Particles production in expanding color flux tube.

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Abstract

This article generalizes Schwinger's mechanism for particles production in the time dependent field volume in quasi static approximation. Solution of DGLAP equations in double leading log approximation for low x gluon distribution function was used to derive the new formula for initial chromofield energy density. This initial chromofield energy is distributed among color neutral clusters or strings. This strings are stretched by receding nucleus. From the proposed mechanism of string fragmentation or color field decay the new formula for the rapidity spectrum of produced partons was derived.

PACS numbers: 12.38.Mh, 25.75.Nq, 25.75.-q

I. INTRODUCTION

The process of partons production can be considered as the tunnelling through the energetic gap of width $2m_\perp$ between the virtual energetic states inside Dirac sea to the real states–produced pairs with nonzero momentum. Semiclassical or WKB consideration of pair production was performed in [1], [15] and in many others. This article generalizes the Schwinger's mechanism for particles production in infinite field volume [2] for the case when caps of color flux tube recede from each other quasistatically. We have used low x behavior of parton distribution function to calculate dispersion of color charge per unit area and hence initial chromofield energy density. Moreover it will be shown how the total probability of string decay is related to the rapidity spectrum of produced partons.

The current work uses the same methods as in [3], where finite field volume effects were considered first, and improves their results. Finite size effects on pair production in transverse direction were taken into account by applying MIT boundary conditions [8]. The other possible but less general way to introduce influence of finite size effects on particles production is based on further development of Green functions method. For instance in [4] finite size effects were incorporated by expansion of Green functions on inverse volume occupied by field.

At RHIC energies ($\gamma \sim 100$) nucleus can be represented as two massive sheets leaving the strong gluon field in their wake. It is instructive to split evolution of the system produced in heavy ion collisions on three characteristic stages. On the first stage immediately after collision the nuclei because of multiple soft gluons exchange acquire stochastic color charge. This color charges produce multitude of color flux tubes occupying space between receding streaks. Shortly afterwards flux tubes decay on prompt partons. In the second stage they lose part of their energy due to gluon radiation in cascade. And finally in the third stage secondary rescatterings drive the system to local thermal equilibrium.

The subject of interest of this article will be the first stage of reaction where the screening of the color field (back reaction of produced plasma) may be disregarded. Parton distribution function calculated on this very initial stage can serve as initial condition for parton cascade model (PCM) [7], where particles are still energetic enough to apply pQCD for calculation of collision integrals and system can be treated as classical.

In this paper probability for each vertex is calculated by solving wave equations where

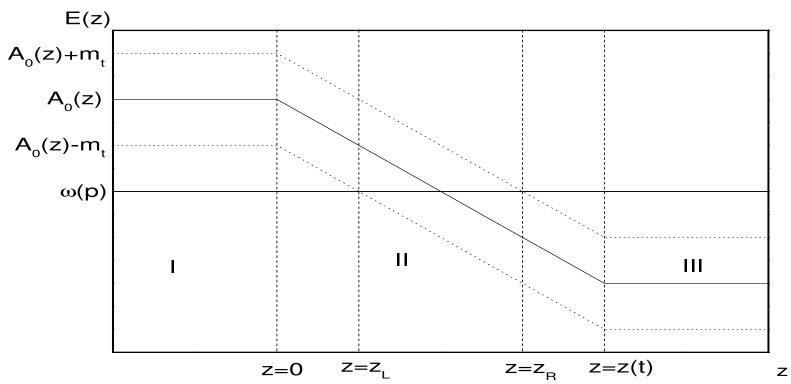

FIG. 1: Energetic gap between positive and negative continuum in the presence of external field (linear string potential) as a function of coordinate.

the volume occupied by field is restricted in transverse direction by the MIT boundary conditions and in the longitudinal direction by the distance L between colliding nuclei or capacitor plates [8].

