

Gravitational Radiation from Ultra High Energy Cosmic Rays in Models with Large Extra Dimensions

Ben Koch ^a, Hans-Joachim Drescher ^{a,b}, Marcus Bleicher ^b

^a*Frankfurt Institute for Advanced Studies (FIAS),
60438 Frankfurt am Main, Germany*

^b*Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität,
60438 Frankfurt am Main, Germany*

Abstract

The effects of classical gravitational radiation in models with large extra dimensions are investigated for ultra high energy cosmic rays (CRs). The cross sections are implemented into a simulation package (SENECA) for high energy hadron induced CR air showers. We predict that gravitational radiation from quasi-elastic scattering could be observed at incident CR energies above 10^9 GeV for a setting with more than two extra dimensions. It is further shown that this gravitational energy loss can alter the energy reconstruction for CR energies $E_{\text{CR}} \geq 5 \cdot 10^9$ GeV.

Key words: extra dimension, cosmic ray, ADD, UHECR

1 Motivation

One of the major problems in modern physics is to combine quantum physics and gravitation. The most promising candidate to solve this problem seems to be string theory because it includes all symmetry groups of the standard model, however, it introduces additional space dimensions [1]. From the present (non-)observation of these dimensions it is concluded that they are compactified and their size was assumed to be on the Planck scale. However, string theory itself does not give any stringent constraints on the size of the extra dimensions other than non-observability so far.

Email address: koch@th.physik.uni-frankfurt.de (Ben Koch).

This opens a possibility for gravity-only extra dimensions, assuming that only gravitons are able to enter these additional dimensions. The size of these dimensions is then primarily constraint by direct measurements of the gravitational inverse square law. However, the strength of gravitational interactions has only been measured down to a scale of some micrometers [2,3]. Below this scale some modification of the gravitational force law is still possible. This allows to introduce a new fundamental scale of gravity $M_f \ll M_{Planck}$ in the TeV-range. These Large Extra Dimensions models (LXDs) have compactification radii up to $\sim \mu\text{m}$ and a new fundamental scale in the TeV range [4,5,6]. If the new scale of gravity is indeed not far beyond the electro-weak scale, it might be reachable in future collider experiments at LHC and beyond or in cosmic ray observatories like AUGER or ICECUBE. At center of mass energies above the new fundamental scale M_f the LXDs become important and first observable effects of quantum gravity might be observable. A multitude of new effects have been predicted in the recent years to obtain deeper insight into this exciting field ranging from Kaluza-Klein graviton production and increased neutrino cross sections to black hole production [7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27].

Apart from these promising phenomena, present day observations also allow for direct and indirect constraints on the new fundamental scale and the size of the LXDs:

- A fundamental scale in the TeV-range does not allow a single LXD as the compactification radius would get in the order of the size of our solar system and we would have noticed.
- For higher numbers of LXDs the compactification radius does not exceed the μm -scale as shown by the mentioned short distance experiments.
- Even smaller distances can be probed in collider experiments. At the moment the highest center of mass energies are reached in hadron-hadron collisions at Tevatron with 2 TeV cm-energy which sets a bound of $M_f \geq 1.0$ TeV for 2 down to 0.7 TeV for 6 LXDs [28,29].
- Other bounds are set by supernova explosions [30,31,32,33]. The cooling of the supernovae is modified by the production of gravitons which is very sensitive to the number of LXDs in the reached energy region. For 2 LXDs a bound of about 500 TeV is obtained. However, for any higher number of LXDs the bounds are lower than from the collider experiments.

In the last years it was recognized that cosmic rays (CRs) provide an excellent laboratory to study the onset of physics beyond the standard model. The initial energies of UHECRs exceeds 10^{11} GeV ($\sqrt{s_{NN}} \sim 100 - 1000$ TeV) and might therefore allow one to probe TeV scale large extra dimensions [34,35,36,37,38,39,40,41,42,43,44,45]. Especially the radiation of gravitational waves in models with large extra dimensions is strongly enhanced compared to standard general relativity. As will be discussed later, this might lead to

observable signatures and modifications of the reconstructed flux and energy estimates for UHECRs.

To explore the effect of gravitational energy loss in cosmic ray air-showers, we use the differential cross section for gravitationally radiated energy in quasi-elastic scatterings as calculated in [46,47] for $2 \rightarrow 2$ processes. Then we apply the result to cosmic ray air shower simulations and extract the impact on cosmic ray observables.

2 Gravitational radiation from quasi-elastic scattering with extra dimensions

First estimates to study effects of gravitational energy loss of CRs due to the presence of extra dimensions were explored by [38]. There, the presence of large extra dimensions was incorporated into the well known results from general relativity [48] by a change of the phase space seen by the emitted gravitational wave. The additional phase space factor for the emitted gravitational wave was given by

$$g_d(k_d) = \frac{(k_d R)^d}{d\Gamma(d/2)\pi^{d/2}2^{d-1}}. \quad (1)$$

Note that $g_0 = 1$. Where R is the compactification radius of the extra dimensions in the ADD scenario given by [4]:

$$R = M_f^{-\frac{d+2}{d}} M_{\text{Pl}}^{\frac{2}{d}}. \quad (2)$$

Here, M_f is the new fundamental scale and M_{Pl} is the four dimensional Planck mass related to the gravitational constant by $G_N = 1/M_{\text{Pl}}^2$.

This method led to a strong modification of the reconstructed energy spectrum and the authors concluded that the steepening of the CR energy spectrum around $10^{15.5}$ eV (the "knee") might be due to gravitational energy loss.

