PROCEEDINGS OF SCIENCE

Investigation of shear stress and shear flow within a partonic transport model

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Starting from a classical picture of shear viscosity we construct a steady velocity gradient in the partonic cascade BAMPS. Using the Navier-Stokes-equation we calculate the shear viscosity coefficient. For elastic isotropic scatterings we find a very good agreement with the analytic values. For both elastic and inelastic scatterings with pQCD cross sections we find good agreement with previously published calculations.

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1. Introduction

Experimental data of ultra-relativistic heavy-ion-collisions at RHIC indicate that the Quark-Gluon-Plasma behaves like an ideal fluid above and close to T_C, and thus can be described by hydrodynamic models. Fundamental for this ansatz is the smallness of the viscosity coefficients. Using the partonic cascade BAMPS (Boltzmann Approach of Multi-Parton Scatterings) [2, 3] we calculate the viscosity-to-entropy-ratio of a gluonic medium with pQCD-cross sections using a steady fluid setup with a flow gradient. Motivated by the classical picture for investigation of viscosity, as introduced in [1], we consider a particle system embedded between two plates, as seen in Fig. 1. The two plates move in opposite directions each with a velocity v_{wall}. In the classical limit the shear stress T^{xz} is proportional to the gradient of the flow velocity.

$$T^{xz} = -\eta \frac{\partial v_z}{\partial x} \tag{1.1}$$

where η is the shear viscosity.

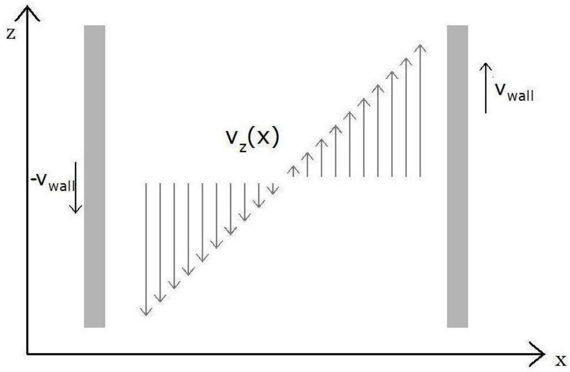

Figure 1: The classical definition of viscosity. Two plates moving in opposite directions with velocity $\pm v_{wall}$. A flow gradient is established between the plates. The viscosity is proportional to the Force.

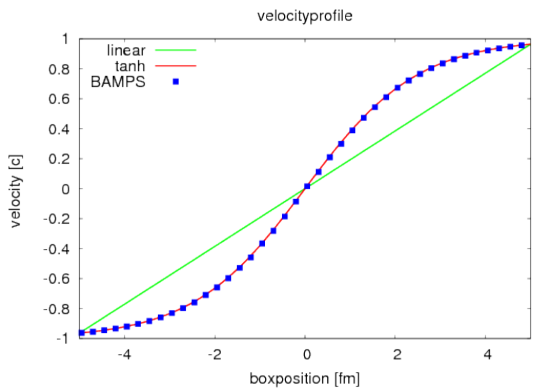

Figure 2: Velocity profile in z-direction. BAMPS results for $v_{wall} = \pm 0.964c$ and mean free path $\lambda_{mfp} = 0.01 fm$.

2. The cascade

We use the partonic cascade BAMPS as introduced in [2, 3] with elastic isotropic crossections σ_{22} .The mean-free-path λ_{mfp} is kept constant using

$$\lambda_{mfp} = \frac{1}{n\sigma_{22}} \tag{2.1}$$

where n is the local particle density.

When a particle hits one of the plates, it will be reflected into the box, but with a new momentum sampled according to the Boltzmann distribution for a massless gas with fixed temperature and velocity v_{wall} in z-direction.

3. Shear flow

A gradient of the flow velocity $v_z(x)$ is established due to the interactions among particles. A naive solution for $v_z(x)$ is a linear function. However, this is an approximate solution, which is only valid for non-relativistic fluids. For a relativistic fluid we claim that difference in velocity between two points in the local rest frame of one of the points should only depend on the space difference of the two points. This is equivalent to the demand that the profile should look the same, if shifted in space and Lorentz boosted.:

$$\Lambda_{v(a)} v(x) = v(x+a) \tag{3.1}$$

We find a differential equation

$$\frac{d^2 v(x)}{dx^2} = 2\alpha^2 (v^3(x) - v(x)) \tag{3.2}$$

which is solved by

$$v(x) = \tanh(\alpha x) \tag{3.3}$$

These assumptions are valid in the ideal case, when effects from the boundaries can be neglected. In the classical limit α becomes small and $\tanh(\alpha x) \approx \alpha x$. With Eq. (3.3) the rapidity $y_z(x)$ becomes:

$$y_z(x) = arctanh(v_z(x)) = \alpha x \tag{3.4}$$

$v_z(x)$ is shown in Fig. 2 as an example with $v_{wall} = 0.964c$.

