac{1}{2}}|^2,$$
  

$$\beta_{BP} = \sqrt{1 - \alpha_{BP}^2} \sin\Delta_{BP},$$
  

$$\gamma_{BP} = \sqrt{1 - \alpha_{BP}^2} \cos\Delta_{BP},$$
(3)

where we take the normalization  $|\mathcal{H}_{\frac{1}{2}}|^2 + |\mathcal{H}_{-\frac{1}{2}}|^2 = 1$ , and  $\Delta_{BP}$  is the phase angle difference between two helicity amplitudes  $\mathcal{H}_{\frac{1}{2}}$  and  $\mathcal{H}_{-\frac{1}{2}}$ .

If the  $\Lambda_c^+$  and  $\bar{\Lambda}_c^-$  decays respect CP symmetry, we have relations between the  $\Lambda_c^+$  and  $\bar{\Lambda}_c^-$  asymmetry parameters

$$\alpha_{\bar{B}\bar{P}} = -\alpha_{BP}, \ \beta_{\bar{B}\bar{P}} = -\beta_{BP}, \ \gamma_{\bar{B}\bar{P}} = \gamma_{BP}.$$

$$\tag{4}$$

# II. JOINT ANGULAR DISTRIBUTION FORMULA FOR THE DECAY $\Lambda_c^+ \to \Xi^0 K^+$

In the helicity frame of the  $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$  system,  $\theta_0$  is the polar angle of the  $\Lambda_c^+$  with respect to the  $e^+$  beam axis in the  $e^+e^-$  CM system. For the  $\Lambda_c^+ \to \Xi^0 K^+$  decay,  $\phi_1$  is the angle between the  $e^+\Lambda_c^+$  and  $\Xi^0 K^+$  planes and  $\theta_1$  is the polar angle of the  $\Xi^0$  with respect to the direction of  $\bar{\Lambda}_c^-$  evaluated in  $\Lambda_c^+$ 's rest frame. For the helicity system describing  $\Xi^0 \to \Lambda \pi^0$  decay,  $\phi_2$  is the angle between the  $\Xi^0 K^+$  and  $\Lambda \pi^0$  planes and  $\theta_2$  is the polar angle of the  $\Lambda$  with respect to the direction of  $K^+$  evaluated in  $\Xi^0$ 's rest frame. For the helicity angles describing the  $\Lambda \to p\pi^-$  decay,  $\phi_3$  is the angle between the  $\Lambda \pi^0$  and  $p\pi^-$  planes and  $\theta_3$  is the polar angle of the proton with respect to the direction of  $\pi^0$  evaluated in  $\Lambda$ 's rest frame. In the  $\Lambda_c^+ \to \Xi^0 K^+$  process, as shown in Table I,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$  and  $\lambda_5$  indicate the helicity of  $\Lambda_c^+$ ,  $\bar{\Lambda}_c^-$ ,  $\Xi^0$ ,  $\Lambda$  and p.  $\mathcal{A}_{\lambda_1,\lambda_2}$ ,  $\mathcal{B}_{\lambda_3}$ ,  $\mathcal{C}_{\lambda_4}$  and  $\mathcal{D}_{\lambda_5}$  are the helicity amplitudes.

TABLE I. Definition of decays, helicity angles and amplitudes, where  $\lambda_i$  indicates the helicity values for the corresponding hadron.

Level	Decay	Helicity angle	Helicity amplitude
0	$e^+e^- \to \Lambda_c^+(\lambda_1) \bar{\Lambda}_c^-(\lambda_2)$	$( heta_0)$	$\mathcal{A}_{\lambda_1,\lambda_2}$
1	$\Lambda_c^+ \to \Xi^0(\lambda_3)  K^+$	$( heta_1,\phi_1)$	$\mathcal{B}_{\lambda_3}$
2	$\Xi^0  o \Lambda(\lambda_4)  \pi^0$	$( heta_2,\phi_2)$	$\mathcal{C}_{\lambda_4}$
3	$\Lambda \to p(\lambda_5)  \pi^-$	$( heta_3,\phi_3)$	$\mathcal{D}_{\lambda_5}$

