Essays on

Consumption, Insurance, and

Portfolio Choice over the Life Cycle

Inaugural-Dissertation

zur Erlangung des Doktorgrades

des Fachbereichs Wirtschaftswissenschaften

der Johann Wolfgang Goethe-Universität

Frankfurt am Main

GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

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Frankfurt am Main 2014

Danksagung

Nach etwas mehr als viereinhalb Jahren an der Goethe-Universität Frankfurt ist diese Arbeit als Ergebnis meiner Forschung entstanden. Im Mai 2014 wurde sie als Dissertationsschrift angenommen.

Ich möchte dieses Vorwort nutzen, um mich bei denjenigen Menschen zu bedanken, ohne die diese Dissertation so nicht möglich gewesen wäre und die mich bei der Anfertigung dieser Arbeit unterstützt haben.

Die Arbeit entstand während meiner Tätigkeit am UBS-Stiftungslehrstuhl für Finance, insbesondere Asset Pricing sowie während meiner Tätigkeit am Center for Financial Studies im Rahmen des Exzellenzzentrum „Sustainable Architecture for Finance in Europe" (SAFE), gefördert von der hessischen Exzellenzinitiative LOEWE. Für diese Finanzierungsmöglichkeiten sowie für die dadurch erhaltene Forschungsunterstützung möchte ich mich herzlich bedanken. Weiterhin möchte ich mich bei der Frankfurt Cloud bedanken. Dadurch, dass ich die Möglichkeit hatte, meine Simulationen auf mehreren virtuellen Hochleistungsservern laufen zu lassen, konnte der Zeitaufwand für die Lösung meiner Modelle stark reduziert werden.

Vor allem möchte ich mich bei meinem Doktorvater, Prof. Dr. Holger Kraft bedanken. Zunächst für die Übernahme der Betreuung meiner Dissertation und das damit im Voraus in mich gesetzte Vertrauen. Außerdem möchte ich mich bedanken für zahlreiche Gespräche, Hinweise und Tipps, wodurch ich noch sehr viel gelernt habe und die die nun vorliegende Qualität der Forschungsarbeit ermöglicht haben. Insbesondere für die sehr gute Zusammenarbeit bei dem gemeinsamen Forschungsprojekt „Life Insurance Demand under Health Shock Risk" möchte ich mich bedanken. Weiterhin möchte ich mich auch für sehr viel Engagement, Ausdauer und Unterstützung bei der Suche nach Finanzierungsmöglichkeiten bedanken, obwohl es diesbezüglich auch viele Rückschläge zu verkraften gab.

Auch bei den anderen Mitgliedern meiner Prüfungskommission möchte ich mich herzlich bedanken. Insbesondere bei Prof. Dr. Helmut Gründl, der das Zweitgutachten übernommen hat und mir noch viele, sehr hilfreiche Kommentare in dem Gutachten insbesondere zu anderen Interpretationsmöglichkeiten der Ergebnisse gegeben hat. Natürlich geht mein Dank auch an die weiteren Mitglieder der Prüfungskommission, Prof. Nicola Fuchs-

Schündeln, Ph.D. und Prof. Dr. Christian Schlag, für die Übernahme der Begutachtung und das Interesse an meiner Forschung.

Weiterhin möchte ich mich bei Prof. Mogens Steffensen, Ph.D. bedanken für die sehr gute Zusammenarbeit in dem Projekt „Life Insurance Demand under Health Shock Risk". Durch viele ausführliche Kommentare und Hinweise habe ich noch sehr viel über den Versicherungsmarkt und die Modellierung von Lebensversicherungen gelernt. Ebenso möchte ich mich bei Prof. Claus Munk, Ph.D. und Farina Weiss für die sehr gute Zusammenarbeit bedanken, auch wenn unser gemeinsames Projekt leider nicht zu einem fertigen Forschungspapier führte.

Ich möchte mich auch bei allen Lehrstuhlmitarbeitern bedanken, für die sehr angenehme Atmosphäre, die Mittags- und Kaffeepausen sowie viele fachliche aber auch private Gespräche. Dies alles hat eine sehr willkommene Abwechslung in den Arbeitsalltag gebracht.

Besonders bedanken möchte ich mich bei Alexander Kohler, Christoph Hambel und Sebastian Wagner, die mir viele hilfreiche Kommentare während der Vorbereitung zur Disputation gaben und sich den ganzen Vortrag zuvor schon anhören durften.

Weiterhin möchte ich mich bei denen bedanken, die mich beim Korrekturlesen der Dissertation unterstützt haben. Farina Weiss, mit der ich vor allem viel über die Forschungsarbeit „Consumption-Investment Problems with Stochastic Mortality Risk" diskutiert habe. Brigitte Groth, von der ich viele, sehr hilfreiche Kommentare zu der korrekten Verwendung von englischen Wörtern sowie der englischen Grammatik bekommen habe. Aber vor allem möchte ich mich bei meinen beiden fleißigsten Korrekturlesern bedanken, die jeweils die komplette Arbeit und die Präsentation gelesen haben. Zum einen bei meiner Schwester, Janina Schendel, die mir als Linguistik-Doktorandin viele, sehr hilfreiche Kommentare gegeben hat zur sprachlichen Verbesserung der Arbeit. Und zum anderen bei Christoph Hambel, mit dem ich viele fachliche Gespräche über die gesamte Arbeit hatte, die mir sehr weiter geholfen haben.

Vor allem möchte ich mich auch bei meinen Eltern bedanken, Egbert und Monika Schendel, die mich immer bedingungslos bei all meinen Vorhaben unterstützt haben. Durch den familiären Rückhalt konnte ich mich auch in finanziell schwierigen Situationen auf die Forschung konzentrieren und daher diesen Karriereweg überhaupt erst einschlagen. Für all die Unterstützung auf meinem gesamten Weg, geht an sie mein größter Dank.

Königstein im Taunus, im Mai 2014 *Lorenz S. Schendel*

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Einleitung

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1 Zusammenhang der Forschungspapiere

Diese Dissertation besteht im Wesentlichen aus den folgenden drei Forschungsarbeiten:

- **Consumption-Investment Problems with Stochastic Mortality Risk** von *Lorenz S. Schendel*,
- **Life Insurance Demand under Health Shock Risk** von *Holger Kraft, Lorenz S. Schendel und Mogens Steffensen*,
- **Critical Illness Insurance in Life Cycle Portfolio Problems** von *Lorenz S. Schendel*.

Alle drei Projekte sind in dem Bereich der zeitstetigen Konsum- und Portfoliooptimierung über den Lebenszyklus angesiedelt und bauen somit auf Merton (1969, 1971) auf. Eine weitere grundlegende Forschungsarbeit ist die von Yaari (1965), die einen stochastischen Todeszeitpunkt einführt. Richard (1975) präsentiert ein Lebenszyklusmodell mit stochastischem Todeszeitpunkt und einer einfachen Lebensversicherung, das er analytisch löst. Auf diesen Arbeiten bauen meine drei Forschungspapiere auf. Aktuellere relevante Papiere, auf denen meine Arbeiten aufbauen, sind Cocco, Gomes und Maenhout (2005) sowie Munk und Sørensen (2010). Erstere lösen ein diskretes Portfoliooptimierungsproblem numerisch und liefern dabei unter anderem eine realistische Einkommenskalibrierung, die dem Einkommensprofil von US-Haushalten über den Lebenszyklus entspricht. Munk und Sørensen (2010) betrachten ein zeitstetiges Modell und übertragen die Einkommenskalibrierung von Cocco, Gomes und Maenhout (2005) auf eine zeitstetige Variante, die ich in meinen Arbeiten nutze. Sie lösen ihr Modell numerisch mittels eines Finite-Differenzen-Verfahrens, das die Grundlage für die in meinen Arbeiten verwendeten numerischen Lösungsmethoden legt.

In meinen Forschungspapieren liegt der Fokus modelltechnisch insbesondere auf den Effekten von stochastischem Sterberisiko mit Sprungkomponenten. In diesen Modellen

	1)	2)	3)
Finanzmarkt mit Aktie und Bond	✓	✓	✓
Nicht-hedgebares stochastisches Einkommen	✓	✓	✓
Sprungkomponente		✓	✓
Nicht-hedgebare stochastische Gesundheitsausgaben			✓
Sprungkomponente			✓
Altersabhängiges Sterberisiko	✓	✓	✓
Diffusionskomponente	✓		
Sprungkomponente	✓	✓	✓
Lebensversicherung	✓	✓	
Realistische Langzeitverträge		✓	
Critical Illness Versicherung			✓
CRRA Präferenzen für Einzelperson	✓		✓
CRRA Präferenzen für Familie		✓	

Tabelle 1: Features der Modelle. Die Tabelle fasst die Features der Modelle in meinen Forschungsarbeiten zusammen. Spalte 1) steht für die Arbeit Consumption-Investment Problems with Stochastic Mortality Risk, Spalte 2) für das Forschungspapier Life Insurance Demand under Health Shock Risk und Spalte 3) für die Forschungsarbeit Critical Illness Insurance in Life Cycle Portfolio Problems.

mit stochastischen Sterberaten werden dann die optimalen Konsum- und Investmentstrategien der Agenten betrachtet. Weiterhin wird die Nachfrage nach verschiedenen Versicherungsarten wie einer Risikolebensversicherung und einer Critical Illness Versicherung untersucht. Zur Lösung der Modelle werden sowohl analytische Verfahren als auch numerische Methoden verwendet. Tabelle 1 liefert einen Überblick über die Features der Modelle in meinen Forschungsarbeiten. Die Arbeiten sind in der oben genannten Reihenfolge entstanden und bauen thematisch aufeinander auf.

Der Fokus der ersten Forschungsarbeit **Consumption-Investment Problems with Stochastic Mortality Risk** liegt auf der Analyse von Sterberisiko in zeitstetigen Portfoliooptimierungsproblemen. Die Relevanz von Sterberisiko allgemein sowie die Effekte von stochastischem Sterberisiko werden untersucht. Ein zentrales Ergebnis ist, dass ein ansonsten äquivalentes Modell mit deterministischem Todeszeitpunkt nahezu identische Ergebnisse liefert, solange das Sterberisiko gering ist. Somit ist ein Fazit, dass Modelle mit Sterberisiko insbesondere dann zu verwenden sind, wenn entweder ältere Agenten im Rentenalter betrachtet werden, da dann das Sterberisiko relevante Auswirkungen hat, oder wenn Finanzprodukte oder Versicherungen betrachtet werden sollen, die mit dem Überleben des Agenten in Zusammenhang stehen. Weiterhin zeige ich, dass eine Sprungkomponente in der Sterberate starke Auswirkungen für den Agenten hat, während eine Diffusionskomponente vernachlässigbar ist.

Direkt auf diese Ergebnisse aufbauend ist das zweite Projekt **Life Insurance Demand under Health Shock Risk** in Zusammenarbeit mit Holger Kraft und Mogens Steffensen entstanden. In der Arbeit betrachten wir ein Modell mit stochastischer Sterberate mit Sprungkomponente und untersuchen die Nachfrage nach einer Risikolebensversicherung einer Familie. Insbesondere achten wir auf eine realistische Modellierung der Risikolebensversicherung mit fixen Versicherungssummen und Gebühren, die bei einem Wechsel der Versicherungshöhe entstehen. Wir kalibrieren den Sterbeprozess mit einer Sterbetafel Deutschlands. Die Sprünge in der Sterberate interpretieren wir als Krebserkrankung und kalibrieren diese entsprechend mit Krebsdaten aus Deutschland. Ein Ergebnis ist, dass ein Sprung in der Sterberate eines Agenten ein einschneidendes Erlebnis für die Familie ist, welches die optimale Konsum- und Investmentstrategie stark beeinflusst. Der Sprung wird als eine Art Sterbeindikator aufgefasst, da durch die erhöhte Sterberate ein Tod in den kommenden Jahren sehr wahrscheinlich ist.

Auf diesem Ergebnis aufbauend ist die Idee zur dritten Arbeit **Critical Illness Insurance in Life Cycle Portfolio Problems** entstanden. Da der Sprung so starke Auswirkungen hat, wäre eine Versicherung sinnvoll, die direkt im Falle eines Sprungs auszahlt und nicht erst später zum Todeszeitpunkt wie die Risikolebensversicherung. Eine solche Critical Illness Versicherung ist in Deutschland bisher kaum verbreitet und wurde wissenschaftlich in Portfoliooptimierungsmodellen noch nicht untersucht. In meinem Modell übernimmt die Versicherung im Falle eines Sprungs die zusätzlichen Kosten, die durch die Krankheit entstehen. Dies kann zum Beispiel Kosten durch teure Therapien oder durch rollstuhlgerechte Ausrüstung enthalten. In dieser Arbeit wird auch auf die Ergebnisse des ersten Forschungspapiers aufgebaut, da wieder ein Modell mit stochastischer Sterberate mit Sprungkomponente genutzt wird und ein Versicherungsprodukt betrachtet wird, das mit der Gesundheit des Agenten in Zusammenhang steht. Auch in diesem Modell lege ich auf eine realistische Kalibrierung Wert. Die Gesundheitskosten kalibriere ich anhand von Gesundheitsausgaben in Deutschland.

Die obigen Ausführungen zeigen, dass ein klarer thematischer Zusammenhang der einzelnen Forschungsarbeiten vorhanden ist.

2 Zusammenfassung der Forschungspapiere

Im Folgenden werde ich die Inhalte der Forschungspapiere zusammenfassen. Dabei werde ich insbesondere auf die Motivation, die wichtigste relevante Literatur, die Modelle sowie die zentralen Ergebnisse eingehen.

Consumption-Investment Problems with Stochastic Mortality Risk In diesem Forschungspapier untersuche ich die Relevanz von Sterberisiko für zeitstetige Portfoliooptimierungsprobleme über den Lebenszyklus. Weiterhin beschäftige ich mich mit der Modellierung von Sterberisiko und untersuche, inwiefern stochastische Sterberaten mit einer Diffusions- oder einer Sprungkomponente Einfluss haben. Motiviert wird diese Arbeit dadurch, dass in der Literatur über zeitstetige Portfoliooptimierungsprobleme häufig die Annahme eines deterministischen Todeszeitpunkts getroffen wird, da dies die analytischen Rechnungen und numerischen Verfahren vereinfacht. Stochastisches Sterberisiko wird ansonsten in der Versicherungsliteratur betrachtet, dort aber aus einer anderen Perspektive. Während für die Versicherung aggregierte Veränderungen der Sterberate relevant sind, wie Kriege, Umweltkatastrophen oder medizinischer Fortschritt, sind für den einzelnen Agenten individuelle Ereignisse relevanter, wie eine schwere Krankheit oder ein Unfall. Daher sind die Sterbeprozessmodellierungen der Versicherungsliteratur häufig nicht geeignet für individuelle Portfoliooptimierungsmodelle.

Cairns, Blake und Dowd (2008) liefern einen detaillierten Überblick über die Modellierung von Sterberisiko in der Versicherungsliteratur. Das von mir verwendete Modell ähnelt dem von Richard (1975). Insbesondere die Modellierung der Versicherung baut auf seiner Forschungsarbeit auf. Er verzichtet jedoch auf Einkommensunsicherheit und stochastische Sterberaten. Huang, Milevsky und Salisbury (2012) vergleichen analytisch stochastische mit deterministischen Sterberaten. Allerdings betrachten sie keine Sprungkomponenten in der Sterberate und betrachten nur den Konsum von Rentnern, jedoch keine Portfolioallokation. Wesentliche Ergebnisse, die sie analytisch zeigen, kann ich in meinem komplexeren Modell numerisch bestätigen.

Ich betrachte einen Agenten mit einer Nutzenfunktion mit konstanter relativer Risikoaversion (CRRA) über Konsum und das Erbe. Der Agent kann in eine Aktie, modelliert als geometrische Brownsche Bewegung, und in einen risikofreien Bond investieren. Der Agent erhält ein variables Einkommen, das ebenfalls als geometrische Brownsche Bewegung modelliert ist mit einem zusätzlichen Wiener Prozess, aber auch korreliert sein kann mit der Diffusionskomponente der Aktie. Somit ist das Einkommen im Allgemeinen nicht replizierbar mit gehandelten Finanzinstrumenten. Die Kalibrierung für den Einkommensprozess sowie die Kalibrierung für den Finanzmarkt und die Präferenzen übernehme ich zu einem großen Teil aus Cocco, Gomes und Maenhout (2005) sowie Munk und Sørensen (2010). Ich vergleiche vier verschiedene Modelle, die unterschiedlich viel Unsicherheit bezüglich des Todeszeitpunkts enthalten. Das Modell D hat einen deterministischen Todeszeitpunkt. In dem Modell S betrachte ich einen stochastischen Todeszeitpunkt mit einer deterministischen Sterberate, welche einem Gompertz Modell folgt. Das Modell SD hat eine stochastische Sterberate, wobei die Sterberate selbst als geometrische Brownsche Bewegung modelliert ist, welche von den anderen Risikoquellen stochastisch unabhängig ist. In dem Modell SDJ füge ich zusätzlich eine Sprungkomponente hinzu. Ich kalibriere alle Modelle mit Sterbedaten aus Deutschland vom Statistischen Bundesamt. Weiterhin betrachte ich eine einfache Versicherung (Leibrente), die zeitstetig gehandelt werden kann und versicherungsmathematisch gerecht bewertet ist. Diese Versicherung zahlt eine Prämie, wenn der Agent überlebt, als Austausch gegen eine Zahlung im Falle des Todes des Agenten an die Versicherung. Dies kann so interpretiert werden, dass der Agent eine Lebensversicherung verkauft. Insgesamt betrachte ich ein Optimierungsproblem über die stochastische Lebensdauer des Agenten mit den drei Kontrollvariablen Konsum, Portfolioallokation und Versicherungsnachfrage. Die vier Zustandsvariablen des Optimierungsproblems sind die Zeit, das finanzielle Vermögen, das Einkommen sowie die Sterberate in den Modellen, in denen diese stochastisch ist. Für ein einfaches Modell mit deterministischem Einkommen und Sterberaten stelle ich die Hamilton-Jacobi-Bellman (HJB) Gleichung auf und löse diese analytisch. Für die komplexeren Modelle mit stochastischem Einkommen und Sterberaten betrachte ich ein äquivalentes Optimierungsproblem mit einer reduzierten Anzahl an Zustandsvariablen. Für dieses Problem stelle ich die vereinfachte HJB auf und löse diese mittels eines numerischen Verfahrens mit finiten Differenzen.

