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Optimal Time-Consistent Monetary and Fiscal Policy

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Dealing with a Liquidity Trap when Government Debt Matters: Optimal Time-Consistent Monetary and Fiscal Policy[*]

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Abstract

How does the need to preserve government debt sustainability affect the optimal monetary and fiscal policy response to a liquidity trap? To provide an answer, we employ a small stochastic New Keynesian model with a zero bound on nominal interest rates and characterize optimal time-consistent stabilization policies. We focus on two policy tools, the short-term nominal interest rate and debt-financed government spending. The optimal policy response to a liquidity trap critically depends on the prevailing debt burden. While the optimal amount of government spending is decreasing in the level of outstanding government debt, future monetary policy is becoming more accommodative, triggering a change in private sector expectations that helps to dampen the fall in output and inflation at the outset of the liquidity trap.

JEL Classification: E31, E52, E62, E63, D11

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1 Introduction

New Keynesian characterizations of optimal time-consistent monetary and fiscal policies in a liquidity trap typically omit government debt from the analysis, assuming that government purchases are financed by lump-sum taxes (e.g. Werning, 2011; Schmidt, 2012; Nakata, 2013). At the same time, the enormous increase in government debt-to-GDP ratios in the course of the recent global financial crisis in major industrialized countries raises important questions about the appropriate stance of monetary and fiscal policy. Should policymakers adhere to fiscal stimulus in the face of a zero lower bound event if the level of government debt is already above its long-run target? How does the need to ensure debt sustainability act upon the effectiveness of monetary policy? In terms of model-based characterizations of optimal policies at the zero lower bound, is the conventional omission of government debt innocuous or do our normative prescriptions change when we account for the fact that lump-sum taxes in general do not adjust one-to-one with other fiscal variables?

We address these questions in a stylized stochastic New Keynesian model with a zero bound on nominal interest rates that accounts for government debt in the form of non-state-contingent, one-period, nominal government bonds as a means of financing government spending. The benevolent government controls the short-term nominal interest rate and the level of government spending, and decides about the supply of government bonds. Hence, in the economy that we consider the central bank and the fiscal authority coordinate their policy measures. We focus on time-consistent policy regimes since it is the absence of a commitment device that renders the zero lower bound detrimental for stabilization policy.[1]

Households appreciate private consumption as well as the provision of public goods and dislike labor. For the ease of exposition, we assign only a very limited role to tax policy in our baseline model. First, private consumption and household labor income are taxed at constant rates, providing revenues to the government. Second, lump-sum taxes are used to finance a constant wage subsidy to ensure that the distortions arising from monopolistic competition in the goods market and from the other taxes are eliminated in the non-stochastic steady state. However, we also present results for the case where the policymaker sets the labor tax rate optimally. Economic uncertainty arises from the presence of a demand shock.

We solve the model using a projection method and then explore how government debt affects optimal policies and stabilization outcomes when the zero bound on nominal interest rates becomes occasionally binding. The presence of government debt makes the optimal time-consistent policy history dependent, that is, the future path of the policy instruments

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[1]For a characterization of optimal monetary policy under commitment see e.g. Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006) and Nakov (2008).

depends on today's level of government debt. We show, that, first, for a given realization of the demand shock, government spending is decreasing in the level of outstanding government debt, i.e. the fiscal stance becomes more contractionary when government debt rises. Second, as long as the zero lower bound is not binding, the nominal interest rate is decreasing in the level of government debt. Real interest rates keep declining as a function of the debt level even if the zero bound on the nominal rate is binding, i.e. the monetary policy stance becomes more expansionary the higher the government debt burden. Third, output and inflation are both increasing in beginning-of-period debt, irrespective of whether the zero bound is binding or not.

How the economy responds to a liquidity trap thus critically depends on the prevailing government debt level. If, for instance, the level of outstanding government debt is high relative to its steady state, then the optimal policy mix will prescribe at most a small government spending stimulus, followed by a spending reversal, and a prolonged period of expansionary monetary policy. The policymaker creates valid expectations of a subsequent boom in inflation and output that help to dampen the economic turmoil at the outset of the liquidity trap. If, on the other hand, the public debt level is low relative to its steady state, government spending is used forcefully to stimulate aggregate demand, when the economy falls into a liquidity trap. In this situation, however, the zero bound episode is not followed by a boom in output and inflation. Absent the expansionary expectations effects of the high debt scenario, the low debt scenario exhibits larger drops in output and inflation.

The ability to issue government debt allows the policymaker to influence private sector expectations without engaging in time-inconsistent policies. As emphasized by Krugman (1998) and Eggertsson and Woodford (2003), during zero lower bound episodes, expectations about future output and inflation can have considerable effects on contemporaneous stabilization outcomes. We demonstrate the powerfulness of government debt-induced history dependence by comparing optimal discretionary policies and stabilization outcomes for a liquidity trap scenario in our baseline economy with those in the conventional model setup that features zero government debt and lump-sum taxes that adjust each period to balance the government budget.

Our paper is closely related to work by Eggertsson (2006), who first showed that the accumulation of government debt allows a discretionary policymaker to influence expectations about the path of monetary policy after the liquidity trap. Our paper differs from this earlier work in several respects. First, the fiscal instrument considered by Eggertsson is a lump-sum tax. In his model, the policymaker lowers lump-sum taxes when the zero bound is binding in order to increase government debt. Tax collection costs make it credible that the increase in government debt will not be solely undone by future tax increases. There is no immediate

trade-off for fiscal policy in a liquidity trap between stimulating the economy and stabilizing government debt. In our paper, the liquidity trap shock reduces the tax base, which may force the policymaker to tighten fiscal policy while the zero lower bound is binding. Second, in Eggertsson's model, the economy starts in a liquidity trap state and returns to the normal state with a constant probability in each subsequent period, where it will stay forever. Instead, in our model, the zero nominal interest rate bound is an occasionally binding constraint. We show that the outstanding amount of government debt prior to the zero bound event critically affects stabilization policies and outcomes in the liquidity trap. For instance, if government debt is low relative to its steady state, then the policymaker may refrain from lowering the nominal interest rate all the way to zero, which exacerbates the fall in output and inflation. Finally, we show, that, unlike in Eggertsson (2006), the optimal discretionary policy is not necessarily associated with a transitory boom in output and inflation after the liquidity trap.

The paper can also be related to studies that investigate optimal monetary and fiscal policy under commitment at the zero lower bound and account for the presence of government debt. Eggertsson and Woodford (2006) determine the optimal nominal interest rate and tax policy mix. Nakata (2011) characterizes the optimal plan for distortionary taxes, government spending and the short-term nominal interest rate.

Finally, several studies have characterized optimal monetary and fiscal policy in New Keynesian models that account for the presence of government debt but abstract from the zero bound on nominal interest rates. Wren-Lewis and Leith (2007) and Vines and Stehn (2007) characterize the optimal policy mix under discretion, whereas Schmitt-Grohe and Uribe (2004) and Adam (2011) analyse optimal commitment policies.

The remainder of the paper is organized as follows. Section 2 presents the model economy. Section 3 specifies the policy problem. Section 4 presents numerical results. Finally, Section 5 concludes.

2   The model

We consider a small monetary business cycle model with nominal rigidities and monopolistic competition. The economy is inhabited by a continuum of identical households of measure one, a final good producer, a continuum of intermediate-goods-producing firms of measure one, and a benevolent policymaker. Following Woodford (2003), the model is treated as a cashless limiting economy. Time is discrete and indexed by t.

2.1 Households and firms

The representative household obtains utility from a private consumption good C_t and the provision of a public consumption good G_t, and dislikes labor $N_t(i), \forall i \in [0,1]$. Expected lifetime utility of the household reads

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t) + g(G_t) - \int_0^1 \nu(N_t(i)) \, di \right), \tag{1}$$

where E_t is the rational expectations operator conditional on information in period t and $\beta \in (0,1)$ is the discount factor. The functions $u(\cdot)$ and $g(\cdot)$ are increasing and concave in their arguments, and $\nu(\cdot)$ is increasing and convex in its argument.

The household enters period t with a degenerate portfolio of non-state-contingent, one-period, nominal government bonds B_{t-1}, paying the household B_{t-1}/P_t units in terms of the final consumption good. For simplicity, we assume that one-period government bonds are the only assets traded in the economy. The household supplies $N_t(i)$ units of labor to the producer of intermediate good i and earns total after-tax labor income $\int_0^1 (1 - \tau^N) W_t(i) N_t(i) \, di$, where $W_t(i)$ denotes the nominal wage rate payed by firm i and τ^N is a constant labor income tax rate. Furthermore, the household receives dividend payments $P_t \Psi_t$ from intermediate-goods-producing firms, which are owned by the household. The household uses her labor income, dividend income and the government's debt repayment to finance purchases of the private consumption good at price $(1 + \tau^C) P_t$ where τ^C is a constant consumption tax rate, to pay lump-sum taxes $P_t T_t$, and to buy newly issued government bonds at price $1/(1 + i_t)$, where $i_t \geq 0$ is the one-period, riskless, nominal interest rate. The flow budget constraint reads

$$\left(1 + \tau^C\right) P_t C_t + \frac{B_t}{1 + i_t} \leq \int_0^1 \left(1 - \tau^N\right) W_t(i) N_t(i) di + B_{t-1} - P_t T_t + \Psi_t. \tag{2}$$

The representative household maximizes her expected lifetime utility (1) by choosing state-contingent plans $\{C_t > 0, N_t(i) > 0, B_t\}_{t=0}^{\infty}$ subject to (2) and a no-Ponzi game condition

$$\lim_{j \to \infty} \mathrm{E}_t \left(\left(\prod_{k=0}^{t+j} \frac{1}{1 + i_k} \right) B_{t+j} \right) \geq 0.$$

The final consumption good is produced under perfect competition using the following technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ and $Y_t(i)$ denotes the intermediate input i. Total demand for the final good consists of household and government demand

$$Y_t = C_t + G_t.$$

The market for intermediate goods features monopolistic competition. Expenditure minimization by the producer of the final good results in the following demand for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t, \tag{3}$$

where $P_t(i)$ denotes the price charged by firm i and $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$ represents the price for the final consumption good. Intermediate goods are produced using labor

$$Y_t(i) = N_t(i).$$

There is no capital. Intermediate-goods firms face price rigidities à la Calvo (1983). In each period, a fraction $1 - \alpha$ of firms is allowed to change prices, whereas the remaining fraction $\alpha \in (0,1)$ of firms keep their price constant at previous period's level. Each intermediate goods firm i that is allowed to reset prices in period t maximizes its expected discounted profits:

$$\max_{P_t(i)} \sum_{j=0}^{\infty} E_t Q_{t,t+j} \alpha^j Y_{t+j}(i) \left[P_t(i) - (1-\tau)W_{t+j}(i)\right],$$

subject to (3). The parameter τ denotes a constant employment subsidy that eliminates the distortions arising from monopolistic competition and distortionary taxes in the non-stochastic steady state and $Q_{t,t+j} = \beta^j \frac{U'(C_{t+j})/P_{t+j}}{U'(C_t)/P_t}$ is the stochastic discount factor between period t and $t+j$.