However, from our present studies (see also [46]) it seems that a calculation of the effects of the gravitational energy loss requires a more elaborate treatment as will be discussed now. The simplified treatment can be improved by direct calculation¹ of the gravitational energy loss in a $N \rightarrow M$ scattering process

¹ We will use the following notations: $4 + d$ space-time vectors will be $x = (x_0, \underline{x})$, where the spatial part can be split again into a three dimensional and a d dimensional part $\underline{x} = (\mathbf{x}, x_\perp)$.

as given by [46,47]:

$$\frac{dE}{d\Omega_{3+d}dk_0} = \frac{1}{M_f^{2+d}} \frac{k_0^{2+d}}{2(\pi)^2(2\pi)^d} \sum_{I,J} \frac{\eta_I \eta_J}{(P_{(I)}k)(P_{(J)}k)} \left[(P_{(I)}^\mu P_{(J)\mu})^2 - \frac{1}{2+d} P_{(I)}^2 P_{(J)}^2 \right], \quad (3)$$

The $P_{(N)}$ are the momenta of the colliding particles and the factors η_N are defined by

$$\eta_N = \begin{cases} -1 & \text{for initial state particles.} \\ +1 & \text{for final state particles.} \end{cases} \quad (4)$$

Thus, in the case of a $2 \rightarrow 2$ collision the index N runs from 1 to 4. Before we continue, we want to point out that equation (3) follows from classical considerations and is not derived from any form of quantum theory of gravity (e.g. loop quantum gravity, SUGRA or string theory). However, we believe that it can account - at least semi-quantitatively - for the major effects of the gravitational energy loss.

Next, we integrate Eq. (3) with the help of the Mathematica package FeynCalc [49,50]. Difficulties for the $d\Omega_{3+d}$ integration arise from the $P \cdot k$ terms in the numerator. The protons are bound to the brane and the product $P \cdot k$ gives for example for one of the incoming protons:

$$P_1 \cdot k = P_1^0 k_0 - \mathbf{p}_1 \mathbf{k} - 0 = |\mathbf{p}| k_0 \left(\sqrt{1 + \frac{m_p^2}{|\mathbf{p}^2|}} - \sqrt{1 - \frac{k_d^2}{k_0^2} \cos \phi_k} \right). \quad (5)$$

For $k_d^2/k_0^2 \approx 0$ and $\phi_k \approx 0$, $P_1 \cdot k$ becomes small and the denominator in Eq. (3) is only regularized by $m_p^2/|\mathbf{p}^2|$.

We introduce the Mandelstam variables s , t and u by

$$s = (P_1 + P_2)^2, \quad t = (P_1 - P_3)^2, \quad u = (P_1 - P_4)^2. \quad (6)$$

It is convenient to perform a coordinate transformation to rewrite Eq. (3) in terms of spherical coordinates in three dimensional space and the d extra-dimensional coordinates separately:

$$\frac{dE}{dk_0 d\Omega_{3+d}} = \frac{k^{d+2} dE}{dk^{3+d}} = \frac{k^{d+2} dE}{dk^d dk^3} = \frac{(\mathbf{k}^2 + k_d^2)^{(d+2)/2} dE}{\mathbf{k}^2 d\mathbf{k} d\Omega_3 |k_d|^{d-1} dk_d d\Omega_d}. \quad (7)$$

Solving this for the new integration variables yields

$$\frac{dE}{d\mathbf{k} d\Omega_3 dk_d d\Omega_d} = \frac{dE}{dk_0 d\Omega_{3+d}} \frac{\mathbf{k}^2 k_d^{d-1}}{(\mathbf{k}^2 + k_d^2)^{(d+2)/2}}. \quad (8)$$

The first term on the right side can be approximated by Eq. (3) as soon as the wavelength of the gravitational wave is smaller than the compactification

radius R of the extra dimensions and the gravitational wave can propagate freely into the bulk. Rephrased as a condition for $|k|$ this constrained becomes

$$|k| > M_f \left(\frac{M_f^2}{M_{\text{Pl}}^2} \right)^{\frac{1}{d}}. \quad (9)$$

A lower bound on $|k|$ is not relevant for the energy loss discussion, because the major contribution to the radiated energy comes from the high energy (i.e. large $|k|$) part. To calculate the energy loss due to the gravitational wave emission one has to perform the $d\Omega_3 = \sin \phi_{kz} d\phi_k d\phi_{kz}$, the $d\Omega_d$, the $d\mathbf{k}$ and the dk_d integrals. However, the rather steep t dependence of the elastic Proton-Proton cross section allows us to simplify these integrals, because the physically relevant processes are dominated by small $|t| < m_p^2$ contributions, with m_p being the mass of the Proton. Thus, one can expand Eq. (3) for small $|t|$. This gives for the part $\sum_{I,J} \dots$ in Eq. (3) containing the sums over external momenta

$$\begin{aligned} \sum_{I,J} \dots = & -8t \left\{ (k_0^2 - k_d^2)^3 (4m_p^2 - s) s^4 \cos(\phi_k)^6 \right. \\ & + k_0^2 (k_0^2 - k_d^2) s (4m_p^2 + s) \cos(\phi_k)^2 \left[k_d^2 s (-8m_p^4 - 4m_p^2 s + s^2) \right. \\ & - 4k_0^2 (32m_p^6 - 14m_p^4 s - 3m_p^2 s^2 + s^3) \\ & + (k_0^2 - k_d^2) s (-8m_p^4 - 4m_p^2 s + s^2) \cos(2\phi_k) \left. \right] \\ & - 0.5 \left[(k_0^2 - k_d^2)^2 s^2 \cos(\phi_k)^4 \left(k_d^2 s (8m_p^4 - 4m_p^2 s + s^2) \right. \right. \\ & + k_0^2 (-128m_p^6 + 88m_p^4 s + 12m_p^2 s^2 - 7s^3) \\ & + (k_0^2 - k_d^2) s (8m_p^4 - 4m_p^2 s + s^2) \cos(2\phi_k) \left. \left. \right] \right. \\ & - 0.5 \left[k_0^4 (4m_p^2 + s)^2 \left(k_d^2 s (8m_p^4 - 4m_p^2 s + s^2) \right. \right. \\ & + k_0^2 (-128m_p^6 + 24m_p^4 s + 12m_p^2 s^2 - 3s^3) \\ & + (k_0^2 - k_d^2) s (8m_p^4 - 4m_p^2 s + s^2) \cos(2\phi_k) \left. \left. \right] \right\} \\ & / \left\{ k_0^8 (-4m_p^2 + s) \left[4m_p^2 + s + \left(-1 + \frac{k_d^2}{k_0^2} \right) s \cos(\phi_k)^2 \right]^4 \right\}. \end{aligned} \quad (10)$$

