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Free Will, Robots, and the Axiom of Choice[1]

Abstract: There are quite a number of similarities between the moral concept of choice and the mathematical axiom of choice. These similarities shed light on how to adapt law to solve cases that arise with the increasing "autonomy" of robots.
Keywords: free will, choice, determinism, indeterminism, axiom of choice, mathematics, robot, law, Immanuel Kant

I. Introduction

Quite a number of humanities discuss choice as characteristic of human action. In many mathematical theories (and thus also in many theories of physics and other sciences), the axiom of choice is essential. Do the moral concept of choice and the mathematical axiom of choice have anything in common? They do. There are actually a multitude of similarities.

These similarities indicate that the traditional discussion of legal rules tends to overestimate the role choice plays and that robots might even legitimately be regarded as subjects of certain legal rules one day, rather than doomed to remain mere legal objects. I will not provide a complete argument that they will because that would entail massive speculation about the future development of robots, their planning skills, and the law. What I will do is explain and analyze the common features of the moral concept of choice and the axiom of choice, first indicating how this observation can serve as a module of an argument suitably included into future reasoning and how this observation relates to robots.

II. The nimbus of choice

In legal contexts, free will and actions are a common basis of arguments. Usually this argument follows the following pattern:

- Some rule presupposes choice.

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[1] This text is based on my presentation on August 16th 2011 in the Special Workshop 33 (Person, Verantwortung, Grenzen des Rechts – alte Debatten im neuen Kontext „Robotik und Künstliche Intelligenz"; organized by Dr. Susanne Beck) of the IVR World Congress 2011 in Frankfurt/Main: "Willensfreiheit, Roboter und Auswahl-axiom". I would like to thank the participants in the discussion for their valuable remarks and Prof. Dr. Sharon Byrd for her help in preparing the English version. A more detailed and referenced German version of this paper will appear in: Beck, Susanne (ed.), *Jenseits von Mensch und Maschine – Moralische und rechtliche Aspekte des Umgangs mit Robotern, Künstlicher Intelligenz und Cyborgs*, Nomos Baden-Baden 2012, 41-73.

- But no free act was committed.
- Thus the rule is not applicable in this situation.

In the same manner even more radical results can be obtained: No object ever exercises choice; objects do not act at all. Thus, rules that presuppose choice never apply to a mere object. Moreover, concepts that presuppose choice can never be used adequately with regard to a mere object. Hence it would seem that rules and concepts like these were always inapt for robots.

The argument is compelling; its application, however, can become quite problematic. The argument refers to choice as if it were a monolithic block. Therefore one can frequently encounter arguments constructed according to this pattern yet oversimplifying the subject matter. For example, penalties require guilt and guilt requires choice. But just like guilt is actually incremental – which the law recognizes, e.g. in § 21 StGB (= German Criminal Code) – so is choice. It is wrong to treat choice and the applicability of rules and concepts that presuppose choice as all-or-nothing issues. Rather differentiated answers are necessary and my observation is meant to motivate them.

There is one issue we handle especially simplistically today: Only human beings can have choice. For them we often recognize different gradations of choice and even circumstances that exclude choice in a given situation. Time and again freedom of human will and actions even gets categorically contested. But no other real beings – and even less inanimate objects – would be said to exercise choice. Only juridical persons are sometimes treated as having choice. Mostly we do not classify them at all in the categories of choice or no-choice but tacitly apply the rules made for natural persons to them. And at times they receive differentiated treatment, e.g. regarding their decision-making.

Sooner or later we might have to consider a similarly differentiated treatment of some robots. For the reason indicated *supra*, this idea will not be developed here any further. Still at least some authority – albeit not considering this specific question, but nonetheless covering the fundamental issue – may be quoted in this respect. Kant holds that in practical interaction we need to presume choice.[2] In his practical philosophy, he examines „reasonable beings," particularly in his Groundwork of the Metaphysics of Morals.[3] Humans are a degenerated form of these beings for they do not always act reasonably.[4] In Kant's view, freedom is always a question of degree and as far as beings act in the real world they can never reach the

[2] Immanuel Kant, *Grundlegung zur Metaphysik der Sitten*, 1785, Akademie-edition vol. IV, 447 f. (and 455).
[3] Cf. Immanuel Kant, *Grundlegung* (Fn. 2), 389, 412
[4] Immanuel Kant, *Grundlegung* (Fn. 2), 407

pure form of freedom. This is especially true for humans. And next to these “only somewhat free beings” there is still plenty of room for some “even less free beings.”

It seems we often quite unconsciously pin a nimbus of uniqueness to the concept of choice. Humans like that nimbus to shine onto, and hope it will be reserved for, them. This tendency makes rational treatment of choice more difficult than it should be. Not only can choice never be proven empirically, it is not a characteristic of “real” events at all and thus cannot establish any “real” quality humans would like to have. Choice is a necessary model assumption in (practical) theories of action. In these theories it serves a technical purpose. The nimbus, however, builds a psychological barrier to considering a degree of freedom for robots, or better, to considering adapting legal constructions, which traditionally presuppose choice, and making them applicable to robots. Countless works of science fiction have documented this barrier at length.

