Scalar mesons and tetraquarks by means of lattice QCD

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We study the light scalar mesons $a_0(980)$ and κ using $N_f = 2+1+1$ flavor lattice QCD. In order to probe the internal structure of these scalar mesons, and in particular to identify, whether a sizeable tetraquark component is present, we use a large set of operators, including diquark-antidiquark, mesonic molecule and two-meson operators. The inclusion of disconnected diagrams, which are technically rather challenging, but which would allow us to extend our work to e.g. the $f_0(980)$ meson, is introduced and discussed.

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1. Introduction

The nonet of light scalar mesons formed by $\sigma \equiv f_0(500)$, $\kappa \equiv K_0^*(800)$, $a_0(980)$ and $f_0(980)$ is poorly understood. Compared to expectation all nine states are rather light and their ordering is inverted. For example in a standard quark antiquark picture the $a_0(980)$ states, which have $I = 1$, must necessarily be composed of two light quarks, e.g. $a_0(980) \equiv \bar{d}u$, while the κ states with $I = 1/2$ are made from a strange and a light quark, e.g. $\kappa \equiv \bar{s}u$. Consequently, κ should be heavier than $a_0(980)$, since $m_s > m_{u,d}$. In experiments, however, the opposite is observed, i.e. $m(\kappa) = 682 \pm 29\,\text{MeV}$, while $m(a_0(980)) = 980 \pm 20\,\text{MeV}$ [1]. On the other hand in a four-quark or tetraquark picture the quark content could be $a_0(980) \equiv \bar{d}u\bar{s}s$ and $\kappa \equiv \bar{s}u(\bar{u}u + \bar{d}d)$ naturally explaining the observed ordering. Moreover, certain decay channels, e.g. $a_0(980) \to K + \bar{K}$, indicate that besides the two light quarks also an $s\bar{s}$ pair is present and, therefore, also support a tetraquark interpretation. A detailed discussion of light scalar mesons can be found e.g. in [2, 3].

In addition to the light scalar mesons there are also various tetraquark candidates among the heavy mesons, e.g. $D_{s0}^*(2317)$ and $D_{s1}(2460)$ (cf. e.g. [4, 5]) or the charmonium states $X(3872)$, $Z(4430)$, $Z(4050)$, $Z(4250)$.

Here we report about the status of an ongoing long-term project with the aim to study possible tetraquark candidates from first principles using lattice QCD. Parts of this work have already been published [6, 7].

2. Lattice setup

We use gauge link configurations with $N_f = 2 + 1 + 1$ dynamical quark flavors generated by the European Twisted Mass Collaboration (ETMC).

We have studied several ensembles with the same rather fine lattice spacing $a \approx 0.086\,\text{fm}$. The ensembles differ in the volume $(L/a)^3 \times (T/a) = 20^3 \times 48, \ldots, 32^3 \times 64$ and the unphysically heavy light quark mass corresponding to $m_\pi \approx 280\,\text{MeV} \ldots 460\,\text{MeV}$. Details regarding these gauge link configurations can be found in [8, 9, 10, 11, 12, 13].

Currently we ignore disconnected diagrams, which are technically rather challenging (cf. the outlook in section 5). An important physical consequence is that the quark number and the anti-quark number are separately conserved for each flavor. Therefore, there is no mixing between $\bar{u}u$, $\bar{d}d$ and $\bar{s}s$ resulting in an η_s meson with flavor structure $\bar{s}s$ instead of η and η'.

Further lattice details and technicalities can be found in [7].

3. Four-quark creation operators

In the following we focus on the $a_0(980)$ sector, which has quantum numbers $I(J^P) = 1(0^+)$. As usual in lattice QCD we extract the low lying spectrum in that sector by studying the asymptotic exponential behavior of Euclidean correlation functions

$$C_{jk}(t) \;=\; \left\langle (\mathscr{O}_j(t))^\dagger \mathscr{O}_k(0) \right\rangle. \tag{3.1}$$

$\mathscr{O}_j$ and $\mathscr{O}_k$ denote suitable creation operators, i.e. operators generating the $a_0(980)$ quantum numbers, when applied to the vacuum state.