2.2   The government

The government issues non-state-contingent, one-period, nominal government bonds and levies lump-sum taxes, labor income taxes and consumption taxes to finance public spending and the provision of a constant wage subsidy τ, and to service the debt incurred from the previous period. We assume that the government can credibly promise to repay its debt each period. The flow budget constraint reads

$$P_t G_t + \tau \int_0^1 W_t(i) N_t(i) \, di + B_{t-1} = \frac{B_t}{1+i_t} + P_t T_t + \tau^C P_t C_t + \tau^N \int_0^1 W_t(i) N_t(i) di.$$

In real terms

$$G_t + \tau \int_0^1 w_t(i) N_t(i) \, di + b_{t-1} \pi_t^{-1} = \frac{b_t}{1 + i_t} + T_t + \tau^C C_t + \tau^N \int_0^1 w_t(i) N_t(i) di,$$

where $b_t = B_t/P_t$, $\pi_t = P_t/P_{t-1}$, and $w_t(i) = W_t(i)/P_t$.

Assumption 1 *Lump-sum taxes are used to finance the wage subsidy. Beyond that, a constant amount of (possibly negative) lump-sum tax revenues T^G is available to finance government spending and to service public debt*

$$T_t = T^G + \tau \int_0^1 w_t(i) N_t(i) \, di.$$

We can then simplify the budget constraint

$$G_t + b_{t-1} \pi_t^{-1} = \frac{b_t}{1 + i_t} + T^G + \tau^C C_t + \tau^N \int_0^1 w_t(i) N_t(i) di. \tag{4}$$

2.3 Equilibrium

An equilibrium consists of paths $\{C_t, N_t(i), Y_t(i), Y_t, B_t, G_t, i_t, P_t(i), P_t, W_t(i)\}_{t=0}^\infty$, given an initial level of government debt B_{-1} and identical initial goods prices $P_{-1}(i) \, \forall i$, such that (i) $\{C_t, N_t(i), B_t\}_{t=0}^\infty$ solves the household optimization problem given prices and policies, (ii) $\{P_t(i)\}_{t=0}^\infty$ solves the optimization problem of producer i, (iii) the government budget constraint and the zero lower bound on the nominal interest rate $i_t \geq 0$ are satisfied, and (iv) the goods market, the labor market, and the government bond market clear.

2.4 Log-linear approximation

The optimization problems of households and firms are standard, we therefore refrain from presenting optimality conditions and directly continue with a log-linear approximation of the resulting behavioral constraints around the non-stochastic steady state with zero inflation

$$\hat{\pi}_t = \kappa \left(\hat{Y}_t - \Gamma \hat{G}_t \right) + \beta E_t \hat{\pi}_{t+1} \tag{5}$$

$$\hat{Y}_t = \hat{G}_t + E_t \hat{Y}_{t+1} - E_t \hat{G}_{t+1} - \frac{1}{\sigma} (i_t - E_t \hat{\pi}_{t+1} - r^*) + d_t. \tag{6}$$

Hat variables denote percentage deviations from the deterministic steady state. $\hat{\pi}_t$ is the inflation rate between period $t-1$ and t, $\hat{Y}_t$ represents real output, and $\hat{G}_t$ denotes government spending expressed as a share of steady state output. The parameter $r^* = \frac{1}{\beta} - 1$

denotes the steady state real interest rate, and $\sigma \equiv -\frac{u''(C)}{u'(C)}Y$ represents the elasticity of the marginal utility of private consumption with respect to total output in the steady state. The parameters κ and Γ are functions of structural parameters

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma+\eta), \quad \Gamma = \frac{\sigma}{\sigma+\eta},$$

where $\eta > 0$ denotes the inverse of the labor supply elasticity, $\alpha \in (0,1)$ represents the share of firms that are unable to change their price in a given period, and $\theta > 1$ is the price elasticity of demand for intermediate goods. We assume that the economy is subject to an exogenous demand shock d_t that follows a stationary autoregressive process

$$d_t = \rho d_{t-1} + \epsilon_t, \tag{7}$$

where ϵ_t is a i.i.d. $N(0, \sigma_\epsilon^2)$ innovation, and $\rho \in [0,1)$.

Finally, the log-linearized government budget constraint (4) reads[2]

$$
\begin{aligned}
\hat{b}_t &= \frac{1}{\beta}\left\{ \hat{b}_{t-1} - \frac{b}{Y}\hat{\pi}_t + \left(1 + \tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N\sigma\right)\hat{G}_t - \left(\tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N(1+\sigma+\eta)\right)\hat{Y}_t \right\} \\
&\quad + \frac{b}{Y}(i_t - r^*),
\end{aligned} \tag{8}
$$

where $\hat{b}_t = \frac{b_t - b}{Y}$.

3   The policy problem

The benevolent policymaker aims to maximize expected lifetime utility (1) of the representative household. We conduct a linear-quadratic approximation to household welfare to obtain a quadratic policy objective function.[3] Each period t, the policymaker minimizes the loss function from period t onwards, taking the decision rules of the private sector and future governments as given. We focus on stationary Markov-perfect equilibria, where the vector of state variables consists of the demand shock and the beginning-of-period government debt

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[2]In the deterministic steady state $\left(1+\tau^C\right)\frac{G}{Y} + (1-\beta)\frac{b}{Y} = \frac{T^G}{Y} + \tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N$.

[3]See Schmidt (2012) for the details of the derivation. We ensure that the non-stochastic steady state of the flexible-price equilibrium is efficient by choosing the constant wage subsidy such that it offsets the distortions arising from monopolistic competition and taxes in the non-stochastic steady state, $\tau = 1 - \frac{\theta-1}{\theta}\frac{1-\tau^N}{1+\tau^C}$.

level, $s_t = (d_t, \hat{b}_{t-1})$. The Bellman equation reads

$$V(s_t) = \min_{\{\hat{\pi}_t, \hat{Y}_t, \hat{G}_t, i_t, \hat{b}_t\}} \left[\frac{1}{2} \left(\hat{\pi}_t^2 + \lambda \left(\hat{Y}_t - \Gamma \hat{G}_t \right)^2 + \lambda_G \hat{G}_t^2 \right) + \beta \mathrm{E}_t V(s_{t+1}) \right]$$

subject to

$$\hat{\pi}_t = \beta \mathrm{E}_t \hat{\pi}(s_{t+1}) + \kappa \left(\hat{Y}_t - \Gamma \hat{G}_t \right)$$

$$\hat{Y}_t = \hat{G}_t + \mathrm{E}_t \hat{Y}(s_{t+1}) - \mathrm{E}_t \hat{G}(s_{t+1}) - \frac{1}{\sigma} (i_t - \mathrm{E}_t \hat{\pi}(s_{t+1}) - r^*) + d_t$$

$$\hat{b}_t = \frac{1}{\beta} \left\{ \hat{b}_{t-1} - \frac{b}{Y} \hat{\pi}_t + \left(1 + \tau^C + \frac{1+\tau^C}{1-\tau^N} \tau^N \sigma \right) \hat{G}_t - \left(\tau^C + \frac{1+\tau^C}{1-\tau^N} \tau^N (1+\sigma+\eta) \right) \hat{Y}_t \right\}$$
$$\quad + \frac{b}{Y} (i_t - r^*)$$

$$i_t \geq 0,$$

and the law of motion for the demand shock (7). The functions $\hat{\pi}(s_{t+1})$, $\hat{Y}(s_{t+1})$ and $\hat{G}(s_{t+1})$ represent the inflation rate, output and government spending that the policymaker expects to be realized in period $t+1$ in equilibrium, contingent on the realization of the demand shock d_{t+1}. The relative weights λ and λ_G in the policymaker's objective function depend on the structural parameters

$$\lambda = \frac{\kappa}{\theta}, \quad \lambda_G = \lambda \Gamma \left(1 - \Gamma + \frac{\omega}{\sigma} \right),$$

where $\omega \equiv -\frac{g''(G)}{g'(G)} Y$ is the elasticity of the marginal utility of public consumption with respect to total output.

The first-order conditions read

$$\lambda_G \hat{G}_t + \left(\frac{1}{\beta} + (1-\Gamma) \frac{b}{Y} \sigma + \frac{1}{\beta} (1-\Gamma) \tau^C - \frac{1}{\beta} \Gamma \frac{1+\tau^C}{1-\tau^N} \tau^N \right) \Phi_t^b - (1-\Gamma) \Phi_t^{\mathrm{zlb}} = 0 \quad (9)$$

$$\mathrm{E}_t \Phi^b(s_{t+1}) - \Omega_{1t} \Phi_t^b + \Omega_{2t} \hat{\pi}_t - \Omega_{3t} \Phi_t^{\mathrm{zlb}} = 0 \quad (10)$$

$$\Phi_t^{\mathrm{zlb}} i_t = 0 \quad (11)$$

$$\Phi_t^{\mathrm{zlb}} \geq 0 \quad (12)$$

$$i_t \geq 0 \quad (13)$$

as well as the New Keynesian Phillips curve, the dynamic IS curve and the government budget constraint, where

$$\Phi_t^{\text{zlb}} \equiv \left(\frac{b}{Y} \left(\sigma + \frac{\kappa}{\beta} \right) + \frac{1}{\beta} \left(\tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N \left(1 + \sigma + \eta \right) \right) \right) \Phi_t^b - \left(\kappa \hat{\pi}_t + \lambda \left(\hat{Y}_t - \Gamma \hat{G}_t \right) \right)$$

$$\Omega_{1t} \equiv 1 - \sigma \frac{b}{Y} \left(\frac{\partial \mathrm{E}_t \hat{Y}(s_{t+1})}{\partial \hat{b}_t} - \frac{\partial \mathrm{E}_t \hat{G}(s_{t+1})}{\partial \hat{b}_t} \right)$$

$$\Omega_{2t} \equiv \beta \frac{\partial \mathrm{E}_t \hat{\pi}(s_{t+1})}{\partial \hat{b}_t}$$

$$\Omega_{3t} \equiv \frac{\partial \mathrm{E}_t \hat{Y}(s_{t+1})}{\partial \hat{b}_t} - \frac{\partial \mathrm{E}_t \hat{G}(s_{t+1})}{\partial \hat{b}_t} + \frac{1}{\sigma} \frac{\partial \mathrm{E}_t \hat{\pi}(s_{t+1})}{\partial \hat{b}_t}.$$

The variable Φ_t^b is the multiplier associated with the government budget constraint and Φ_t^{zlb} represents the (normalized) multiplier associated with the zero lower bound constraint. Solving condition (9) for government spending, we get

$$\hat{G}_t = \frac{1}{\lambda_G} \left[(1 - \Gamma) \Phi_t^{\text{zlb}} - \left(\frac{1}{\beta} + (1 - \Gamma) \frac{b}{Y} \sigma + \frac{1}{\beta} (1 - \Gamma) \tau^C - \frac{1}{\beta} \Gamma \frac{1 + \tau^C}{1 - \tau^N} \tau^N \right) \Phi_t^b \right]. \quad (14)$$

Assumption 2 *The parameters satisfy* $\left(1 + \Gamma + \tau^C \right) \tau^N \leq 1 + (1 - \Gamma) \tau^C$.

