Comparison of Nuclear Transport Models with 800A-MeV La+La Data

J. Aichelin,(1) J. Cugnon,(2) Z. Fraenkel,(3) K. Frankel,(4) C. Gale,(5) M. Gyulassy,(6) D. Keane,(7) C. M. Ko,(8) J. Randrup,(6) A. Rosenhauer,(9) H. Stöcker,(10) G. Welke,(11) and J. Q. Wu(8)

(1)*Institut für Theoretische Physik, Universität Heidelberg, D-6900 Heidelberg, Federal Republic of Germany*
(2)*Institut de Physique Sart Tilman, Université de Liège, B-4000 Liège 1, Belgium*
(3)*Nuclear Physics Department, Weizmann Institute, Rehovot 76100, Israel*
(4)*Research Medicine and Biophysics Division, Lawrence Berkeley Laboratory, Berkeley, California 94720*
(5)*Theoretical Physics Branch, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada KOJ 1J0*
(6)*Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*
(7)*Physics Department, Kent State University, Kent, Ohio 44242*
(8)*Cyclotron Institute and Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843*
(9)*School of Physics and Astronomy, Tel Aviv University, 69978 Tel Aviv, Israel*
(10)*Institut für Theoretische Physik, J. W. Goethe Universität, D-6000 Frankfurt, Federal Republic of Germany*
(11)*Department of Physics, State University of New York, Stony Brook, New York 11794*
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Nuclear transport models including density- and momentum-dependent mean-field effects are compared to intranuclear-cascade models and tested on recent data on inclusive p-like cross sections for 800A-MeV La+La. We find a remarkable agreement between most model calculations but a systematic disagreement with the measured yield at 20°, possibly indicating a need for modification of nuclear transport properties at high densities.

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Since the discovery of collective-nuclear-flow phenomena[1] in high-energy nuclear collisions, there has been an intensified effort to develop microscopic nuclear transport models including effects due to nuclear mean fields. Up to that time, intranuclear-cascade models,[2,3] which include only the effects of incoherent nucleon-nucleon scattering, could reproduce most features of double-differential inclusive cross sections.[4] While there were earlier hints of a possible breakdown of cascade models,[4] collective flow could only be confirmed after it became possible to measure triple-differential inclusive cross sections for collisions of heavy nuclei with $A > 100$. Such nuclear flow was first predicted in terms of hydrodynamical models,[5] but the directed in-plane flow momenta were typically overestimated by a factor of 2. On the other hand, the flow momenta were typically underestimated by a factor of 2 by cascade modes.[5,6] The extra "side splash" has been interpreted as evidence for extra nuclear repulsion due to the stiffness of nuclear matter at high densities, while the relative smallness of the flow momenta shows the importance of nonequilibrium transport effects in finite nuclei. In terms of transport theory, these observed flow patterns motivated the addition of a nuclear Vlasov term to the Boltzmann collision term.

Several groups have developed transport models including such a nuclear Vlasov term.[5−12] The essential new input in this class of models is the nucleon optical potential $U(\rho,p)$, which depends not only on density but also on the momentum of the nucleon. The goal of such approaches is to constrain the possible form of U up to several times normal nuclear density by fitting triple-differential data. In this way, it is hoped that high-energy heavy-ion collisions will eventually lead to reliable experimental constraints on the nuclear equation of state. In addition, by studying the effect of varying the effective nucleon-nucleon cross sections in the Boltzmann term, it is hoped that information on the nuclear transport coefficients in dense, highly excited nuclear matter can also be extracted from the data.

While most of the new transport models can fit the observed in-plane flow momenta by adjusting the nuclear potential $U(\rho,p)$, the form of U that leads to the best fit of the data differs substantially from one model to the next. Expressed in terms of the nuclear incompressibility modulus, the results from the various approaches range between $K=200$–400 MeV. These differences are due to differences in the dynamical implementation of Pauli blocking and binding effects, the momentum dependence of U, and differences between numerical techniques. At present, considerable controversy still surrounds the validity of particular model assumptions and the correct self-consistent formulation of high-energy nuclear transport theory remains under active debate.[6,7] It is therefore essential that all models be tested on the data other than just the moments of the high-multiplicity-selected triple-differential yields.