II. THE MODEL

QCD Lagrangian and corresponding field equations read:

$$\mathcal{L}_{QCD}(x) = \bar{\Psi}(x)[i\gamma^{\mu}\mathcal{D}_{\mu} - m]\Psi(x) - \frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu,a} \quad \text{(sum over repeated indices) } a = 1\dots 8 \tag{1}$$

$$[i\gamma^{\mu}\mathcal{D}_{\mu} - m]\Psi(x) = 0; \tag{2}$$

$$\mathcal{D}_{\mu}F^{a}_{\mu\nu} = \bar{\Psi}(x)\gamma_{\nu}\frac{\lambda^{a}}{2}\Psi; \tag{3}$$

$$\mathcal{D}_{\mu} = \partial_{\mu} - ig_{s}\frac{\lambda^{a}}{2}A^{a}_{\mu}. \tag{4}$$

Gluon field is assumed gradually converting to plasma. This external field corresponds to the string potential $g_s A_{\mu} = (A_0(z), 0)$. The energy stored in the color flux tube or gluon bag is regulated by its length L and its radius r_0, see below. Thus linear vector potential is given by

$$A_0 = \begin{cases} 0 \quad \text{for } z \leq 0 \text{ (region I)} \\ -\sigma z \quad \text{for } 0 \leq z \leq L \text{ (region II)} \\ -\sigma L \quad \text{for} \quad z \geq L \quad \text{(region III)} \end{cases} \tag{5}$$

which is shown on Fig.1.

In MIT bag model the energy stored in color flux tube is constrained by its volume $\mathcal{A}L$, where $\mathcal{A} = \pi r_0^2$ is the string cross section. Therefore energy per unit length or string tension σ can be expressed though the profile of the initial chromofield energy density $\epsilon_f(b, s)$ which will be estimated below:

$$\sigma = (\epsilon_f(b, s) + B)\mathcal{A}, \tag{6}$$

where $B = \Lambda_{QCD}^4$ is the bag constant, and $\Lambda_{QCD} = 200MeV$. As it is shown on Fig.2 the larger color flux tube radius the larger density of probability of pair production. This is easily explained by increasing string tension and therefore density of probability of string decay.

In accordance with [9] color charge per unit area ρ acquired by nucleus after collision is random variable with zero mean value. It adds together by random walk Ref.[5],[6]. Therefore color charge can be treated as Gaussian random variable:

$$P(\rho) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \tag{7}$$

with dispersion σ.

To calculate chromoelectric field energy density $\epsilon_f = \frac{1}{2}\langle \mathbf{E}^2 \rangle$ we should know expectation value of the chromoelectric field strength squared. To perform this task it should be noted that resulting electric field is vector sum of electric fields induced by projectile and target color charges: $\mathbf{E} = \mathbf{E}_p + \mathbf{E}_t$. Color field strength and generated stochastic color charge are related by simple Gauss law (Abelian approximation is assumed valid): $E_a = \frac{Q_a}{\mathcal{A}} = \rho_a$. Chromoelectric field here is assumed uniform on elementary transverse area $\mathcal{A}$ which can be for instance associated with color flux tube cross section. More accurate formulas for continuous charge distribution will be shown later on. By taking into account $\langle \mathbf{E}_p \mathbf{E}_t \rangle = 0$ and $\langle \mathbf{E}_a^2 \rangle = \langle \rho_a^2 \rangle = \sigma_a^2$ chromofield energy density can be expressed through dispersions of color charges on opposite sides of colliding nuclei at given coordinate in transverse plane (b, s):

$$\epsilon_f(b, s) = \frac{1}{2}(\langle E_p^2 \rangle + \langle E_t^2 \rangle) = \frac{1}{2}(\sigma_p^2 + \sigma_t^2). \tag{8}$$

Hence chromoelectric field energy density is gaussian random variable:

$$P_\epsilon(\rho) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon(b, s)} \exp\left(-\frac{\rho^2}{2\sigma_\epsilon^2(b, s)}\right) \tag{9}$$

with dispersion $\sigma^2_\epsilon(b,s) = \sigma^2_p + \sigma^2_t$. Averaging color field energy density with above distribution yields:

$$\epsilon_f(b,s) = \frac{1}{2}\langle\rho^2\rangle = \frac{1}{2}\int\limits_{-\infty}^{\infty} \rho^2 P_\epsilon(\rho) d\rho. \tag{10}$$

Fast-moving gluons and valence quarks act as sources of chromofield. Therefore to calculate mean squared deviation of color charge per unit area of nucleus a we should add their contributions:

$$\sigma^2_a = \mu^2_q + \mu^2_g. \tag{11}$$

Quark color charge squared in a tube of transverse area $d^2\mathbf{s}$ is the color charge squared per unit quark g^2C_F times the number of quarks in the tube:

