

Bound states of anti-nucleons in finite nuclei

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Abstract

We study the bound states of anti-nucleons emerging from the lower continuum in finite nuclei within the relativistic Hartree approach including the contributions of the Dirac sea to the source terms of the meson fields. The Dirac equation is reduced to two Schrödinger-equivalent equations for the nucleon and the anti-nucleon respectively. These two equations are solved simultaneously in an iteration procedure. Numerical results show that the bound levels of anti-nucleons vary drastically when the vacuum contributions are taken into account.

PACS number(s): 21.10.-k; 21.60.-n; 03.65.Pm

In spite of the great successes of the relativistic mean field (RMF) theory [1, 2, 3, 4] and the relativistic Hartree approach (RHA) [5, 6] in describing the ground states of nuclei, the arguments of introduction of strong Lorentz scalar (S) and time-component Lorentz vector (V) potential in the Dirac equation are largely indirect. So far, no evidence from experiments ensures the physical necessity. One usually compares the theoretical predictions only with the experimental data of the nucleon sector (i.e., the shell-model states), which is subject to a relatively small quantity stemming from the cancellation of two potentials $S + V$ (V is positive, S is negative.). While the dynamical content of the Dirac picture certainly lies with both the nucleon and the anti-nucleon sector, the study of the anti-nucleon sector enjoys an additional bonus: it provides us with a chance to determine the individual S and V ! Due to the G -parity, the vector potential changes its sign in the anti-nucleon sector. The bound states of anti-nucleons are sensitive to the sum of the scalar and vector field $S - V$. Combining with the information from the nucleon sector, one may fix the individual values of the scalar and vector field.

The study of the anti-nucleon bound states is extremely interesting for modern nuclear physics. If the potential of anti-nucleons is much weaker than what one expects or predicts by means of the RMF/RHA models, that is, the strong scalar and vector field are not necessary, one may question the validity of the models since some important physical ingredients, such as quantum corrections, correlation effects, three-body forces et al., are still missing in these phenomenological approaches. One may think about constituting a more elaborate model. Alternatively, if a deep potential of anti-nucleons is indeed observed, that is, the strong scalar and vector potential are realistic, an interesting phenomenon is that at certain density the energy of anti-nucleons may turn out to be larger than the free nucleon mass, the system becomes unstable with respect to the nucleon-anti-nucleon pair creation [7]. On the other hand, as pointed out in Ref. [8], in high-energy relativistic heavy-ion collisions, the nucleons may be emitted from the deep bound states emerging from the Dirac sea due to dynamics. These can create a great number of anti-nucleons in bound states. Such collective creation processes of anti-matter clusters have a large probability for the production of anti-nuclei, – and analogously also for multi- Λ , multi- $\bar{\Lambda}$ nuclei. These open two fascinating directions to extend the periodic system, i.e., to extend into the anti-nucleon sector and into the multi-strangeness dimension, in addition to the islands of super-heavy nuclei. In order to reach the quantitative study of the above theoretical conjecture, a prerequisite is to know the exact potential depth of the bound states of anti-nucleons. Up to now, no answers from experimental side or theoretical side are available.

This is the aim of the present work. We study the problem within the relativistic Hartree approach including the vacuum contributions. The starting point is the following effective Lagrangian for nucleons interacting through the exchange of mesons [1, 2, 3]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[i\gamma_{\mu}\partial^{\mu} - M_N]\psi + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\ & + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_{\rho}^2\mathbf{R}_{\mu}\cdot\mathbf{R}^{\mu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} \end{aligned}$$

$$+g_\sigma\bar{\psi}\psi\sigma - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \frac{1}{2}g_\rho\bar{\psi}\gamma_\mu\boldsymbol{\tau}\cdot\psi\mathbf{R}^\mu - \frac{1}{2}e\bar{\psi}(1+\tau_0)\gamma_\mu\psi A^\mu, \quad (1)$$

here $U(\sigma)$ is the self-interaction part of the scalar field [9]