This assumption is sufficient to ensure that the coefficient on Φ_t^b in (14) is negative.

Note, first, that the second term in equation (14) would vanish if we assumed that government spending is financed by lump-sum taxes. In this case, government spending would only be used as a stabilization tool if the zero lower bound were binding, and the fiscal policy stance during zero bound events would be unequivocally expansionary, see Schmidt (2012).

In the model with government debt, however, public spending may have to deviate from its steady state level even if the economy is away from the zero lower bound so that $\Phi_t^{\text{zlb}} = 0$. Intuitively, whenever monetary policy is unable to stabilize government debt as well as inflation and output simultaneously, government spending will be used as an additional stabilization tool.

Furthermore, from (14) it is not clear whether fiscal policy in a liquidity trap should be expansionary, $\hat{G}_t > 0$, or contractionary, $\hat{G}_t < 0$. Specifically, if $\Phi_t^b > 0$, the zero bound multiplier and the government budget constraint multiplier have opposite implications for the sign of the fiscal policy response. As we will show below, stabilization outcomes and policies in a liquidity trap critically depend on the amount of outstanding government debt when hitting the zero bound.

4 Numerical results

In this section, we characterize the optimal time-consistent policy mix numerically. The policy functions are approximated using a projection method with finite elements. The procedure is described in the Appendix. The baseline calibration is presented in Table 1, where the period length is one quarter. The steady state real interest rate and the law of motion of the demand shock are calibrated based on U.S. data for 1983 to 2010. We set the ratio of government spending to total output in the deterministic steady state equal to 0.2. The labor income tax rate is set to 0.3 and the consumption tax rate to 0.1, as in Denes, Eggertsson, and Gilbukh (2013). In the baseline, the steady state government debt to annualized output ratio is set to 0.5 but we also consider lower and higher values in the sensitivity analysis. All other structural parameters assume standard values from the literature.

Table 1: Calibration

Parameter	Value	Interpretation
r^*	2.6/4	Steady state natural real rate of interest (in %)
β	0.9950	Discount factor
G/Y	0.2	Steady state share of government spending in total output
τ^N	0.3	Labor income tax rate
τ^C	0.1	Private consumption tax rate
$b/(4Y)$	0.5	Steady state government debt to output ratio
α	0.66	Share of firms per period that keep prices unchanged
θ	7.66	Price elasticity of demand in the steady state
η	1	Inverse of labor supply elasticity
σ	$0.5/(1-G/Y)$	Elasticity of marginal utility of private consumption w.r.t. total output
ω	$0.5/(G/Y)$	Elasticity of marginal utility of public consumption w.r.t. total output
κ	0.0333	Slope parameter in New Keynesian Phillips curve
ρ	0.77	AR-coefficient demand shock
σ_ϵ	0.72	Standard deviation demand shock innovation (in %)
λ	0.0043	Loss function weight I
λ_G	0.0077	Loss function weight II

4.1 Optimal time-consistent policy in a liquidity trap

We begin our discussion of the optimal time-consistent policy with an experiment where the occurrence of a large negative demand shock pushes the economy for several quarters into a liquidity trap. Figure 1 shows impulse responses of output, inflation, government spending, the nominal interest rate, government debt and the real interest rate to a negative demand shock of -3 unconditional standard deviations when the economy is initially in the risky steady state. The realized paths in the absence of any further shocks are represented by solid lines, the expected paths as of period 0 are represented by dashed lines and blue-shaded

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areas represent confidence intervals. The demand shock materializing in period 0 drives the

Figure 1: Impulse responses to a negative demand shock
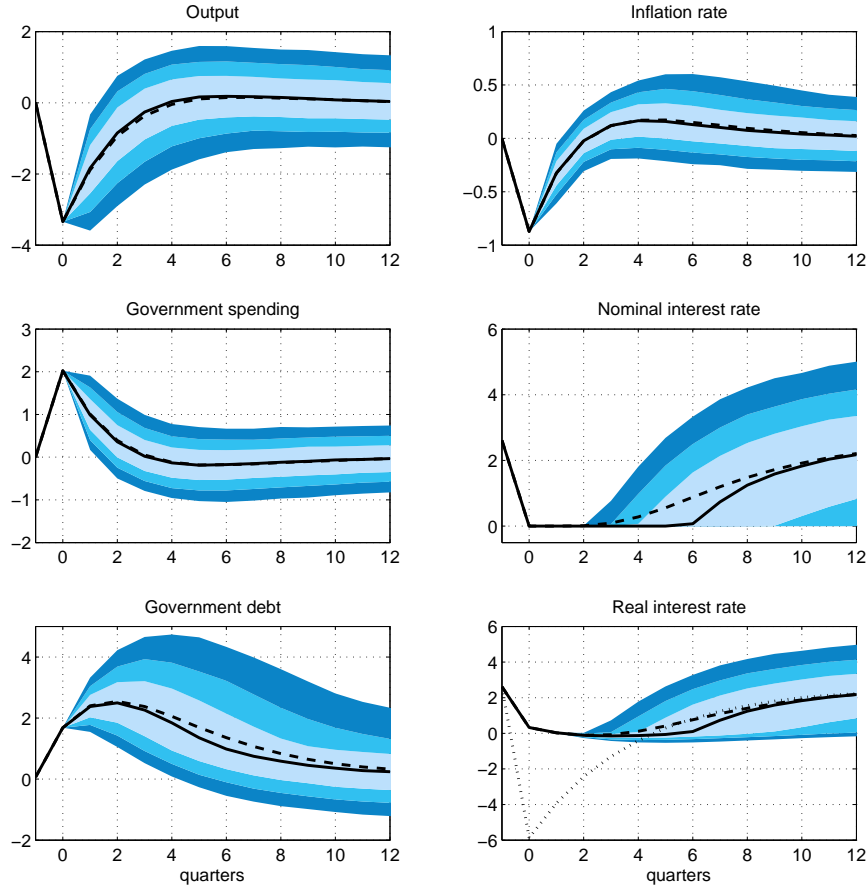
Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the baseline calibration. Realized path (solid line), path expected in period 0 (dashed line), 50%, 75% and 90% confidence intervals (shaded areas). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

natural real rate of interest $r_t^* = r^* + \sigma d_t$ into negative territory and forces the policymaker to lower the short-term nominal interest rate to zero where it stays for several periods. The economy starts to contract, both output and inflation drop below their target levels, and the reduction in the tax base leads to an increase in government debt. The fiscal policy response is first expansionary, contributing to the accumulation of government debt, but turns slightly contractionary before the zero bound episode ends. Figure 2 decomposes the response of government spending into the response of the zero lower bound multiplier component (dashed line) and the response of the budget constraint multiplier component (dashed-dotted line)

as in equation (14). Initially, both components exhibit a positive sign, which means that

Figure 2: Decomposition of government spending response
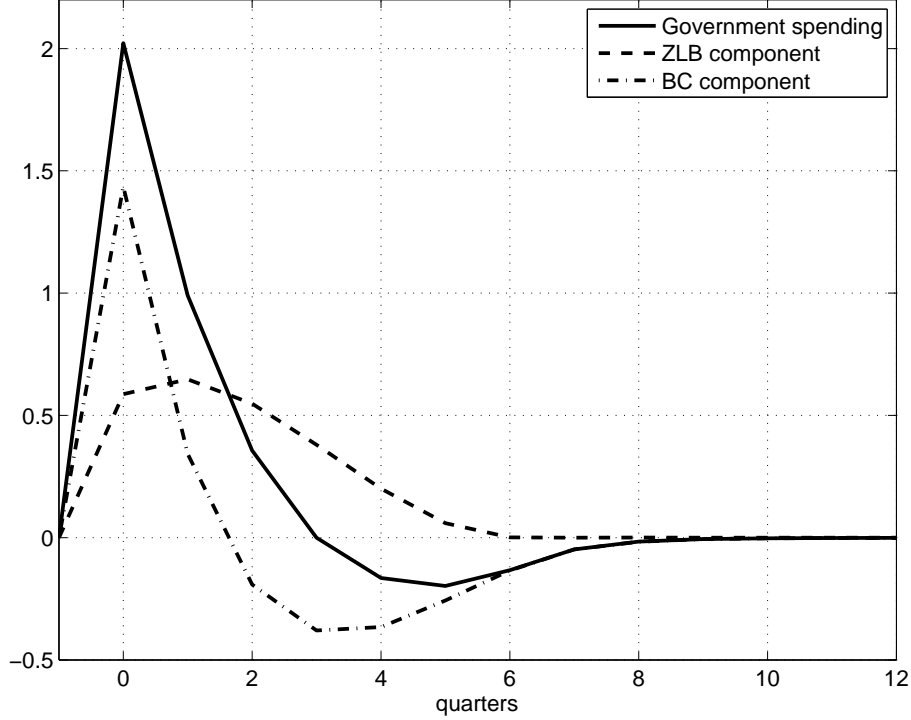
Notes: Impulse responses of government spending (solid line), the zero lower bound multiplier component $\frac{1-\Gamma}{\lambda_G}\Phi_t^{\mathrm{zlb}}$ (dashed line) and the budget constraint multiplier component $-\frac{1}{\lambda_G}\left(\frac{1}{\beta}+(1-\Gamma)\frac{b}{Y}\sigma+\frac{1}{\beta}(1-\Gamma)\tau^C-\frac{1}{\beta}\Gamma\frac{1+\tau^C}{1-\tau^N}\tau^N\right)\Phi_t^b$ (dashed-dotted line) to a demand shock of -3 unconditional standard deviations.

$\Phi_t^{\mathrm{zlb}} > 0$ and $\Phi_t^b < 0$. The negative budget constraint multiplier implies that it would have been desirable from a welfare perspective if the government had entered the period with a somewhat higher debt level. However, in subsequent periods when the government debt burden has become more elevated the budget constraint multiplier component switches signs, turning from positive to negative, and government spending declines below its pre-shock level. Coming back to Figure 1, while the increased debt burden narrows the room for expansionary fiscal policy, it facilitates the implementation of an expansionary monetary policy stimulus. Once the natural real rate has reentered positive territory, the nominal interest rate remains transitorily below the level that would be warranted by output and inflation stabilization considerations alone in order to contribute to the stabilization of government debt. As a consequence, the economy experiences a small boom in output and inflation. Importantly, private agents attach positive probabilities to positive future realizations of output and inflation and (correctly) expect both variables to move temporarily above their target levels,

which attenuates the drop at the outset of the zero bound event.