Ein zentrales Ergebnis der Kalibrierung ist, dass mit einer Sprungkomponente in der Sterberate die Sterbedaten Deutschlands deutlich besser erklärt werden können als mit einem Gompertz Modell alleine. Die Diffusionskomponente hat hingegen keinen relevanten Einfluss. Auch bei den numerischen Simulationen zeigt sich, dass die Modelle S und SD weitestgehend ähnliche Ergebnisse liefern und die Existenz der Diffusionskomponente einen geringen Einfluss für den Agenten hat. Die Sprungkomponente hingegen ist relevant für den Agenten. Der Sprung agiert als Indikator für einen bevorstehenden Tod, da das Sterberisiko deutlich erhöht ist nach einem Sprung. Der Agent passt seine Konsum- und Investitionsstrategie an die neue erwartete Restlebensdauer an. Er steigert seinen Konsum und reduziert den Anteil des Vermögens, der risikoreich angelegt ist. Die Wichtigkeit der Sprungkomponente zeigt sich auch dadurch, dass das Sicherheitsäquivalent im Modell SDJ für 50-jährige Agenten um mehr als 25% höher ist als das im Modell S. Dies lässt sich dadurch erklären, dass der Agent durch den Sprung seinen erwarteten Todeszeitpunkt deutlich besser einschätzen kann. In einem Modell ohne Versicherung sorgt die Sprungkomponente zudem dafür, dass Agenten ein unbeabsichtigt hohes Erbe deutlich reduzieren können. Ein Vergleich der Modelle S und D zeigt, dass sich eine deterministische Sterberate sehr gut als Approximation verwenden lässt, solange das Sterberisiko gering ist. Soll also nur die Arbeitsphase des Lebens betrachtet werden und spielen Finanzinstrumente und Versicherungen, die mit dem Gesundheitszustand oder der Sterberate in Zusammenhang stehen, keine Rolle, so kann zur Vereinfachung ein Modell mit deterministischem Todeszeitpunkt betrachtet werden. In anderen Fällen sollte jedoch ein Modell mit stochastischem Todeszeitpunkt gewählt werden, in welchem die Sterberate stochastisch ist und durch einen Sprungprozess getrieben wird. Die Analyse der Versicherung zeigt, dass diese verwendet wird, um das optimale Erbe im Falle des Todes sicherzustellen. Das heißt, in einem Modell mit der Versicherung ist das komplette Erbe beabsichtigt, während der Großteil des Erbes in einem Modell ohne Versicherung unbeabsichtigt ist. Ohne Versicherung spart der Agent mehr Vermögen an und konsumiert weniger im Rentenalter, um im Falle eines sehr langen Überlebens nicht ohne finanzielle Mittel dazustehen. Tritt dann der Tod ein, hinterlässt der Agent mehr als optimalerweise gewollt.

Life Insurance Demand under Health Shock Risk In dieser Forschungsarbeit betrachten wir die Risikolebensversicherungsnachfrage einer Familie, wobei der Alleinverdiener stochastischem Sterberisiko mit einer Sprungkomponente ausgesetzt ist. Dabei achten wir insbesondere auf eine realistische Modellierung der Versicherung, indem wir längerfristige Verträge modellieren, sodass eine zeitstetige Anpassung der Versicherungsnachfrage nicht optimal ist. Eine Motivation für diese Arbeit sind die Ergebnisse der vorherigen Forschungsarbeit, die die Relevanz einer Sprungkomponente in der Sterberate hervorheben. Weiterhin ist das Arbeitseinkommen die wichtigste Vermögensquelle für die Agenten. Ein frühzeitiger unerwarteter Tod des Alleinverdieners sollte darum sehr starke Auswirkungen für die ganze Familie haben. Daher ist es interessant, zu betrachten, inwiefern die Familie bereit ist, sich gegen dieses Risiko mit einer Risikolebensversicherung abzusichern. In der bisherigen Literatur wird dafür zumeist auf eine stark vereinfachte Versicherung zurückgegriffen. Das heißt, die Versicherungssumme ist frei wählbar, der Beitrag ist versicherungsmathematisch gerecht berechnet und die Versicherungssumme ist zeitstetig veränderbar ohne zusätzliche Kosten. Diese Annahmen entsprechen jedoch nicht der Realität in dem Versicherungsmarkt. Versicherungen erheben Verwaltungsgebühren sowie Abschlussgebühren. Der Agent kann aus einer gewissen Anzahl von Versicherungssummen wählen und ein nachträglicher Wechsel ist häufig mit zusätzlichen Kosten verbunden. Die Verträge sind auf längere Laufzeiten ausgelegt. Weiterhin wird bei bereits vorhandenen Krankheiten ein Vertragsabschluss oder eine Erhöhung der Versicherungssumme verweigert. Alle diese Punkte berücksichtigen wir in unserem Modell, um eine realistischere Modellierung der Versicherung zu erreichen.

Eine ähnliches numerisches Forschungspapier von Huang, Milevsky und Wang (2008) betrachtet auch die Konsum-, Investment- und Lebensversicherungsentscheidung einer Familie. Ein wichtiges Ergebnis ist, dass die Versicherungsnachfrage stark von der Einkommensvolatilität abhängt, jedoch kaum von der relativen Risikoaversion. Die Autoren erlauben jedoch eine stetige Anpassung der Versicherungssumme und verzichten auf eine realistische Modellierung der Versicherung. Weiterhin verzichten sie auf stochastische Sterberaten. Love (2010) betrachtet ebenfalls eine einfache Lebensversicherung. Sein Modell untersucht insbesondere den Einfluss von demographischen Schocks auf die

Konsum-, Investment- und Versicherungsentscheidung. Diese exogenen Schocks bedeuten zum Beispiel, dass der Agent heiratet, sich scheiden lässt oder Kinder bekommt. Dadurch werden die Präferenzen, und somit auch die optimalen Entscheidungen, stark beeinflusst. Auch hier wird jedoch nicht auf eine realistische Modellierung der Versicherung und auf stochastische Sterberaten eingegangen. Weitere Forschungspapiere betrachten ähnliche Modelle mit analytischen Ansätzen. Dazu zählen unter anderem Bruhn und Steffensen (2011) sowie Kwak, Shin und Choi (2011). Erstere betrachten die Konsum-, Investment- und Versicherungsentscheidung eines Mehrpersonenhaushalts, wobei die einzelnen Haushaltsmitglieder verschiedenen Sterberisikoprozessen ausgesetzt sind. Letztere untersuchen die Lebensversicherungsnachfrage von Eltern, die ihre Kinder dadurch im eigenen Todesfall vor einem Einkommensverlust schützen können. Aufgrund der analytischen Lösungstechnik können diese Modelle einige realistische Elemente, wie zum Beispiel ein stochastisches Einkommen, nicht darstellen. Die Versicherungsnachfrage wird auch empirisch untersucht. Eine Arbeit von Hong und Ríos-Rull (2012) betrachtet die Lebensversicherungsnachfrage von Haushalten unter anderem in Abhängigkeit von der Haushaltsgröße.

In unserem Modell betrachten wir eine Familie, die Konsum-, Investment- und Risikolebensversicherungsentscheidungen über den Lebenszyklus trifft. Die verwendete CRRA Nutzenfunktion wird dabei an die Familiengröße angepasst. Dazu wird das Konsumlevel durch einen Faktor geteilt, der von der Anzahl der Erwachsenen und der Kinder im Haushalt abhängt. Der Faktor berücksichtigt, dass zwei Personen im selben Haushalt nicht die doppelte Menge an Konsumgütern benötigen, um denselben Nutzen zu haben wie bei einer separaten Betrachtung. Wie im vorherigen Forschungspapier kann die Familie in eine Aktie und eine risikofreie Anleihe investieren. Das Einkommen der Familie wird von einem Alleinverdiener erwirtschaftet, ist stochastisch und die Kalibrierung orientiert sich wieder an Cocco, Gomes und Maenhout (2005) sowie Munk und Sørensen (2010). Der Alleinverdiener hat ein stochastisches Sterberisiko mit Sprungkomponente. Den Sprung interpretieren wir als Gesundheitsschock und kalibrieren ihn mit Krebsdaten aus Deutschland vom Zentrum für Krebsregisterdaten und Robert Koch Institut. Die Sterberaten werden erneut mit Daten aus Deutschland vom Statistischen Bundesamt kalibriert. Im Falle eines Gesundheitsschocks kann die Familie keine Risikolebensversicherung mehr abschließen oder verändern und das Einkommen der Familie reduziert sich. Stirbt der Alleinverdiener, erhält die Familie gar kein Einkommen mehr. Solange der Alleinverdiener am Leben ist, gesund ist und jünger als 70 ist, kann die Familie eine Risikolebensversicherung abschließen oder die Versicherungssumme verändern. Die Versicherung wird mit zehn verschiedenen fixen Versicherungssummen angeboten, die im Todesfall an die Familie ausgezahlt werden. Im Gegenzug muss die Familie einen jährlichen Beitrag zahlen sowie eine Einmalzahlung bei Abschluss. In den Beiträgen werden Verwaltungsgebühren und Transaktionsgebühren berücksichtigt. Der Versicherungsvertrag endet im Alter von 75 oder mit dem Tod des Alleinverdieners. Solange der Alleinverdiener gesund ist, kann die Familie den Versicherungsschutz erhöhen oder verringern, dabei entstehen jedoch zusätzliche Kosten. Die Kosten kalibrieren wir mit Daten von deutschen Lebensversicherern von map-report. Unter Berücksichtigung dieser Kosten berechnen wir die Versicherungsbeiträge versicherungsmathematisch gerecht. Insgesamt betrachten wir ein Optimierungsproblem mit drei Kontrollvariablen: dem Konsum, der Portfolioallokation und der Versicherungsstrategie. Die Versicherungsstrategie besteht dabei aus den Zeitpunkten einer Änderung des Vertrags und der Höhe der Veränderung der Versicherungssumme. Das Problem wird durch fünf Zustandsvariablen charakterisiert: der Zeit, dem Vermögen, dem Einkommen, der aktuellen Versicherungssumme und dem Gesundheitszustand (gesund, krank, tot) des Alleinverdieners. Zur numerischen Lösung der Optimierung teilen wir das Problem auf in ein Impulssteuerungsproblem für die Versicherungsstrategie und in ein stochastisches Kontrollproblem für die Konsum- und Investmentstrategie. Für letzteres stellen wir die HJB auf und lösen diese mit einem Finite-Differenzen-Verfahren. Dies tun wir für alle möglichen Versicherungsentscheidungen und erhalten so die Lösung für die wertefunktionmaximierende Intervention.

Im Verlauf des Lebenszyklusses erhöhen die Familien ihre Versicherungssummen. Die Versicherung sowie die Ersparnisse der Familie ermöglichen den Erhalt eines nutzenäquivalenten Konsumniveaus im Falle des Todes eines älteren Alleinverdieners. Allerdings reduziert sich das durchschnittliche Konsumwachstum nach dem Tod des Alleinverdieners deutlich. Stirbt der Alleinverdiener jedoch sehr früh, zum Beispiel im Alter von 30, reduziert sich sowohl das Konsum-Level als auch das Konsumwachstum drastisch. In einer solchen frühen Phase des Lebenszyklusses hat die Familie noch nicht genug finanzielle Ersparnisse und keine Risikolebensversicherung mit einer ausreichend hohen Versicherungssumme abgeschlossen. Somit kann sie den immensen Einkommensverlust nicht ausgleichen. Die Familie wäre dann auf soziale Transferzahlungen angewiesen. Aus ökonomischer Sicht sollte die Familie daher einen Anreiz haben, auch in jungen Jahren eine Risikolebensversicherung abzuschließen. In dem Modell hingegen bleiben die Familien bis zu einem Alter von 30 Jahren dem Versicherungsmarkt fern. Wir identifizieren mehrere Aspekte, die für die geringe Versicherungsnachfrage, insbesondere in frühen Jahren, verantwortlich sind. Der Gesundheitsschock wird als Indikator für einen bevorstehenden wahrscheinlichen Tod gesehen und daher kann durch optimale Reaktion teilweise auf die Versicherung verzichtet werden. Folglich ist in einem äquivalenten Modell ohne Gesundheitsschocks die Versicherungsnachfrage höher. Für eine geringe Versicherungsnachfrage sorgen außerdem ein hohes Einkommen, eine hohe relative Risikoaversion, eine hohe Einkommensvolatilität und hohe Gebühren der Versicherung. Insbesondere die Einkommensvolatilität hat einen sehr starken Effekt auf die Versicherungsnachfrage, da die Versicherung die negativen Effekte von Einkommensschocks verstärkt. In einem ohnehin schon negativen Zustand nach einem Einkommensschock hat die Familie zusätzlich eine Versicherung, deren Beiträge höher als optimal sind unter Berücksichtigung der neuen finanziellen Situation. Nun kann die Familie entweder die zu hohen Beiträge bezahlen und dafür auf Konsum oder Ersparnisse verzichten oder sie muss den Versicherungsvertrag ändern, was zu zusätzlichen Kosten führt. Hier zeigt sich, dass die realistische Modellierung der Versicherung mit langfristigen Verträgen und Gebühren bei einem Wechsel essenziell ist für die qualitativen Ergebnisse, da dieser Effekt sonst nicht beobachtbar ist. Ein ähnlicher Effekt tritt bei der Risikoaversion auf. Während bei Huang, Milevsky und Wang (2008) die Versicherungsnachfrage kaum auf Änderungen der Risikoaversion reagiert, gilt dies in unserem Modell nur, wenn der Alleinverdiener bereits Rentner ist und somit das Einkommen nicht mehr volatil ist. Vor dem Rentenalter sorgt der verstärkende Effekt der Versicherung dafür, dass risikoaversere Familien eine geringere Versicherungssumme nachfragen.

Critical Illness Insurance in Life Cycle Portfolio Problems In dem letzten Forschungspapier betrachte ich eine Critical Illness (CI) Versicherung in einem Modell, das zusätzlich exogene Gesundheitskosten enthält. Im Deutschen ist diese Versicherung auch unter dem Namen Dread-Disease-Versicherung bekannt. Im Gegensatz zur Risikolebensversicherung zahlt die CI Versicherung bereits die Prämie, sobald eine schwere Krankheit diagnostiziert wird und nicht erst im Todesfall. Daher eignet sich die CI Versicherung besonders, um die zusätzlichen Gesundheitskosten zu kompensieren, die durch die Krankheit entstehen. Dies können zum Beispiel teure Krebsmedikamente sein, die nicht von der Krankenkasse übernommen werden, oder auch Kosten für den Umbau zu einem rollstuhlgerechten Haus. In Ländern ohne ein entsprechendes Gesundheitssystem kann die Versicherung nötig sein, um überhaupt die Möglichkeit zu haben, eine lebensrettende Operation zu erhalten. In solchen Ländern ist die Versicherung weiter verbreitet, während sie in Deutschland vergleichsweise unbekannt ist. Die Bekanntheit und Verbreitung der CI Versicherung steigen stetig. Allerdings wurde sie bisher noch nicht im Rahmen eines Konsum- und Portfoliooptimierungsproblems über den Lebenszyklus betrachtet. Ich untersuche, inwieweit Agenten die CI Versicherung nutzen möchten, um sich gegen zusätzliche Gesundheitskosten im Falle einer schweren Krankheit abzusichern.