For $k_d \approx 0$ the radiation does not propagate into the extra dimensions and Eq. (10) reduces to the well known classical limit. From Eq. (10) one can see that for $k_d^2/k_0^2 s \geq 4m_p^2$ the regularising part in the denominator is not m_p^2/s any more and a Taylor expansion of Eq. (10) around $m_p^2/s = 0$ is allowed. This expansion has a large validity region for ultra high energy collisions because it just demands that

$$\frac{\sqrt{s}}{2} > k_d^2 > \mathbf{k}^2 \frac{4m_p^2}{s}. \quad (11)$$

This approximation also fulfils the condition in Eq. (9). After performing the

integration $d\phi_{kz}$, this series gives

$$\frac{dE}{dk_d d\mathbf{k} d\phi_k d\Omega_d} = \frac{t}{(2\pi)^{d+1} M_f^{d+2}} \frac{2k_d^{d-1} \mathbf{k}^2 [(k_0^2 - k_d^2) \cos(2\phi_k) - k_0^2]}{[k_0^2 + (k_d^2 - k_0^2) \cos(\phi_k)]^2}. \quad (12)$$

Next we perform the ϕ_k integration,

$$\frac{dE}{dk_d d\mathbf{k} d\Omega_d} = \frac{t}{(2\pi)^d M_f^{d+2}} \frac{k_d^{d-2} \mathbf{k}^2 (2k_d^2 + 3\mathbf{k}^2)}{(k_d^2 + \mathbf{k}^2)^2}. \quad (13)$$

The integration over the d dimensional unit sphere Ω_d gives a factor $2\pi^{d/2}/\Gamma(d/2)$.

$$\frac{dE}{dk_d d\mathbf{k}} = \frac{t}{2^{d-1} \pi^{d/2} \Gamma(d/2) M_f^{d+2}} \frac{k_d^{d-2} \mathbf{k}^2 (2k_d^2 + 3\mathbf{k}^2)}{(k_d^2 + \mathbf{k}^2)^2}. \quad (14)$$

Next, the k_d and the $|\mathbf{k}|$ integration can be performed with respect to the integration limits $k_d^2 + \mathbf{k}^2 < k_{max}^2$ and Eq. (11). This calculation can be done explicitly for two, four and six extra dimensions:

$$\begin{aligned} E(t, d=2) &= -k_{max}^3 t [5\sqrt{2} - \log(1 + \sqrt{2})] / (12\pi M_f^4), \\ E(t, d=4) &= -k_{max}^5 t [\sqrt{2} - \log(1 + \sqrt{2})] / (16\pi^2 M_f^6), \\ E(t, d=6) &= -k_{max}^7 t [11\sqrt{2} - 13\log(1 + \sqrt{2})] / (1792\pi^3 M_f^8). \end{aligned} \quad (15)$$

Let us now discuss the relation between this result and those obtained in earlier publications:

- In Ref. [46] the gravitational wave was assumed to have a momentum vector only in the direction out of the brane, thus the denominator in Eq. (3) simplifies to $P_I k = E_I^0 k_0$. After integrating over k_0 (which is not strictly correct, because the problem is not spherically symmetric in $3+d$ spatial dimensions any more) the result shows the same t and k_{max} dependence as Eq. (15). The different factors are due to the simplification in the integration.
- The phase space argument used in Ref. [38] leads to the same k_{max} dependence. However, the pre-factors differ and even more striking the result derived in [38] has no t dependence (but s instead). Therefore, this approach leads to drastic overestimation of the gravitational energy loss in high energy cosmic rays, as we will see in the following sections.

3 Quasi-Elastic hadron-nucleus scattering

In order to calculate the energy loss due to gravitational wave emission in air showers at high energies one has to know the elastic scattering cross section $d\sigma_{\text{elastic}}/dt$. We construct it from the hadron-nucleon scattering cross section.

In the impact parameter representation, the profile function is defined as

$$\Gamma(s, b) = \frac{\sigma_{\text{total}}}{2} \frac{1}{2\pi B_{\text{elastic}}} \exp\left(-\frac{b^2}{2B_{\text{elastic}}}\right) \quad , \quad (16)$$

where B is the elastic scattering slope and σ_{total} is the total hadron-nucleon cross section. The profile function is related to the amplitude,

$$\Gamma(s, b) = -\frac{i}{8\pi} A(s, b) \quad . \quad (17)$$

The profile function Eq. (16) can be expressed via the eikonal [51] function χ by

$$\Gamma(s, \mathbf{b}) = 1 - \exp[i\chi(\mathbf{b})] \quad , \quad (18)$$

and is related to the phase shift of the scattered wave.

The Fourier transform relates impact parameter space to momentum space by

$$A(s, t) = \frac{s}{4\pi} \int d^2\mathbf{b} A(s, \mathbf{b}) \exp(-i\mathbf{b}\mathbf{q}) \quad , \quad (19)$$

with $t = -q^2 \approx -\mathbf{q}^2$. The amplitude gives the differential elastic cross section:

$$\frac{d\sigma_{\text{elastic}}}{dt} = \frac{1}{16\pi s^2} |A(s, t)|^2 = \frac{d\sigma_{\text{elastic}}}{dt} \Big|_{t=0} \exp(B_{\text{elastic}} t) \quad . \quad (20)$$

From the last expression we can define the elastic scattering slope

$$B \equiv \left[\frac{d}{dt} \left(\ln \frac{d\sigma_{\text{elastic}}}{dt} \right) \right]_{t=0} \quad . \quad (21)$$