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3!}b\sigma^3 + \frac{1}{4!}c\sigma^4. \quad (2)$$

Based on this Lagrangian, we have developed a relativistic model describing the bound states of both nucleons and anti-nucleons in finite nuclei [10]. Instead of directly searching for two solutions of the Dirac equation in a finite many-body system, we reduce the Dirac equation to two Schrödinger-equivalent equations for the nucleon and the anti-nucleon respectively. These two equations can be solved simultaneously with the numerical technique of the relativistic mean-field theory for finite nuclei. The contributions of the Dirac sea to the source terms of the meson fields are evaluated by means of the derivative expansion [11] up to the leading derivative order for the one-meson loop and one-nucleon loop. Thus, the wave functions of anti-nucleons, which are used to calculate the single-particle energies, are not involved in evaluating the vacuum contributions to the scalar and baryon density which are, in turn, expressed by means of the scalar and vector field as well as their derivative terms [10]. The Schrödinger-equivalent equation of the nucleon and the equations of motion of mesons (containing the densities contributed from the vacuum) are solved within a self-consistent iteration procedure [2]. Then, the equation of the anti-nucleon is solved with the known mean fields to obtain the wave functions and the single-particle energies of anti-nucleons. The space of anti-nucleons are truncated by the specified principal and angular quantum numbers n and j with the guarantee that the calculated single-particle energies of anti-nucleons are converged when the truncated space is extended. We find that the results are insensitive to the exact values of n and j provided large enough numbers are given. We have used $n = 4$, $j = 9$ for ^{16}O ; $n = 5$, $j = 11$ for ^{40}Ca ; and $n = 9$, $j = 19$ for ^{208}Pb .

In the previous RHA calculations for the bound states of nucleons [5, 6], the parameters of the model are fitted to the saturation properties of nuclear matter as well as the *rms* charge radius in ^{40}Ca . The *best-fit* routine within the RHA to the properties of spherical nuclei has not been performed yet. Thus, we first fit the parameters of Eqs. (1) and (2) within the RHA to the empirical data of binding energy, surface thickness and diffraction radius of eight spherical nuclei ^{16}O , ^{40}Ca , ^{48}Ca , ^{58}Ni , ^{90}Zr , ^{116}Sn , ^{124}Sn , and ^{208}Pb as has been done in Ref. [2] for the RMF model. We distinguish two different cases with (RHA1) and without (RHA0) nonlinear self-interaction of the scalar field. The obtained parameters and the corresponding saturation properties are given in Table I. For the sake of comparison, two sets of the linear (LIN) and nonlinear (NL1) RMF parameters from Ref. [2] are also presented. One can see that the RHA gives a larger effective nucleon mass than the RMF does, which is mainly caused by the feedback of the vacuum to the meson fields, as can be seen from Eqs. (71) ~ (74) of Ref. [10]. When the effective nucleon mass decreases, the scalar density originated from the Dirac sea ρ_S^{sea} increases. It is negative and cancels part of the scalar density contributed from the valence nucleons ρ_S^{val} , which causes the effective nucleon mass to increase again. At the end, it reaches a

balance value. In the fitting procedure, we have tried different initial values giving smaller effective nucleon mass. After running the code many times, all of them slowly converge to a large m^* . The larger effective nucleon mass explains why a larger χ^2 value is obtained for the RHA1 compared to the NL1. If one uses the current nonlinear RMF/RHA models to fit the ground-state properties of spherical nuclei, an effective nucleon mass around 0.6 is preferred. The situation, however, might be changed when other physical ingredients, e.g., tensor couplings, correlation effects, three-body forces, are taken into account, which warrants further investigation. On the other hand, in the case of linear model, the RHA0 gives a better fit than the LIN does. This is mainly due to the vacuum contributions which improve the theoretical results of the surface thickness substantially, and finally improve the total χ^2 value. An interesting quantity is the shell fluctuation which can be best expressed via the charge density in ^{208}Pb as

$$\delta\rho = \rho_C(1.8 \text{ fm}) - \rho_C(0.0 \text{ fm}). \quad (3)$$

The empirical value is -0.0023 fm^{-3} [2], which is nicely reproduced in the RHA (see Table I) while the RMF overestimates $\delta\rho$ by a factor of 3, sharing the same disease with the non-relativistic mean field theory [12].