Die empirischen Arbeiten von Rosen und Wu (2004), Berkowitz und Qiu (2006) sowie Fan und Zhao (2009) zeigen einen starken Zusammenhang zwischen dem Gesundheitszustand eines Agenten und den Investmententscheidungen auf. In einem anderen Forschungspapier kommen Love und Smith (2010) hingegen zu dem Ergebnis, dass der Gesundheitszustand keine signifikanten Auswirkungen auf die Investmententscheidung hat. Unabhängig von diesen Ergebnissen, gibt es einen intuitiv klaren Zusammenhang zwischen dem Gesundheitszustand eines Agenten und seinen Gesundheitskosten. Unsichere Gesundheitskosten haben in dem Optimierungsproblem einen ähnlichen Einfluss wie ein unsicheres Einkommen, da beide durch einen permanenten, exogenen und stochastischen Zahlungsstrom beschrieben werden. Vor dem Hintergrund der hohen Relevanz von unsicherem Einkommen für die Investmententscheidung rechne ich mit einem signifikanten Einfluss der Gesundheitskosten auf die Entscheidungen des Agenten. Pang und Warshawsky (2010) betrachten ein diskretes Modell mit stochastischen Gesundheitsausgaben für einen Rentner. Unter anderem zeigen sie, dass risikobehaftete Gesundheitsausgaben zu weniger riskanten Investments führen. Deren Arbeit, genau wie andere Forschungsarbeiten in diesem Bereich, fokussiert sich jedoch vollständig auf den Rentenzeitraum und erlaubt keine Reduzierung der Gesundheitsausgaben durch eine CI Versicherung. Auch auf stochastische Sterberaten, deren Relevanz in meinem ersten Projekt gezeigt wird, verzichten sie.

Ich betrachte erneut ein Konsum- und Portfoliooptimierungsproblem über den Lebenszyklus. Der Agent optimiert den Nutzen von Konsum und Erbe mit einer CRRA Nutzenfunktion. Sein finanzielles Vermögen kann er in eine risikolose Anleihe oder in eine Aktie investieren. Die Sterberate ist stochastisch mit einer Sprungkomponente, die als kritische Krankheit interpretiert wird. Die Kalibrierung dafür übernehme ich aus Kraft, Schendel und Steffensen (2014). Weiterhin muss der Agent exogene Gesundheitskosten bezahlen, die als geometrische Brownsche Bewegung mit Sprungkomponente modelliert sind. Ich kalibriere die Gesundheitskosten mit Daten aus Deutschland vom Statistischen Bundesamt. Die Gesundheitskosten springen, sobald der Agent eine kritische Krankheit bekommt. Weiterhin können die Gesundheitskosten sprunghaft ansteigen durch eine Krankheit oder einen Unfall. In diesem Fall erhöhen sich die Gesundheitskosten, aber die Sterberate wird nicht beeinflusst. Ursachen dafür können zum Beispiel eine Behinderung in Folge eines Unfalls oder psychische Erkrankungen sein. Um diese Sprünge in den Gesundheitskosten zu umgehen, kann der Agent eine CI Versicherung abschließen. Die Versicherung übernimmt die Mehrkosten bei den Gesundheitsausgaben, welche bei Eintritt einer schweren Erkrankung oder eines Unfalls anfallen. Für den Versicherungsschutz muss der Agent eine Prämie zahlen, die vom Alter und den aktuellen Gesundheitskosten abhängt. Die Versicherungsentscheidung kann kontinuierlich verändert werden. Ich kalibriere die Versicherung so, dass sie im Erwartungswert approximativ versicherungsmathematisch gerecht ist. Anschließend berücksichtige ich Gebühren für Verwaltungs- und Transaktionskosten sowie einen Gewinn der Versicherung. Zur Finanzierung der Ausgaben erhält der Agent ein stochastisches Arbeitseinkommen, welches ebenfalls als geometrische Brownsche Bewegung mit Sprungkomponente modelliert wird. Die Sprungkomponente berücksichtigt dabei, dass nach einer schweren Krankheit oder einem Unfall die Arbeitsfähigkeit des Agenten beeinträchtigt sein kann und sich daher das Einkommen reduziert. Der Einkommensprozess wird erneut analog zu Cocco, Gomes und Maenhout (2005) sowie Munk und Sørensen (2010) kalibriert. Somit habe ich ein zeitstetiges Portfoliooptimierungsproblem mit fünf Zustandsvariablen: der Zeit, dem finanziellen Vermögen, dem Einkommen, den Gesundheitsausgaben sowie dem aktuellen Gesundheitszustand des Agenten. Zur Nutzenmaximierung über den Lebenszyklus wählt der Agent drei Kontrollvariablen: den Konsum, die Entscheidung, welcher Anteil des Vermögens riskant investiert werden soll, und die Entscheidung, ob er die CI Versicherung abschließt. Zur Lösung des Optimierungsproblems betrachte ich ein äquivalentes Problem mit einer Zustandsvariablen weniger. Für dieses Problem stelle ich die dazugehörige HJB auf und löse sie numerisch mittels eines Finite-Differenzen-Verfahrens.

Zunächst zeige ich, dass der Sprung in den Gesundheitsausgaben einen großen Einfluss auf die aggregierten Ergebnisse hat sowie auf die optimalen Kontrollvariablen eines individuellen Agenten. Ein Vergleich eines Sicherheitsäquivalents zeigt, dass besonders in mittleren Jahren, im Alter von 45 Jahren, der Agent mehr als 40% bessergestellt ist, wenn er kein Sprungrisiko in den Gesundheitsausgaben hat. Konsequenterweise ist die Nachfrage nach der CI Versicherung sehr hoch. Bis zu einem Alter von 50 Jahren schließen fast alle Agenten die Versicherung ab, selbst wenn dies sehr kostspielig ist mit einem Versicherungsprofit von 200%. In späteren Jahren, insbesondere im Rentenalter, sinkt die Versicherungsnachfrage deutlich. Mit einem realistischen Versicherungsprofit von 20%, wobei davon noch Verwaltungs- und Transaktionskosten abgezogen werden müssen, schließen jedoch immer noch mehr als 50% der Agenten die Versicherung während des Rentenalters ab. Agenten mit Zugriff auf die CI Versicherung haben im Alter von 45 Jahren ein ca. 18% höheres Sicherheitsäquivalent als Agenten ohne Zugriff auf die Versicherung. Eine genauere Untersuchung der Versicherungsnachfrage zeigt, dass insbesondere das Einkommen eine wichtige Rolle für die Versicherungsentscheidung spielt. In jungen Jahren haben die Agenten eine starke Unsicherheit bezüglich des zukünftigen Einkommens. Daher kann dieses schlecht zur Absicherung gegen hohe Gesundheitsausgaben verwendet werden. Je älter der Agent wird, desto sicherer wird das zukünftige Einkommen bis zum Rentenalter. Ab diesem Zeitpunkt wird das Einkommen deterministisch modelliert. Ein sichereres Einkommen kann besser zur Absicherung gegen hohe Gesundheitsausgaben verwendet werden und reduziert daher die Versicherungsnachfrage. Dies erklärt sowohl eine sinkende Versicherungsnachfrage mit steigendem Alter als auch eine sinkende Versicherungsnachfrage bei einer Reduzierung der Einkommensvolatilität. In jungen Jahren schließen die Agenten die Versicherung insbesondere dann ab, wenn sie ein hohes Einkommen und geringe aktuelle Gesundheitskosten haben. Dies macht die Versicherung vergleichsweise günstig. Im Rentenalter verzichten die Agenten auf die Versicherung, wenn sie ein sehr geringes Einkommen haben, da die Versicherung dann relativ gesehen zu teuer ist. Ebenso schließen die Agenten keine Versicherung ab, wenn die aktuellen Gesundheitsausgaben sehr gering sind, da in dem Fall der Effekt eines Sprunges verhältnismäßig gering ist. Unabhängig vom Alter ist das finanzielle Vermögen bei der Versicherungsentscheidung nahezu irrelevant. Weiterhin untersuche ich die Auswirkungen einer Unterschätzung der Wahrscheinlichkeit einer schweren Krankheit sowie einer Unterschätzung der durch einen Sprung entstehenden zusätzlichen Kosten. Eine solche Fehleinschätzung reduziert die Versicherungsnachfrage maßgeblich und kann als eine Erklärung dafür dienen, warum diese Art von Versicherung in der Realität kaum abgeschlossen wird trotz ihrer hohen Relevanz im Modell.

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Part II

Research Paper

Consumption-Investment Problems with Stochastic Mortality Risk

Lorenz S. Schendel*

Abstract: I numerically solve realistically calibrated life cycle consumption-investment problems in continuous time featuring stochastic mortality risk driven by jumps, unspanned labor income as well as short-sale and liquidity constraints and a simple insurance. I compare models with deterministic and stochastic hazard rate of death to a model without mortality risk. Mortality risk has only minor effects on the optimal controls early in the life cycle but it becomes crucial in later years. A diffusive component in the hazard rate of death has no significant impact, whereas a jump component is desired by the agent and influences optimal controls and wealth evolution. The insurance is used to ensure optimal bequest such that there is no accidental bequest. In the absence of the insurance, the biggest part of bequest is accidental.

Keywords: Stochastic mortality risk, Health jumps, Labor income risk, Portfolio choice, Insurance

JEL-Classification: D91, G11

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I gratefully acknowledge research and financial support from the Center of Excellence SAFE, funded by the State of Hessen initiative for research LOEWE. I thank the Frankfurt Cloud for access to high-performance virtual servers. All remaining errors are of course my own.

1 Introduction

Life cycle consumption-investment models often assume a deterministic time of death or, if at all, include deterministic mortality risk given by mortality tables. Only a few recent papers allow for stochastic mortality risk driven by a diffusive component. In reality, cancer and other critical illnesses suggest that there is a significant jump component in the individual hazard rate of death that is not captured by the life cycle consumption-investment models. This raises the question of the importance of mortality risk in life cycle consumption-investment models, and what impact a jump component in the hazard rate of death has. My paper aims to close this gap in the literature.

I analyze the impact of mortality risk in a life cycle consumption-investment model. Further model features are unspanned labor income risk, short-sale and liquidity constraints and a simple insurance. The mortality process is calibrated to mortality data for Germany and allows uncertainty driven by a diffusive and jump component. I compare results with deterministic time of death and stochastic time of death. Furthermore, I provide sensitivity analyses with respect to the agent's characteristics and the financial market. I also analyze the impact of the insurance and of a bequest motive.

The mortality calibration shows that allowing for jumps in the hazard rate of death significantly increases the fit to the data. Considering the results, the model with deterministic time of death and the model with stochastic time of death produce nearly identical optimal consumption and portfolio holdings as long as mortality risk is very low. This highlights that early in the lifetime mortality risk can be neglected for simplicity. The endogenous insurance is used by the agents to allocate optimal bequest such that all bequest is intended. In a model without insurance, agents leave more than twice as much bequest on average if they face mortality risk. Most bequest is accidental then. Without insurance, the consumption profile over the life cycle shows a hump-shaped pattern. A diffusive component in the hazard rate of death has almost no influence, whereas a jump component has a significant impact. The agent is better off since a mortality jump acts as a death indicator. A jump in mortality risk also crucially affects the consumption and investment decision. Together with the importance in the mortality calibration, this highlights that mortality risk should be modeled with a jump component in life cycle consumption-investment models. In contrast, a diffusive component can be left out. In general, mortality risk should not be neglected when considering the retirement phase of the life or health related insurance products.

The remaining paper is organized as follows. In Section 2, I give an overview of the related literature considering life cycle consumption-investment models especially with a focus on mortality risk and unspanned labor income. Section 3 introduces the general model setup of a life cycle consumption-investment problem with stochastic mortality risk.

In Section 4, I provide analytical results for a complete market case with deterministic labor income. I compare the results with deterministic mortality risk to results without mortality risk. Section 5 provides the calibration of the models with a special focus on the mortality risk calibration using German mortality data. In Section 6, I illustrate numerical results with unspanned labor income and short-sale and liquidity constraints for the model with deterministic hazard rate of death. I provide sensitivity analyses regarding the preference, labor income and asset parameters. Section 7 compares the numerical results with deterministic hazard rate of death with the results of an equivalent model without mortality risk. Especially, I consider the impact of the insurance and the bequest motive. In Section 8, I allow for a stochastic hazard rate of death driven by a diffusive and jump component. I comment on the importance of the insurance in the different models and consider the impact of the diffusive and jump component. Finally, Section 9 concludes and gives an outline for further research.

2 Literature

The main feature of my model is the uncertain time of death due to mortality risk. Mortality risk is rarely considered in continuous-time life cycle models, whereas it is often analyzed in the insurance literature. Cairns, Blake, and Dowd (2008) review several approaches for modeling mortality risk, both in discrete time and in continuous time. They present and compare different models and interpret mortality patterns from an insurance point of view. Especially, they stress the importance of a diffusive component since mortality rates fluctuate from year to year with a high volatility. They interpret these fluctuations as weather dependent, e.g. a hot summer or a cold winter increases mortality risk significantly especially for old agents. The models that they review do not include jumps. The main difference between the life cycle and the insurance perception of mortality risk is that the insurance literature considers aggregate mortality rates. In contrast, in the individual consumption-investment decision, I focus on an agent's perspective and consider the individual mortality risk of an agent. This difference affects both interpretation and modeling. If the actuarial literature considers jumps, it focuses mostly on negative jumps since these yield decreased profits of annuities. Furthermore, it considers jumps in aggregate mortality rates. Negative jumps in aggregate mortality rates are mainly interpreted as medical progress. Since there are little explanations for the interesting case of negative jumps, most models neglect the jump component. Positive aggregate jumps can occur due to catastrophic events like war, earthquake, tsunami, pandemic or epidemic. These jumps are mainly transitory and affect mortality rates for few years only. After the catastrophic event, the old mortality pattern returns for survivors. These positive jumps have a higher magnitude but also a low intensity and are mainly considered in life insurance as well as property or casualty insurance. In my model, individual positive shocks that occur with higher intensity are more important. These can be interpreted as health problems with a permanent impact, e.g. medical disasters like a cancer detection or an accident.

There are several papers that study the effect of mortality risk analytically. The first paper that analyzes the uncertain lifetime analytically in a continuous-time framework is Yaari (1965). Closely related to my model is the setup of Richard (1975) who solves the portfolio problem with several risky assets in continuous time with mortality risk, deterministic labor income and an endogenous insurance in closed form. He deduces that efficient portfolios are identical compared to a setup without mortality risk if the time of death is independent of other sources of risk. Ye (2006) and Pliska and Ye (2007) provide closely related extensions to the model. Amongst other things, they overcome problems with the terminal condition and present the resulting life insurance rules. Other papers studying the effect of mortality risk are Blanchard (1985), Blanchet-Scalliet et al. (2008) and Blanchet-Scalliet, Karoui, and Martellini (2005). Blanchet-Scalliet et al. (2008) extend the model setup of Merton (1971) with a stochastic time horizon. They show that the optimal portfolio decisions are influenced by the stochastic time of death if randomness in stocks and time of death are related. They derive their main results using a martingale approach. Kraft and Steffensen (2008) extend the setup with a state switch between "alive" and "dead" to a Markov chain with more states, interpreted as "unemployed" or "disabled". They focus on the optimal consumption pattern and neglect the asset allocation dimension. Bruhn and Steffensen (2011) also use the Markov chain approach and focus on a family with several mortality processes. They calculate the corresponding optimal consumption, investment and insurance decision. Kwak, Shin, and Choi (2011) consider the life insurance purchase of a family as well. They use a HARA utility as a weighted average from parents' and children's utility. The parents can buy life insurance to protect their children from an income loss that occurs if the parents suffer an early death. Huang, Milevsky, and Salisbury (2012) use the basic model from Yaari (1965) for a comparison of optimal consumption in two frameworks: one with deterministic mortality rates and another one with stochastic mortality rates modeled with a diffusive component. In the simple model with deterministic mortality rates, their results coincide with the results from Yaari (1965). Their analysis focuses on the effect of mortality risk on consumption for retired agents. They do not include an asset allocation dimension and do not model income. However, all papers mentioned in this paragraph analyze the uncertainty with respect to the time of death analytically and none of them provides results for a calibrated life cycle in continuous time or includes jumps in the hazard rate of death.

Huang, Milevsky, and Wang (2008) solve a continuous-time life cycle model with deterministic mortality risk numerically. They include a life insurance and focus especially on a correlation between labor income and the stock market. To the best of my knowledge, Kraft, Schendel, and Steffensen (2014) is the only continuous-time paper with a realistically calibrated life cycle and stochastic mortality risk. They include health jumps which increase the mortality risk significantly. Their focus lies on the analysis of the optimal term life insurance demand of a family that faces the risk of an unexpected early death of the wage earner.