$$dn_q = N_c N_a(\mathbf{s},\mathbf{b}) d^2\mathbf{s}. \tag{12}$$

Therefore $\mu^2_q = g^2 C_F \frac{dn_q}{d^2\mathbf{s}}$, where C_F is Casimir operator for quarks: $C_F = \frac{N_c^2-1}{2N_c}$, and $N_a(\mathbf{s},\mathbf{b})$ is the number of participants from nucleus a. Gluon color charge squared μ^2_g in a tube of transverse area $d^2\mathbf{s}$ is the color charge squared per gluon g^2C_A times the number of gluons in the tube:

$$dn_g = \left\{ N_a(\mathbf{s},\mathbf{b}) \int\limits_{x_0}^{1} G(x,Q^2_s) dx \right\} d^2\mathbf{s}, \tag{13}$$

where lower integration limit is defined from relation between Bjorken's variable x and c.m. beam energy $\sqrt{s}$:

$$x_0 = \frac{Q_0^2}{s - m_N^2}, \tag{14}$$

Minimal transfered four momentum Q_0 was taken as a starting scale for Q^2 evolution: $Q_0^2 = 1GeV^2$, see below. Therefore expression for gluon color charge squared per unit takes the form $\mu^2_g = g^2 C_A \frac{dn_g}{d^2\mathbf{s}}$, where $C_A = N_c$ is the gluon Casimir operator.

Gluons inside nucleus before collision form very dense system- CGC (see [10] and refs therein) characterized by saturation momentum Q_a. By matching gluon distribution functions in low density(DGLAP) [11] and dense regimes we obtain the following equation for Q_a:

$$N_a(b,s) x G(x,Q_a^2) = \frac{C_F Q_a^4 (1-x)}{4\pi^2 \alpha_s(Q_a^2)(Q_a^2 + 4m_N^2 x^2)}, \tag{15}$$

where subscript a corresponds to nucleus $a = p, t$. The same argumentation for deriving similar equation is based on McLerran-Venugopalan model, [10],[12]. This equation can be solved iteratively. Q^2-evolution of low x gluon distribution function $G(x, Q^2)$ is governed by the DGLAP equation. Then in so-called double leading log approximation [11], where only terms proportional to $ln\frac{1}{x}lnQ^2$ are taken, it has the form

$$G(x,Q^2) = \frac{1}{x}exp\left(\sqrt{\frac{48}{11-\frac{2}{N_c}N_f}ln\left(\frac{ln\frac{Q^2}{\Lambda^2}}{ln\frac{Q_0^2}{\Lambda^2}}\right)ln\frac{1}{x}}\right)+\dots \tag{16}$$

Running coupling constant is defined as $g_s(Q^2) = \sqrt{4\pi\alpha_s(Q^2)}$, where

$$\alpha_s(Q^2) = \frac{4\pi}{(11-\frac{2}{N_c}N_f)ln\frac{Q^2}{\Lambda^2}} \tag{17}$$

is the fine structure constant.

Finally in this dense regime number of gluons per unit area of nucleus a is calculated by

$$\frac{dn_g}{d^2\mathbf{s}} = \int\limits_{x_0}^{1} \frac{C_F Q_a^4(1-x)}{4\pi^2\alpha_s(Q_a^2)(Q_a^2+4m_N^2x^2)x}dx \tag{18}$$

Therefore we have the following formula for initial chromofield energy density:

$$\epsilon_f = \frac{1}{2}(\sigma_p^2+\sigma_t^2), \tag{19}$$

where dispersion σ_a^2 depends on only saturation momentum and number of participants from nucleus a exclusively:

$$\sigma_a^2 = 4\pi\alpha_s(Q_a^2)C_F N_c\left(N_a + \frac{Q_a^4}{4\pi^2\alpha_s(Q_a^2)}\int\limits_{x_0}^{1}\frac{1-x}{x(Q_a^2+4m_N^2x^2)}dx\right) \tag{20}$$

III. PROBABILITY OF STRING DECAY

In this section we calculate total probability of string decay. Complicated further evolution of cascade related with numerous branchings of secondaries is out of scope of our consideration. Due to the three possible interactions which follow from $\mathcal{L}_{QCD}$ string can decay into quark-antiquark, gluon pair or tree gluons. Then the total density of probability is

$$W_{tot}(p,z,L) = W_{q\bar{q}}(p,z,L) + W_{gg}(p,z,L) + W_{ggg}(p,z,L) \tag{21}$$

Last term will be disregarded due to lack of analytic solution for 3-body problem.

The density of probability of gluon pair production is calculated by the formula

$$W_{gg}(p,z,L) = \nu_g \check{W}_g(p,z,L), \tag{22}$$

where $\nu_g = 2(N_c^2 - 1)$; $\check{W}_g = \Psi^*\Psi$, and Ψ is solution of Klein-Gordon equation for massless particles (48).