4.2   Equilibrium responses to government debt

To provide a more general characterization of the optimal time-consistent policy and how it is affected by the level of outstanding government debt, Figure 3 displays equilibrium responses to the beginning-of-period government debt level. We consider two alternative realizations of the demand shock, $d = 0$ (solid line) and $d = -3$ unconditional standard deviations (dashed line). For both values of d, the optimal amount of government spending

Figure 3: Equilibrium responses to previous period's government debt
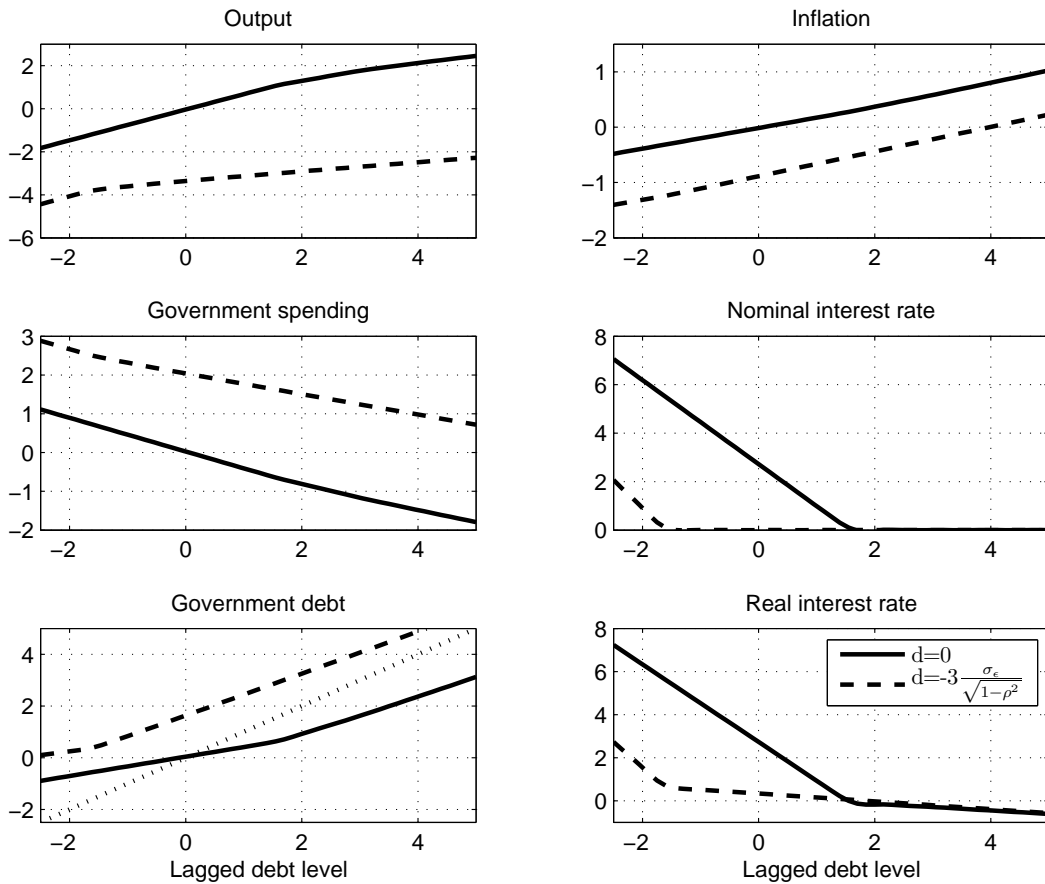
Notes: Equilibrium responses to the beginning-of-period government debt level. The demand shock is either set equal to zero (solid line) or to -3 unconditional standard deviations (dashed line). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

is decreasing in the public debt burden. Thus, if the beginning-of-period level of government debt is high, the fiscal policy response to a liquidity trap becomes considerably muted. At the same time, inflation and output are both increasing in the level of outstanding public

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debt. Monetary policy turns out to be crucial to understand this result. The higher the level of government debt incurred from the previous period the lower the nominal interest rate, as long as the zero lower bound is not binding. In other words, under the optimal policy mix, monetary policy bears part of the responsibility to stabilize government debt. Moreover, the real interest rate keeps decreasing in the level of outstanding debt even if the zero nominal interest rate bound is binding, reflecting the positive effect of government debt on expected future inflation. Intuitively, if monetary policy is unable to lower the current nominal interest rate further, because the zero bound is binding, future monetary policy will have to stabilize government debt, thereby stimulating future output and inflation. If, on the contrary, the level of outstanding government debt is low relative to the steady state, when a large negative demand shock hits the economy, then the policymaker may even refrain from lowering the nominal interest rate immediately all the way to zero. While the government spending stimulus in such states is particularly large, the decline in output and inflation is more pronounced than in states with higher levels of outstanding government debt.

4.3 Comparison to the case without government debt

In the previous parts, we have characterized the optimal time-consistent policy mix in the presence of government debt. We now compare this optimal policy mix to the one in an economy where government debt is zero, distortionary taxes are zero as well, and lump-sum taxes are free to adjust each period to balance the budget. Figures 4 and 5 show impulse responses to a negative demand shock for the two economies.[4] The policymaker in the model without government debt engages in a more pronounced fiscal stimulus and implements a lower nominal interest rate path than his counterpart in the economy with government debt. Nevertheless, the drop in output and inflation turns out to be considerably larger. The comparison shows how powerful history dependence is in affecting stabilization outcomes by shaping private sector expectations. In the economy without government debt, agents anticipate that the policymaker will never allow inflation to rise above target, which is reflected in the corresponding confidence intervals. In contrast, agents in the model with government debt anticipate that *if* current conditions got worse so that government debt increased, *future* monetary policy would respond to today's economic conditions by becoming more accommodative than would be warranted by future inflation and output gap dynamics alone. Hence, they attach positive probabilities to above-target future output and inflation, and the expected real interest rate path lies below its counterpart in the model without

---

[4]For the comparison we set $\sigma = 3^{-1}/(1 - G/Y)$ and $\omega = 3^{-1}/(G/Y)$, since the model without government debt cannot be solved for the baseline calibration. The effect of this change in parameter values is addressed in the sensitivity analysis.

Figure 4: Impulse responses with and without government debt
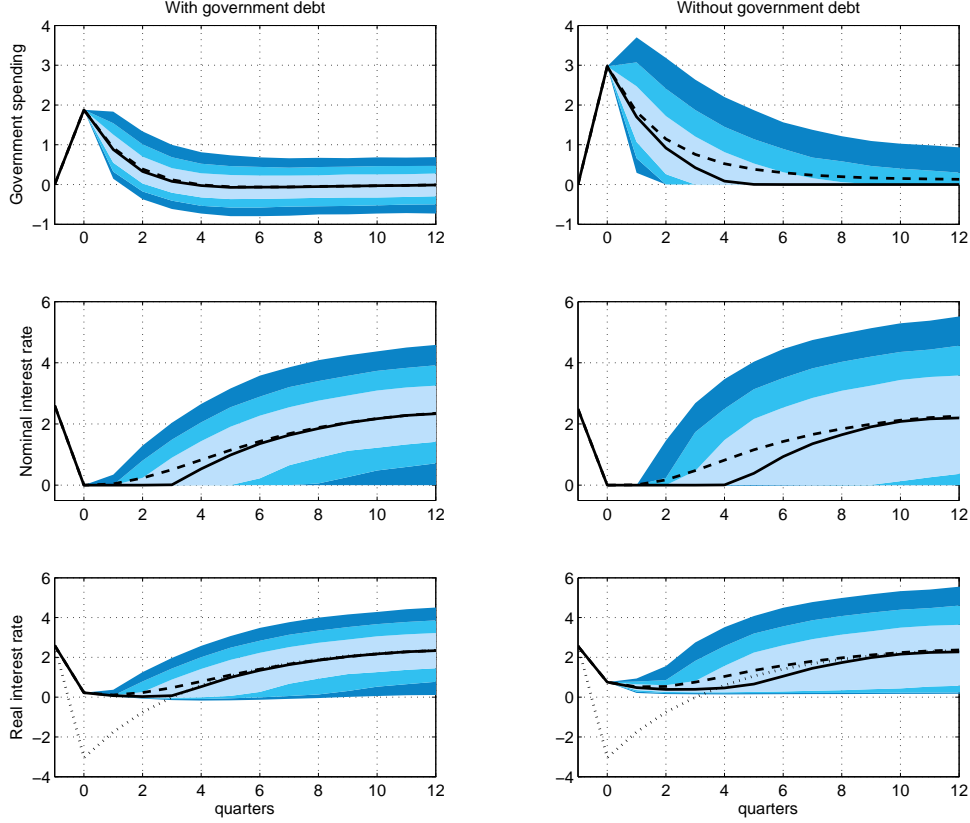
Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the economy with positive government debt (left) and for an economy with zero government debt (right). Except for $\sigma = 3^{-1}/(1 - G/Y)$ and $\omega = 3^{-1}/(G/Y)$, the baseline calibration is used. Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

government debt.

4.4   Sensitivity analysis

In this section, we investigate the sensitivity of results to preference parameters, the degree of price stickiness and the choice of the steady state debt-to-output ratio.