According to the total amplitude (M), the differential dacay rate  $(d\Gamma)$  can be expressed as  $d\Gamma \propto |M(\vec{\xi}_i; \vec{\eta})|^2$ , where  $\vec{\xi}_i$  denotes the kinematic angular observables  $(\theta_{0,1,2,3} \text{ and } \phi_{1,2,3})$  and  $\vec{\eta}$  denotes the free parameters  $(\alpha_{\Xi^0 K^+} \text{ and } \phi_{1,2,3})$ 

 $\Delta_{\Xi^0 K^+}$ ). The joint angular distribution can be calculated as

$$\frac{d\Gamma}{d\cos\theta_{0} \ d\cos\theta_{1} \ d\cos\theta_{2} \ d\cos\theta_{3} \ d\phi_{1} \ d\phi_{2} \ d\phi_{3}}} \propto \sum_{\substack{\lambda_{1},\lambda_{1}',\lambda_{2},\lambda_{2}',\lambda_{3},\lambda_{4}',\lambda_{4}',\lambda_{5}}} \rho^{\lambda_{1}-\lambda_{2},\lambda_{1}'-\lambda_{2}}(\theta_{0}) D_{\lambda_{1},\lambda_{3}}^{\frac{1}{2}*}(\theta_{1},\phi_{1},0) D_{\lambda_{1}',\lambda_{3}'}^{\frac{1}{2}}(\theta_{1},\phi_{1},0) \mathcal{B}_{\lambda_{3}} \mathcal{B}_{\lambda_{3}'}^{*}}$$
(5)  
$$D_{\lambda_{3},\lambda_{4}}^{\frac{1}{2}*}(\theta_{2},\phi_{2},0) D_{\lambda_{3}',\lambda_{4}'}^{\frac{1}{2}}(\theta_{2},\phi_{2},0) \mathcal{C}_{\lambda_{4}} \mathcal{C}_{\lambda_{4}'}^{*} D_{\lambda_{4},\lambda_{5}}^{\frac{1}{2}*}(\theta_{3},\phi_{3},0) D_{\lambda_{4}',\lambda_{5}}^{\frac{1}{2}}(\theta_{3},\phi_{3},0) |\mathcal{D}_{\lambda_{5}}|^{2},$$

where  $\rho^{\lambda_1 - \lambda_2, \lambda_1' - \lambda_2}(\theta_0) = \sum_{\lambda_0 = \pm 1} d^1_{\lambda_0, \lambda_1 - \lambda_2}(\theta_0) d^1_{\lambda_0, \lambda_1' - \lambda_2}(\theta_0) A_{\lambda_1, \lambda_2} A^*_{\lambda_1', \lambda_2}$  corresponds to the  $\Lambda_c^+$  spin density matrix,  $\lambda_0$  is the helicity of virtual photon, and  $D^J_{m,n}(\phi, \theta, \gamma) = e^{-im\phi} d^J_{m,n}(\theta) e^{-in\gamma}$  is Wigner-D function [2]. The helicity amplitudes  $\mathcal{A}_{\lambda_1, \lambda_2}$  are related to the asymmetry parameters  $\alpha_0 = \frac{\left|\mathcal{A}_{\frac{1}{2}, -\frac{1}{2}}\right|^2 - 2\left|\mathcal{A}_{\frac{1}{2}, \frac{1}{2}}\right|^2}{\left|\mathcal{A}_{\frac{1}{2}, -\frac{1}{2}}\right|^2 + 2\left|\mathcal{A}_{\frac{1}{2}, \frac{1}{2}}\right|^2}$ , and helicity  $\mathcal{B}_{\lambda_3}$  is

related to the asymmetry parameter  $\alpha_{\Xi^0 K^+} = \frac{\left|\mathcal{B}_{\frac{1}{2}}\right|^2 - \left|\mathcal{B}_{-\frac{1}{2}}\right|^2}{\left|\mathcal{B}_{\frac{1}{2}}\right|^2 + \left|\mathcal{B}_{-\frac{1}{2}}\right|^2}$ , helicity  $\mathcal{C}_{\lambda_4}$  is related to the asymmetry parameter