My observations should undermine that nimbus.

III. Qualities of the moral concept of choice

The idea of choice includes a couple of elements that nourish its nimbus. In my observations, I will show that these elements can also be detected for the mathematical axiom of choice. While moral philosophers and lawyers will certainly not get excited about that idea, they also should not get so excited about these aspects of choice in the first place:

1. On the face of it, the assumption of choice seems clear at first blush. We experience ourselves as deciding on our own actions and perceive other people in the same way. Our decisions even seem to be what defines us as human. Not until we give deeper thought to this assumption does it become problematic.

2. We picture an act of choice as an active selection between different options.

3. A complete explanation of actions is impossible. This is because it is impossible to give a detailed reconstruction of the “choosing” involved. It cannot be analyzed into elementary steps that would necessarily be repeated in the same way under similar circumstances at another time.

4. Hence choices are ultimately always unpredictable. They can never be reconstructed in a deductive system based on empirical premises.

5. It is not even possible to provide a precise formulation of the options that existed before the choice or at least the option that was finally chosen. The options and the act actually performed (which is the result of the choosing) can only be described in an approximate manner.

6. Consistent models of human actions can be based on the assumption of choice. However, deterministic models – meaning models which exclude choice – can be just as consistent.[5]

7. Theories based on the assumption of choice can usually be much more comprehensive with regard to the moral and especially legal status of human actions.

8. Realizing that both the assumption of choice and the assumption of determinism can each be part of consistent theories (not both at the same time of course) gives rise to very productive questions: What really depends on the assumption of choice or determinism respectively? And what can be established independently of these assumptions, i.e. consistently with both of them?

These elements are important characteristics of freedom of will and actions, even though they are certainly not exhaustive. They seem, however, to contain the essential basis for the nimbus – especially those mentioned first.

IV. Concordance with the axiom of choice

The axiom of choice is a thought construct of pure mathematics. It does not concern actions, evaluations, or descriptions of the real world. Accordingly, it is neither the subject of a normative nor of an empirical theory. In a manner of speaking, it belongs to the chamber of horrors for everyone trying to focus on more practical matters.

The axiom of choice applies to sets. It assumes an arbitrary amount of sets (but at least one) as given. None of these sets may be empty, but each can be arbitrarily large. They need not match in any way – neither in size nor content. The axiom of choice consists of the assumption that there always is a mapping – called the choice function – that maps each set to one of its own elements.[6] (For example, given the sets {1, 2, 3}, {11, 12} and {21}, the first set could be mapped to 2, the second set to 12, and the third set here must be mapped to 21, for this is its only element.)

In modern mathematics, axioms do not contain any claim to truth.[7] Thus the axiom of choice is only an assumption, indispensable in some mathematical theories, axiomatically established to obtain its own consequences, which the theory treats. It simply formulates a

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[5] Cf. e.g. Michael S. Moore, *Placing Blame – A General Theory of the Criminal Law*, 1997, esp. 504 ff., 617 f.

[6] Cf. Imre Leader, The axiom of choice, in: *The Princeton Companion to Mathematics*, ed. by T. Gowers et al., 2008, III.1, p. 158.

[7] See Hans Freudenthal, »Axiomatik«, in: *Historisches Wörterbuch der Philosophie*, ed. by J. Ritter, K. Gründer, G. Gabriel, vol. 1 (1971); H. C. Kennedy, The Origins of Modern Axiomatics, *The American Mathematical Monthly*, 79 (1972), 133 ff. concerning the development of Hilbert's and Ackermann's concept of an axiom and its deviation from Aristotle's and Euklid's concept. For the formulation of that concept see David Hilbert/ Wilhelm Ackermann, *Grundzüge der theoretischen Logik*, 6th ed. (1972), § 8, p. 22 (just as in the 1st ed., 1928).

logical precondition – if it is fulfilled the desired consequences follow, if not the theory is inapplicable to the situation. Pure mathematics is pure exactly because it does not consider the applicability of its theories to pose a mathematical problem. In fact mathematical theories using the axiom of choice find very broad application[8] – in the end mostly without even the possibility to prove that the preconditions of the axiom of choice are met in any strict sense.

The axiom of choice shares this characteristic with the moral concept of choice. But for neither the mathematical axiom nor the moral concept is this similarity specific. It merely results from the fact that both have an axiomatic character and the users of theories usually are more interested in their results than in paying attention to their preconditions. Furthermore, the mathematical axiom of choice shares all the characteristics listed above with the moral concept:

1. On the face of it, the axiom of choice seems self-evident: Since the sets are given, why should it be a problem to select elements? Indeed there is no problem as long as only a finite number of sets are involved. Yet arbitrarily many sets may be given – especially infinitely many, a simple example being that only countably infinitely many sets are given (i.e. only as many as there are natural numbers). Even then the selection of elements would take much too long for any human to actually perform all necessary choices during one lifetime. At least a selection process would be "construable" in the following way: Imagine someone who processes set by set and picks one element from each. This someone would never get finished with all sets. However, (if that someone had eternal life) every set would be processed after a finite period of time (that differs from set to set, exceeds every possible boundary, yet always remains finite). Yet these are only the "harmless" cases. In the world of mathematics, finite or countably infinite sets are pretty small. Even the real numbers are no longer countable. Therefore it would suffice to take as many sets as there are real numbers, and it would be absolutely impossible for humans to imagine how the choice function could ever be construed. On second thought, choice in mathematics is therefore just as mysterious as in moral philosophy.