Inserting Eqs. (19) and (20) into Eq. (21), the scattering slope becomes

$$\begin{aligned} B &= \frac{2 \frac{d}{dt} \left| \int d^2\mathbf{b} A(s, \mathbf{b}) e^{-i\mathbf{b}\mathbf{q}} \right|}{\left| \int d^2\mathbf{b} A(s, \mathbf{b}) e^{-i\mathbf{b}\mathbf{q}} \right|} \Big|_{t=0} \\ &= \frac{2 \frac{d}{2q dq} \left| \int d^2\mathbf{b} A(s, \mathbf{b}) (1 + (-i\mathbf{b}\mathbf{q}) + \frac{1}{2}(-i\mathbf{b}\mathbf{q})^2 - \dots) \right|}{\left| \int d^2\mathbf{b} A(s, \mathbf{b}) \right|} \Big|_{t=0} \\ &= \frac{\left| \int d^2\mathbf{b} A(s, \mathbf{b}) (b^2 \cos^2 \phi) \right|}{\left| \int d^2\mathbf{b} A(s, \mathbf{b}) \right|} = \frac{\int db b^3 \Gamma(s, b)}{2 \int db b \Gamma(s, b)} \quad , \end{aligned} \quad (22)$$

where we have expanded the exponential and kept only the third term. The first term does not depend on q and the second term in the expansion vanishes due to symmetry.

To obtain the scattering slope of a hadron-nucleus collision we replace the hadron-nucleon scattering profile function by

$$\Gamma_{\text{hA}}(s, \mathbf{b}) = 1 - \exp\left[-t \sum_{i=1}^A \chi_i(s, \mathbf{b}_i)\right] = 1 - [1 - \tilde{T}_A(\mathbf{b})]^A \quad (23)$$

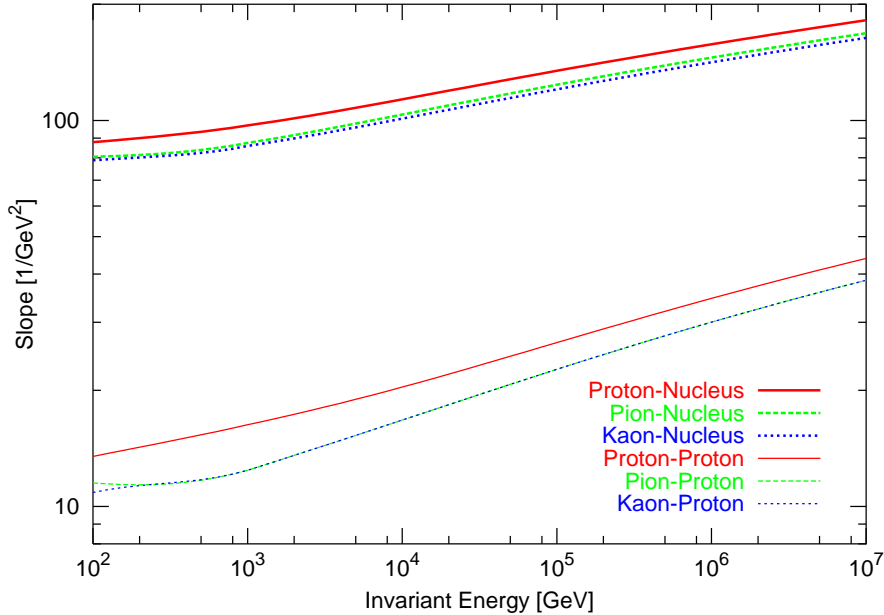


Fig. 1. The hadron-nucleus slopes (thin lines) and the hadron-nucleon slopes (thick lines) as a function of the collision energy in the center of mass frame. Primary particles are Protons (full lines), π s (dashed lines), and Kaons (dotted lines).

where \tilde{T}_A is obtained from the Glauber-Gribov formalism [52,53,54] by convolution of the thickness function with the hadron-nucleon profile,

$$\begin{aligned}\tilde{T}_A(\mathbf{b}) &= \int d^2\mathbf{c} T_A(\mathbf{b}) \Gamma(\mathbf{b} - \mathbf{c}) \quad , \\ T_A(\mathbf{b}) &= \frac{1}{A} \int dz \rho(z, \mathbf{b}).\end{aligned}\tag{24}$$

Using Eq. (24), the scattering slope for elastic hadron-nucleus collisions Eq. (22) is calculated as a function of the collision energy. The underlying hadron-nucleon scattering slopes are taken from the SIBYLL model [55,56]. Fig. (1) depicts the underlying hadron-nucleon slopes (thin lines) and the calculated hadron-nucleus slopes (thick lines). The hadron-nucleus slopes are clearly higher than the hadron-nucleon slopes at the same energy. However, the ratios of the two slopes decreases with increasing energy.

4 Gravitational radiation from high energy cosmic rays

After the derivation of the basic equations in the previous sections, we are now ready to calculate the amount of energy that is emitted into gravitational radiation by a high energy proton propagating through the atmosphere.