In contrast to continuous-time models, there is a vast literature about discrete-time models that analyze life cycle models with deterministic mortality risk using numerical methods. The seminal paper of Cocco, Gomes, and Maenhout (2005) provides numerical results for a realistically calibrated lifetime optimization problem which also features mortality risk, unspanned labor income as well as short-sale and borrowing constraints. Especially, they analyze how unspanned labor income can be calibrated to fit real data. In a similar way, Cocco (2005) analyzes the effect of housing on asset allocation in a calibrated model. Recent research like Horneff et al. (2008), Horneff et al. (2009), Horneff, Maurer, and Stamos (2008), Maurer et al. (2013) and Chai et al. (2011) mostly focuses on different types of annuities in realistic setups considering the impact on retirement planning and the possibility to hedge longevity risk. Horneff et al. (2008) compare different strategies for retirees to invest their wealth with focus on annuities and phased withdrawal plans. Additionally, they provide a comprehensive literature overview regarding the retirement payout research. Horneff, Maurer, and Stamos (2008) numerically solve a life cycle model for an agent with Epstein-Zin preferences who can invest in illiquid life annuities with non-zero initial cost. This implies that insurance decisions cannot be revised and the insurance decision is not independent from the asset allocation decision. Furthermore, they explore the welfare impact of an incomplete insurance market that allows for gradual investment. Horneff et al. (2009) consider a different type of annuities, called survival-contingent investment-linked annuities. These products have the advantage of participating in the stock market and simultaneously pooling longevity risk. They give the surviving agents the extra return from dying agents (survival credit) at the cost of illiquidity. In contrast to classical annuities that can be considered as an implicit investment in a bond, these investment-linked annuities deliver an implicit investment in a specific portfolio. They also use Epstein-Zin preferences and allow for differences in believes considering mortality risk. Chai et al. (2011) make the model more realistic by allowing the agent to endogenously choose if he wants to work, how much hours he wants to work and when he wants to retire. They use power utility and a modified Cobb-Douglas function for the trade-off between leisure and consumption. They also allow for illiquid life annuities (not investment-linked) and differences in believes with regard to survival probabilities. Love (2010) considers the modeling of a family together with demographic shocks that allow for a varying family size. The model also features mortality risk and a simple term life insurance. Hubener, Maurer, and Rogalla (2013) focus on a couple with two mortality processes in the retirement state. They do not consider a working stage with labor income and especially analyze the optimal life insurance and annuity demand.

Recent discrete-time life cycle models also include stochastic mortality risk, however there are only few published papers. Cocco and Gomes (2012) model mortality rates as a random walk with drift. They consider the effect of stochastic mortality rates by allowing to choose the date of retirement endogenously and they analyze longevity bonds as investment products. Maurer et al. (2013) allow for systematic longevity risk, which they model with stochastic mortality tables. They make the annuity product more complex and analyze the impact of variable investment-linked deferred annuities. These products allow the payout period to begin at some predetermined specific age. As far as I know, there is no discrete-time life cycle portfolio choice paper that allows for jumps in the mortality rate.

Besides mortality risk, another important feature of my model is unspanned labor income. Merton (1971) analyzes the effect of deterministic labor income that can be summarized as an implicit investment in the risk-free asset. Bodie, Merton, and Samuelson (1992) focus on spanned labor income in a life cycle model in which agents can decide about their work effort. However, the importance of unspanned labor income is well known in the literature and outlined e.g. by Viceira (2001) as well as Cocco, Gomes, and Maenhout (2005). In continuous time, Munk and Sørensen (2010) provide results for a realistically calibrated life cycle with unspanned labor income risk, but with deterministic time of death. They analyze the optimal consumption and investment decisions in a setup with power utility, unspanned labor income and a stochastic risk-free rate.

3 Model Setup

I introduce a general model and the corresponding life cycle consumption-investment problem. I focus on four specifications of the general model that differ with respect to the mortality structure.

Financial Assets I consider a continuous-time model. The economy consists of two assets. The risk-free rate is constant and denoted by r. The agent can invest in the risky stock S with dynamics

$$\begin{aligned} dS_t &= S_t \left[\mu_S \, dt + \sigma_S \, dW_t^S \right] \\ &= S_t \left[(r + \sigma_S \lambda) \, dt + \sigma_S \, dW_t^S \right] \end{aligned}$$

with constant parameters μ_S, σ_S and a standard Brownian motion $W^S = (W_t^S)$. The parameter $\lambda = \frac{\mu_S - r}{\sigma_S}$ denotes the market price of risk. The second asset is a riskless bond B which yields the risk-free rate. The price dynamics are given by

$$dB_t = B_t r\,dt.$$

Labor Income The agent receives a stream of income Y until his time of death. Before retirement, Y is interpreted as labor income. After retirement, the payment stream can be interpreted as pension that is payed by the government and is related to earnings before retirement. Y is modeled with dynamics

$$dY_t = Y_t\left[\mu_Y(t)\,dt + \sigma_Y(t)\left(\rho(t)\,dW_t^S + \sqrt{(1-\rho(t)^2)}\,dW_t^Y\right)\right], \tag{1}$$

for $t \in [0, \tau)$ with volatility $\sigma_Y(t)$ and a standard Brownian motion $W^Y = (W_t^Y)$ that is independent of W^S. The correlation between Y and the stock S is captured by $\rho(t)$. The drift $\mu_Y(t)$ allocates the expected labor income over the life cycle.

Mortality In the general model with stochastic time of death, the agent faces mortality risk. He has an uncertain lifetime which is modeled by a doubly stochastic stopping time τ. The time of death is the first jump of the jump process $N^D = (N_t^D)$ with time-dependent and stochastic intensity π_t. The jump process N^D is independent of all other sources of risk, i.e. the Brownian motions W^S and W^Y. The hazard rate of death π_t follows

$$d\pi_t = \mu_\pi(t)\pi_t\,dt + \sigma_\pi(t)\pi_t\,dW_t^\pi + \beta(t)\,dN_t^\pi \tag{2}$$

with another Brownian motion $W^\pi = (W_t^\pi)$ and a jump process $N^\pi = (N_t^\pi)$ with time-dependent intensity $\kappa(t)$ and magnitude $\beta(t)$. These sources of risk are independent from the other Brownian motions and the death jump process. I use the drift parameter to model that the risk of dying increases when the agent gets older ($\mu_\pi(t) > 0$). The diffusion parameter captures small fluctuations in the hazard rate of death, e.g. given by a common cold. The jump term accounts for catastrophic events, like a critical illness ($\beta(t) > 0$).

Insurance The agent has the possibility to contract a simple insurance as in Richard (1975). The insurance can be interpreted as a life insurance. However, the results show that agents optimally sell the life insurance to the insurance company and therefore, I construct the insurance in the inverse way (as an annuity). For a notional of 1, the insurance pays $\pi_t\,dt$ if the insured person stays alive. If the agent dies, the insurance receives the notional. The insurance is continuously traded without transaction costs such that the agent can adjust the notional continuously. The agent chooses η_t which is the fraction of his financial wealth he wants to insure. The financial wealth of the agent is denoted by X_t. Thus, as long as the agent lives, he continuously receives $\eta_t \pi_t X_t \, dt$ from the insurance and has to pay $\eta_{\tau^-} X_{\tau^-}$ to the insurance when he dies. The insurance is actuarially fair by construction.

The assumption of such an insurance can be justified as follows: If one considers a world with a large amount of identical agents facing the same mortality risk that are all equally insured, the insurance breaks even almost surely. Thus, assuming a competitive insurance market with risk neutral insurance companies would lead to such an insurance contract in the absence of administrative fees.[1] Due to the unrealistic features of the insurance and the non-availability of such an insurance in the real world, I also consider a model setup where agents are not allowed to contract the insurance.

Preferences of the Agent The agent lives from time 0 to τ. I consider an agent with utility of the constant relative risk aversion (CRRA) type with risk aversion parameter γ. The agent wants to maximize lifetime utility of consumption and terminal wealth at every point in time $t \in [0, \tau)$. The expected utility at time t is given by

$$\mathrm{E}_{t,x,y,\pi}\left[\int_t^\tau \mathrm{e}^{-\delta(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} ds + \epsilon(t) \mathrm{e}^{-\delta(\tau-t)} \frac{X_\tau^{1-\gamma}}{1-\gamma}\right], \tag{3}$$

where δ is the time preference rate and ϵ captures the weight of the bequest motive. X_τ is the financial wealth that the agent leaves as bequest. Due to the insurance choice, the financial wealth for bequest is calculated as

$$X_\tau = (1 - \eta_{\tau^-}) X_{\tau^-},$$

since the remaining financial wealth has to be payed to the insurance.

To gain intuition, I convert the model with uncertain but finite horizon to a model with infinite horizon. For a moment, I drop the bequest motive and assume a deterministic hazard rate of death. Then, I rewrite the expected utility as

$$\mathrm{E}_{t,x,y,\pi}\left[\int_t^\infty \mathrm{e}^{-\int_t^v \delta + \pi(s) ds} u(c_v) dv\right]. \tag{4}$$

Considering (4) in detail, we see that the setup is like an infinite horizon model but with a new time preference rate $\delta + \pi(t)$. Furthermore, the time preference rate is not constant, which might yield severe problems considering model solving since this may lead to

[1] Risk averse insurance companies or insurance companies facing administrative costs would demand a higher payment in the case of death or pay a lower rate if the agent survives.

time-inconsistent behavior according to Marín-Solano and Navas (2009, 2010).[2] However, Bommier (2006b) disentangles uncertainty with respect to death and time by assuming that the agent knows that he cannot die in a certain interval. Then, he analyzes conditions under which the agent has time-consistent preferences. According to his calculations, the preferences I use in this paper are time-consistent.

Financial Wealth Dynamics and the Optimization Problem The financial wealth of the agent is denoted by X. The agent chooses optimal consumption c and optimal portfolio holdings θ, where θ denotes the fraction of financial wealth invested into the stock. The remaining financial wealth $(1-\theta)X$ is invested into the bond. Furthermore, the agent decides about the fraction of financial wealth that is insured η. Given the consumption, portfolio holdings and insurance decision, the financial wealth evolution for $t \in [0,\tau]$ has the dynamics

$$dX_t = X_{t^-}\left[\left(r+\pi_{t^-}\eta_{t^-}+\theta_{t^-}\lambda\sigma_S\right)dt+\theta_{t^-}\sigma_S\,dW_t^S\right]+(Y_{t^-}-c_{t^-})\,dt-\eta_{t^-}X_{t^-}\,dN_t^D. \quad (5)$$

The objective of the agent is maximizing utility from intermediate consumption and terminal wealth. Hence, the optimization problem of the agent is given by

$$\begin{aligned}\max_{c,\theta,\eta}\quad & \mathrm{E}_{t,x,y,\pi}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}du+\epsilon(t)\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]\\ \text{s.t.}\quad & dX_t = X_{t^-}\left[\left(r+\pi_{t^-}\eta_{t^-}+\theta_{t^-}\lambda\sigma_S\right)dt+\theta_{t^-}\sigma_S\,dW_t^S\right]+(Y_{t^-}-c_{t^-})\,dt-\eta_{t^-}X_{t^-}\,dN_t^D. \quad (6)\end{aligned}$$

I define the value function (indirect utility) J as

$$J(t,x,y,\pi)=\sup_{\{c_s,\theta_s,\eta_s\}_{s\in[t,\tau)}}\mathrm{E}_{t,x,y,\pi}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}du+\epsilon(t)\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]. \quad (7)$$

Calculating the corresponding Hamilton-Jacobi-Bellman equation (HJB) yields

$$\begin{aligned}\delta J = \sup_{c,\theta,\eta}\quad \Bigg\{ & \frac{c^{1-\gamma}}{1-\gamma}+J_t+J_x\left[x\left(r+\eta\pi+\theta\lambda\sigma_S\right)+y-c\right]+\frac{1}{2}J_{xx}x^2\theta^2\sigma_S^2\\ & +J_y y\mu_Y+\frac{1}{2}J_{yy}y^2\sigma_Y^2+J_{xy}xy\sigma_S\sigma_Y\rho\theta\\ & +J_\pi\pi\mu_\pi+\frac{1}{2}J_{\pi\pi}\pi^2\sigma_\pi^2\\ & +\pi\left[J(\tau,(1-\eta)x,0,0)-J(t,x,y,\pi)\right]\end{aligned}$$

[2] Marín-Solano and Navas (2009) comment on the problems occurring with time-dependent time preference rates and illustrate how to derive Hamilton-Jacobi-Bellman equations for pre-commited, naive and sophisticated agents with time-inconsistent preferences. Marín-Solano and Navas (2010) apply the approach in a Merton (1969, 1971) setup to derive optimal consumption and portfolio rules for these agents.

$$+\kappa\left[J(t,x,y,\pi+\beta)-J(t,x,y,\pi)\right]\Bigg\}, \tag{8}$$

where I use subscripts for partial derivatives of J, for example: $J_x = \frac{\partial J}{\partial x}$.

Special Cases In the following sections, I consider four model specifications with different mortality structure. Increasing in uncertainty with respect to mortality risk, the four specifications are derived from the general model as follows:

In model D, the time of death is deterministic and known to the agent. I get this model from the above setup by setting τ equal to a constant which implicates $\pi = \kappa = 0$. This case leads to a classical finite horizon model. The insurance has no effect, π is not needed as state variable and the HJB (8) gets the terminal condition $J(\tau,x,y) = \epsilon(\tau)\frac{X_\tau^{1-\gamma}}{1-\gamma}$.

The model S has a stochastic time of death, but the hazard rate of death itself is not stochastic. I get the setup by setting $\sigma_\pi = \kappa = 0$. Furthermore, π as state variable is redundant in this case, since there is no additional uncertainty captured by π due to the time state variable t.

In the model SD the agent faces stochastic mortality risk with a diffusion component. I set $\kappa = 0$.

The SDJ model with most sources of risk additionally includes the jump component in the hazard rate of death. This setup is exactly as presented above.

4 Analytical Results

This section provides analytical results for a special case with deterministic labor income for the models D and S. However, analytical results cannot account for important realistic features like a non-negativity constraint on financial wealth and a short-sell constraint. Furthermore, I simplify the problem by setting $\sigma_Y = 0$. In this case, the market is complete and the labor income can be replicated using the riskless bond. Alternatively, I could set $\sigma_Y \neq 0$ but $\rho = \pm 1$ instead. Then, labor income is spanned (it can be replicated using the stock and the bond), the market is complete and analytical results can be derived. However, a perfect correlation of the stock and labor income is highly unrealistic, which is why I stick to the assumption of $\sigma_Y = 0$ here.

First, I present the results for the model with deterministic time of death D in the special case of deterministic labor income. The results are summarized in the following proposition.

Proposition 1. *For a complete market case with deterministic labor income, $\sigma_Y = 0$, the optimization problem (6) of the model D can be solved analytically. The indirect utility (7) can be expressed as*

$$J^D(t,x,y) = \frac{1}{1-\gamma}\left(x + y f^D(t)\right)^{1-\gamma} g^D(t)^\gamma,$$

for $t \in [0,\tau]$ with

$$\begin{aligned} f^D(t) &= \int_t^\tau \mathrm{e}^{\int_t^s \mu_Y(u) - r\,du}\,ds, \\ g^D(t) &= \int_t^\tau \mathrm{e}^{\left(\frac{1-\gamma}{\gamma}r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(s-t)}\,ds + \epsilon(\tau)^{\frac{1}{\gamma}}\,\mathrm{e}^{\left(\frac{1-\gamma}{\gamma}r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(\tau-t)} \\ &= \frac{1}{\frac{1-\gamma}{\gamma}r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}}\left(\mathrm{e}^{\left(\frac{1-\gamma}{\gamma}r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(\tau-t)} - 1\right) \\ &\quad + \epsilon(\tau)^{\frac{1}{\gamma}}\,\mathrm{e}^{\left(\frac{1-\gamma}{\gamma}r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(\tau-t)}. \end{aligned}$$

For $t \in [0,\tau)$, the optimal controls for consumption and portfolio holdings are

$$\begin{aligned} c^D(t,x,y) &= \frac{x + y f^D(t)}{g^D(t)}, \\ \theta^D(t,x,y) &= \frac{1}{\gamma}\frac{x + y f^D(t)}{x}\frac{\lambda}{\sigma_S}. \end{aligned}$$

Proof. The formulas can be verified by substituting the results for J, c, θ into the HJB (8). The derivation follows the lines of the proof of Proposition 2 and is therefore skipped here. □

Next, I consider the model S with stochastic time of death. In the case of deterministic labor income ($\sigma_Y = 0$), the optimization problem (6) can be solved analytically using a similar approach as above. The results are summarized below.