First, we consider how the intertemporal elasticity of substitution in private and public spending affects stabilization policies and outcomes in a liquidity trap. Figure 6 shows impulse responses for the baseline calibration and for the case of somewhat higher intertemporal elasticities, $\sigma = 3^{-1}/(1 - G/Y)$ and $\omega = 3^{-1}/(G/Y)$. The change in σ increases the interest elasticity of aggregate demand, rendering monetary policy more effective in stabilizing output

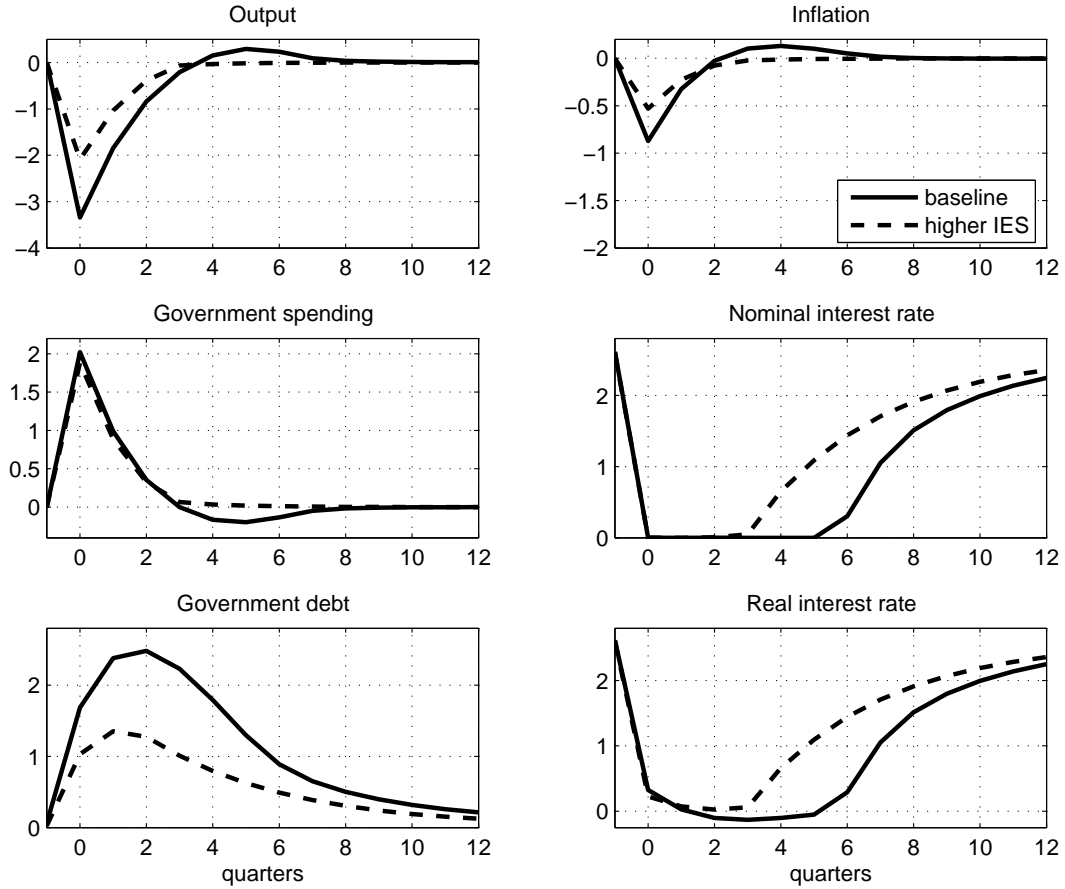Figure 5: Impulse responses with and without government debt
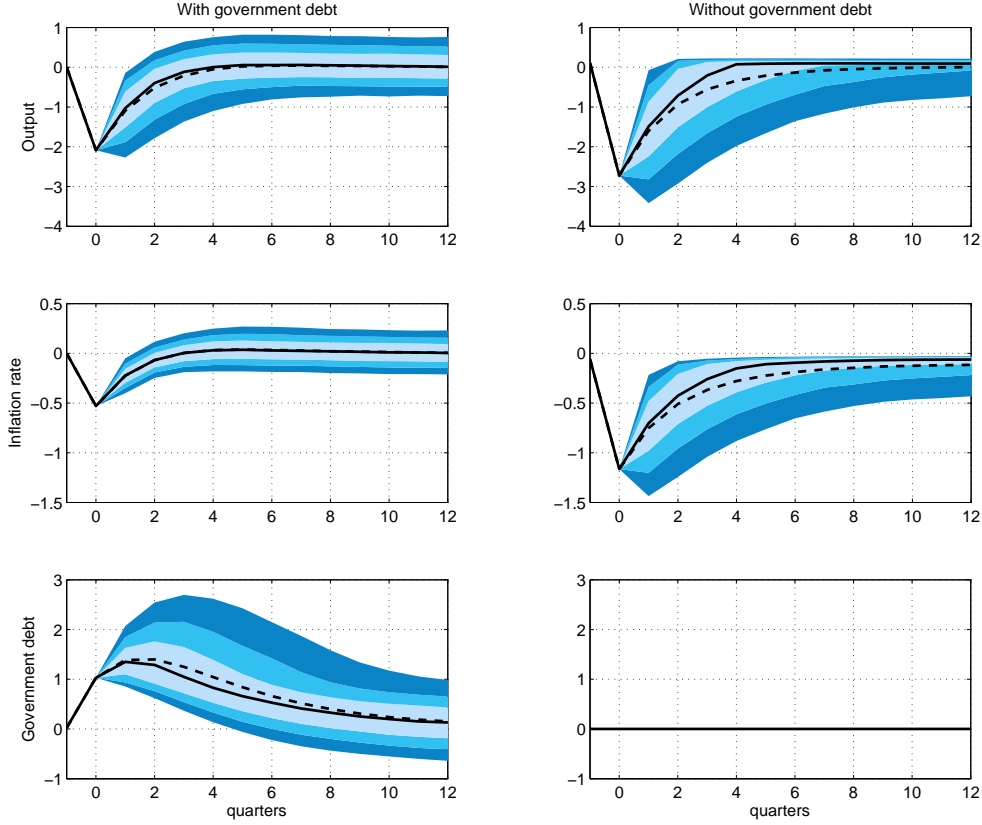
Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the economy with positive government debt (left) and for an economy with zero government debt (right). Except for $\sigma = 3^{-1}/(1 - G/Y)$ and $\omega = 3^{-1}/(G/Y)$, the baseline calibration is used. Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

and inflation than under the baseline calibration. Consequently, we observe smaller declines of output and inflation. Government debt increases by less so that no fiscal retrenchment is necessary to stabilize the economy, and output and inflation do not overshoot their target levels.

Figure 7 shows impulse responses for two alternative degrees of price stickiness, the baseline case with a Calvo parameter of $\alpha = 0.66$ (solid line) and an alternative case with more price rigidities, $\alpha = 0.75$ (dashed line). The policy responses for the two calibrations are very similar. In case of a higher degree of price stickiness, however, inflation is less responsive to variations in current real activity so that the initial drop in the inflation rate as well as the subsequent boom are more muted than under the baseline calibration.

Finally, Figure 8 compares impulse responses for the baseline calibration to the case of a
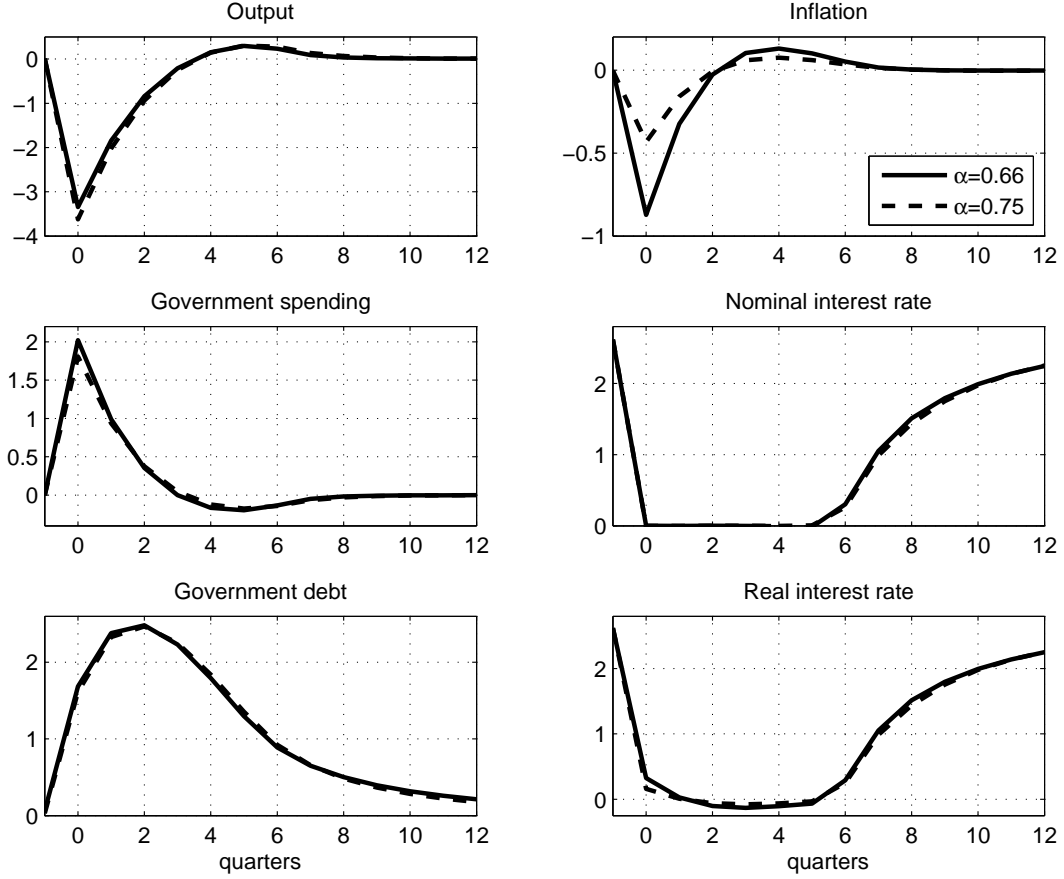
Figure 6: Impulse responses for alternative intertemporal elasticities of substitution

Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the baseline calibration (solid line) and in case of higher intertemporal elasticities of substitution (dashed line). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

lower (30%) and a higher (65%) steady state government debt-to-output ratio. The higher the steady state debt ratio, the more leverage do changes in the nominal interest rate have over the government's interest rate payments. Hence, in case of the low steady state debt ratio, monetary policy is less effective in stabilizing government debt and therefore keeps nominal interest rates low for longer than in the baseline case. At the same time, fiscal policy is less expansionary and engages in a stronger subsequent retrenchment in order to stabilize government debt and to mitigate the boom in future output and inflation. Nevertheless, output and inflation decline less on impact than in the baseline case. Conversely, in the case of the high steady state government debt-to-output ratio the policymaker implements a bigger spending stimulus but keeps monetary policy less accommodative, resulting in somewhat larger drops in output and inflation.