 $\alpha_{\Lambda\pi^{0}} = \frac{\left|\mathcal{C}_{\frac{1}{2}}\right|^{2} - \left|\mathcal{C}_{-\frac{1}{2}}\right|^{2}}{\left|\mathcal{C}_{\frac{1}{2}}\right|^{2} + \left|\mathcal{C}_{-\frac{1}{2}}\right|^{2}} \text{ and helicity } \mathcal{D}_{\lambda_{5}} \text{ is related to the asymmetry parameter } \alpha_{p\pi^{-}} = \frac{\left|\mathcal{D}_{\frac{1}{2}}\right|^{2} - \left|\mathcal{D}_{-\frac{1}{2}}\right|^{2}}{\left|\mathcal{D}_{\frac{1}{2}}\right|^{2} + \left|\mathcal{D}_{-\frac{1}{2}}\right|^{2}}.$  Then the joint angular distribution becomes

$$\begin{aligned} \frac{a}{d\cos\theta_{0} d\cos\theta_{1} d\cos\theta_{2} d\cos\theta_{3} d\phi_{1} d\phi_{2} d\phi_{3}} \\ &\propto 1 + \alpha_{0} \cos^{2}\theta_{0} \\ &+ (1 + \alpha_{0} \cos^{2}\theta_{0}) \alpha_{\Xi^{0}K^{+}} \alpha_{\Lambda^{0}} \cos\theta_{2} \\ &+ (1 + \alpha_{0} \cos^{2}\theta_{0}) \alpha_{\Xi^{0}K^{+}} \alpha_{\mu^{-}} \cos\theta_{2} \cos\theta_{3} \\ &+ (1 + \alpha_{0} \cos^{2}\theta_{0}) \alpha_{\Xi^{0}K^{+}} \sqrt{1 - \alpha_{\Lambda^{\pi^{0}}}^{2}} \alpha_{\mu^{\pi^{-}}} \sin\theta_{2} \sin\theta_{3} \cos(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \alpha_{\Xi^{0}K^{+}} \sin\theta_{1} \sin\phi_{1} \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \alpha_{\Xi^{0}K^{+}} \sin\theta_{1} \sin\phi_{1} \cos\theta_{2} \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \alpha_{\Xi^{0}K^{+}} \alpha_{\Lambda^{\pi^{0}}} \alpha_{\mu^{\pi^{-}}} \sin\theta_{1} \sin\phi_{1} \cos\theta_{3} \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \alpha_{\Xi^{0}K^{+}} \alpha_{\Lambda^{\pi^{0}}} \alpha_{\mu^{\pi^{-}}} \sin\theta_{1} \sin\phi_{2} \sin\theta_{3} \cos(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \alpha_{\mu^{\pi^{-}}} \sin\theta_{1} \sin\phi_{1} \sin\theta_{2} \sin\theta_{3} \cos(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \alpha_{\Lambda^{\pi^{0}}} \cos\theta_{1} \sin\theta_{1} \sin\theta_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \alpha_{\Lambda^{\pi^{0}}} \cos\theta_{1} \sin\theta_{2} \sin(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \alpha_{\mu^{\pi^{-}}} \cos\theta_{1} \sin\theta_{1} \sin\theta_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \cos\theta_{3} \\ &- \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \alpha_{\mu^{\pi^{-}}} \cos\theta_{1} \sin\theta_{1} \sin\theta_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \cos\theta_{3} \\ &+ \sqrt{1 - \alpha_{0}^{2}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \alpha_{\mu^{\pi^{-}}} \cos\theta_{1} \sin\theta_{1} \sin\theta_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \sin\theta_{3} \sin(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \sqrt{1 - \alpha_{\Lambda^{\pi^{0}}}^{2}} \alpha_{\mu^{\pi^{-}}} \cos\theta_{1} \sin\theta_{1} \sin\theta_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \sin\theta_{3} \sin(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \sqrt{1 - \alpha_{\Lambda^{\pi^{0}}}^{2}} \alpha_{\mu^{\pi^{-}}} \cos\theta_{1} \sin\phi_{1} \sin\phi_{2} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \sin\theta_{3} \sin(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\ &+ \sqrt{1 - \alpha_{0}^{2}}} \sin\Delta_{0} \sin\theta_{0} \cos\theta_{0} \sqrt{1 - \alpha_{\Xi^{0}K^{+}}} \sqrt{1 - \alpha_{\Lambda^{\pi^{0}}}^{2}} \alpha_{\mu^{\pi^{-}}} \cos\phi_{1} \cos(\Delta_{\Xi^{0}K^{+}} + \phi_{2}) \sin\theta_{3} \sin(\Delta_{\Lambda^{\pi^{0}}} + \phi_{3}) \\$$