Proposition 2. *With deterministic labor income, $\sigma_Y = 0$, the optimization problem (6) of the model S can be solved analytically and the indirect utility (7) is given by*

$$J^S(t,x,y) = \mathbb{1}_{\{t<\tau\}}\left(\frac{1}{1-\gamma}\left(x + y f^S(t)\right)^{1-\gamma} g^S(t)^\gamma\right) + \mathbb{1}_{\{t=\tau\}}\left(\epsilon(t)\frac{x^{1-\gamma}}{1-\gamma}\right),$$

for $t \in [0,\tau]$ where

$$\begin{aligned} f^S(t) &= \int_t^\infty \mathrm{e}^{\int_t^s \mu_Y(u) - r - \pi(u)\,du}\,ds, \\ g^S(t) &= \int_t^\infty \mathrm{e}^{\int_t^s \frac{1-\gamma}{\gamma}r - \frac{1}{\gamma}\delta - \pi(u) + \frac{1}{2}\frac{1-\gamma}{\gamma^2}\lambda^2\,du}\left(1 + \pi(s)\epsilon(s)^{\frac{1}{\gamma}}\right)ds. \end{aligned}$$

The optimal controls for $t \in [0,\tau)$ are

$$c^S(t,x,y) = \frac{x + yf^S(t)}{g^S(t)},$$
$$\theta^S(t,x,y) = \frac{1}{\gamma}\frac{x + yf^S(t)}{x}\frac{\lambda}{\sigma_S},$$
$$\eta^S(t,x,y) = 1 - \frac{x + yf^S(t)}{g^S(t)x}\epsilon(t)^{\frac{1}{\gamma}}.$$

Proof. The formulas can be verified by substituting the results for J, c, θ, η into the HJB (8). A derivation is given in Appendix A. □

Insurance Demand Note that η can be rewritten as

$$\eta^S(t,x,y) = 1 - \frac{c^S(t,x,y)}{x}\epsilon(t)^{\frac{1}{\gamma}}. \tag{9}$$

The optimal fraction of insured wealth depends only on the actual consumption-to-wealth ratio, the weighting of the bequest motive and the risk aversion parameter. But, it is independent of the hazard rate of death. This is worth noting since the mortality risk is the intensity that triggers the event of death and is therefore crucial for insurance related payoffs. However, due to the insurance being actuarially fair and the agent being risk neutral with respect to the time of death, the fraction insured is independent of the mortality risk. Substituting (9) into the bequest if death occurs yields

$$\left(1 - \eta^S(t,x,y)\right)x = \left(1 - 1 + \frac{c^S(t,x,y)}{x}\epsilon(t)^{\frac{1}{\gamma}}\right)x = c^S(t,x,y)\epsilon(t)^{\frac{1}{\gamma}}.$$

The amount of bequest is expressed as the actual consumption level weighted with the bequest importance parameter to the power of the intertemporal elasticity of substitution. This indicates that in the complete market the insurance is only used to ensure optimal bequest if death occurs. Furthermore, the bequest is intended and not accidental due to the insurance.

Comparing the Complete Market Formulas I start the comparison with a simpler model without bequest motive. It follows immediately that it is optimal for the agent to insure his whole wealth: If his wealth is positive and the agent survives, he receives an insurance premium which is a benefit for the agent. If the agent dies, he loses all his wealth. However, the agent is indifferent between leaving money on the table and paying the notional to the insurance since he has no bequest motive. Furthermore, one can prove the intuitive statement that the agent wants to fully insure his financial wealth if there is no bequest motive via substituting $\epsilon = 0$ in the results from Proposition 2. This yields $\eta(t,x,y) = 1$ and hence, the whole financial wealth is insured.

I consider the analytical results for the complete market of the Propositions 1 and 2 and substitute $\epsilon = 0$. The structure of the indirect utility and optimal controls is identical in both setups. The effect of mortality risk is embedded in the functions g and f. Writing the results from Proposition 1 and 2 in a slightly different manner (using $\epsilon = 0$), I get

$$
\begin{aligned}
f^D(t) &= \int_t^\tau \mathrm{e}^{\int_t^s \mu_Y(u) - r\,du}\, ds, \\
f^S(t) &= \int_t^\infty \mathrm{e}^{\int_t^s \mu_Y(u) - r\,du}\, \mathrm{e}^{-\int_t^s \pi(u)\,du}\, ds, \\
g^D(t) &= \int_t^\tau \mathrm{e}^{\left(\frac{1-\gamma}{\gamma} r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(s-t)}\, ds, \\
g^S(t) &= \int_t^\infty \mathrm{e}^{\left(\frac{1-\gamma}{\gamma} r - \frac{\delta}{\gamma} + \frac{1}{2}\frac{(1-\gamma)\lambda^2}{\gamma^2}\right)(s-t)}\, \mathrm{e}^{-\int_t^s \pi(u)\,du}\, ds.
\end{aligned}
$$

The only difference is that the integrals of the model D run to infinity instead to τ and the term $\mathrm{e}^{-\int_t^s \pi(u)\,du}$, which equals the survival probability from time t to s, is added. Thus, the effect of added mortality risk can be interpreted as a different discounting here. Nothing else changes.

Now adding the bequest motive, the formulas for f stay the same and the g formulas get the additional bequest term as in the Propositions. The bequest term enters weighted with the hazard rate of death in the S model for each possible time of death to capture the uncertainty with respect to death. In contrast, the bequest occurs at the fixed time τ in the D model and is discounted. Hence, the different weighting of the bequest can also be interpreted as a different discounting.

Whereas the formulas are useful to provide a rough intuition, analytical solutions are not possible for realistically calibrated setups in which labor income uncertainty is an important source of risk.

5 Calibration

For a numerical analysis of the effects of mortality risk, I calibrate the models. The calibration and the parameters that I choose for assets, labor income and preferences are closely related to the calibration used by Munk and Sørensen (2010) and Cocco, Gomes, and Maenhout (2005). For calibrating the mortality risk I use German mortality data.

Assets I choose the parameters for the assets as

$$\mu_S = 0.06,$$

$$\sigma_S = 0.2,$$
$$r = 0.02,$$

which are similar to the values of Munk and Sørensen (2010) and Cocco, Gomes, and Maenhout (2005).

Labor Income Calibrating the labor income process, I mainly follow Munk and Sørensen (2010). They adapt the calibration from Cocco, Gomes, and Maenhout (2005) to a continuous time model. Cocco, Gomes, and Maenhout (2005) model the drift of the labor income process before retirement as a polynomial of the age of the agent for different education groups (no high school, high school, college) using data from the Panel Study of Income Dynamics (PSID). After retirement, the drift is set to zero and the agent receives a fixed fraction of the income before retirement as pension. The continuous-time adaption from Munk and Sørensen (2010) yields for the labor income drift

$$\mu_Y(t) = \begin{cases} \xi_0 + b + 2ct + 3dt^2 & \text{for } t < T_{ret}, \\ -(1-P) & \text{for } T_{ret} \le t \le T_{ret}+1, \\ 0 & \text{for } t > T_{ret}+1, \end{cases} \tag{10}$$

where b, c, d are constant parameters dependent on the education level, originally estimated by Cocco, Gomes, and Maenhout (2005). ξ_0 captures an real wage increase that is independent of the age and the education. P determines the income reduction when going into retirement. T_{ret} denotes the age at which the agent retires. I take the parameter calibration from Munk and Sørensen (2010) (Table 4) for college graduates

$$\begin{aligned} T_{ret} &= 65, & b &= 0.3194, \\ \xi_0 &= 0.02, & c &= -0.00577, \\ P &= 0.93887, & d &= 0.000033, \\ Y_0 &= 13\,912. \end{aligned}$$

Y_0 denotes the starting income at the age of 20 which depends on the education. The volatility function that I use allows different values for the working period and the retirement period with linear interpolation when the status changes, hence

$$\sigma_Y(t) = \begin{cases} \sigma_Y^w & \text{for } t < T_{ret}, \\ \sigma_Y^w - \left(\sigma_Y^w - \sigma_Y^r\right)(t - T_{ret}) & \text{for } T_{ret} \le t \le T_{ret}+1, \\ \sigma_Y^r & \text{for } t > T_{ret}+1. \end{cases} \tag{11}$$

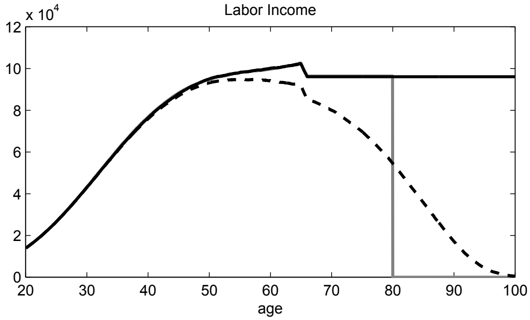

Figure 1: Average Labor Income Profile over the Life Cycle. The black solid line depicts the average labor income of living agents in a model with stochastic time of death (S). The dashed line shows the expected labor income with stochastic time of death. The grey line corresponds to the model with deterministic time of death (D). The labor income process is calibrated with the values from Munk and Sørensen (2010) for college graduates. The parameter calibration is stated in Section 5.

I assume the same structure for the correlation with the risky asset

$$\rho(t) = \begin{cases} \rho^w & \text{for } t < T_{ret}, \\ \rho^w - \left(\rho^w - \rho^r\right)(t - T_{ret}) & \text{for } T_{ret} \le t \le T_{ret} + 1, \\ \rho^r & \text{for } t > T_{ret} + 1. \end{cases} \tag{12}$$

In the benchmark calibration, I stick to the values used by Munk and Sørensen (2010) for the volatility and correlation:

$$\sigma_Y^w = 0.2, \qquad \rho^w = 0,$$
$$\sigma_Y^r = 0, \qquad \rho^r = 0.$$

The resulting pattern for expected labor income is shown in Figure 1.

Mortality Risk To calibrate the mortality process, I use mortality data for Germany. I calibrate the models D, S, SD and SDJ separately. In the deterministic time of death model D, I set $\tau = 80$, which is the average life expectancy in Germany for newborns in 2010.[3] For calibrating the stochastic time of death models, I use age-dependent mortality

[3] The number is estimated using life expectancy at birth, total (years). "Life expectancy at birth indicates the number of years a newborn infant would live if prevailing patterns of mortality at the time of its birth were to stay the same throughout its life." Source: WDI and GDF 2010. The estimate for a child in Germany at 2010 is given by 79.988. The source for this information, from which I directly cite here, is The World Bank Group. The data is available online at: `http://search.worldbank.org/data?qterm=SP.DYN.LE00.IN`, last access: January 21, 2014.

	b	m	σ_π
S	8.8	84.56	0.00
SD	8.3	83.68	0.05
SDJ	4.7	87.55	0.10

Table 1: Mortality Process Parameters. The table gives the drift and diffusion parameter calibration of the mortality process that I use in Section 8.

data.[4] The mortality data is given for males and females separately. I weight both genders equally and calculate the corresponding average values. Considering the drift component of the hazard rate of death, I rewrite the initial value and drift parameter to the classical Gompertz form. The initial value and drift are then expressed as

$$\mu_\pi = \frac{1}{b},$$
$$\pi_0 = \frac{1}{b}\,\mathrm{e}^{\frac{x-m}{b}}$$

with constant parameters x, m, b. The interpretation of x is the age at $t = 0$. m sets the x-axis displacement and the growth rate b influences the steepness of the curve. I set the starting age $x = 20$ in all models. In Section 6 and 7, I show results for the model S and compare them to the deterministic time of death model D. In this case, when I have no diffusive and jump component, I can directly express the hazard rate of death and get the standard Gompertz form

$$\pi(t) = \frac{1}{b}\,\mathrm{e}^{\left(\frac{x+t-m}{b}\right)}.$$

In the model S in Section 6 and 7, I set $b = 8.9$ and $m = 85.1$, which leads to an average age of death of $\mathrm{E}[\tau] = 80$. Considering the comparison of the S, SD and SDJ model in Section 8, I have to stick to the stochastic differential equation definition (2) of the hazard rate of death. I calibrate the models such that the average age of death equals $\mathrm{E}[\tau] = 80$ in all models. Furthermore, the parameters are set such that the time of death distribution fits the empirical values of Germany. The calibration of the parameters m, b and the diffusive component σ_π are given in Table 1. Note that the model S is calibrated differently in

[4] Mortality data is taken of a Life table for Germany "Sterbetafel 2009/11, Statistisches Bundesamt, 2013", available online at: `https://www.destatis.de/DE/ZahlenFakten/GesellschaftStaat/Bevoelkerung/Sterbefaelle/Tabellen/SterbetafelDeutschland.html`, last access: January 21, 2014.

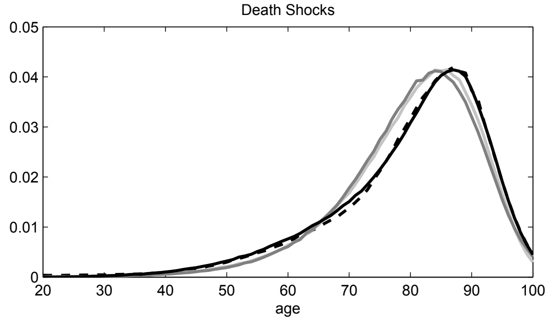

Figure 2: Biometric Risk Calibration Results. The graph compares the gender-averaged German mortality data (dashed line) with my model calibrations (solid lines). The figure depicts the death distribution for a normalized population of size 1. My simulated death shocks are averaged after simulating 1 000 000 times of death. The light line represents the model S, the grey line the model SD and the black line the model SDJ. The calibration can be found in Section 5.

Section 8 compared to Section 6 and 7.[5] Comparing the S, SD and SDJ model calibrations, we can see that the model S has no uncertainty about future mortality patterns, whereas the model SD has little uncertainty about future mortality rates, and the model SDJ has plenty of uncertainty considering the future mortality situation. Next, I give the results for the calibration of the jump term of the hazard rate of death in the SDJ model. I interpret a jump in the hazard rate of death as a critical illness and use gender-averaged cancer data for Germany for the calibration.[6] The health jump calibration is taken from Kraft, Schendel, and Steffensen (2014) where it is described in detail. Thus, I set the health jump intensity to

$$\kappa(t) = 0.02489\,\mathrm{e}^{-\left(\frac{\min(t,65)-66.96}{29.42}\right)^2},$$

and the corresponding jump magnitude to

$$\beta(t) = 0.048 + 0.0008t.$$

Figure 2 depicts the empirical mortality pattern, compared to the resulting mortality patterns of the S, SD and SDJ calibration. We directly see that the calibration of the model

[5] The different calibration results from the differences in the calculation method. The first method directly expresses π, whereas the second method expresses the increase of π via a differential equation. Numerically small deviations are not avoidable although both expressions are analytically identical. The difference between both techniques decreases with a decreasing numerical choice of the length of dt. Although the effect is very small, I use slightly different calibrations here to capture the impact.

[6] The German cancer data is taken from “Cancer in Germany 2007/2008, German Centre for Cancer Registry Data & Robert Koch Institute, 8th Edition 2012”, available online at: `http://www.rki.de/EN/Content/Health_Monitoring/Cancer_Registry/cancer_registry_node.html`, last access: January 21, 2014.

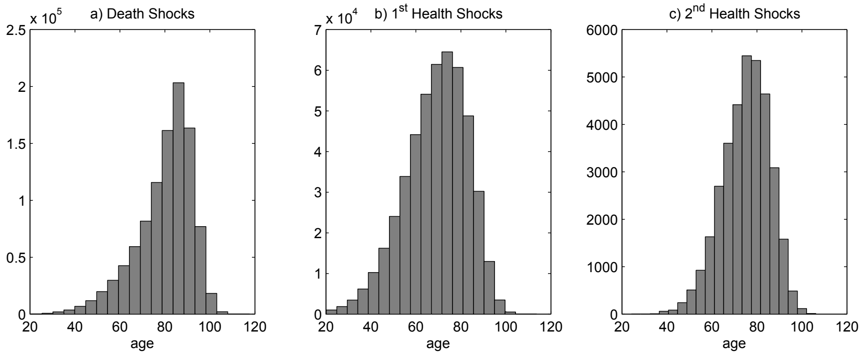

Figure 3: Histogram of Death and Health Shocks in the SDJ Model. The graphs depict the histogram of death and health jumps after 1 000 000 simulations in the model SDJ with the calibration of Section 5. a) shows the death jump distribution, b) presents the histogram of the first health shock and c) depicts the second health shocks.

SDJ fits the empirical mortality pattern pretty well, whereas the S and SD calibration do significantly worse. In early years for the age of 20 until 35, none of the three models is able to capture the relatively high empirical number of deaths. Then up to the age of 63, the models S and SD underestimate the number of deaths significantly, whereas the model SDJ only slightly overestimates the empirical values. Afterwards roughly until the age of 85, the models S and SD consequently overestimate death rates. Especially, the peak appears too early. After the age of 85, the models S and SD underestimate the empirical values again. The model SDJ and the data show a strong co-movement, especially after the age of 75. The calibration results highlight that the jump component is essential to explain the empirical mortality pattern.

Figure 3 depicts histograms of death and health shocks for the SDJ model after 1 000 000 simulations. In the sample the average time of death is 80. The average time of the first health shock is 68.9, where 47.8% of the population face health shocks during their lifetime. 3.5% face at least two health shocks with the second one at the average age of 75.1. Only 0.2% of the population face three health shocks or more.

Preferences I choose the risk aversion, the time preference rate and the weight of the bequest motive according to

$$\delta = 0.03,$$
$$\gamma = 4,$$
$$\epsilon = 3.$$

These values are taken from Munk and Sørensen (2010). I set the initial financial wealth equal to the first labor income $X_0 = Y_0 = 13\,912$.

Unfortunately, there is no clear evidence about the impact of a bequest motive in the literature.[7] Cocco, Gomes, and Maenhout (2005) model a bequest motive between 0 and 5. Munk and Sørensen (2010) use values of 1 or 3 for the weight of the bequest motive. In my model, agents face mortality risk throughout their life which raises the question how a bequest motive changes over time. On the one hand, young people normally do not think about a potential death and bequest, whereas the topic is more present to older people. This might indicate an increasing bequest motive over the lifetime. On the other hand, young people contract term life insurance to protect their partner and children in the case of death. In later years, the children are grown up and there is no substantial need for a bequest. These thoughts indicate a decreasing bequest motive. Since there is no clear evidence how the bequest motive should behave over the life cycle, I stick to a constant bequest motive here although the model setup allows a time-dependent bequest motive.