Assuming that the experimentally measured $a_0(980)$ with mass $980 \pm 20\,\text{MeV}$ is a rather strongly bound four quark state, suitable creation operators to excite such a state are

$$\mathcal{O}^{K\bar{K}\ \text{molecule}}_{a_0(980)} \ = \ \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \Big) \Big(\bar{d}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \Big) \tag{3.2}$$

$$\mathcal{O}^{\text{diquark}}_{a_0(980)} \ = \ \sum_{\mathbf{x}} \Big(\varepsilon^{abc} \bar{s}^b(\mathbf{x}) C \gamma_5 \bar{d}^{c,T}(\mathbf{x}) \Big) \Big(\varepsilon^{ade} u^{d,T}(\mathbf{x}) C \gamma_5 s^e(\mathbf{x}) \Big). \tag{3.3}$$

The first operator has the spin/color structure of a $K\bar{K}$ molecule ($\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})$ and $\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})$ correspond to a kaon K and an antikaon $\bar{K}$ at the same position $\mathbf{x}$). The second resembles a bound diquark antidiquark pair, where spin coupling via $C\gamma_5$ corresponds to the lightest diquarks/antidiquarks (cf. e.g. [2, 14, 15]).

Further low lying states in this sector are the two particle states $K + \bar{K}$ and $\eta_s + \pi$. Suitable creation operators to resolve these states are

$$\mathcal{O}^{K+\bar{K}\ \text{two-particle}}_{a_0(980)} \ = \ \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 u(\mathbf{x}) \right) \left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 s(\mathbf{y}) \right) \tag{3.4}$$

$$\mathcal{O}^{\eta_s+\pi\ \text{two-particle}}_{a_0(980)} \ = \ \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x}) \gamma_5 s(\mathbf{x}) \right) \left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y}) \gamma_5 u(\mathbf{y}) \right). \tag{3.5}$$

4. Numerical results an their interpretation

We first discuss numerical results for the ensemble with the smallest volume, $(L/a)^3 \times (T/a) = 20^3 \times 48$, which corresponds to a spatial extension of $L \approx 1.72\,\text{fm}$. This ensemble is particularly suited to distinguish two-particle states with relative momentum from states with two particles at rest and from possibly existing $a_0(980)$ tetraquark states (two-particle states with relative momentum have a rather large energy because one quantum of momentum $p_{\text{min}} = 2\pi/L \approx 720\,\text{MeV}$).

Figure 1a shows effective mass plots from a 2×2 correlation matrix with a $K\bar{K}$ molecule operator (3.2) and a diquark-antidiquark operator (3.3). The corresponding two plateaus are around $1100\,\text{MeV}$ and, therefore, consistent both with the expectation for possibly existing $a_0(980)$ tetraquark states and with two-particle $K + \bar{K}$ and $\eta_s + \pi$ states, where both particles are at rest ($m(K + \bar{K}) \approx 2m(K) \approx 1198\,\text{MeV}$; $m(\eta_s + \pi) \approx m(\eta_s) + m(\pi) \approx 1115\,\text{MeV}$ in our lattice setup).

Increasing this correlation matrix to 4×4 by adding two-particle $K + \bar{K}$ and $\eta_s + \pi$ operators (eqs. (3.4) and (3.5)) yields the effective mass results shown in Figure 1b. Two additional states are observed, whose plateaus are around $1500\,\text{MeV}\ldots 2000\,\text{MeV}$. From this 4×4 analysis we conclude the following:

- We do not observe a third low-lying state around $1100\,\text{MeV}$, even though we provide operators, which are of tetraquark type as well as of two-particle type. This suggests that the two low-lying states are the expected two-particle $K + \bar{K}$ and $\eta_s + \pi$ states, while an additional stable $a_0(980)$ tetraquark state does not exist.