The purpose of this Letter is to report the results of a new test of competing nuclear transport models. We compare calculated double-differential p-like inclusive cross sections to data on La+La at 800A MeV.[13] Recall that the p-like inclusive cross section is defined as

$$\sigma_{\rm inv} \equiv \sum_f Z_f A_f^2 E_f \frac{d^3\sigma^f}{d^3k_f}\,, \qquad (1)$$

where the sum extends over all nuclear fragments with charge and mass number (Z_f,A_f), and $E_f d^3\sigma^f/d^3k_f$ is the invariant fragment cross section with $(E_f,\mathbf{k}_f)$ denoting the energy and momentum per nucleon of the fragment. This reaction was considered because this is the only one involving heavy nuclei with $A>100$ for which the absolute differential fragment cross sections for $f=p$, d, t, ^{3}He, and ^{4}He have been measured. This represents therefore the most severe absolute test of the models at this time. Since these data are not multiplicity selected, an unrestricted impact-parameter average is involved, and possible trigger biases are thereby minimized.

Before discussing the results, we first describe briefly each transport model. In the intranuclear-cascade models,[2,3] nuclear transport is described by straight-line propagation of nucleons to potential-scattering points defined by the distance d of closest approach of two nucleons. If $d<(\sigma_{\rm NN}/\pi)^{1/2}$, then a binary scattering is assumed to take place. The NN cross section $\sigma_{\rm NN}$ is taken from free-space NN data, and scattering is treated as a stochastic process with final momenta selected randomly according to the measured differential cross sections. Differences between the Fraenkel-Yariv (FY) cascade model,[2] the original Cugnon (CG1) cascade model,[3] and the latest Cugnon version (CG2)[14] arise due to different prescriptions adopted to simulate Pauli blocking, initial Fermi motion, and nuclear binding effects.

To incorporate nuclear mean-field effects in addition to Pauli-blocked collision dynamics, several versions of the Vlasov-Uehling-Uhlenbeck (VUU) transport theory[15] were developed. We consider here two versions, VUU[8] and BUU (Boltzmann, Uehling, and Uhlenbeck).[7,9] In each event, particles propagate on curved trajectories as determined by the nuclear mean field. In order to reduce fluctuations, the mean field is calculated by averaging over an ensemble of synchronously calculated events. Binary collisions between nucleons and Δ resonances are processed as in intranuclear-cascade models, using experimental scattering cross sections and including Pauli-blocking factors.

In VUU,[8] the isospin of each particle is explicitly incorporated. The mean field is assumed to be given by a local *momentum-independent* potential, with a functional form

$$U(x)=a\rho(x)+b\rho^{\gamma}(x)\,.$$

The local density of nucleons $\rho(x)$ is determined by an ensemble average, taking a spherical volume of radius 2 fm. The parameter γ fixes the incompressibility K and the remaining two parameters are constrained by nuclear equilibrium conditions. In this work a "stiff" nuclear equation of state corresponding to $\gamma=2$ and $K=380$ MeV was considered. In the special case in which $\partial U/\partial\rho=0$ above $\rho=\rho_0$ (equilibrium nuclear density) VUU reduces essentially to CG2.

In BUU,[9] the momentum dependence of the nuclear potential is considered explicitly, and each parallel ensemble contains fifty events, as opposed to fifteen in the case of VUU. It is important to emphasize that both VUU and BUU are one-body transport theories[7] because the ensemble average washes out many-body correlations. While pion production is incorporated, modifications for pion propagation in the nuclear medium are neglected, as in all present nuclear transport models. In this model the nuclear potential is parametrized as

$$U(\rho,\mathbf{p})=a\left(\frac{\rho}{\rho_0}\right)+b\left(\frac{\rho}{\rho_0}\right)^{\sigma}+2\frac{c}{\rho_0}\int d^3p'\frac{f(\mathbf{r},\mathbf{p}')}{1+[(\mathbf{p}-\mathbf{p}')/\Lambda]^2}\,, \qquad (2)$$