The density of probability of quark-antiquark pair production is defined as

$$W_{q\bar{q}}(p,z,L) = \nu_q \check{W}_{q\bar{q}}(p,z,L), \tag{23}$$

where $\nu_q = N_c N_f$; $\check{W}_{q\bar{q}} = \Psi^\dagger\Psi$, and Ψ is solution of Dirac equation (27).

Solution of the Dirac equation can be represented by the following series:

$$\Psi(x) = \sum_p \tilde{\Psi} \exp[i(p_x x + p_y y - Et)], \tag{24}$$

where

$$\tilde{\Psi} = \sum_{r=1,2} \Psi_r. \tag{25}$$

Eigenvectors Ψ_r are Dirac spinors corresponding to the different spins [17]:

$$\Psi_r = a_r \begin{pmatrix} \psi_1 + A_r\psi_2 \\ B_r(\psi_1 + A_r\psi_2) \\ A_r\psi_2 - \psi_1 \\ -B_r(A_r\psi_2 - \psi_1) \end{pmatrix}, \tag{26}$$

making of the eigenfunctions of the squared Dirac equation:

$$\left(\left(i\frac{\partial}{\partial t} + A_0\right)^2 - \left(i\frac{\partial}{\partial r}\right)^2 - m^2 \pm i\sigma\right)\psi_r = 0. \tag{27}$$

Cylindrical boundary conditions applied to the bag surface discretize transverse momentum inside the bag. The ground state of transverse momentum $p_\perp^0 = c_1/r_0$ is included to effective parton mass m_t [8]; $m_t = \sqrt{m^2 + (p_\perp^0)^2}$, where m is the current parton mass; $c_1 = 1.4347$. The spin coefficients were calculated in [17]:

$$A_{1,2} = \frac{m_\perp \pm ip_\perp}{m_\perp}, \qquad B_{1,2} = \pm i\frac{p_x + ip_y}{p_\perp}, \tag{28}$$

where $m_\perp = \sqrt{m_t^2 + p_\perp^2}$; $p_\perp = \sqrt{p_x^2 + p_y^2}$. Therefore the decomposition (25) in accordance with (26) is transformed to:

$$\tilde{\Psi} = \mu_1\psi_1 + \mu_2\psi_2, \tag{29}$$

where $\mu_1 = 2a_1 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$; $\mu_2 = 2a_2 \begin{pmatrix} \frac{m_t}{m_\perp} \\ \frac{p_x+ip_y}{m_\perp} \\ -\frac{m_t}{m_\perp} \\ \frac{p_x+ip_y}{m_\perp} \end{pmatrix}$

Spinor normalization constants are defined from condition $\mu_r^\dagger \mu_r = 1$: $a_r = \frac{1}{2\sqrt{2}}$, $r = 1, 2$.

After separation of variables (27) takes Schrodinger-type form:

$$\left(\frac{\partial^2}{\partial z^2} + p_r^2 \right) \psi_r = 0, \tag{30}$$

where

$$p_r^2 = \begin{cases} p_L^2 \quad \text{for } z \leq 0 \text{ (region I)} \\ (\omega(p) + \sigma z)^2 - m_\perp^2 \pm i\sigma \quad \text{for } 0 \leq z \leq L \text{ (region II)} \\ p_R^2 \quad \text{for} \quad z \geq L \quad \text{(region III)}, \end{cases} \tag{31}$$

where $E_L = \omega(p)$; $E_R = \omega(p) + \sigma L$; $p_{R,L} = \sqrt{E_{R,L}^2 - m_\perp^2}$ and $\omega(p) \leq -m_\perp$. Solutions in regions I, III are the following:

$$\psi_r = \begin{cases} Ie^{ik_I r} + R(L)e^{ik_R r} \quad \text{for} \quad z \leq 0 \quad \text{(region I)} \\ T(L)e^{ik_T r} \quad \text{for} \quad z \geq L \quad \text{(region III)}, \end{cases} \tag{32}$$

where R, T are amplitudes of the reflected, and transmitted waves, respectively and four-momenta in this regions are