where the  $\alpha_0$  is the angular distribution parameter of  $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$  and  $\Delta_0$  is the transverse polarization for  $\Lambda_c^+$ . The decay asymmetry parameters  $\alpha_{\Lambda\pi^0}$  and  $\alpha_{p\pi^-}$  are taken from PDG [3]. The  $\Delta_{\Xi^0K^+} = \delta_{\frac{1}{2}}^{\mathcal{B}} - \delta_{-\frac{1}{2}}^{\mathcal{B}}$  and  $\Delta_{\Lambda\pi^0} = \delta_{\frac{1}{2}}^{\mathcal{C}} - \delta_{-\frac{1}{2}}^{\mathcal{C}}$  are the difference of the phase of the helicity amplitude B and C, respectively. For the charge conjugate mode,  $\bar{\Lambda}_c^- \to \bar{\Xi}^0 K^-$ , the formula of angular distribution is same.

### **III. SYSTEMATIC UNCERTAINTY**

The systematic uncertainties arise mainly from the reconstruction of final states,  $\Delta E$  requirement,  $M_{\rm BC}$  signal selection, the background subtraction, the uncertainties from the quoted values of  $\alpha_0$ ,  $\Delta_0$ ,  $\alpha_{\Lambda\pi^0}$ ,  $\alpha_{\bar{\Lambda}\pi^0}$ ,  $\Delta_{\Lambda\pi^0}$ ,  $\Delta_{\bar{\Lambda}\pi^0}$ ,  $\Delta_{\bar{\Lambda}\pi^0}$ ,  $\alpha_{\bar{\mu}\pi^-}$ , and  $\alpha_{\bar{p}\pi^+}$ , and the fit bias. Systematic uncertainties from various sources are combined in quadrature to calculate the total systematic uncertainties.

The reconstruction efficiency of charged kaon is studied with the control sample of  $J/\psi \to K_S^0 K^{\pm} \pi^{\mp}$  events, that for  $\pi^0$  with  $\psi(3686) \to \pi^0 \pi^0 J/\psi$  and  $e^+e^- \to \omega \pi^0$ , and that for  $\Lambda$  with  $J/\psi \to \bar{p}K^+\Lambda$  and  $J/\psi \to \Lambda\bar{\Lambda}$  [4]. The signal MC samples are re-weighted based on the data-MC differences in various momentum ranges resulting in new MC integration and new fitting parameters. The uncertainties related to the  $\Delta E$  and  $M_{\rm BC}$  requirement are evaluated by smearing the signal MC sample with a Gaussian resolution function. The changes of the fit results based on new accepted signal MC events are taken as the systematic uncertainties. All effects mentioned above are negligible except for  $\Lambda$  reconstruction.

For the background subtraction, we consider both background size and the background modelling. The background size including combinational background and mis-reconstructed component is obtained from the fit to the  $M_{\rm BC}$  spectrum. The relevant systematic uncertainties are examined by repeating the fits with a alternative background size obtained from the Gaussian sampling of the fitted parameters. The ensemble of fitted parameters obtained will be fit to a Gaussian and the sum of the fitted Gaussian resolution and the difference between the fitted Gaussian mean and the nominal result are assigned as the systematic uncertainty for background size. The uncertainty of the background modelling also considers both combinational background and the mis-reconstructed component. The mis-reconstructed model is examined by an alternative signal MC sample produced with the new input parameters  $\alpha_{\Xi^0K^+}$  which are changed within  $\pm 1\sigma$ . The uncertainty due to the combinational background model is estimated by varying the relative weights between  $\Lambda_c^+ \bar{\Lambda}_c^-$  pairs and other hadronic events based on the uncertainties of their cross section ratio.