6 Results with Stochastic Time of Death

This section provides numerical results and interpretation for the model S with stochastic time of death but a deterministic hazard rate of death. I show policy functions, the optimal controls and the wealth evolution over the life cyle and provide sensitivity analyses.

Starting from now, I present numerical results for models with unspanned labor income, short-sale and liquidity constraints. This means, I omit the restriction of $\sigma_Y = 0$ from Section 4 to make labor income stochastic and unspanned. Furthermore, agents are restricted to portfolio holdings $\theta_t \in [0,1], \forall t$, which avoids short selling. Additionally, agents have to choose optimal controls such that $X_t > 0, \forall t$. Thus, for low levels of financial wealth, agents have to consume less than their income and are not allowed to invest in the risky asset if financial wealth could go negative. I also restrict the insurance decision to $\eta_t \in [0,1], \forall t$ such that only available financial wealth can be insured and negative insurance holdings are forbidden.

[7] Abel (1985) argues that agents leave unintended bequest due to mortality risk although agents do not have a bequest motive. Bernheim, Shleifer, and Summers (1985) consider a strategic bequest motive. Parents use bequest to influence their children's behavior. They claim that agents have a strategic bequest motive and provide empirical evidence for their results. Hurd (1989) analyzes empirically whether agents have a bequest motive. According to his results, the motive for intended bequest is nearly zero and observed bequest is mostly accidental. Laitner (2002) provides a comparison of different models with bequest using calibrated simulations. In order to match the data, the models need a positive bequest motive and he gets best results if agents care roughly the same about themselves and their children. Lockwood (2012) argues that the bequest motive is crucial for agents not to insure wealth. He uses the bequest motive as a possible explanation for the annuity puzzle. In his simulations, he gets best results (realistic amounts of insured wealth) if he assumes a bequest motive.

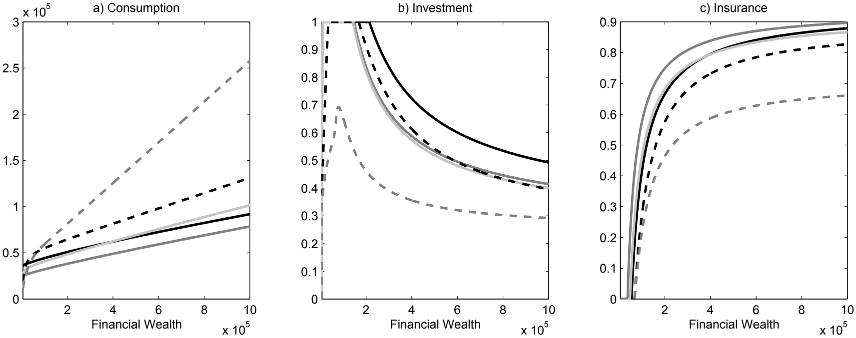

Figure 4: Policy Functions for the S Model. The graphs depict policy functions with varying financial wealth for differently aged agents for fixed labor income and age. The labor income is fixed to 50 000 for all lines. The solid lines are for an agent in the working phase of the age of 25 (dark line), age 40 (grey line) and age 60 (light line). The dashed lines represent a retired agent of the age of 70 (dark line) and age 90 (grey line). a) plots consumption for varying financial wealth, b) shows the corresponding portfolio holdings and c) depicts the insurance decision. The model S is calibrated with the parameters of Section 5.

In the following, I show results that are based on the numerical procedure described in Appendix B for different calibrations. All models include the constraints introduced here. The benchmark calibration is as presented in Section 5 for the model S. The figures depict averaged results after 100 000 simulations. Especially, I analyze the effect of the bequest motive, the driver of the optimal insurance decision and the impact of the labor income parameters. Furthermore, I provide sensitivity analyses with respect to preference and asset parameters.

Policy Functions First, I present policy functions for different ages and a fixed income of 50 000 in Figure 4. On the x-axis is financial wealth. The graphs illustrate the optimal portfolio holdings, fraction of insured wealth and consumption for the working phase (age 25, 40 and 60, solid lines) and for the retirement phase (age 70 and 90, dashed lines). Lemma 3 in Appendix B shows that the optimal controls only depend on the fraction $\frac{x}{y}$. Therefore, it is sufficient to restrict the consideration on the policy functions for financial wealth. Policy functions for labor income would deliver the inverse results.

The consumption graph increases in financial wealth, as usual. Consumption steepness is increasing in age, which can be interpreted as an increase in the time preference rate due to the increased mortality risk. Older agents with more financial wealth want to use their excess wealth before they die and, thus, consume more. Poor old agents consume less than younger ones since they want to accumulate at least a bit financial wealth for bequest.

The amount riskily invested is generally decreasing in financial wealth and decreasing in age. The effect of the secure labor income in retirement is an increase of risky invest-

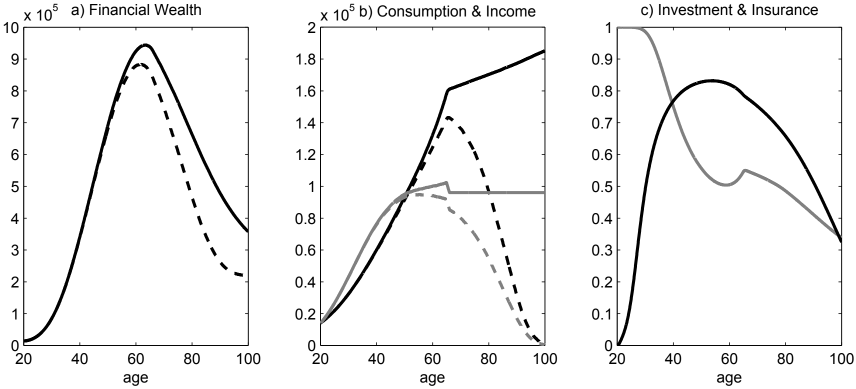

Figure 5: Optimal Wealth and Controls over the Life Cycle in the Model S. a) depicts the average optimal financial wealth over the life cycle. The solid line represent the average financial wealth of living agents. The dashed line shows the average financial wealth of all agents (dead agents are counted with their amount of bequest). b) depicts average optimal consumption (dark lines) and labor income (grey lines) over the life cycle. Solid lines indicate averages for living agents only, whereas dashed lines also include dead agents. c) presents the average optimal fraction riskily invested (grey line) and the average optimal fraction insured (dark line). The model calibration is given in Section 5.

ments. Since labor income changes from being risky to being risk-free, the implicit risky investment due to labor income is gone and, hence, the optimal explicit risky investment increases to offset this effect. With more financial wealth, labor income becomes less important. For diversification purposes, the agent reduces the risky investment and the effect of the change in volatility is also less pronounced. For very low levels of financial wealth, the risky investment goes to zero due to the introduced liquidity constraint.

The insurance offers mainly a choice between actual wealth and bequest. Contracting the insurance means more wealth while alive but less bequest. The insurance holdings are increasing in financial wealth for all ages. More financial wealth means more wealth for bequest. This allows the agent to increase the fraction insured since the bequest motive is already fulfilled. One can observe that the insurance holdings for the ages 25 and 60 intersect at the same level of financial wealth as the corresponding consumption lines. This indicates a relation between the insurance choice and the consumption level, as in the complete market in Section 4.

Benchmark Results Figure 5 depicts the average optimal controls and average wealth evolution over the life cycle for the benchmark calibration.

The average financial wealth of living agents and the expected financial wealth of all agents are increasing until shortly before retirement. This is due to excess income over consumption in early years, a positive average return of the invested financial wealth and

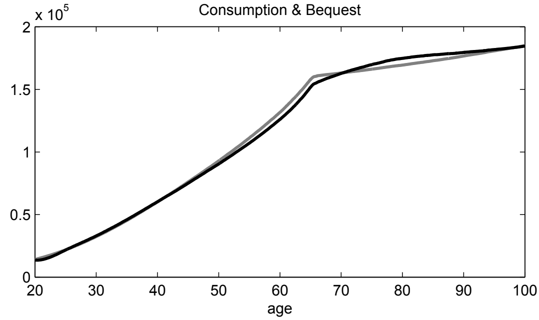

Figure 6: Consumption and Expected Bequest over the Life Cycle. The grey line depicts the average optimal consumption of living agents in the model S with a bequest motive of $\epsilon = 1$. The dark line corresponds to the average optimal bequest that would occur if the agent died (calculated as $(1-\eta_t)X_t$). The bequest motive is changed to $\epsilon = 1$, the remaining calibration is as presented in Section 5 for the model S.

the insurance premium for living agents. Afterwards, the financial wealth decreases due to excess consumption over income. However, the positive return from investment and the increasing insurance premium mitigate the decrease. The expected financial wealth of all agents approaches the average amount of bequest.

The consumption of the living agents is increasing throughout the lifetime with a jump in consumption growth at retirement. The expected consumption of all agents (where dead agents consume zero, i.e. $c_t = 0$ for $t \geq \tau$) is increasing until retirement and decreases to zero afterwards due to mortality. Comparing consumption with labor income, we can see that labor income exceeds consumption until the age of 50 in order to accumulate financial wealth. Afterwards, consumption exceeds labor income to ensure an increasing consumption path over the life cycle and dissave excess financial wealth.

The fraction riskily invested is decreasing over the lifetime, as usual. With mortality risk, the risky investment decreases with increasing risk of dying. At retirement, the labor income volatility decreases which affects the implicit risky investment. Therefore, we observe the slight increase in risky investment at retirement to offset this effect.

The insurance holdings show a hump-shaped pattern. The next paragraph explains that the agent chooses the insurance holding at every point in time such that if death occurs, he realizes the desired amount of financial wealth for bequest. With the optimal insurance decision the agent avoids accidental bequest. The insurance is the best way for the agent to allocate wealth to bequest since other controls are not affected and the financial wealth evolution only slightly depends on the insurance holdings as long as mortality risk is low. Hence, the hump-shaped insurance choice over the life cycle is explained by the hump-shaped financial wealth graph.

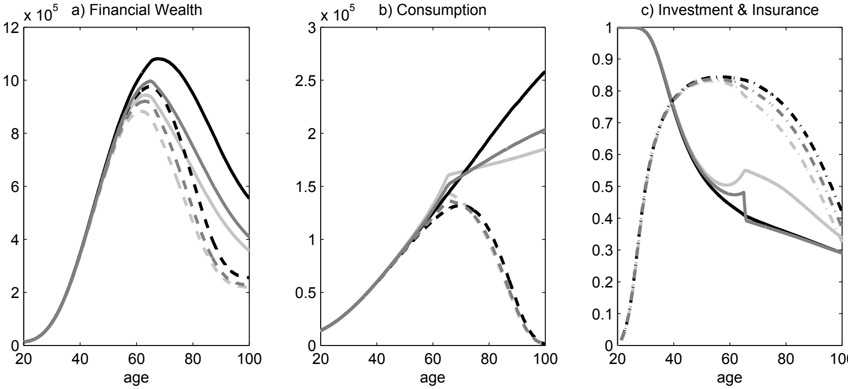

Figure 7: Sensitivity Analysis for Retirement Income. The graphs compare the average optimal controls and financial wealth evolution for different calibrations of the retirement income in the S model: $\sigma_Y^r = 0$ and $\rho^r = 0$ (light lines), $\sigma_Y^r = 0.1$ and $\rho^r = 0.2$ (grey lines), $\sigma_Y^r = 0.2$ and $\rho^r = 0$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

The Impact of the Insurance To shed light on how the agent uses the insurance and to identify the main driver of the insurance decision, I consider the optimal insurance decision together with the amount of bequest in detail here. Figure 6 depicts the optimal average consumption (grey line) in a model where the bequest motive is set to $\epsilon = 1$. The black line represents the amount of bequest that the agent would leave if he would die at the age specified on the x-axis. The bequest is determined by the insurance decision and the actual financial wealth, calculated as $(1-\eta_t)X_t$. The utility function (3) indicates that in a model with a bequest motive of $\epsilon = 1$, the bequest is valued equal to one year consumption. The figure depicts a strong co-movement of the consumption and the bequest line. Thus, in the model the insurance is used by the agents to allocate optimal bequest and not to capture other investment or hedging motives. Furthermore, the graph shows that there should be approximately no accidental bequest in the presence of the insurance. Hence, the agents use the insurance as outlined in the complete market setup in Section 4 where I show this result analytically.

The Impact of Income The most important source of wealth in this setup is income. Here, I consider the impact of varying the parameters that drive the income process.

Figure 7 depicts the effects of changing the retirement income. The income drift remains unchanged, i.e. equals zero during the retirement phase, but I allow for a risky retirement income. I compare the results with the benchmark calibration with a secure retirement income, given by the light lines.

The grey lines depict results for a risky retirement income ($\sigma_Y^r = 0.1$) and a positive correlation with the stock ($\rho^r = 0.2$). This parametrization can be justified by the assumption that the retirement income depends on the financial situation of the country which is positively linked to a stock market index. Alternatively, one could argue that the agent contracted retirement products with a stock-related return. The main change appears at the portfolio holdings when going into retirement. Portfolio holdings decrease since retirement income now has an implicit investment in the risky asset due to the positive correlation with the stock. To incorporate this effect, the explicit investment decreases for diversification purposes. Thus, retirement portfolio holdings are lower with risky retirement income and a positive correlation with the stock. Furthermore, retirement consumption growth is larger and the agent accumulates more financial wealth to capture retirement income risk.

The dark lines show a setup where I assume that labor income and pension payments have the same patterns of risk. Hence, I use $\sigma_Y^r = 0.2$ and $\rho^r = 0$ during retirement. This pattern might be more realistic if there is no social security system that accounts for a certain retirement income. The graphs for consumption and portfolio holdings have no significant change in steepness when retirement occurs. Thus, the changing consumption and portfolio holdings during retirement in the benchmark model occur due to a change in volatility and correlation and not due to the changing drift at retirement. Compared with the benchmark calibration, retirement portfolio holdings are lower due to the implicit risky investment of retirement income. Retirement consumption growth and the amount of accumulated financial wealth increases further.

Up to now, I examined changes in retirement income. In the following, I vary the income volatility during the working phase $\sigma_Y^w \in \{0.15, 0.2, 0.25\}$ in Figure 8. The less risky the income, the more similar are the working and the retirement phase. For a lower income volatility, the consumption increases in early years since the agent needs less financial wealth as buffer for negative income shocks. Due to more consumption in early years, the agent accumulates less financial wealth and has less bequest on average as well as less consumption in later years. With a less risky income, there is more investment in the risky asset since the implicit risky investment due to labor income is lower. Furthermore, the wealth effect increases the risky investment as well. The fraction insured decreases due to the wealth effects. After retirement, the setups are identical. Hence, the observed differences in consumption, portfolio holdings and insurance holdings after retirement are all due to the wealth effect.