- The effective masses of the two low-lying states are of much better quality in Figure 1b than in Figure 1a. We attribute this to the two-particle $K + \bar{K}$ and $\eta_s + \pi$ operators, which
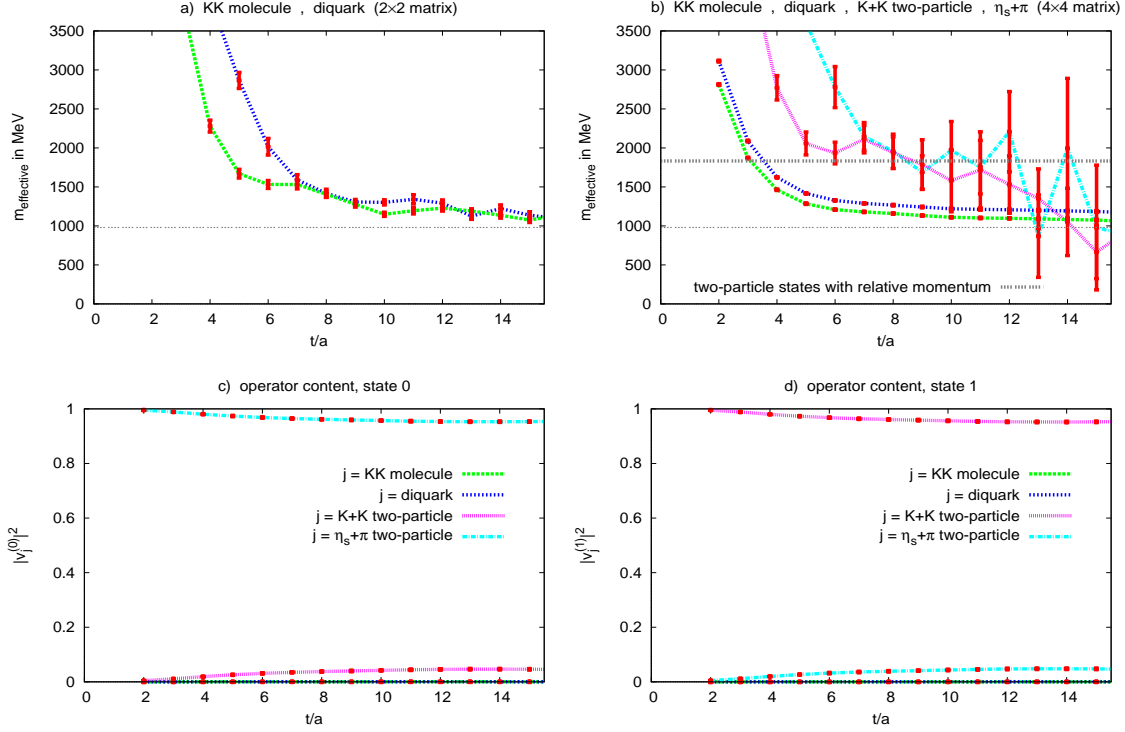
Figure 1: $a_0(980)$ sector, $(L/a)^3 \times (T/a) = 20^3 \times 48$. **a)** Effective masses as functions of the temporal separation, 2×2 correlation matrix (operators: $K\bar{K}$ molecule, diquark-antidiquark, eqs. (3.2) and (3.3)). **b)** 4×4 correlation matrix (operators: $K\bar{K}$ molecule, diquark-antidiquark, two-particle $K + \bar{K}$, two-particle $\eta_s + \pi$, eqs. (3.2) to (3.5)). **c), d)** Squared eigenvector components of the two low-lying states from **b)** as functions of the temporal separation.

presumably create larger overlap to those states than the tetraquark operators. This in turn confirms the interpretation of the two observed low-lying states as two-particle states.

- To investigate the overlap in a more quantitative way, we show the squared eigenvector components of the two low-lying states in Figure 1c and Figure 1d (cf. [13] for a more detailed discussion of such eigenvector components). Clearly, the lowest state is of $\eta_s + \pi$ type, whereas the second lowest state is of $K + \bar{K}$ type. On the other hand, the two tetraquark operators are essentially irrelevant for resolving those states, i.e. they do not seem to contribute any important structure, which is not already present in the two-particle operators. These eigenvector plots give additional strong support of the above interpretation of the two observed low lying states as two-particle states.

- The energy of two-particle excitations with one relative quantum of momentum can be estimated according to

$$m(1+2, p = p_{\min}) \approx \sqrt{m(1)^2 + p_{\min}^2} + \sqrt{m(2)^2 + p_{\min}^2} \quad , \quad p_{\min} = \frac{2\pi}{L}. \quad (4.1)$$

Inserting the meson masses corresponding to our lattice setup, $m(K) \approx 599\,\text{MeV}$, $m(\eta_s) \approx 774\,\text{MeV}$ and $m(\pi) \approx 341\,\text{MeV}$, yields $m(K + \bar{K}, p = p_{\min}) \approx 1873\,\text{MeV}$ and

$m(\eta_s + \pi, p = p_{\min}) \approx 1853\,\mathrm{MeV}$. These numbers are consistent with the effective mass plateaus of the second and third excitation in Figure 1b. Consequently, we also interpret them as two-particle states.

We obtained qualitatively identical results, when varying the light quark mass and the space-time volume, as discussed in section 2. Corresponding plots are shown in [7].

Using exactly the same techniques, i.e. four-quark operators of tetraquark and of two-particle type, we also studied the κ-sector (for details cf. [7]). Again we find no sign of any four-quark bound state besides the expected two-particle spectrum (in this case $K + \pi$ states). Note that this result is in contradiction to a very similar recent lattice study of the κ meson [16], where an additional low lying four-quark bound state has been observed.

5. Inclusion of singly disconnected diagrams

As mentioned in section 2 disconnected diagrams have been ignored for the results presented so far. In this section we briefly discuss our strategy for computing such diagrams for specific four-quark operators.