Figure 7: Impulse responses for alternative degrees of price stickiness
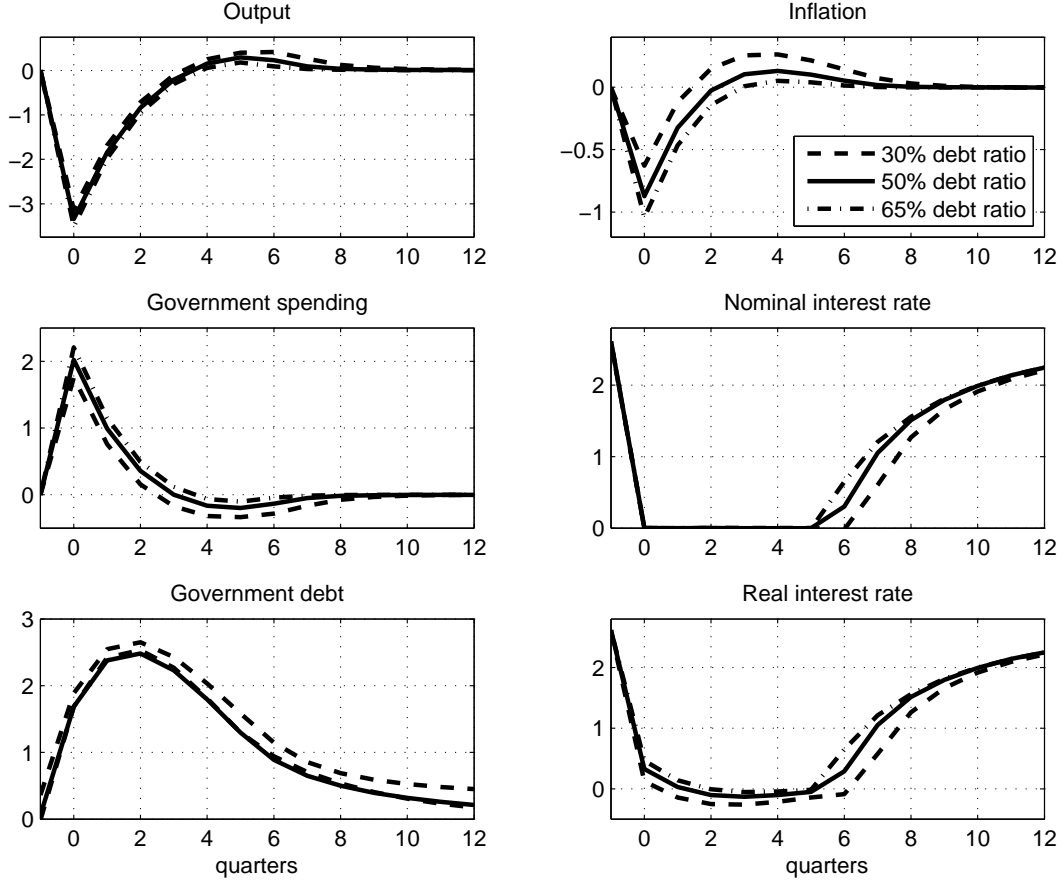

Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the baseline calibration (solid line) and for a higher degree of price stickiness of $\alpha = 0.75$ (dashed line). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

4.5 Labor income tax as additional policy instrument

In this section, we relax the assumption of a constant labor income tax rate, endowing the policymaker with an additional fiscal instrument. The modified Bellman equation then reads as follows:

$$V(s_t) = \min_{\{\hat{\pi}_t, \hat{Y}_t, \hat{G}_t, i_t, \hat{b}_t, \hat{\tau}_t^N\}} \left(\frac{1}{2} \left(\hat{\pi}_t^2 + \lambda \left(\hat{Y}_t - \Gamma \hat{G}_t \right)^2 + \lambda_G \hat{G}_t^2 \right) + \beta \mathrm{E}_t V(s_{t+1}) \right)$$

Figure 8: Impulse responses for alternative steady state government debt ratios
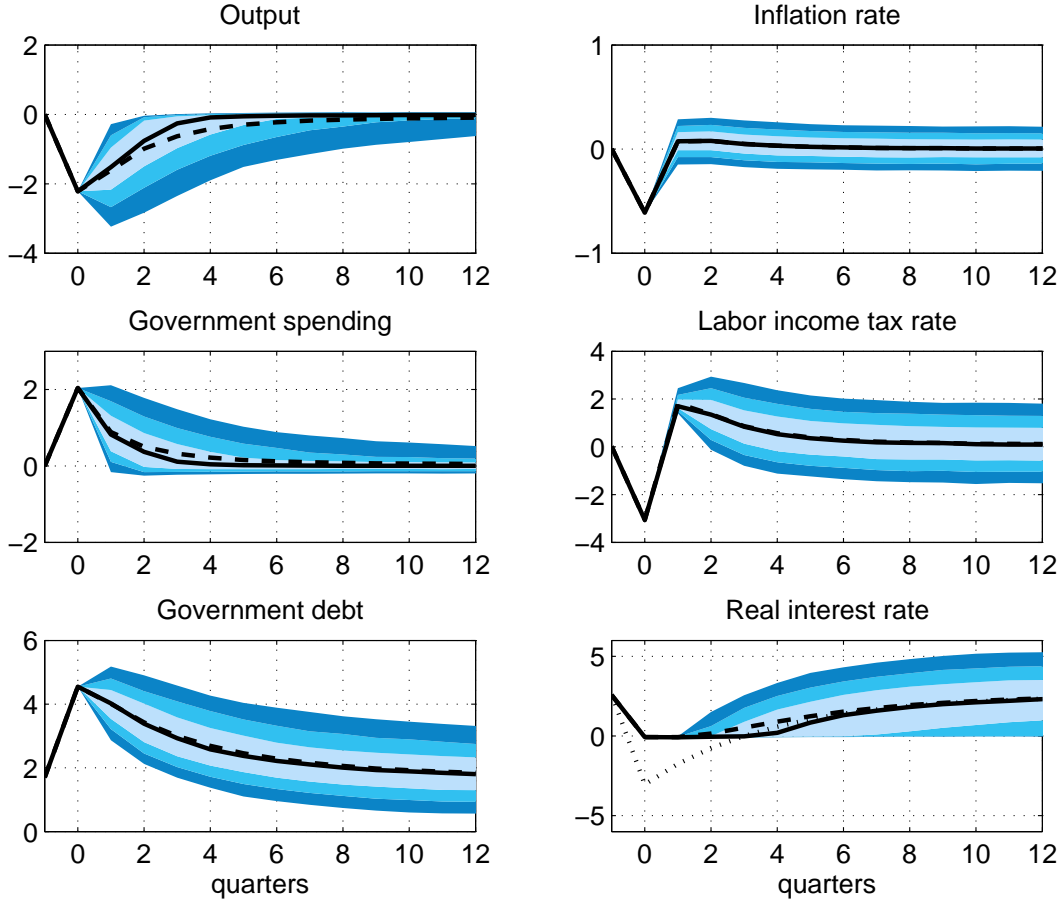

Notes: Impulse responses to a -3 unconditional standard deviation demand shock in case of steady state debt-to-output ratios of 30% (dashed lines), 50% (solid lines) and 65% (dashed-dotted lines). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output.

subject to

$$\hat{\pi}_t = \beta E_t \hat{\pi}(s_{t+1}) + \kappa \left(\hat{Y}_t - \Gamma \hat{G}_t + \frac{(\sigma + \eta)^{-1}}{1 - \tau^N} \hat{\tau}_t^N \right)$$

$$\hat{Y}_t = \hat{G}_t + E_t \hat{Y}(s_{t+1}) - E_t \hat{G}(s_{t+1}) - \frac{1}{\sigma} \left(i_t - E_t \hat{\pi}(s_{t+1}) - r^* \right) + d_t$$

$$\hat{b}_t = \frac{1}{\beta} \left(\hat{b}_{t-1} - \frac{b}{Y} \hat{\pi}_t + \left(1 + \tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N \sigma \right) \hat{G}_t - \left(\tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N (1 + \sigma + \eta) \right) \hat{Y}_t$$

$$- \frac{1 + \tau^C}{(1 - \tau^N)^2} \hat{\tau}_t^N \right) + \frac{b}{Y} (i_t - r^*)$$

$$i_t \geq 0,$$

and the law of motion for the demand shock (7), where $\hat{\tau}_t^N = \tau_t^N - \tau^N$. All parameters are defined as before. The first-order optimality conditions are provided in the Appendix. Figure 9 displays impulse responses to a negative demand shock that drives the economy into a liquidity trap[5]. Upon occurrence of the shock, the labor income tax rate is initially

Figure 9: Impulse responses - variable labor income tax rate

Notes: Impulse responses to a -3 unconditional standard deviation demand shock for the baseline calibration. Realized path (solid line), path expected in period 0 (dashed line), 50%, 75% and 90% confidence intervals (shaded areas). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output. Labor income tax rate is expressed as percentage point deviation from its steady state level.

lowered considerably and then raised above its steady state level in subsequent periods, before gradually returning to its long-run level. The responses of the real interest rate and government spending are similar to those in the baseline setup. Government debt increases, starting however from a higher stochastic steady state than in the baseline model. While

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[5]We again set $\sigma = 3^{-1}/(1 - G/Y)$ and $\omega = 3^{-1}/(G/Y)$, since the model with variable labor income tax rate could not be solved for the baseline calibration.

agents continue to attach a positive probability to positive future inflation rates, they now do not attach much weight on the possibility of future output being above target.

To understand how the additional policy instrument affects the optimal policy mix and stabilization outcomes, Figure 10 displays equilibrium responses to the beginning-of-period government debt level. We again focus on two alternative realizations of the demand shock, $d = 0$ (solid line) and $d = -3$ unconditional standard deviations (dashed line). There are

Figure 10: Equilibrium responses to previous period's government debt - variable labor income tax rate
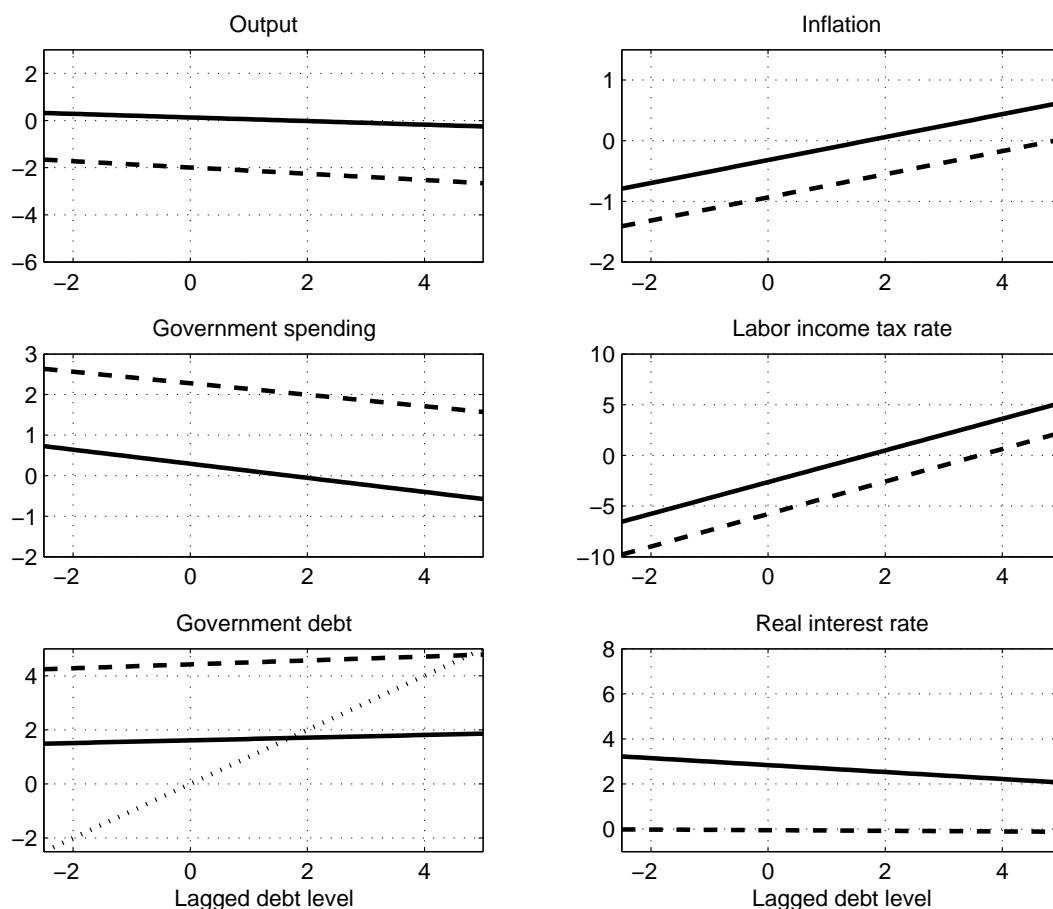
Notes: Equilibrium responses to the beginning-of-period government debt level. The demand shock is either set equal to zero (solid line) or to -3 unconditional standard deviations (dashed line). Inflation and interest rates are expressed in annualized terms. Government debt is expressed as a share of annualized steady state output, government spending is expressed as a share of steady state output. Labor income tax rate is expressed as percentage point deviation from its steady state level.