where $f(\mathbf{r},\mathbf{p})$ is the one-body phase-space density of nucleons. The five constants above are fixed by requiring that $E/A=-16$ MeV, $\rho_0=0.16$ fm^{-3}, $K=215$ MeV, $U(\rho_0,p=0)=-75$ MeV, and $U(\rho_0,p^2/2m=300\text{ MeV})=0$. Their values are then $a=-110.44$ MeV, $b=140.9$ MeV, $c=-64.95$ MeV, $\sigma=1.24$, and $\Lambda=1.58p_F^{(0)}$, and yield an effective mass at the Fermi surface of $m^*=0.67m$. With these parameters, the potential becomes repulsive for cold nuclear matter at normal density for kinetic energy E_k greater than 300 MeV. For much higher kinetic energies, the potential reaches an asymptotic value of 30.5 MeV. These features are in accord with optical-model-potential fits to nucleon-nucleus scattering. Unlike VUU,[8] this BUU calculation assumes isospin degeneracy. The p-like fragments are obtained by summing over all nucleons and scaling by Z/A.

The relativistic Vlasov-Uehling-Uhlenbeck (RVU) model considered here is the one based on Refs. 10 and 11. It follows in the semiclassical and local approximation from the extended quantum hadrodynamics (QHD)[16] with scalar meson self-interaction. The parameters are the same as in Ref. 11, corresponding to $K=380$ MeV and a nucleon effective mass of $0.83m$ at normal nuclear-matter density. The RVU model is solved with the method of test particles[10,11] and the results are obtained with fifty test particles for each physical nucleon.

Unlike VUU and BUU which are one-body transport models, the quantum-molecular-dynamics (QMD) model[12] follows the evolution of the A-body phase-space distribution. It goes beyond classical-molecular-dynamics models, which solve the A-body Newtonian equations of motion numerically, by incorporating quantal stochasticity through random two-body scattering as in intranuclear-cascade models. It evolves the particles in a Gaussian-smoothed mean field between two-body collisions. The Gaussian smoothing is taken to simulate finite wave-packet effects with a FWHM taken to be $\Delta r=1.8$ fm. This smoothing of the nuclear field reduces the fluctuations and gradients of the mean field. The present results are not sensitive to the exact value of Δr. In the present calculation, the potential density is taken as a sum of a Skyrme-type local two- and three-body potential, an effective Yukawa one-pion exchange potential, and a Coulomb potential. However, the momentum dependence of the optical potential was neglected. The parameters for the potentials were chosen to correspond to the stiff nuclear equation of state with $K=380$ MeV.

We now turn to the comparison of the calculated results. In Fig. 1, the inclusive p-like data at laboratory angles 20°, 40°, and 60° are compared to the various calculations. In 1(a), results of cascade models are compared. Note that the FY cascade model significantly overpredicts the cross sections although the shapes are roughly reproduced. This problem was also observed in earlier comparisons[2] on lighter nuclear reactions such as Ne+U at 400A MeV. The dashed curves show that the original Cugnon code, CG1, converges to the same results as FY at high momentum but differs substantially at low momentum. At low momentum, the difference between FY and CG1 is presumably due to the different nuclear-binding prescriptions. The solid curves in Fig. 1(a) show the effect of an improved Pauli-blocking algorithm in CG2. The high-momentum yield is reduced by