2. We picture the content of the axiom of choice as a step-by-step selection process. For each set all its elements are options to select from and only one gets chosen. Accordingly, we actually picture the choice function as act-decisions (which also is why the axiom is called axiom of choice).

[8] For an overview over equivalent assumptions in very different parts of mathematics see Horst Herrlich, *Axiom of Choice*, 2006, esp. 9 ff. and Herman Rubin, *Equivalents of the axiom of choice*, II, 1985, passim, both with further references.

3. Only in special cases can the choice function be explicitly formulated (esp. when merely a small amount of sets is given or special characteristics of the given sets make a description of the selection possible – e.g. in case each set contains a zero the choice function can be described as “always select the zero”). In the general case, however, formulating the choice function is impossible; being “non-constructive” is just what the axiom of choice is about. Its demand for mathematical existence of the choice function only means that (1) the idea of such a function does not contradict the other axioms and that (2) all considerations based on the axiom of choice may presuppose a choice function. In contrast that does not mean that a process can be defined which would actually effect the choosing of an element from each set – just like no process can be formulated, which would fully explain human decisions to act.
4. Just like actions are generally unpredictable there is no way to tell what the choice function (or the set of all selected elements) would actually look like.
5. In the analogy explained above a single act-decision corresponds to the selection of one element from one of the given sets. In every set each of its elements could be selected, thus it is a choice-option and corresponds to an option for the act-decision. Just like an option to act can never be described in an exhaustive manner neither can (in the general case) the choice-option. That is because the meaning of “given sets” is inherently problematic. The sets do not need to be listed anywhere; oftentimes it would not even be possible to list them since there can be uncountably – and therefore “unlistably” – many sets. The same is especially true for the elements of those sets. Consequently, the requirement that sets need to be given is very abstract. Still nobody who is used to talking about options and decisions to act – which are presumed despite the fact that they can never be expressed in an exact manner – should feel confused by that abstractness.
6. Many mathematical theories are consistent with both the axiom of choice and its negation, just like both the assumption of choice and the assumption of determinism can be part of theories of actions (not both at the same time, of course).
7. So many important mathematical theories contain the axiom of choice that few of today's mathematicians would seriously think about sacrificing that axiom. Core parts of analysis and algebra, for example, are based on the axiom of choice or equivalent premises.

8. Mathematicians have put great effort into analyzing which theorems need the axiom of choice or an equivalent assumption and which theorems are also consistent with its negation.[9]

This list shows a complete correspondence of the characteristics first explained for the moral concept of choice and now for the mathematical axiom. It would therefore be a fundamental error to attribute those characteristics to the nature of humans. Of course in some sense even the axiom of choice depends on human nature, because this axiom only "exists" in their minds. But it still is just a more or less arbitrary assumption made for the benefit of its consequences. It is a technical means to construct desired theories. But it has no ontological basis – especially not in humanity.

The observed parallels suggest that the same might be true for the moral concept of choice. Simultaneously it suggests that there is no reason to abolish the moral assumption of choice, but rather that it is necessary to handle it without emotional baggage.

I do not want to claim that my observations are new. Although I do not know of any thorough analysis of the similarities between the moral concept of choice and the mathematical axiom, the parallels are widely understood in a more general sense. The fact that both are named "choice" already indicates this similarity.

V. Outlook

Back to the robots! They are machines constructed by humans, and thus they – like their creators – will most certainly never be able to define the choice function in the general case. Their capabilities may exceed those of humans with regard to the speed and resilience with which they can select from defined options, but it is practically certain that they will never be able to handle uncountably infinite selections in any way that would principally exceed the capabilities of humans. This is simply due to the fact that it would not even be possible to define the options they would have to choose from.

Nevertheless one day we might have robots whose behavior can only be explained on the basis of a theory that contains the axiom of choice (e.g. because the physical theory that explains one of the robot's components needs mathematical methods which depend on that axiom). Today's robots, with huge but still finite memory, processing states, and possibilities of input given at any time, do not come near that threshold and I do not want to make any guesses regarding the likelihood that this will ever happen. But if it happens, then those robots

[9] With the words of Saharon Shelah/ Alexander Soifer, Axiom of choice and chromatic number of the plane, *JCTA* (Journal of combinatorial theory, Ser. A) 103 (2003), 387, 388: „So, it is natural to ask, what if we have no choice? Absence of choice – in mathematics as in life – may affect outcome."

would have reached the point where their behavior matches human actions in all of the eight characteristics discussed here.

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