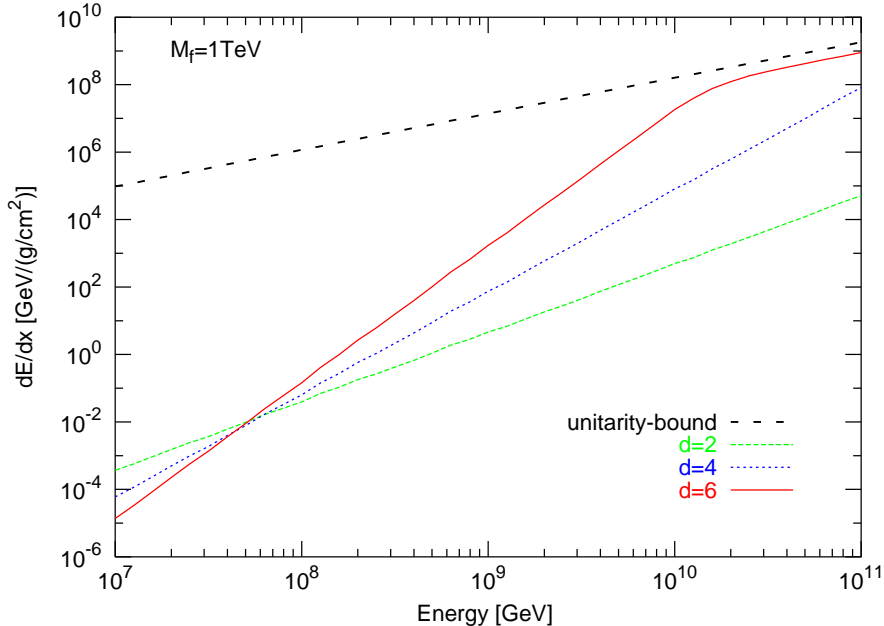


Fig. 2. Energy loss (in $\text{GeV}/(\text{g}/\text{cm}^2)$) of a proton propagating through the atmosphere as a function of the lab-frame energy for $M_f = 1$ TeV and $d = 2, 4, 6$.

The differential energy loss is given by

$$\frac{dE}{dx}(s, d) = \frac{\int_0^{\sqrt{s}/2} dt \frac{d\sigma_{hA}^0}{dt} E(t, s, d)}{\lambda \int_0^{\sqrt{s}/2} dt \frac{d\sigma_{hA}^0}{dt}}, \quad (25)$$

where λ is the mean free path for elastic scattering of the projectile in units of g/cm^2 and $d\sigma_{hA}^0/dt$ is the differential hadron-nucleus cross section. For cosmic ray calculations it is convenient to calculate the energy loss E in the laboratory frame. The corresponding Lorentz transformations are given in App. A.

Figs. 2 (for $M_f = 1$ TeV) and 3 (for $M_f = 2$ TeV) show the differential energy loss of a Proton propagating through the atmosphere as a function of the initial energy in the laboratory frame. The short dashed, dotted and full lines give the results for two, four and six extra dimensions, the long dashed lines show the unitarity bounds. For large initial energies, a higher number of extra dimensions leads to an enhancement of the gravitational energy loss. However, with increasing fundamental scale M_f the effect is much weaker as shown in Fig. 3. Note, that the result is cut-off dependent as k_{max} is not determined from first principles. For the present study, we have chosen $k_{\text{max}} = \sqrt{s}/2$, which is the maximal value consistent with energy conservation in the picture of a gravitational wave being emitted by one of the outgoing states. The comparison of these results with [38] shows that an approximation of the effects of extra dimension with a simple phase space argument does yield a

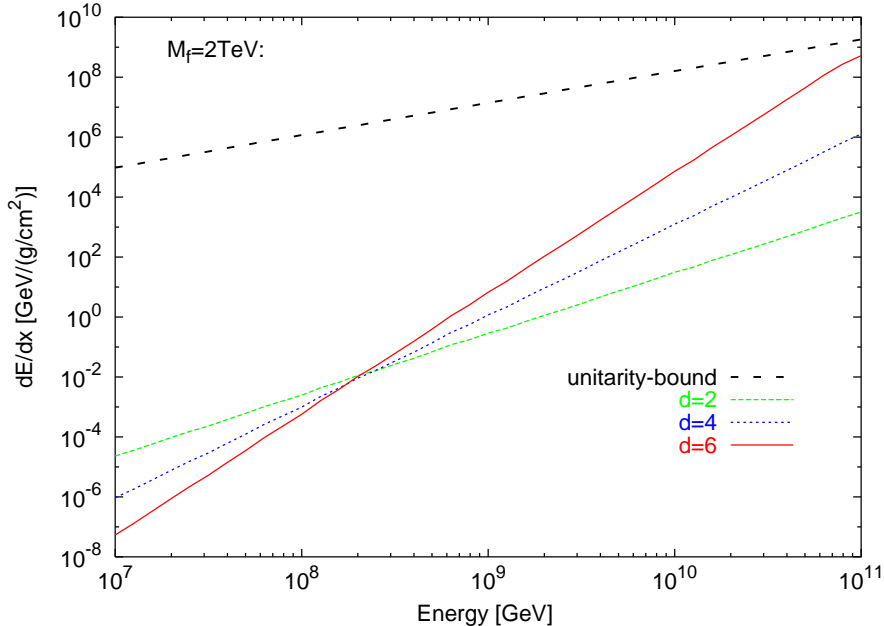


Fig. 3. Energy loss (in $\text{GeV}/(\text{g}/\text{cm}^2)$) of a proton propagating through the atmosphere as a function of the lab-frame energy for $M_f = 2$ TeV and $d = 2, 4, 6$.

similar shape for the energy loss as those shown in Figs. 2 and 3. However, the omission of the correct kinematics of the energy loss Eq. (15) results in a dramatic overestimation of the gravitational energy loss effect by several orders of magnitude. In addition, the simple extension of the standard formula with a modified phase space factor on the integrated cross sections results in a violation of the unitarity bound.

Even though the energy loss into gravitational waves in our (very optimistic) scenario is much lower than expected from previous approximations, it might still have observable consequences for very high energy cosmic rays. Therefore we implemented Eq. (15) and the elastic cross sections into a complete cosmic ray air shower simulation (SENECA) [57,58] to study the modifications of the shower properties in detail.