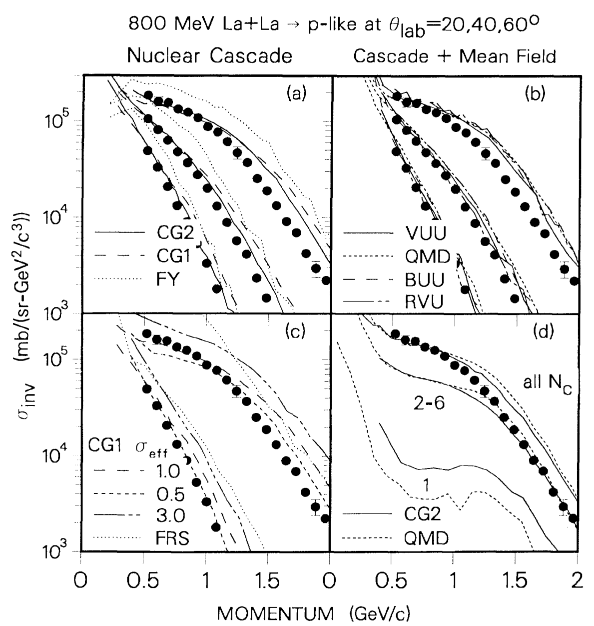

FIG. 1. Comparison of nuclear transport calculations to data (Ref. 13). (a) Comparison of Cugnon cascade model versions CG1 (Ref. 3) and CG2 (Ref. 14) with the Fraenkel-Yariv cascade model FY (Ref. 2). (b) Comparison of momentum-independent VUU (Ref. 8) and QMD (Ref. 12) with $K=380$ MeV, to momentum-dependent BUU (Ref. 9) with $K=210$ MeV, and relativistic RVU (Ref. 11). (c) Effects at 20° and 60° of rescaling the free-space NN cross sections in CG1 by factors of 0.5, 1.0, and 3.0. The dotted curves show results of the FREESCO fireball model FRS (Ref. 17). (d) The contributions to the 20° yield for QMD and CG2 from single-collision ($N_c=1$) and multiple-collision ($N_c=2$–6) components.

this effect. The difference between CG1 and CG2 illustrates the magnitude of uncertainties associated with different Pauli-blocking algorithms.

In Fig. 1(b), the models incorporating the nuclear mean fields are compared. Recall that the incompressibility modulus varies by a factor of 2 between the various models. We note the remarkable insensitivity of the results to variations in the nuclear equation of state and to the details of the transport methods. In fact VUU, BUU, QMD, and RVU give results within 20% of CG2 in Fig. 1(a). This shows that even for very heavy nuclear collisions, the double-differential cross sections cannot be used to constrain the nuclear equation of state.

On the other hand, Fig. 1(c) shows that the results are sensitive to variations of a factor of 2 in the nucleon-nucleon cross sections. Using the CG1 code with all cross sections scaled by 0.5, 1.0, and 3.0, we see that an improved agreement with data at high momentum with a reduced cross section can only be achieved at the expense of underpredicting the low-momentum yield at 20°. The results for the three-times free-space cross sections are obtained with the additional constraint that the scattering is repulsive. From previous studies,[18] we know that this case corresponds closely to the predictions of ideal hydrodynamics. We see that this simulated hydrodynamics badly overpredicts the data in this reaction. The same is true for the statistical FREESCO model FRS,[17] which considers the microcanonical explosion and subsequent evaporation from fully equilibrated participant and spectator sources.

The important point we emphasize in Fig. 1 is the failure of all models to reproduce the low-cross section yields at 20°. To provide a better understanding of the physics associated with that region of momentum space where the discrepancies between the models and the data are the largest, we show in Fig. 1(d) a breakdown of the QMD and CG2 calculations into components involving nucleons that have suffered a particular range of two-body scattering. The $N_c=1$ curve shows the contribution from nucleons suffering only one hard nucleon-nucleon collision. We see that this is a negligible contribution to the 20° yield. Even the intermediate component corresponding to 2–6 collisions only accounts for about half the yield at high momentum. This region of momentum space is then strongly influenced by the reaction zone in which the largest number of binary interactions occurred. The discrepancy is therefore of interest, since the highest nuclear densities are likely to be produced there.

The common feature of all models is the assumption that the NN cross sections can be taken from free-space data. However, many-body effects can modify the in-medium cross sections.[6,19] The results in Fig. 1(c) show that no simple rescaling of those cross sections is satisfactory. It is possible that momentum-dependent effective cross sections, reducing from free-space values for low-momentum nucleons to about half that value for the higher-momentum nucleons, could lead to better agreement with the data. However, such corrections for time-dependent in-medium effects would require substantial modifications of the present models. If the present data are free from additional systematic errors, then a better understanding of nuclear transport at high densities is called for. We note that in a similar study[20] on rapidity distributions, the free-space cross sections gave the best agreement; however, the data in that case were dominated by particles at angles beyond 20°.

We conclude that further tests of the nuclear collision term via double-differential data on heavy nuclear collisions are urgently needed. Uncertainties in nuclear transport properties suggested by this study could obscure the effects due to the sought-after equilibrium equation of state. For example, one study[21] indicated that the in-plane flow momenta may be just as sensitive to the effective NN cross sections as to the nuclear incompressibility. Especially important would be a systematic measurement of absolute p-like cross sections in $A+A$ collisions ranging from Ne+Ne to Au+Au in the entire energy range $(0.2-1.0)A$ GeV.

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