For correlation functions of four-quark operators with flavor structure $\bar{q}_1 q_1 \bar{q}_2 q_3$ a so-called singly-disconnected diagram has to be computed. Tetraquark candidates with this flavor structure are e.g. the previously discussed $a_0(980) \equiv \bar{s}s\bar{d}u$ (cf. Figure 2) or $D_{s0}^*(2317), D_{s1}(2460) \equiv (\bar{u}u + \bar{d}d)\bar{c}s$.

While for connected four-quark diagrams (i.e. all four quark propagators connect the timeslices at time 0 and time t) standard point-to-all propagators can be used, applying exclusively such propagators is not possible in practice for singly disconnected diagrams. The reason is that one has to include a sum over space, $\sum_{\mathbf{x}}$, at least on one of the two timeslices (wlog. at time t in Figure 2), to project to zero momentum. This in turn requires a all-to-all propagator of quark flavor q_1 on that timeslice, due to $\sum_{\mathbf{x}} \bar{q}_1(\mathbf{x}, t) q_1(\mathbf{x}, t) \dots$ (the s quark propagator at timeslice t represented by the solid red line in Figure 2).

Since all-to-all propagators are prohibitively expensive to compute, they are typically estimated stochastically. While using a single stochastic propagator for a specific diagram typically results in a favorable or at least acceptable signal-to-noise ratio, using a larger number of such propagators drastically increases statistical errors. Therefore, we decided for the following strategy: three quark propagators (the q_1-loop at timeslice 0 and the q_2 and q_3 propagators connecting the timeslices 0 and t) are realized by exact point-to-all propagators, while the remaining propagator (the q_1-loop at timeslice t) is estimated stochastically, using random $Z_2 \times Z_2$ timeslice sources.

First numerical tests of this strategy performed for both $a_0(980)$ and $D_{s0}^*(2317)$ have been promising in the sense that the statistical errors of the singly disconnected diagrams for the molecule type operator (3.2) are of the same order of magnitude as the statistical errors of the corresponding connected diagrams, when investing a comparable amount of HPC resources.

6. Conclusions and future plans

We have studied the $a_0(980)$ and the κ channel by means of $N_f = 2 + 1 + 1$ flavor lattice QCD using four-quark operators of molecule, diquark and two-particle type. Besides the expected
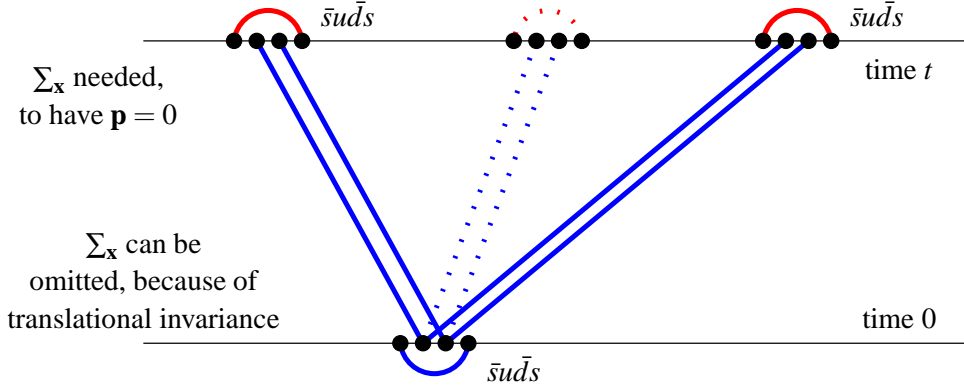
Figure 2: The singly disconnected diagram of the $a_0(980) \equiv \bar{s}s\bar{d}u$ correlator.

two-particle spectrum (two essentially non-interacting pseudoscalar mesons) no indication of any additional low lying state, in particular no sign of a four-quark bound state could be observed. This suggests that both the $a_0(980)$ and κ meson have either no sizeable tetraquark component or they are rather weakly bound unstable states.

To investigate the latter one needs to study the volume dependence of the two-particle spectrum in the corresponding sectors ("Lüscher's method", cf. e.g. [17, 18, 19]). Such computations are very challenging using lattice QCD, but first results have recently been published (cf. [20, 21]). We plan to perform similar computations with our setup in the near future.

Moreover, certain possibly present systematic errors need to be studied, quantified and removed: (1) disconnected diagrams have to be computed and included (cf. section 5); (2) lattice discretization errors and the continuum limit has to be studied; (3) computations at even lighter and, therefore, more realistic u/d quark masses would be desirable.

Acknowledgments

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