$$k_I = (E_L, p_x, p_y, -p_L) \tag{33}$$

$$k_R = (E_L, p_x, p_y, p_L) \tag{34}$$

$$k_T = (E_R, p_x, p_y, p_R) \tag{35}$$

$$kr = E - p_x x - p_y y - p_z z \tag{36}$$

Solution in the region II is represented as a sum of the linear independent parabolic cylinder function of order $\nu_r = i\rho_r - 1/2$; $\rho_r = -\frac{m_\perp^2}{2\sigma} \pm s$; $s = \frac{i}{2}$ [18]:

$$\psi_r = A(L) D_{\nu_r}^0 (e^{i\frac{\pi}{4}} \zeta(p, z)) + B(L) D_{\nu_r}^1 (e^{i\frac{\pi}{4}} \zeta(p, z)) \tag{37}$$

where $\zeta(p, z) = \sqrt{\frac{2}{\sigma}} (\omega(p) + \sigma z)$. Coefficients $A(L), B(L)$ are found by matching the wave function at the boundaries. The boundary conditions implement the wave function continuity at $z = 0$ and $z = L$:

$$\begin{cases} (I + R(L))\mu_r = (A(L)D_L^0 + B(L)D_L^1)\mu_r \\ ip_L(-I + R(L))\mu_r = (A(L)D_L^{0'} + B(L)D_L^{1'})\mu_r \\ T(L)e^{ip_R L}\mu_r = (A(L)D_R^0 + B(L)D_R^1)\mu_r \\ ip_R T(L)e^{ip_R L}\mu_r = (A(L)D_R^{0'} + B(L)D_R^{1'})\mu_r, \end{cases} \tag{38}$$

where $D_{L,R}^{0,1} = D_{\nu_r}^{0,1}(E_{L,R}e^{i\frac{\pi}{4}})$, and amplitude of incident wave is $I = 1$. Therefore

$$B(L) = \frac{2}{q_1}\frac{D_R^{0'} - ip_L D_R^0}{ip_R(D_R^1 - qD_R^0) + qD_R^{0'} - D_R^{1'}}; \tag{39}$$

$$A(L) = \frac{2}{q_1} - B(L)q; \tag{40}$$

$$R(L) = B(L)(D_L^1 - qD_L^0) + \frac{2}{q_1}D_L^0 - 1; \tag{41}$$

$$T(L) = e^{-ip_R L}\left[B(L)(D_R^1 - qD_R^0) + \frac{2}{q_1}D_R^0\right], \tag{42}$$

where

$$q_1 = D_L^0 - \frac{D_L^{0'}}{ip_L}; q = \frac{D_L^1 - \frac{D_L^{1'}}{ip_L}}{q_1} \tag{43}$$

Gluon production is described by Klein-Gordon equation:

$$\left(\left(i\frac{\partial}{\partial t} + A_0\right)^2 - \left(i\frac{\partial}{\partial r}\right)^2\right)\Psi = 0 \tag{44}$$

Solution of this equation is the following:

$$\Psi = \sum_p \sum_{\lambda=\pm 1} \tilde{\Psi}\exp[i(p_x x + p_y y - Et)]e_\lambda, \tag{45}$$

where e_λ polarizations of vector particles.

After separating of variables (44) takes the Schrodinger-type form:

$$\left(\frac{\partial^2}{\partial z^2} + p^2\right)\tilde{\Psi} = 0, \tag{46}$$

where

$$p^2 = \begin{cases} p_L^2 \quad \text{for } z \le 0 \text{ (region I)} \\ (\omega(p) + \sigma z)^2 - m_\perp^2 \quad \text{for } 0 \le z \le L \text{ (region II)} \\ p_R^2 \quad \text{for} \quad z \ge L \quad \text{(region III)}, \end{cases} \tag{47}$$

and $m_\perp = \sqrt{(\frac{c_1}{r_0})^2 + p_\perp^2}$ is the gluon transverse mass. In the region II solution of this equation is

$$\tilde{\Psi} = A(L)D_\nu^0(e^{i\frac{\pi}{4}}\zeta(p,z)) + B(L)D_\nu^1(e^{i\frac{\pi}{4}}\zeta(p,z)) \tag{48}$$

where $\rho = -\frac{m_\perp^2}{2\sigma}$.