Fig. 4 gives the relative energy loss as a function of the incident energy E . The calculation is averaged over incident zenith angles $d\cos(\theta)$ in the range $0^\circ \leq \theta \leq 60^\circ$. The full lines indicate the calculations with six extra dimensions, while the dotted lines show the results for four extra dimensions ($M_f = 1$ TeV is shown by thick lines, $M_f = 2$ TeV is shown by thin lines). For the case of two extra dimensions, deviations from the non-modified shower properties are very small even for the most optimistic cases. However, for four extra dimensions first deviations from the standard calculation become visible at energies higher than $5 \cdot 10^{10}$ GeV. For $d = 6$ the gravitational radiation becomes sizeable and already leads to deviations around $5 \cdot 10^9$ GeV. At the highest energies, the integrated relative energy loss due to gravitational radiation might even exceed

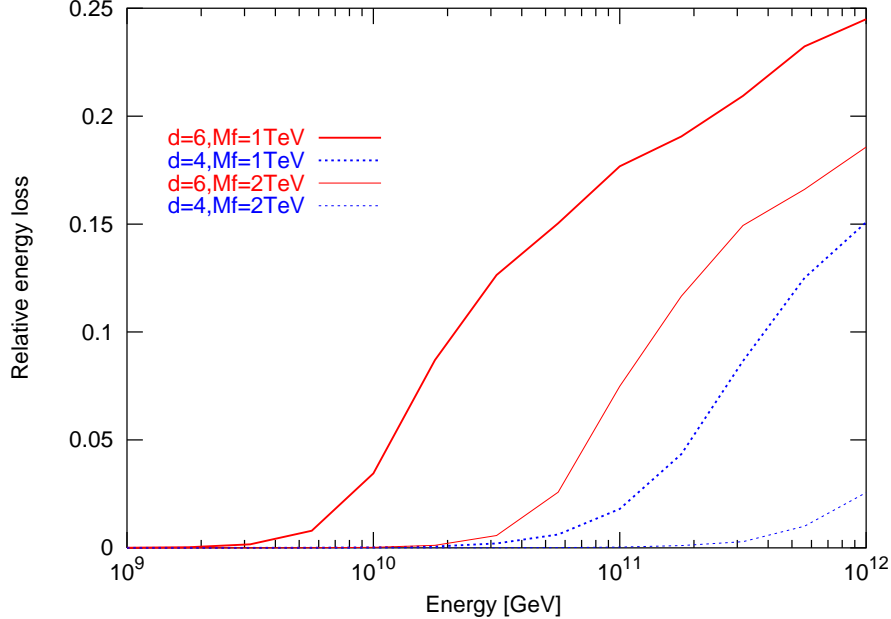


Fig. 4. Relative energy loss into gravitational radiation as a function of the incident cosmic ray energy E for $d = (4, 6)$ and $M_f = (1, 2)$ TeV.

20% of the initial particle energy.

In present day experiments, e.g. AUGER, this gravitational energy loss would show up as a decrease in the number of observed secondary particles. The multiplicity of secondary particles $N_{\text{sec}}(E, x)$ is directly observable in fluorescence experiments and is a key observable to estimate the cosmic ray's initial energy. Any non-visible energy emission results in an underestimation of the initial energy in the energy reconstruction procedure. Thus, it has an impact on the interpretation of the measured cosmic ray flux in dependence of the incoming particle energy.

How big is the distortion of the reconstructed flux due to graviton emission quantitatively? Neglecting fluctuations, for a given incoming flux $F = dN/dE$, the measured flux $F' = dN'/dE'$ depends on the reconstructed energy $E'(E)$. By identifying the integrated fluxes $N'(E') = N(E)$ one finds

$$F'(E') = \frac{dN'}{dE'} = \frac{dN(E)}{dE} \cdot \frac{dE}{dE'} = F(E) \cdot \frac{dE}{dE'} . \quad (26)$$

For an incoming flux $F = kE^{-3}$ the flux reconstruction is shown in Fig. 5. In all scenarios ($d \geq 4$) gravitational wave emission might indeed influence the energy reconstruction above $5 \cdot 10^9$ GeV. For ultra high energy cosmic rays even an apparent cut-off seems possible², because the relative amount of non-

² Note the linear scale on the y-axis, thus the suppression does not mimic the GZK

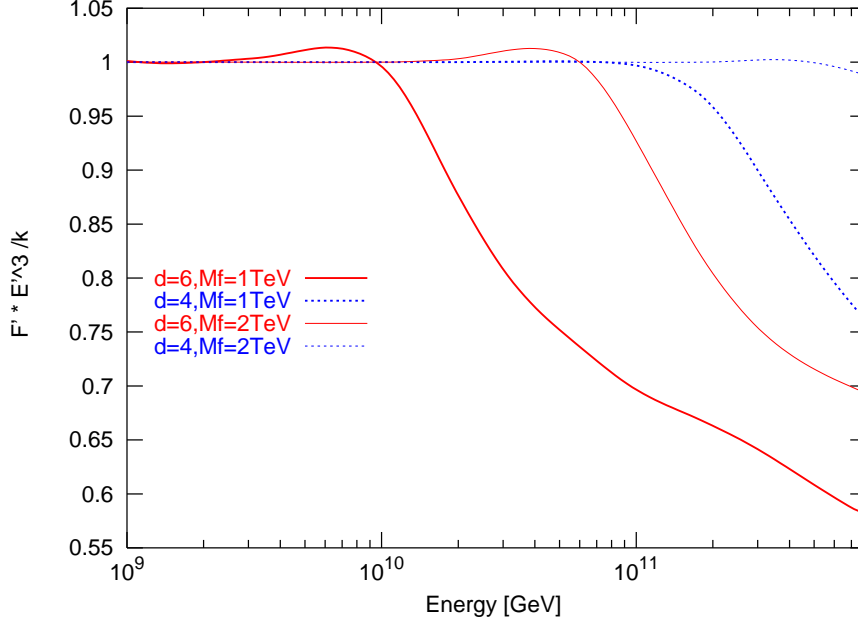


Fig. 5. Reconstructed flux $F'E'^3/k$ as a function of the incident cosmic ray energy E' for $d = (4, 6)$ and $M_f = (1, 2)$ TeV.

visible energy increases strongly with increasing energy. Hence, for UHECRs the interpretation of experimental data might have to be modified in scenarios with large extra dimensions.