In Table II we present the results of both the proton and the anti-proton spectra of ^{16}O , ^{40}Ca and ^{208}Pb . The binding energy per nucleon and the *rms* charge radius are given too. The numerical calculations are performed within two frameworks, i.e., the RHA including the contributions of the Dirac sea to the source terms of the meson fields and the RMF taking into account only the valence nucleons as the meson-field sources. The experimental data are taken from Ref. [13]. From the table one can see that all four sets of parameters can reproduce the empirical values of the binding energies, the *rms* charge radii and the single-particle energies of protons fairly well. For the E/A and the r_{ch} , the agreement between the theoretical predictions and the experimental data are improved from the LIN to the RHA0, RHA1 and NL1 set of parameters. For the spectra of protons, due to large error bars, it seems to be difficult to queue up the different sets of parameters. However, because of the large effective nucleon mass, the RHA has a smaller spin-orbit splitting (see $1p_{1/2}$ and $1p_{3/2}$ state) compared to the RMF. This situation can be improved through introducing a tensor coupling for the ω meson [2] which will be investigated in the future studies. For the anti-nucleon sector, no experimental data are available. In all four cases, the potential of anti-protons is much deeper than the potential of protons. On the other hand, one can notice the drastic difference between the RHA and the RMF calculations – the single-particle energies of anti-protons calculated from the RHA are about half of that from the RMF, exhibiting the importance of taking into account the Dirac sea effects. It demonstrates that the anti-nucleon spectra deserve a sensitive probe to the effective interactions. The spin-orbit splitting of the anti-nucleon sector is so small that one nearly can not distinguish the $1\bar{p}_{1/2}$ and the $1\bar{p}_{3/2}$ state. This is because the spin-orbit potential is related to $d(S + V)/dr$ in the anti-nucleon sector and two fields cancel each other to a large extent. Nevertheless, the space between the $1\bar{s}$ and the $1\bar{p}$ state is still evident, especially for lighter nuclei. This might be helpful to separate the process of knocking out a $1\bar{s}_{1/2}$ nucleon from the background – a promising

way to measure the potential of the anti-nucleon in laboratory.

In summary, we have proposed to study the bound states of anti-nucleons in finite nuclei which will provide us with a chance to judge the physical necessity of introducing strong scalar and vector potential in the Dirac picture. Due to the feedback of the vacuum to the meson fields, the scalar and vector fields decrease in the RHA. Numerical calculations show that the single-particle energies of anti-nucleons change drastically in the RMF and the RHA approach for different sets of parameters, while the single-particle energies of nucleons remain in a reasonable range. It is very important to have experimental data to check the theoretical predicted bound levels of anti-nucleons. If the Dirac picture with the large potentials is valid for nucleon-nucleus interactions, a fascinating direction of future studies is to investigate the vacuum correlation and the collective production of anti-nuclei in relativistic heavy-ion collisions. Experimental efforts in this direction are presently underway [14].

The authors thank P.-G. Reinhard, Zhongzhou Ren, J. Schaffner-Bielich and C. Beckmann for stimulating discussions. G. Mao gratefully acknowledges the Alexander von Humboldt-Stiftung for financial support and the people at the Institut für Theoretische Physik der J. W. Goethe Universität for their hospitality. This work was supported by DFG-Graduiertenkolleg Theoretische und Experimentelle Schwerionenphysik, GSI, BMBF, DFG and A.v.Humboldt-Stiftung.