So far, I restricted my analysis to a change in the volatility and the correlation of income. Figure 9 analyzes the effect of different expected earnings profiles over the life cycle. In

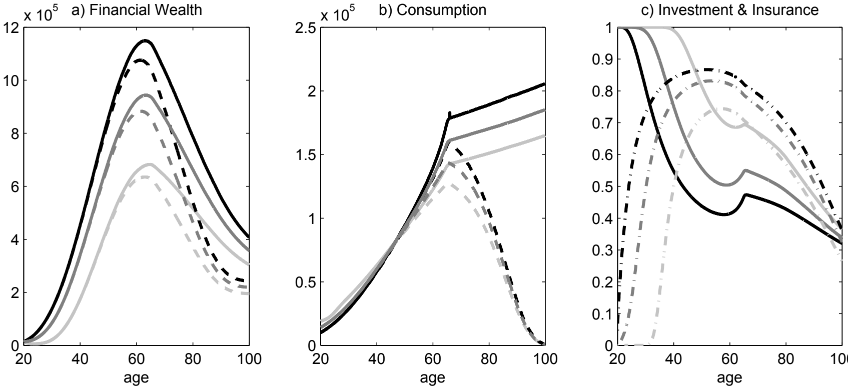

Figure 8: Sensitivity Analysis for Labor Income Volatility. The graphs compare the average optimal controls and financial wealth evolution for different values of labor income volatility in the working phase of the S model: $\sigma_Y^w = 0.15$ (light lines), $\sigma_Y^w = 0.2$ (grey lines) and $\sigma_Y^w = 0.25$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

addition to the college degree calibration, I use the parameter calibration from Munk and Sørensen (2010) for agents with high school degree and no high school education. The income graph shows that the college graduate starts with less initial labor income but has a huge expected increase in income, whereas the loss due to retirement is rather small. Agents with high school education start with the highest initial labor supply but have a huge decrease in labor income when getting retired. The agents without high school education are on average worse off throughout their lifetime but income does not decrease as much when going into retirement. The consumption graph shows that the college graduate consumes on average more throughout his lifetime although he starts with less initial labor supply. This is due to the high expected labor income in the future and his willingness to smooth consumption. Therefore, financial wealth is lower for college graduates in early years. At the age of 40, college graduates have the most wealth, due to the labor income increase. The average bequest is highest for college graduates and lowest for agents with no high school education. The portfolio holdings graph shows that a higher education translates into more risky investment. However, at the age of 60, the agent with high school degree invests less riskily compared to the agent without high school education. The intuition is taken from Cocco, Gomes, and Maenhout (2005). They state that due to the large income decrease when going into retirement, the high school agent shifts the portfolio to be less risky since labor income is an implicit riskless investment during retirement. Considering the insurance, the agent with college degree always has

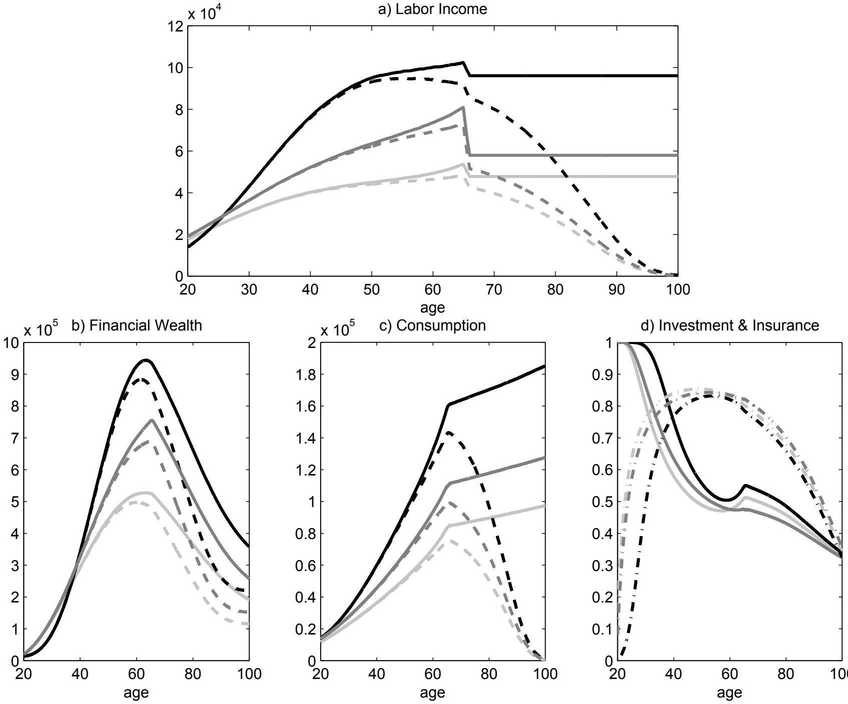

Figure 9: Sensitivity Analysis for the Earnings Profile over the Life Cycle. The graphs compare the average optimal controls, financial wealth evolution and labor income for different calibrations of the earnings profile over the life cycle in the S model: The black lines depict the benchmark results with a college graduate calibration. The grey lines correspond to the calibration for agents with a high school degree: $b = 0.1682$, $c = -0.00323$, $d = 0.000020$, $P = 0.68212$, $Y_0 = 19\,107$. The light lines represent results for agents without high school degree: $b = 0.1684$, $c = -0.00353$, $d = 0.000023$, $P = 0.88983$, $Y_0 = 17\,763$. These values are taken from Munk and Sørensen (2010). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) depicts the average earnings over the life cycle, b) shows the average financial wealth evolution, c) the average optimal consumption and d) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

the smallest demand for insurance. In early years, this is due to little wealth that yields low insurance demand. In later years, the college graduate has a huge amount of labor income. Since insurance holdings decrease with increasing labor income, I deduce that the labor income effect dominates the financial wealth effect here. The high school and no high school graphs again intersect around the age of 60. Since the agent with high school degree has a more reduced labor income in retirement, he increases insurance holdings to anticipate the effect and to be able to leave the desired amount of bequest.

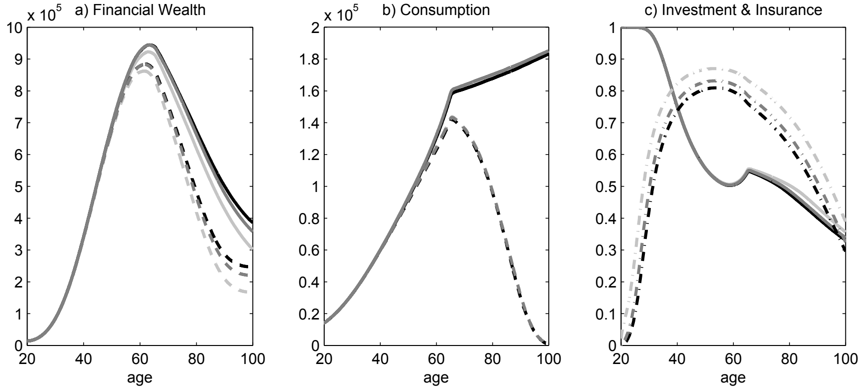

Figure 10: Sensitivity Analysis for the Bequest Motive. The graphs compare the average optimal controls and financial wealth evolution for different values of the importance of the bequest motive in the S model: $\epsilon = 1$ (light lines), $\epsilon = 3$ (grey lines) and $\epsilon = 5$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

The Impact of a Bequest Motive Previously, I highlighted the strong relation between the optimal insurance choice and the bequest motive. Here, I examine the effect of the bequest motive in detail. I illustrate the results for different importance of the bequest motive $\epsilon \in \{1,3,5\}$ in Figure 10. The consumption graph depicts that optimal consumption is nearly unaffected. The lower the bequest motive, the higher is the consumption since agents want less financial wealth as bequest and have more to consume. However, the magnitude of this effect is very low. The amount riskily invested is almost unchanged as well. A small decrease in later years for an increasing bequest motive can be observed due to the wealth effect. As expected, the higher the bequest motive is the higher is the average bequest. Furthermore, living agents accumulate more financial wealth if they face a higher bequest motive. Considering the insurance holdings, we observe that a higher bequest motive goes along with less insurance. Less insurance means more bequest if death occurs, which gives the result an intuition. Note that the change in optimal consumption and portfolio holdings is very small, whereas there is a larger effect on optimal insurance holdings. These results also support the previous statement that mainly the insurance is used to realize the optimal financial wealth for bequest.

Sensitivity Analyses for Preference Parameters I provide further sensitivity analyses and investigate the impact of the preference parameters.

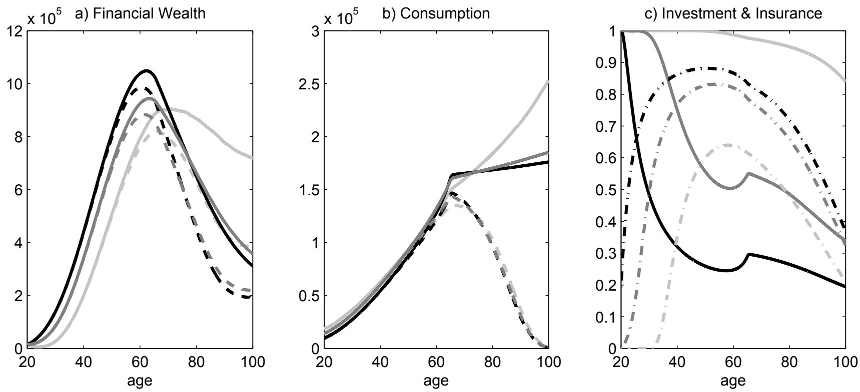

Figure 11: Sensitivity Analysis for Risk Aversion. The graphs compare the average optimal controls and financial wealth evolution for different values of the relative risk aversion coefficient of the S model: $\gamma = 1.5$ (light lines), $\gamma = 4$ (grey lines) and $\gamma = 7$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

I start with the sensitivity with respect to the relative risk aversion and provide results for $\gamma \in \{1.5, 4, 7\}$ in Figure 11. Intuitively, the portfolio holdings are lower throughout the lifetime for a higher level of relative risk aversion. Furthermore, the amount of accumulated financial wealth is higher for a high value of relative risk aversion. These changes reflect the willingness of the investor to decrease risk. Agents that are less risk averse invest more in the risky asset which has a higher expected return. Therefore, average consumption is higher for agents that are less risk averse. For the same reason, less risk averse investors have on average more wealth in later years and also more bequest. This comes at the cost of a higher variation in financial wealth evolution and optimal consumption compared to agents that are more risk averse (not shown in the figure). In early years, more risk averse agents accumulate more financial wealth due to less consumption and their willingness to get a buffer in order to handle future shocks in labor income. After retirement when labor income is certain, the agent does not need that buffer anymore and dissaves. The consumption graph highlights that the risk aversion crucially influences the valuation of labor income risk by the agents. For the less risk averse agent the consumption steepness is nearly unaffected by the different income volatility in the working and retirement stage, whereas the more risk averse agent has a very pronounced change in steepness. The insurance holdings are adjusted to ensure optimal bequest at each point in time. This yields less insurance for the less risk averse investor and more insurance for the more risk averse investor.

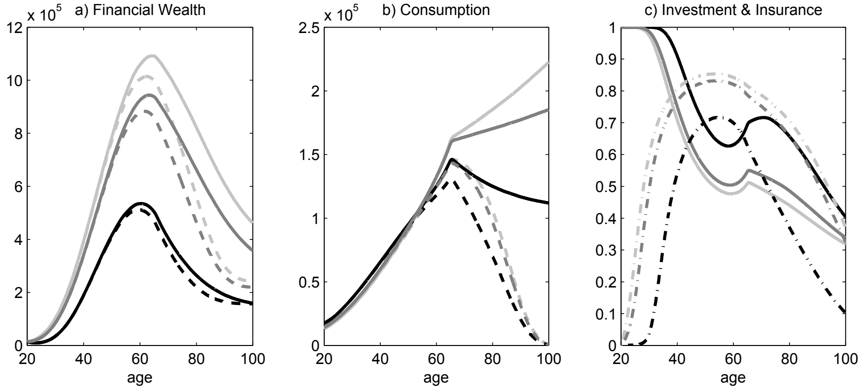

Figure 12: Sensitivity Analysis for the Time Preference Rate. The graphs compare the average optimal controls and financial wealth evolution for different values of the time preference rate of the S model: $\delta = 0.01$ (light lines), $\delta = 0.03$ (grey lines) and $\delta = 0.1$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

Now, I consider the effect of changing the time preference rate $\delta \in \{0.01, 0.03, 0.1\}$. The results are depicted in Figure 12. Intuitively, I expect more consumption today and less tomorrow with a higher time preference rate. This is exactly what the consumption graph shows. Especially after retirement, there is a huge effect with regard to consumption growth. Actually, consumption is decreasing when agents have a high time preference rate. For high values of δ, the fraction insured is lower. Insuring the financial wealth means getting a payoff today but having to pay something later when death occurs. Since future payoffs decrease in value and present payments increase for a higher time preference rate, I would expect a higher fraction of insured wealth. However, the financial wealth of the agent decreases compared to a lower δ which is due to the high consumption in early years. In order to leave the desired bequest, the agent has to decrease his insurance holdings. This effect dominates the payment time effect and overall, the insurance holdings are lower for a higher δ. Although the insurance holdings are lower, the agent with high δ leaves less bequest on average. With a high time preference rate, agents accumulate less financial wealth since they consume more. This explains the low bequest on average, which is intuitive since bequest occurs in the future compared to actual consumption. The fraction riskily invested increases with increasing time preference rate due to the wealth effect.

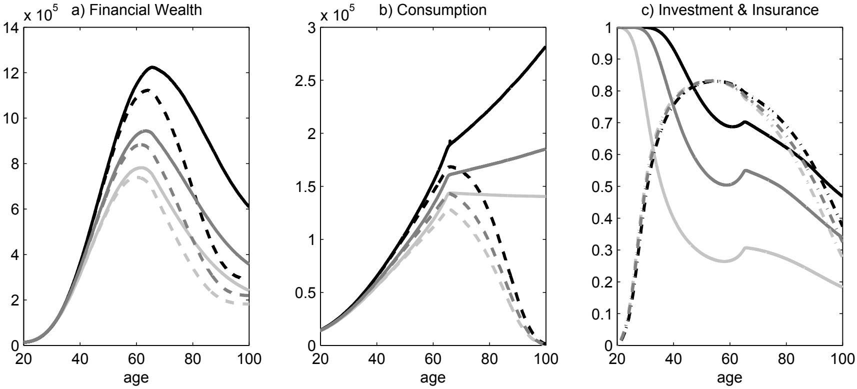

Figure 13: Sensitivity Analysis for the Stock Drift. The graphs compare the average optimal controls and financial wealth evolution for different values of the stock drift in the S model: $\mu_S = 0.04$ (light lines), $\mu_S = 0.06$ (grey lines) and $\mu_S = 0.08$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

Sensitivity Analyses for Asset Parameters I analyze the effect of a changing investment opportunity set. In particular, I vary μ_S, σ_S and r and show the corresponding graphs.

I investigate the stock drift and vary $\mu_S \in \{0.04, 0.06, 0.08\}$. The results are depicted in Figure 13. Intuitively, there is more risky investment with a higher stock drift. Furthermore, average wealth and bequest increase. The same holds true for consumption. These effects are due to a better investment opportunity set since agents are strictly better off with a higher stock drift. In later years, the fraction insured is higher due to more available financial wealth. In early years, the fraction insured is slightly lower. This can be explained by actually relatively low financial wealth but the willingness to leave more bequest in the case of death, due to the anticipated future wealth and consumption evolution. However, these thoughts can be summarized by stating that the insurance decision is corrected to ensure the optimal amount of bequest throughout the life cycle.

In Figure 14, I vary $\sigma_S \in \{0.05, 0.2, 0.4\}$. A higher volatility yields less risky investment due to the increased risk. Furthermore, a higher stock volatility yields less consumption, less financial wealth and less bequest on average since the investment opportunity set worsens. Due to less risky investment, the agent has less financial wealth since the risky asset has a higher expected return compared to the risk-free asset. For a higher volatility, the fraction insured increases in early years and decreases in later years. The interpretation is identical to the previous passage. Changing the stock volatility has a

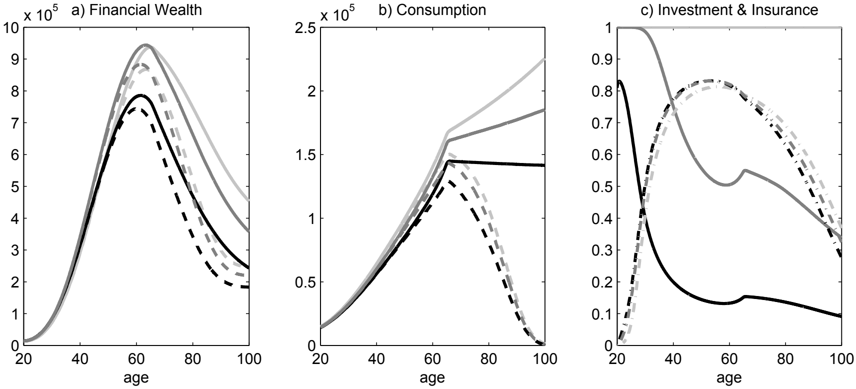

Figure 14: Sensitivity Analysis for the Stock Volatility. The graphs compare the average optimal controls and financial wealth evolution for different values of the stock volatility of the S model: $\sigma_S = 0.05$ (light lines), $\sigma_S = 0.2$ (grey lines) and $\sigma_S = 0.4$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

similar effect like varying the stock drift. It changes the investment opportunity set in a way that the agent is strictly better off or strictly worse off.

I vary the risk-free rate $r \in \{0.008, 0.02, 0.04\}$ and present the results in Figure 15. Increasing r yields decreasing portfolio holdings since the attractiveness of the risk-free asset increased in comparison to the risky asset. However, average financial wealth and average consumption are lower for a high r since the expected return of the risky asset is higher than the return of the risk-free asset. Although the investment opportunity set improved, the agent consumes less, accumulates less financial wealth and has less bequest on average. However, the variation of optimal consumption, financial wealth evolution and bequest is smaller (not shown in the figure) due to the less risky investment. This is more important for the agent than the small reduction of expected consumption and bequest.

7 Deterministic vs Stochastic Time of Death

This section compares the stochastic time of death model S, presented in detail in the previous section, with the standard model D with deterministic time of death. The comparison highlights the effects of considering deterministic mortality risk instead of a fixed time of death. Especially, I comment on the role of the insurance and the bequest motive. The benchmark calibration of the models can be found in Section 5 and details on

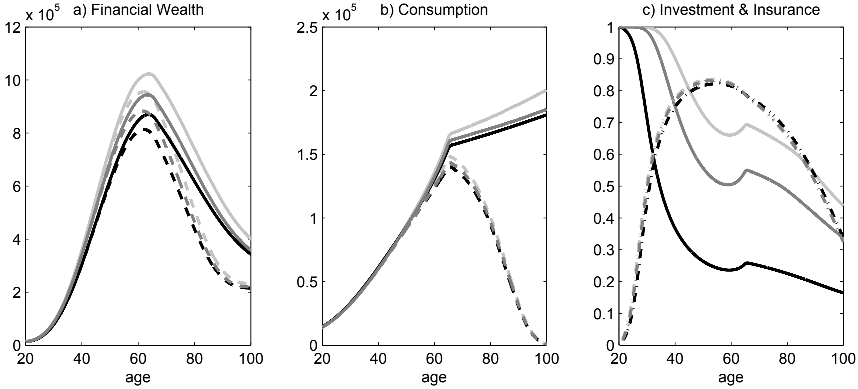

Figure 15: Sensitivity Analysis for the Risk-Free Rate. The graphs compare the average optimal controls and financial wealth evolution for different values of the risk-free rate in the S model: $r = 0.008$ (light lines), $r = 0.02$ (grey lines) and $r = 0.04$ (dark lines). The solid lines represent results for living agents, whereas the dashed lines are average results including dead agents. a) shows the average financial wealth evolution, b) the average optimal consumption and c) the average optimal risky investment (solid lines) and the average optimal insurance decision (dash-dotted lines). The other parameters are calibrated as described in Section 5.

the numerical approach are given in the Appendix B. All figures depict means calculated from 100 000 simulations.