IV. TRANSMISSION AMPLITUDE OF PAIR PRODUCTION IN INFINITE VOLUME

In this section we will show that in infinite volume occupied by field the density of probability or transmission amplitude has the proper Schwinger asymptotic. Transmission amplitude is expressed by the formula (42) which is transformed to the following

$$T(L)=\frac{2p_L(D^0_R D^{1'}_R - D^{0'}_R D^1_R)e^{-ip_R L}}{p_L(D^{1'}_R D^0_L - D^{0'}_R D^1_L) + i(D^{1'}_R D^{0'}_L - D^{0'}_R D^{1'}_L) + p_R D^1_R(D^{0'}_L - ip_L D^0_L) + p_R D^0_R(ip_L D^1_L - D^{1'}_L)} \quad (49)$$

At large arguments parabolic cylinder function [18] takes the following form $D^0_R \to e^{-\frac{\pi m_\perp^2}{2\sigma}}$, $D^{0,1'}_{R,L} \to \pm ip_{R,L} D^{0,1}_{R,L}$. Therefore asymptotic of transmission amplitude reads:

$$T(L)e^{ip_R L} \sim \frac{-4ip_L p_R D^0_R D^1_R}{-2ip_L p_R D^0_R D^1_L} = 2\frac{D^1_R}{D^1_L} \sim e^{-\frac{\pi m_\perp^2}{2\sigma}}. \quad (50)$$

From latter it follows that amplitude of pair production in infinite field volume approaches the following limit:

$$\varpi = \lim_{L\to\infty} |T(L)|^2 \sim \exp\left(-\frac{\pi m_\perp^2}{\sigma}\right). \quad (51)$$

V. PARTONS RAPIDITY DISTRIBUTION AND EXPANDING CHROMOFIELDS

In the initial stage of reaction all the kinetic energy lost by baryons is transformed to chromofield:

$$E_f(b) = \int d^2 s(\epsilon_f(b,s) + B)\bar{z}(b,s), \quad (52)$$

where average string length is

$$\bar{z}(b,s) = \sum_{L=L_{min}}^{\infty} \int_{-\infty}^{\infty} d^3p \int_{-\infty}^{\infty} dz z W_{tot}(p,z,L), \quad (53)$$

where $L_{min} = \frac{2m_\perp}{\sigma}$. On the other hand finally all chromofield energy is transformed to partonic plasma

$$E_f(b) = \int d^3p\Gamma(p) = \int d^3p\omega(p)\frac{dN}{d^3p} \quad (54)$$

Therefore rapidity distribution of partons generated in the slab-slab collision is calculated by the formula:

$$\frac{dN}{dy}(b) = \int_{-\infty}^{\infty} d^2p_\perp \Gamma(p) \quad (55)$$

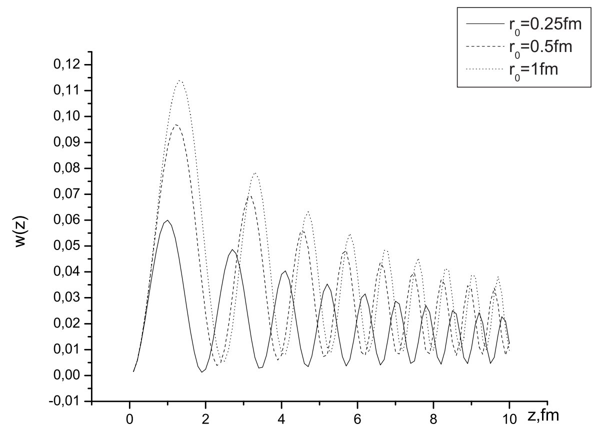

FIG. 2: Density of probability of quark-antiquark pair production $W^f_{q\bar{q}}(z, p_\perp, y, L)$ as a function of coordinate inside the color flux tube of length $L = 10fm$ at the fixed rapidity $y = 0.21$ and average transverse momentum $< p_\perp >= 0.5GeV$.

VI. CONCLUSIONS

The results of this article are following: 1)the total density of probability of partons production is obtained by the exact solutions of squared Dirac and Klein-Gordon Eqs. in the vector linear potential; 2)The time evolution of density of probability of the produced partons have been introduced by time-dependent boundary conditions (quasistatic approximation); 3)Rapidity spectra of the field energy and produced particles are calculated from the energy conservation law; 3)It was shown that the density of probability of particles production in the infinite field volume approaches the classical Schwinger's result.

Numerical simulations were performed for most central $Au+Au$ collisions with c.m.energy $\sqrt{s} = 200AGeV$. The coordinate dependencies of a $q\bar{q}$–pair production density of probability at the different color flux tubes radii ($r_0 = 0.25fm, 0.5fm, 1fm$) are shown on Fig.2. Quarks have current mass $m_q \sim 8MeV$ and gluons are assumed massless.

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