several important differences to the case with constant tax rates. First, the equilibrium responses of the end-of-period government debt level to beginning-of-period debt are almost flat. In addition, also government spending and the interest rate vary much less with out-

standing government debt than in the baseline setup. Instead, the labor income tax rate now becomes strongly increasing in the beginning-of-period debt level. Essentially, the policymaker uses the labor income tax rate to implement the desired level of government debt. For a given beginning-of-period government debt level, the optimal amount of debt is higher when there is a negative demand shock, and hence the optimal tax rate is lower. Since the labor tax rate affects marginal costs, the inflation rate is also increasing in the government debt level. In contrast, output is now slightly decreasing in the debt level, and remains always close to its target level as long as the economy is not in a liquidity trap. Since the policymaker can always use the labor income tax rate to achieve the desired government debt level, there is now a much weaker link between the current debt level and future monetary policy. Consequently, in case of the large negative demand shock the equilibrium response of the real interest rate is essentially flat. In this respect, the use of the labor income tax rate as an additional policy instrument reduces the amount of history dependence in monetary policy.

5   Conclusion

How does the need to preserve government debt sustainability affect the optimal, time-consistent monetary and fiscal policy response to a liquidity trap? We address this question using a small, stochastic New Keynesian model with a zero bound on nominal interest rates and focusing on two policy instruments, the short-term nominal interest rate and government spending financed by non-state-contingent, nominal government bonds. Under the optimal time-consistent policy mix, government spending is a decreasing function of the level of outstanding government debt. Whereas in models with freely adjusting lump-sum taxes it is optimal for a discretionary policymaker to raise government spending in a liquidity trap, in our model a high government debt level might force the policymaker to lower government spending despite a binding zero bound. At the same time, the monetary policy stance becomes more expansionary the higher the level of government debt. Crucially, the real interest rate keeps declining as a function of government debt when the nominal interest rate is constrained by the zero bound. Hence, the lack of fiscal stimulus in a liquidity trap characterized by a high government debt burden is compensated by a more accommodative future nominal interest rate policy.

Appendix

A Numerical algorithm

Let $Z = \begin{bmatrix} \hat{\pi} & \hat{Y} & \hat{G} & i & \Phi^b \end{bmatrix}'$ and $\tilde{Z} = \begin{bmatrix} \hat{\pi} & \hat{Y} & \hat{G} & \Phi^b \end{bmatrix}'$. We approximate Z by a linear combination of n basis functions ψ_j, $j = 1, ..., n$. In matrix notation

$$Z\left(d, \hat{b}_{-1}\right) \approx C\Psi\left(d, \hat{b}_{-1}\right), \tag{A.1}$$

where

$$C = \begin{pmatrix} c_1^\pi & \cdots & c_n^\pi \\ c_1^Y & \cdots & c_n^Y \\ c_1^G & \cdots & c_n^G \\ c_1^i & \cdots & c_n^i \\ c_1^\Phi & \cdots & c_n^\Phi \end{pmatrix}, \qquad \Psi\left(d, \hat{b}_{-1}\right) = \begin{pmatrix} \psi_1\left(d, \hat{b}_{-1}\right) \\ \vdots \\ \psi_n\left(d, \hat{b}_{-1}\right) \end{pmatrix}.$$

The coefficients c_j^h, $j = 1, 2, ..., n$; $h \in \{\pi, Y, G, i, \Phi\}$, are set such that (A.1) holds exactly at n selected collocation nodes

$$Z\left(X_{(k,:)}\right) = C\Psi\left(X_{(k,:)}\right), \tag{A.2}$$

for $k = 1, ..., n$, where

$$X = \begin{bmatrix} \left(\iota_b \otimes \bar{d}\right)' \\ \left(\bar{b}_{-1} \otimes \iota_d\right)' \end{bmatrix}'$$

is a $n \times 2$ matrix, and $X_{(k,:)}$ refers to the elements in row k of matrix X. ι_p is a column vector of ones with length n_p, $p \in \{d, b\}$. The column vectors $\bar{d}$ and $\bar{b}_{-1}$ contain the grid points of the demand shock and the lagged level of government debt, respectively. The vectors have length n_p. It holds $n = n_d \cdot n_b$.

The iterative solution algorithm is based on two nested loops: one outer loop (counter s_1) targeted at the convergence of the derivatives of the expectations functions and an inner loop (counter s_2) seeking convergence of policy function coefficients. The algorithm then works as follows:

1. At the initial iteration step, we start with a guess for the coefficient matrix $C^{(0,0)}$ and for the partial derivatives of the expectation functions $\frac{\partial E\hat{\pi}}{\partial \hat{b}}^{(0)}$, $\frac{\partial E\hat{Y}}{\partial \hat{b}}^{(0)}$, $\frac{\partial E\hat{G}}{\partial \hat{b}}^{(0)}$.

2. At iteration step s_1 of the **outer loop**, we proceed as follows.

   (a) At iteration step s_2 of the **inner loop**, we use the guess $C^{(s_1, s_2)}$ to determine the

level of government debt $\hat{b}$ at the n collocation nodes. Using the budget constraint:

$$
\begin{aligned}
\hat{b}^{(s_1,s_2)}\left(X_{(k,:)}\right) &= \frac{1}{\beta}\Bigg(X_{(k,2)} - \frac{b}{Y} C^{(s_1,s_2)}_{(1,:)} \Psi\left(X_{(k,:)}\right) \\
&\quad + \left(1 + \tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N \sigma\right) C^{(s_1,s_2)}_{(3,:)} \Psi\left(X_{(k,:)}\right) \\
&\quad - \left(\tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N (1+\sigma+\eta)\right) C^{(s_1,s_2)}_{(2,:)} \Psi\left(X_{(k,:)}\right) \Bigg) \\
&\quad + \frac{b}{Y}\left(C^{(s_1,s_2)}_{(4,:)}\Psi\left(X_{(k,:)}\right) - r^*\right),
\end{aligned}
$$

for $k = 1, ..., n$.

(b) Next, we update the expectation functions:

$$
\mathrm{E}\hat{\pi}^{(s_1,s_2)}\left(X_{(k,:)}\right) = \sum_{l=1}^{m} \varpi_l C^{(s_1,s_2)}_{(1,:)} \Psi\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,s_2)}\left(X_{(k,:)}\right)\right)
$$

$$
\mathrm{E}\hat{Y}^{(s_1,s_2)}\left(X_{(k,:)}\right) = \sum_{l=1}^{m} \varpi_l C^{(s_1,s_2)}_{(2,:)} \Psi\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,s_2)}\left(X_{(k,:)}\right)\right)
$$

$$
\mathrm{E}\hat{G}^{(s_1,s_2)}\left(X_{(k,:)}\right) = \sum_{l=1}^{m} \varpi_l C^{(s_1,s_2)}_{(3,:)} \Psi\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,s_2)}\left(X_{(k,:)}\right)\right)
$$

$$
\mathrm{E}\hat{\Phi}^{b(s_1,s_2)}\left(X_{(k,:)}\right) = \sum_{l=1}^{m} \varpi_l C^{(s_1,s_2)}_{(5,:)} \Psi\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,s_2)}\left(X_{(k,:)}\right)\right),
$$

for $k = 1, ..., n$. A Gaussian quadrature scheme is used to discretize the normally distributed random variable, where ϵ is a vector of quadrature nodes with length m and ϖ is a vector of length m containing the weights.

(c) Assuming first, that the zero bound is not binding at any collocation node, the optimality conditions for the discretionary policy regime imply

$$
\begin{aligned}
Z^{(s_1,s_2)}\left(X_{(k,:)}\right) &= \left(A^{(s_1)}\left(X_{(k,:)}\right)\right)^{-1} \cdot B + \left(A^{(s_1)}\left(X_{(k,:)}\right)\right)^{-1} \cdot F \cdot \mathrm{E}\tilde{Z}^{(s_1,s_2)}\left(X_{(k,:)}\right) \\
&\quad + \left(A^{(s_1)}\left(X_{(k,:)}\right)\right)^{-1} \cdot D \cdot X_{(k,1)},
\end{aligned}
$$

(A.3)

for $k = 1, ..., n$, where

$$A_k^{(s_1)} = \begin{pmatrix} 1 & -\kappa & \kappa\Gamma & 0 & 0 \\ 0 & 1 & -1 & 1/\sigma & 0 \\ \kappa & \lambda & -\lambda\Gamma & 0 & a_{35} \\ 0 & 0 & 1 & 0 & a_{45} \\ -\Omega_{2k}^{(s_1)} & 0 & 0 & 0 & \Omega_{1k}^{(s_1)} \end{pmatrix},$$

where

$$a_{35} = -\left(\frac{\kappa}{\beta} + \sigma\right)\frac{b}{Y} - \frac{1}{\beta}\left(\tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N(1+\sigma+\eta)\right)$$

$$a_{45} = \frac{1}{\lambda_G}\left(\frac{1}{\beta} + (1-\Gamma)\frac{b}{Y}\sigma + \frac{1}{\beta}(1-\Gamma)\tau^C - \frac{1}{\beta}\Gamma\frac{1+\tau^C}{1-\tau^N}\tau^N\right)$$

$$B = \begin{pmatrix} 0 \\ r^*/\sigma \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad F = \begin{pmatrix} \beta & 0 & 0 & 0 \\ 1/\sigma & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

and

$$\Omega_{1k}^{(s_1)} \equiv 1 - \sigma\frac{b}{Y}\left(\frac{\partial E\hat{Y}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right) - \frac{\partial E\hat{G}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right)\right)$$

$$\Omega_{2k}^{(s_1)} \equiv \beta\frac{\partial E\hat{\pi}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right)$$

$$\Omega_{3k}^{(s_1)} \equiv \frac{\partial E\hat{Y}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right) - \frac{\partial E\hat{G}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right) + \frac{1}{\sigma}\frac{\partial E\hat{\pi}^{(s_1)}}{\partial\hat{b}}\left(X_{(k,:)}\right).$$