Presently available data from Hires and AGASA do not allow one to observe the predicted suppression pattern, because even in our most optimistic scenario the flux is reduced only by a factor of 0.5 for the highest energies. However, with the expected high statistics data from the Pierre Auger Observatory a detailed exploration of this phenomenon might be possible.

As a remark, we want to point out that in our calculation, gravitational wave emission does not give new insights into phenomena at lower energies ($E \leq 10^{18}$ eV) and can not be considered as a candidate to explain the famous knee in the cosmic ray spectrum.

5 Conclusion

The energy loss into gravitational waves is calculated for ultra high energy cosmic rays. In contrast to previous estimates, quasi-elastic particle scattering in the ADD scenario with 4 or 6 extra dimensions has no observable influence on the properties of cosmic ray air showers at incident energies below $5 \cdot 10^{18}$ eV.

cut-off.

Thus, the emission of gravitational radiation can not be used to explain the steepening of the cosmic ray spectrum at the "knee" ($E \sim 10^{15.5}$ eV). For two large extra dimensions, the studied effects are generally too small to lead to any observable effect.

However, for energies above $5 \cdot 10^{18}$ eV and $M_f \leq 2$ TeV, $d \geq 4$ gravitational energy loss during the air shower evolution can be sizeable. This might result in an underestimation of the reconstructed energy for ultra high energy cosmic rays as studied by Hires, AGASA and the Pierre Auger Observatory.

Acknowledgements

The authors thank Drs. S. Hossenfelder, S. Hofmann and S. Ostapchenko for fruitful discussions. B.K. thanks the Frankfurt International Graduate School of Science (FIGSS) for financial support through a PhD fellowship. This work was supported by GSI and BMBF.

All numerical calculations have been performed at the Frankfurt Center for Scientific Computing (CSC).

A Energy loss in the lab system

Equation (15) provides the gravitationally radiated energy in the centre of mass frame of the reaction. To transform the kinematic variables to the laboratory frame with a target Proton at rest one has to apply the Lorentz transformation matrix

$$\Lambda = \begin{pmatrix} \text{Cosh}(\eta) & \text{Sinh}(\eta) \\ \text{Sinh}(\eta) & \text{Cosh}(\eta) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{s}}{2m_p} & \frac{\sqrt{s-16m_p^2}}{2m_p} \\ \frac{\sqrt{s-16m_p^2}}{2m_p} & \frac{\sqrt{s}}{2m_p} \end{pmatrix}, \quad (\text{A.1})$$

which acts on the t and the z (i.e. longitudinal) component of the $4 + d$ dimensional vector. All the other (transverse) components remain unchanged. Eq. (15) gives the energy E and momentum \underline{k} of the gravitational radiation emitted from one of the interacting particles (p_1, p_2). For different momentum directions $\underline{k}/|\underline{k}|$ the Lorentz transformation Eq. (A.1) gives different energy losses in the lab-frame. To avoid this complication we use a mean value of the left over four momentum $\overline{p'}$ of the scattering particles. If the energy is radiated away from particle i we define $p'_i = p_i - k$. Averaging over these cases yields

$$\overline{p'} = \sum_i^N \frac{p'_i + \sum_{l \neq i}^N p_l}{N}. \quad (\text{A.2})$$

Using the symmetry of a $2 \rightarrow 2$ scattering in the centre of mass system we find

$$\vec{p}_{\text{CM}} = (\sqrt{s} - k_0, 0, 0, 0, \dots). \quad (\text{A.3})$$

Because \vec{p}_{CM} has no z component, the mean left over energy in the laboratory system becomes

$$\vec{p}_{\text{lab}} = \Lambda \cdot \vec{p}_{\text{CM}} = \left(\frac{\sqrt{s}}{2m_p} (\sqrt{s} - k_0), 0, 0, \dots \right). \quad (\text{A.4})$$

From Eq. (A.4) the mean energy loss in the lab system is obtained as

$$\overline{E}_{\text{lab}}^{\text{loss}} = \frac{s}{2m_p} - \vec{p}_{0\text{lab}} = \frac{\sqrt{s}}{2m_p} E_{\text{CM}}^{\text{loss}}. \quad (\text{A.5})$$

References

- [1] M. B. Green, J. H. Schwarz, E. Witten *Superstring Theory* vol 1, Cambridge University Press, 1987
- [2] C. D. Hoyle, D. J. Kapner, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt and H. E. Swanson, Phys. Rev. D **70**, 042004 (2004) [arXiv:hep-ph/0405262].
- [3] E. G. Adelberger, B. R. Heckel and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. **53**, 77 (2003) [arXiv:hep-ph/0307284].
- [4] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].
- [5] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999) [arXiv:hep-ph/9807344].
- [6] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].
- [7] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B **441**, 96 (1998) [arXiv:hep-th/9808138].
- [8] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **544**, 3 (1999) [arXiv:hep-ph/9811291].
- [9] T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D **59**, 105006 (1999) [arXiv:hep-ph/9811350].
- [10] E. A. Mirabelli, M. Perelstein and M. E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999) [arXiv:hep-ph/9811337].
- [11] L. Vacavant and I. Hinchliffe, J. Phys. G **27**, 1839 (2001).