The systematic uncertainty due to the input parameters is evaluated by varying these parameters within  $\pm 1\sigma$  using a Gaussian sampling method. For each parameter, the obtained results are fit to a Gaussian function and the sum of the fitted Gaussian resolution and the difference between the fitted Gaussian mean and the nominal result is taken as the systematic uncertainty. The systematic uncertainties arising from fit bias are investigated via pull distribution checks. The mean values of the pull distributions are used to correct the nominal results. The differences between the uncorrected and corrected results are assigned as systematic uncertainties.

All systematic uncertainties discussed above are summarized in Table II.

Source	$\alpha_{\Xi^0K^+}$	$\Delta_{\Xi^0 K^+}$ [rad]
Tracking and PID efficiencies	negligible	negligible
$\pi^0$ reconstruction	negligible	negligible
$\Lambda$ reconstruction	0.01	0.01
$\Delta E$ and $M_{\rm BC}$ signal regions	negligible	negligible
Background subtraction	0.03	0.08
Input parameters	0.01	0.14
Fit bias	negligible	0.05
Total	0.03	0.17

TABLE II. Systematic uncertainties in  $\alpha_{\Xi^0 K^+}$  and  $\Delta_{\Xi^0 K^+}$ .

#### IV. RELATION WITH WEAK DECAY AMPLITUDE

In the Standard Model, the amplitude for the two-body weak decay  $\Lambda_c^+ \to \Xi^0 K^+$  can be parameterized as  $\mathcal{M} = i\bar{u}_{\Xi^0}(A - B\gamma_5)u_{\Lambda_c^+}$ , based on the S-wave and P-wave amplitudes, s = A and  $p = \kappa B$  [3]. Combination the Eq. (1) and the relationship between  $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$  and the amplitude, the partial decay width  $(\Gamma_{\Xi^0 K^+})$  and decay asymmetry

can be obtained as follows:

$$\Gamma_{\Xi^{0}K^{+}} = \frac{\mathcal{B}(\Lambda_{c}^{+} \to \Xi^{0}K^{+})}{\tau_{\Lambda_{c}^{+}}} = \frac{|\vec{p}_{c}|}{8\pi} \Big[ \frac{(m_{\Lambda_{c}^{+}} + m_{\Xi^{0}})^{2} - m_{K^{+}}^{2}}{m_{\Lambda_{c}^{+}}^{2}} |A|^{2} + \frac{(m_{\Lambda_{c}^{+}} - m_{\Xi^{0}})^{2} - m_{K^{+}}^{2}}{m_{\Lambda_{c}^{+}}^{2}} |B|^{2} \Big],$$

$$\alpha_{\Xi^{0}K^{+}} = \frac{2\kappa |A| |B| \cos(\delta_{p} - \delta_{s})}{|A|^{2} + \kappa^{2} |B|^{2}},$$

$$\Delta_{\Xi^{0}K^{+}} = \arctan\frac{2\kappa |A| |B| \sin(\delta_{p} - \delta_{s})}{|A|^{2} - \kappa^{2} |B|^{2}},$$
(7)

with  $\kappa = |\vec{p}_c|/(E_{\Xi^0} + m_{\Xi^0}) = \sqrt{(E_{\Xi^0} - m_{\Xi^0})/(E_{\Xi^0} + m_{\Xi^0})} \approx 0.234582$  and  $\vec{p}_c$  is the momentum of the  $\Xi^0$  baryon in the rest frame of  $\Lambda_c^+$  particle [5]. Upon solving the system of equations presented in Eq. (7), an analytic solution of the amplitude magenitudes |A| and |B|, as well as the strong phase difference  $(\delta_p - \delta_s)$  can be obtained.

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