For those k for which the zero lower bound is violated, i.e. $i^{(s_1,s_2)}\left(X_{(k,:)}\right) < 0$, $A^{(s_1)}\left(X_{(k,:)}\right)$ in (A.3) is replaced by

$$\widehat{A_k^{(s_1)}} = \begin{pmatrix} 1 & -\kappa & \kappa\Gamma & 0 & 0 \\ 0 & 1 & -1 & 1/\sigma & 0 \\ (1-\Gamma)\kappa & (1-\Gamma)\lambda & (\lambda_G - (1-\Gamma)\lambda\Gamma) & 0 & \hat{a}_{35} \\ 0 & 0 & 0 & 1 & 0 \\ -\Omega_{2k}^{(s_1)} - \kappa\Omega_{3k}^{(s_1)} & -\lambda\Omega_{3k}^{(s_1)} & \lambda\Gamma\Omega_{3k}^{(s_1)} & 0 & \hat{a}_{55} \end{pmatrix},$$

where

$$\widehat{a}_{35} = \frac{1}{\beta} - (1-\Gamma)\frac{\kappa}{\beta}\frac{b}{Y} - \frac{1}{\beta}(1+\eta)\frac{1+\tau^C}{1-\tau^N}\tau^N$$

$$\widehat{a}_{55} = \Omega_{1k}^{(s_1)} + \left(\frac{b}{Y}\left(\frac{\kappa}{\beta}+\sigma\right) + \frac{1}{\beta}\left(\tau^C + \frac{1+\tau^C}{1-\tau^N}\tau^N(1+\sigma+\eta)\right)\right)\Omega_{3k}^{(s_1)}.$$

(d) Let

$$\bar{C}^{(s_1,s_2+1)} = \left[Z^{(s_1,s_2)}\left(X_{(1,:)}\right) \quad \cdots \quad Z^{(s_1,s_2)}\left(X_{(n,:)}\right)\right]\Psi(X)^{-1},$$

where the element in the vth row and wth column of the $n \times n$ matrix $\Psi(X)$ equals $\psi_v\left(X_{(w,:)}\right)$. For given s_1, we then update

$$C^{(s_1,s_2+1)} = \zeta_2\bar{C}^{(s_1,s_2+1)} + (1-\zeta_2)C^{(s_1,s_2)},$$

where $\zeta_2 \in (0,1]$, and continue iterating on the inner loop until

$$\left\|vec\left(\bar{C}^{(s_1,s_2+1)} - C^{(s_1,s_2)}\right)\right\|_\infty < \delta.$$

After convergence of the inner loop, we update the derivatives of the expectations functions with respect to $\hat{b}$. Let

$$\overline{\frac{\partial \mathrm{E}\hat{\pi}}{\partial\hat{b}}}^{(s_1+1)}\left(X_{(k,:)}\right) \equiv \sum_{l=1}^m \varpi_l C_{(1,:)}^{(s_1,\bar{s}_2)}\Psi_b\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,\bar{s}_2)}\left(X_{(k,:)}\right)\right)$$

$$\overline{\frac{\partial \mathrm{E}\hat{Y}}{\partial\hat{b}}}^{(s_1+1)}\left(X_{(k,:)}\right) \equiv \sum_{l=1}^m \varpi_l C_{(2,:)}^{(s_1,\bar{s}_2)}\Psi_b\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,\bar{s}_2)}\left(X_{(k,:)}\right)\right)$$

$$\overline{\frac{\partial \mathrm{E}\hat{G}}{\partial\hat{b}}}^{(s_1+1)}\left(X_{(k,:)}\right) \equiv \sum_{l=1}^m \varpi_l C_{(3,:)}^{(s_1,\bar{s}_2)}\Psi_b\left(\rho X_{(k,1)} + \epsilon_{(l)}, \hat{b}^{(s_1,\bar{s}_2)}\left(X_{(k,:)}\right)\right),$$

where $\Psi_b(\cdots)$ represents the first derivative of the basis functions with respect to the second argument $\hat{b}$, and $\bar{s}_2$ represents the last iteration step in the inner loop before convergence. The guess for the partial derivatives of the expectations functions is then

updated as follows

$$\frac{\partial \mathrm{E}\hat{\pi}^{(s_1+1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) = \zeta_1 \overline{\frac{\partial \mathrm{E}\hat{\pi}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) + (1-\zeta_1)\frac{\partial \mathrm{E}\hat{\pi}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right)$$

$$\frac{\partial \mathrm{E}\hat{Y}^{(s_1+1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) = \zeta_1 \overline{\frac{\partial \mathrm{E}\hat{Y}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) + (1-\zeta_1)\frac{\partial \mathrm{E}\hat{Y}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right)$$

$$\frac{\partial \mathrm{E}\hat{G}^{(s_1+1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) = \zeta_1 \overline{\frac{\partial \mathrm{E}\hat{G}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) + (1-\zeta_1)\frac{\partial \mathrm{E}\hat{G}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right),$$

for $k = 1, ..., n$, with updating parameter $\zeta_1 \in (0, 1]$. Finally, we set $C^{(s_1+1,0)} = C^{(s_1, \bar{s}_2)}$.

3. The algorithm ends when:

$$\left\| vec \left(\begin{array}{c} \overline{\frac{\partial \mathrm{E}\hat{\pi}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) - \frac{\partial \mathrm{E}\hat{\pi}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) \\ \overline{\frac{\partial \mathrm{E}\hat{Y}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) - \frac{\partial \mathrm{E}\hat{Y}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) \\ \overline{\frac{\partial \mathrm{E}\hat{G}^{(s_1+1)}}{\partial \hat{b}}}\left(X_{(k,:)}\right) - \frac{\partial \mathrm{E}\hat{G}^{(s_1)}}{\partial \hat{b}}\left(X_{(k,:)}\right) \end{array} \right) \right\|_{\infty} < \delta,$$

for some $\delta > 0$.

The collocation nodes are distributed with a support covering $\pm$ 4 unconditional standard deviations of the exogenous state variable and the realizations of the endogenous state variable when simulating the model. We use MATLAB routines from the CompEcon toolbox of Miranda and Fackler (2002) to obtain the Gaussian quadrature approximation of the innovations to the demand shock, and to evaluate the spline functions and their first-order derivatives.

B Optimal time-consistent policy with a variable labor income tax rate

The Bellman equation reads:

$$V(s_t) = \min_{\{\hat{\pi}_t, \hat{Y}_t, \hat{G}_t, i_t, \hat{b}_t, \hat{\tau}_t^N\}} \left(\frac{1}{2}\left(\hat{\pi}_t^2 + \lambda \left(\hat{Y}_t - \Gamma \hat{G}_t \right)^2 + \lambda_G \hat{G}_t^2 \right) + \beta \mathrm{E}_t V(s_{t+1}) \right)$$

subject to

$$\hat{\pi}_t = \beta E_t \hat{\pi}(s_{t+1}) + \kappa \left(\hat{Y}_t - \Gamma \hat{G}_t + \frac{(\sigma + \eta)^{-1}}{1 - \tau^N} \hat{\tau}_t^N \right)$$

$$\hat{Y}_t = \hat{G}_t + E_t \hat{Y}(s_{t+1}) - E_t \hat{G}(s_{t+1}) - \frac{1}{\sigma} (i_t - E_t \hat{\pi}(s_{t+1}) - r^*) + d_t$$

$$\hat{b}_t = \frac{1}{\beta} \left(\hat{b}_{t-1} - \frac{b}{Y} \hat{\pi}_t + \left(1 + \tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N \sigma \right) \hat{G}_t - \left(\tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N (1 + \sigma + \eta) \right) \hat{Y}_t \right.$$

$$\left. - \frac{1 + \tau^C}{(1 - \tau^N)^2} \hat{\tau}_t^N \right) + \frac{b}{Y} (i_t - r^*)$$

$$i_t \geq 0,$$

and the law of motion for the demand shock (7).

The consolidated first-order conditions read

$$\left\{ (1 - \Gamma) \kappa + \left[1 - \frac{\tau^N}{1 - \tau^N} (1 + \tau^C) (1 + \eta) - (1 - \Gamma) \kappa \frac{b}{Y} \right] \left(\frac{b}{Y} + \frac{1 + \tau^C}{1 - \tau^N} \frac{\sigma + \eta}{\kappa} \right)^{-1} \right\} \hat{\pi}_t$$

$$+ (1 - \Gamma) \lambda \hat{Y}_t + (\lambda_G - (1 - \Gamma) \lambda \Gamma) \hat{G}_t = 0$$

$$E_t \hat{\pi}(s_{t+1}) + \left(\left(\frac{b}{Y} + \frac{1 + \tau^C}{1 - \tau^N} \frac{\sigma + \eta}{k} \right) \frac{1}{\beta} \Omega_{2t} - \Omega_{1t} \right) \hat{\pi}_t - \Omega_{3t} \Phi_t^{\text{zlb}} = 0$$

$$\Phi_t^{\text{zlb}} i_t = 0$$

$$\Phi_t^{\text{zlb}} \geq 0$$

$$i_t \geq 0,$$

as well as the New Keynesian Phillips curve, the dynamic IS curve and the government budget constraint, where

$$\Phi_t^{\text{zlb}} \equiv \left(\frac{b}{Y} \left(\sigma + \frac{\kappa}{\beta} \right) + \frac{1}{\beta} \left(\tau^C + \frac{1 + \tau^C}{1 - \tau^N} \tau^N (1 + \sigma + \eta) \right) \right) \hat{\pi}_t$$

$$- \frac{1}{\beta} \left(\frac{b}{Y} + \frac{1 + \tau^C}{1 - \tau^N} \frac{\sigma + \eta}{k} \right) \left(\kappa \hat{\pi}_t + \lambda \left(\hat{Y}_t - \Gamma \hat{G}_t \right) \right)$$

$$\Omega_{1t} \equiv 1 - \sigma \frac{b}{Y} \left(\frac{\partial E_t \hat{Y}(s_{t+1})}{\partial \hat{b}_t} - \frac{\partial E_t \hat{G}(s_{t+1})}{\partial \hat{b}_t} \right)$$

$$\Omega_{2t} \equiv \beta \frac{\partial E_t \hat{\pi}(s_{t+1})}{\partial \hat{b}_t}$$

$$\Omega_{3t} \equiv \frac{\partial E_t \hat{Y}(s_{t+1})}{\partial \hat{b}_t} - \frac{\partial E_t \hat{G}(s_{t+1})}{\partial \hat{b}_t} + \frac{1}{\sigma} \frac{\partial E_t \hat{\pi}(s_{t+1})}{\partial \hat{b}_t}.$$

The variable Φ_t^{zlb} is the (normalized) multiplier associated with the zero lower bound constraint.

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