- [12] T. Banks and W. Fischler, arXiv:hep-th/9906038.
- [13] D. Atwood, S. Bar-Shalom and A. Soni, Phys. Rev. D **61**, 054003 (2000) [arXiv:hep-ph/9906400].
- [14] D. Atwood, S. Bar-Shalom and A. Soni, Phys. Rev. D **61**, 116011 (2000) [arXiv:hep-ph/9909392].
- [15] D. Atwood, S. Bar-Shalom and A. Soni, arXiv:hep-ph/9903538.
- [16] T. G. Rizzo, arXiv:hep-ph/9910255.
- [17] S. C. Ahern, J. W. Norbury and W. J. Poyser, Phys. Rev. D **62**, 116001 (2000) [arXiv:gr-qc/0009059].
- [18] S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D **62**, 055012 (2000) [arXiv:hep-ph/0001166].
- [19] S. B. Giddings, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, eConf **C010630**, P328 (2001) [arXiv:hep-ph/0110127].
- [20] S. Hossenfelder, S. Hofmann, M. Bleicher and H. Stoecker, Phys. Rev. D **66**, 101502 (2002) [arXiv:hep-ph/0109085].
- [21] S. Hofmann, M. Bleicher, L. Gerland, S. Hossenfelder, S. Schwabe and H. Stoecker, arXiv:hep-ph/0111052.
- [22] M. Bleicher, S. Hofmann, S. Hossenfelder and H. Stoecker, Phys. Lett. **548**, 73 (2002) [arXiv:hep-ph/0112186].
- [23] T. G. Rizzo, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, eConf **C010630**, P301 (2001) [eConf **C010630**, E3021 (2001)] [arXiv:hep-ph/0108235].
- [24] E. Dvergsnes, P. Osland and N. Ozturk, Phys. Rev. D **67**, 074003 (2003) [arXiv:hep-ph/0207221].
- [25] S. Hossenfelder, M. Bleicher, S. Hofmann, H. Stoecker and A. V. Kotwal, Phys. Lett. B **566**, 233 (2003) [arXiv:hep-ph/0302247].
- [26] S. Hossenfelder, M. Bleicher, S. Hofmann, J. Ruppert, S. Scherer and H. Stoecker, Phys. Lett. B **575**, 85 (2003) [arXiv:hep-th/0305262].
- [27] T. Buanes, E. W. Dvergsnes and P. Osland, Eur. Phys. J. C **35**, 555 (2004) [arXiv:hep-ph/0403267].
- [28] B. Abbott *et al.* [D0 Collaboration], Phys. Rev. Lett. **86**, 1156 (2001) [arXiv:hep-ex/0008065].
- [29] D. Acosta [CDF Collaboration], Phys. Rev. Lett. **92**, 121802 (2004) [arXiv:hep-ex/0309051].
- [30] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999) [arXiv:hep-ph/9903422].

- [31] S. Hannestad and G. G. Raffelt, Phys. Rev. D **67**, 125008 (2003) [Erratum-ibid. D **69**, 029901 (2004)] [arXiv:hep-ph/0304029].
- [32] C. Hanhart, D. R. Phillips, S. Reddy and M. J. Savage, Nucl. Phys. B **595**, 335 (2001) [arXiv:nucl-th/0007016].
- [33] C. Hanhart, J. A. Pons, D. R. Phillips and S. Reddy, Phys. Lett. B **509**, 1 (2001) [arXiv:astro-ph/0102063].
- [34] G. Domokos and S. Kovesi-Domokos, Phys. Rev. Lett. **82**, 1366 (1999) [arXiv:hep-ph/9812260].
- [35] R. Emparan, M. Masip and R. Rattazzi, Phys. Rev. D **65**, 064023 (2002) [arXiv:hep-ph/0109287].
- [36] A. Ringwald and H. Tu, Phys. Lett. B **525**, 135 (2002) [arXiv:hep-ph/0111042].
- [37] L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, Phys. Rev. D **65**, 124027 (2002) [arXiv:hep-ph/0112247].
- [38] D. Kazanas and A. Nicolaidis, Gen. Rel. Grav. **35**, 1117 (2003) [arXiv:hep-ph/0109247].
- [39] M. Kowalski, A. Ringwald and H. Tu, Phys. Lett. B **529**, 1 (2002) [arXiv:hep-ph/0201139].
- [40] J. Alvarez-Muniz, J. L. Feng, F. Halzen, T. Han and D. Hooper, Phys. Rev. D **65**, 124015 (2002) [arXiv:hep-ph/0202081].
- [41] G. Sigl, arXiv:hep-ph/0207254.
- [42] L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, Phys. Rev. D **68**, 104025 (2003) [arXiv:hep-ph/0307228].
- [43] L. A. Anchordoqui, Z. Fodor, S. D. Katz, A. Ringwald and H. Tu, JCAP **0506**, 013 (2005) [arXiv:hep-ph/0410136].
- [44] L. A. Anchordoqui, H. Goldberg, M. C. Gonzalez-Garcia, F. Halzen, D. Hooper, S. Sarkar and T. J. Weiler, Phys. Rev. D **72**, 065019 (2005) [arXiv:hep-ph/0506168].
- [45] L. Anchordoqui and F. Halzen, arXiv:hep-ph/0510389.
- [46] V. Cardoso, O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D **67**, 064026 (2003) [arXiv:hep-th/0212168].
- [47] B. Koch and M. Bleicher, arXiv:hep-th/0512353.
- [48] S. Weinberg, Gravitation and Cosmology, ISBN 0-471-92567-5, 1972
- [49] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
- [50] S. Wolfram, The Mathematica Book, Cambridge University Press, 1996
- [51] M. Levy and J. Sucher, Phys. Rev. **186**, 1656 (1969).

- [52] V. N. Gribov, Sov. Phys. JETP **29**, 483 (1969) [Zh. Eksp. Teor. Fiz. **56**, 892 (1969)].
- [53] R. J. Glauber and G. Matthiae, Nucl. Phys. B **21**, 135 (1970).
- [54] R. J. Glauber, "High-Energy Collision Theory", in Lo, S.Y. (ed.): Geometrical pictures in hadronic collisions, 83-182.
- [55] R. S. Fletcher, T. K. Gaisser, P. Lipari and T. Stanev, Phys. Rev. D **50**, 5710 (1994).
- [56] R. Engel, T. K. Gaisser, T. Stanev and P. Lipari, *Prepared for 26th International Cosmic Ray Conference (ICRC 99), Salt Lake City, Utah, 17-25 Aug 1999*
- [57] G. Bossard *et al.*, Phys. Rev. D **63**, 054030 (2001) [arXiv:hep-ph/0009119].
- [58] H. J. Drescher and G. R. Farrar, Phys. Rev. D **67**, 116001 (2003) [arXiv:astro-ph/0212018].