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Table 1: Parameters of the RMF and the RHA models as well as the corresponding saturation properties. The results of shell fluctuation and the χ^2 values of different sets of parameters are also presented.

	RMF		RHA	
	LIN	NL1	RHA0	RHA1
M_N (MeV)	938.000	938.000	938.000	938.000
m_σ (MeV)	615.000	492.250	615.000	458.000
m_ω (MeV)	1008.00	795.359	916.502	816.508
m_ρ (MeV)	763.000	763.000	763.000	763.000
g_σ	12.3342	10.1377	9.9362	7.1031
g_ω	17.6188	13.2846	11.8188	8.8496
g_ρ	10.3782	9.9514	10.0254	10.2070
b (fm $^{-1}$)	0.0	24.3448	0.0	24.0870
c	0.0	-217.5876	0.0	-15.9936
ρ_0 (fm $^{-3}$)	0.1525	0.1518	0.1513	0.1524
E/A (MeV)	-17.03	-16.43	-17.39	-16.98
m^*/M_N	0.533	0.572	0.725	0.788
K (MeV)	580	212	480	294
a_4 (MeV)	46.8	43.6	40.4	40.4
$\delta\rho$ in ^{208}Pb (fm $^{-3}$)	-0.0075	-0.0070	-0.0016	-0.0030
χ^2	1773	66	1040	812

Table 2: The single-particle energies of both protons and anti-protons as well as the binding energy per nucleon and the *rms* charge radius in ^{16}O , ^{40}Ca and ^{208}Pb .

	RMF		RHA		EXP.
	LIN	NL1	RHA0	RHA1	
^{16}O					
E/A (MeV)	7.80	8.00	8.01	8.00	7.98
r_{ch} (fm)	2.59	2.73	2.62	2.66	2.74
PROTONS					
$1s_{1/2}$ (MeV)	42.99	36.18	32.21	30.68	40 ± 8
$1p_{3/2}$ (MeV)	20.71	17.31	16.09	15.23	18.4
$1p_{1/2}$ (MeV)	10.85	11.32	12.98	13.24	12.1
ANTI-PRO.					
$1\bar{s}_{1/2}$ (MeV)	821.30	674.11	413.62	299.42	
$1\bar{p}_{3/2}$ (MeV)	754.62	604.70	369.78	258.40	
$1\bar{p}_{1/2}$ (MeV)	755.43	605.77	370.36	258.93	
^{40}Ca					
E/A (MeV)	8.38	8.58	8.65	8.73	8.55
r_{ch} (fm)	3.36	3.48	3.39	3.42	3.45
PROTONS					
$1s_{1/2}$ (MeV)	51.21	46.86	38.64	36.58	50 ± 11
$1p_{3/2}$ (MeV)	35.05	30.15	27.11	25.32	
$1p_{1/2}$ (MeV)	29.25	25.11	25.17	24.03	34 ± 6
ANTI-PRO.					
$1\bar{s}_{1/2}$ (MeV)	840.76	796.09	456.58	339.83	
$1\bar{p}_{3/2}$ (MeV)	792.36	706.36	424.85	309.24	
$1\bar{p}_{1/2}$ (MeV)	792.75	707.86	425.14	309.52	
^{208}Pb					
E/A (MeV)	7.83	7.89	7.96	7.93	7.87
r_{ch} (fm)	5.34	5.52	5.43	5.49	5.50
PROTONS					
$1s_{1/2}$ (MeV)	58.71	50.41	44.43	40.80	
$1p_{3/2}$ (MeV)	52.74	44.45	39.87	36.45	
$1p_{1/2}$ (MeV)	51.83	43.75	39.49	36.21	
ANTI-PRO.					
$1\bar{s}_{1/2}$ (MeV)	830.16	717.01	476.61	354.18	
$1\bar{p}_{3/2}$ (MeV)	819.15	705.20	466.08	344.48	
$1\bar{p}_{1/2}$ (MeV)	819.22	705.28	466.13	344.52	