4. Finite-size-effects

For imperfect fluids there exists a relation between the value of a gradient at the points x and $x\prime$. We assume that, as long as all particle scatterings are elastic and isotropic, the probability of one particle moving from $x\prime$ to x without interacting with the medium decreases exponentially with the distance. Thus we assume that $I_x(x\prime)$ the influence of $x\prime$ on x, decreases exponentially, too:

$$I_x(x\prime) \propto y_z(x\prime) \times exp^{-\frac{|x-x\prime|}{\lambda_{mfp}}} \tag{4.1}$$

Taking the normalized integral of (4.1) over all space points we obtain:

$$y_z(x) = c_{norm} \int_{-\infty}^{\infty} dx\prime I_x(x\prime) = \frac{1}{2\lambda_{mfp}} \int_{-\infty}^{\infty} dx\prime y_z(x\prime) \times exp^{-\frac{|x-x\prime|}{\lambda_{mfp}}} \tag{4.2}$$

One can show, that for $y_z(x)$ in Eq. (4.2) the following equation holds:

$$\frac{d^2 y_z(x)}{dx^2} = 0 \tag{4.3}$$

This implies, that only linear functions of the form $y_z(x) = mx + n$ solve Eq. (4.2). As this equation should also be valid in the ideal limit where $\lambda_{mfp} \to 0$, we use rapidity instead of velocity. Fixing the value of $y_z(x)$ outside the interval $[-L/2, L/2]$ to $\pm y_{wall}$, we obtain the solution for Eq. (4.2):

$$y_z(x) = \frac{2y_{wall}}{L + 2\lambda_{mfp}} x = \alpha x \tag{4.4}$$

Results from BAMPS show a very good agreement with Eq. (4.4), as demonstrated in Fig.3.

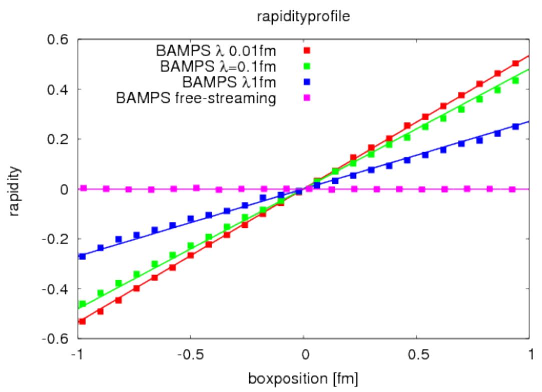

Figure 3: rapidity gradient in BAMPS with $y_{wall} = 0.55$, $Boxlength = 2fm$ and variable λ_{mfp}. Comparison with results from Eq. (4.4).

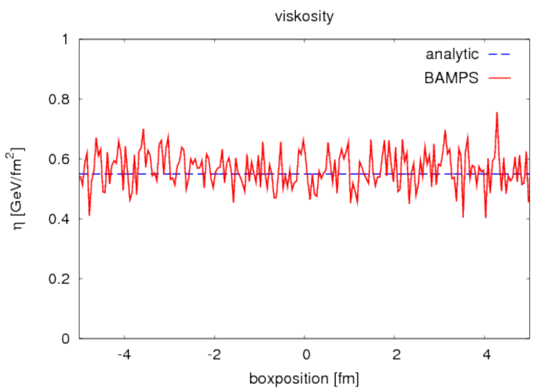

Figure 4: Viscosity over box position for $\lambda = 0.08fm$. Analytic fit with Eq. (5.6) reproduces BAMPS very good.

5. Shear viscosity

To calculate the shear viscosity η we use the Navier-Stokes-Approximation:

$$\pi^{\mu\nu} = 2\eta\nabla^{<\mu}u^{\nu>} \tag{5.1}$$

where u^{μ} is the four-velocity and $\pi^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{eq}$. With

$$u^{\mu} = (\gamma, 0, 0, \gamma v) = (\cosh(\alpha x), 0, 0, \sinh(\alpha x)) \tag{5.2}$$

Eq. (5.1) becomes

$$\pi^{xz} = -\eta\gamma\alpha \tag{5.3}$$

Since $T^{xz}_{eq} = 0$, Eq. (5.3) reduces to Eq. (1.1) in the non-relativistic case.
In comparison to our calculations we use the relation from deGroot [6] and Huovinen, Molnar [7]

$$\eta^{NS} \approx 0.8436\frac{T}{\sigma_{tr}} \tag{5.4}$$

for isotropic $2 \Leftrightarrow 2$ cross sections where σ_{tr} is given by [5]

$$\sigma_{tr} = \frac{2}{3}\sigma_{22} = \frac{2}{3}\frac{1}{n\lambda} \tag{5.5}$$

Finally we obtain

$$\eta^{NS} \approx 1.2654 T n\lambda \tag{5.6}$$

This we refer to as the analytic value.

For given temperature T, particle density n and mean-free-path λ_{mfp} one can calculate η^{NS} according to Eq. (5.6). The result of such calculation employing BAMPS is shown in Fig. 4, where we observe a perfect agreement with the analytic value.

Implementing elastic and inelastic pQCD-cross sections in BAMPS [2, 3, 8], we calculate the shear viscosity to entropy density ratio for a gluonic gas. Using the entropy density of an ideal ultra-relativistic gas, $s = 4n$, we obtain for $\alpha_s = 0.3$:

$$\frac{\eta}{s} = 0.118 \tag{5.7}$$

which is in very good agreement with dynamical calculations by Xu, Greiner [5] and Muronga, El [8] and the static setup calculations by Wesp [9]

6. Summary

The gradient of a shear flow is analytically derived and confirmed numerically within BAMPS. The influence of finite-size effects to the flow velocity is analysed. Using the Navier-Stokes approximation we calculated viscosity η and the viscosity to entropy ratio $\frac{\eta}{s}$ for both isotropic elastic scattering and inelastic scattering with pQCD-cross sections.

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