Benchmark Comparison Figure 16 depicts the corresponding graphs for a comparison of the models D and S with the benchmark calibration.

The expected financial wealth looks similar in both models and mainly co-moves until the age of 55. Surprisingly, the average bequest is identical in both models. In the deterministic time of death setup the bequest motive only matters at the time of death and the amount of bequest can be planned, whereas the bequest motive is important throughout the lifetime in the setup with mortality risk. With a deterministic time of death, all bequest is intended. In the setup with mortality risk, one would expect that total bequest splits into an intended part (due to the bequest motive) and an accidental part (due to death occurring while having more or less wealth as the agent wants to allocate as bequest). However, the average bequest is almost identical in both setups. The previous section shows that the endogenous insurance partially explains this issue: The agent chooses the insurance holding at every point in time such that if death occurs, he realizes the desired financial wealth for bequest. In this way, the agent avoids accidental bequest. The same level of bequest is nevertheless surprising. Although both models have the same expected time of death, the distribution of deaths is completely different. Hence,

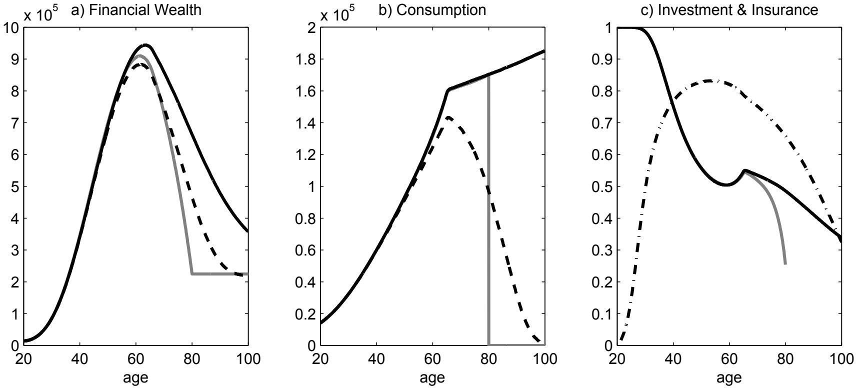

Figure 16: Model Comparison (D,S). The graphs compare the deterministic time of death model (grey lines) with the stochastic time of death model (dark lines). a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed line includes all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed line). c) presents the optimal fraction riskily invested (solid lines) and for the S model the average optimal fraction insured (dash-dotted line). The model calibrations are given in Section 5.

the identical average bequest shows that agents are risk neutral with respect to the time of death as they only care about the expected time of death and not about higher moments.

Considering the graph for consumption, there is a nearly identical pattern for living agents in both models. Consumption is increasing throughout the lifetime with a change in consumption growth at retirement. Hence, the insurance premium in the S model is not used to consume more but for accumulating more financial wealth.

The fraction riskily invested is decreasing over the lifetime in both models and almost identical until retirement. Then, the portfolio holdings decrease faster in the deterministic time of death model. Since the agent wants to reach the desired amount of bequest, he reduces risk. With certain time of death, this can be realized via a low risky investment shortly before the time of death. With mortality risk, the risky investment decreases as the hazard rate of death increases.

Details about the insurance holdings in the S model are given in the previous section. In the model D, there is no insurance decision due to the certain time of death.

The results highlight that mortality risk together with an actuarially fair insurance has nearly no impact until the age of 60. As long as the hazard rate of death is low, the differences between both models are negligible. The resulting financial wealth evolution and optimal controls over the life cycle are similar considering living agents. The differences between both models become important if one considers the retirement stage in detail, or considers mortality risk related assets or insurance products. In contrast, the assumption

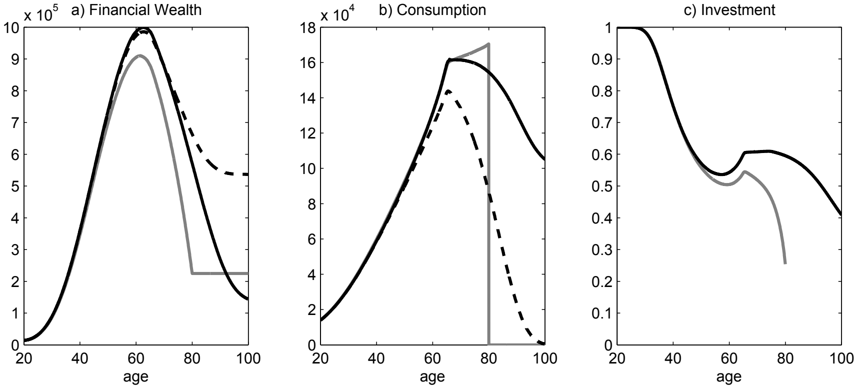

Figure 17: Model Comparison (D,S) without Insurance. The graphs compare the deterministic time of death model (grey lines) to the stochastic time of death model without insurance (dark lines). a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed line includes all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed line). c) presents the optimal fraction riskily invested. The models are calibrated as presented in Section 5.

of a deterministic time of death can be used to simplify computations if the focus is the working period of the life cycle.

The Impact of the Insurance Here, I examine the impact of the absence of the insurance. The previous sections show that the insurance is used to ensure the optimal financial wealth for bequest. Thus, the insurance takes out a crucial implication of mortality risk, namely accidental bequest. Furthermore, the structure of the insurance is rather unrealistic. No insurance company allows to contract such an insurance in the real world. Therefore, I consider the implications of a setup where the agents are not allowed to insure their financial wealth and compare it to the D model with deterministic time of death. Figure 17 depicts the results.

Considering the bequest first, the intuition presented before gets verified now. Without insurance, the average bequest is more than two times higher in the model with mortality risk compared to the model without mortality risk or the S model with insurance. Hence, the major part of bequest is accidental now. The financial wealth graph of living agents shows that agents save more in early years. In particular, the peak is higher. The absence of the insurance amplifies the risk of outliving available financial wealth. Longevity risk becomes important here. The older the agent gets, the higher is the sure payout that he receives from the insurance in the benchmark model. In the absence of the insurance, the agent has to accumulate more wealth to encounter longevity risk. However, this increases accidental bequest in the case of an early death. In later years, the financial wealth of living agents is significantly lower without insurance. On the one hand, this is due to the increasing mortality risk. The agent wants to use his wealth before he dies. On the other hand, the financial wealth is lower due to the missing insurance premium. Agents that are older than 95 leave on average less bequest compared to agents in the deterministic time of death model.

The consumption graph for the model S now has a hump-shaped pattern. Due to the missing insurance premium and the fear of outliving available wealth, the agent is not able to afford that much consumption when getting older. With insurance, he expects a high sure cash flow if he survives. Without insurance, he faces longevity risk as he has to use his own accumulated financial wealth to finance consumption. Therefore, consumption decreases in the absence of the insurance. However, consumption is on average still above the retirement income. Actually, an agent at the age of 100 dissaves. Comparing the models S and D, the optimal consumption is again almost identical until retirement. Afterwards, the differences are crucial with a different consumption growth sign.

Considering portfolio holdings, we see again more risky investment in the model S in later years. Without insurance, the agent has less financial wealth in later years which results in more risky investment. This effect increases with increasing mortality rates when the agent gets older as the insurance premium increases.

The results show more realistic patterns in the absence of an insurance. The agents face accidental bequest and the consumption graph depicts a hump-shaped pattern. These two effects are not observable in the presence of the insurance. Furthermore, the annuity puzzle and the lack of existence of an insurance like the one assumed here indicate that the simple insurance should be omitted in order to get realistic life cycle results. Without insurance, there are more differences between the models S and D after the age of 50. For younger agents, mortality risk is negligible again.

The Impact of a Bequest Motive I consider the models S and D without bequest motive in Figure 18. In the S model, the insurance holdings are trivial: It is always optimal for all agents to insure all the financial wealth. In both models, agents leave no bequest: in the deterministic time of death model due to the certainty of death whereas in the model with mortality risk due to the insurance holdings. The financial wealth of living agents is again almost identical in the S and D model in early years. In later years, financial wealth is reduced to capture the absence of the bequest motive. The optimal consumption is nearly identical in both models until short before the time of death in the model without mortality risk. Then, the agent increases consumption in order to get a very low level of financial wealth since there is no benefit from leaving money on the table. The portfolio holdings are nearly identical until retirement as well. In the model D, the risky investment is lower during retirement and decreases extremely shortly before the

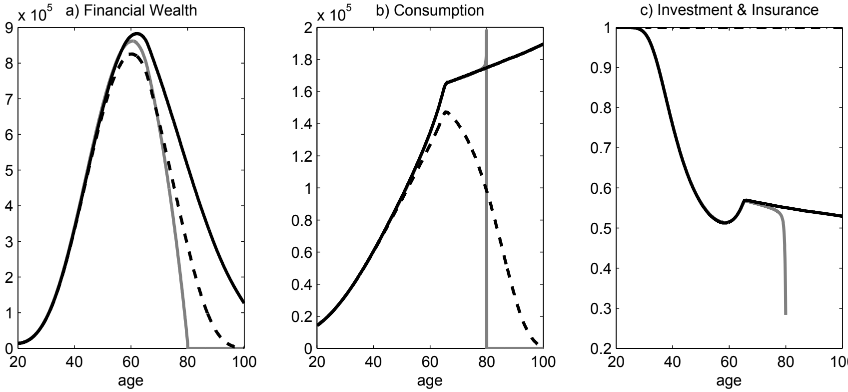

Figure 18: Model Comparison (D,S) without Bequest Motive. The graphs compare the deterministic time of death model (grey lines) to the stochastic time of death model (dark lines) with $\epsilon = 0$ in both models. a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed line includes all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed line). c) presents the optimal fraction riskily invested (solid lines) and for the S model the average optimal fraction insured (dash-dotted line). The remaining parameters are calibrated as described in Section 5 for the models S and D.

certain time of death in order to avoid risk and reach the desired amount of bequest which equals zero. The absence of the bequest motive yields a higher risky investment in later years in the S model due to less financial wealth. Comparing the models S and D, the previous results with insurance remain valid. There are no significant differences until retirement and mortality risk can be neglected in the working phase of the life cycle.

Figure 19 depicts a comparison of the models S and D without bequest motive and without insurance. The financial wealth graph highlights again the effect of accidental bequest in the absence of the insurance. Although the financial wealth of living agents approaches zero when the agents get close to the age of 100, the average expected financial wealth, and thus the bequest, is more than four times of last year's consumption. This effect is driven by agents that die during a phase of the life cycle in which they have a huge amount of financial wealth in order to save for the later years and to capture the impact of longevity risk. The consumption graph shows again a hump-shaped pattern without insurance. Until retirement, the consumption mainly co-moves in both models however, afterwards consumption increases in the model with certain time of death and decreases in the model with mortality risk. The portfolio holdings co-move in early years. Without insurance and bequest motive, the portfolio holdings increase in the S model between the age of 60 and 90 due to the fast decreasing financial wealth. Afterwards, the average portfolio holdings decrease due to the constraint that ensures positive wealth. Risky investment is forbidden for very low levels of financial wealth, which reduces average

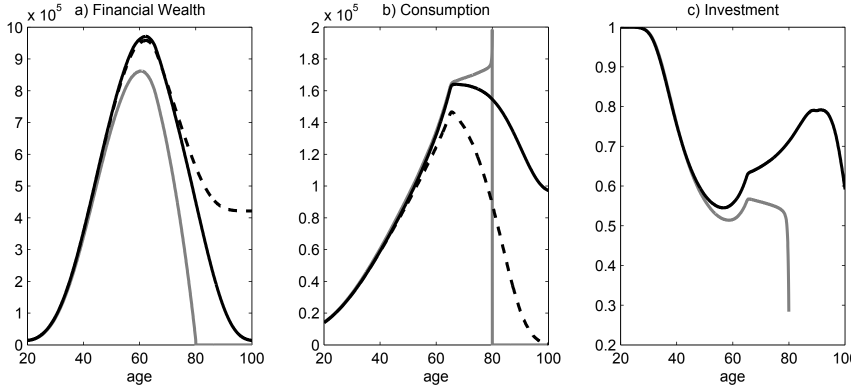

Figure 19: Model Comparison (D,S) without Insurance and without Bequest Motive. The graphs compare the deterministic time of death model (grey lines) with the stochastic time of death model (dark lines) where both models are without insurance and without bequest motive ($\epsilon = 0$). a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed line includes all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed line). c) presents the optimal fraction riskily invested. The remaining calibration of the models S and D is given in Section 5.

risky investment. Comparing both models without bequest motive and without insurance, the previous intuition of the comparison without insurance remains unaffected. Until the age of 50, mortality risk is negligible, whereas it has a crucial impact afterwards, especially during retirement.

8 Deterministic vs Stochastic Hazard Rate of Death

In this section, I allow the hazard rate of death to have a diffusive and a jump component. I analyze the impact of the stochastic hazard rate of death and compare the results with the deterministic hazard rate model. In order to avoid any impact of the numerical calculation technique, I use the same approach for all models. The solution method differs from the algorithm used before due to the hazard rate of death as additional state variable. Details considering the numerical approach are given in Appendix B. In the following, I present results from the S, SD and SDJ model with the calibration of Section 5. The figures depict averaged results after 100 000 simulations.

Benchmark Comparison Figure 20 compares the optimal controls and the wealth evolution of the three models. The S and SD results are nearly identical for all optimal controls and for the financial wealth evolution. This is not surprising since the SD model has only little uncertainty with respect to future mortality rates. In detail, we observe

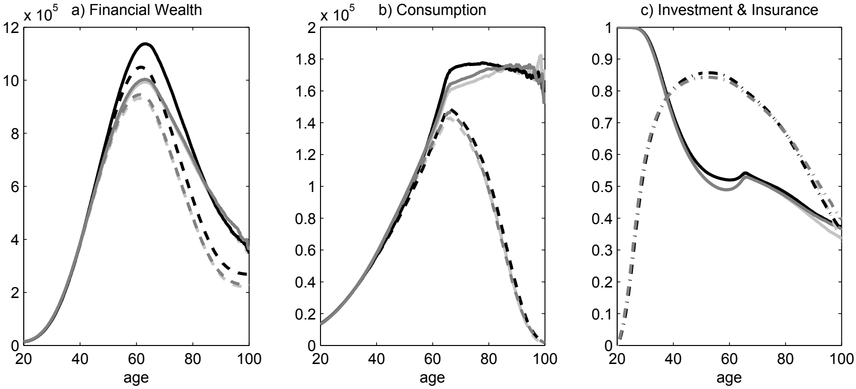

Figure 20: Model Comparison (S,SD,SDJ). This figure compares the model S with deterministic hazard rate of death (light lines) with stochastic hazard rate of death models. The SD model (grey lines) has little uncertainty with respect to future mortality risk modeled with a diffusive component in the hazard rate. The SDJ model (dark lines) additionally has a jump component and more uncertainty with respect to future mortality risk. a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed lines include all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed lines). c) presents the optimal fraction riskily invested (solid lines) and the optimal insurance decision (dash-dotted lines). The parameters are calibrated as stated in Section 5 for the models S, SD and SDJ.

that in the SD model the agent has a little more financial wealth, more consumption and more risky investment over the life cycle compared to the S model.

Comparing the S and SD model with the SDJ model shows more differences. The SDJ model delivers more financial wealth, more bequest, a higher amount of consumption and more risky investment. These results indicate that the agent is better off when he faces jump risk in the hazard rate of death. To verify the intuition and to evaluate

	age 20	age 50	age 80
SD with Insurance	0.02	1.38	0.82
SD without Insurance	0.03	1.38	0.88
SDJ with Insurance	8.45	28.82	16.33
SDJ without Insurance	8.41	28.61	16.80

Table 2: Welfare Impact of the Stochastic Mortality Risk. The table compares the certainty equivalent of the S model with the SD and SDJ model with insurance and without insurance. The table gives the percentage gain in the certainty equivalent induced by the stochastic hazard rate of death. For the agent with age 20, I use the values $t = 0$, $x = 13912$, $y = 13912$, $\pi = 0.000074$. For the agent aged 50, the values are $t = 30$, $x = 750000$, $y = 92500$, $\pi = 0.0022$. For the agent at the age of 80, I have $t = 60$, $x = 690000$, $y = 91000$, $\pi = 0.0635$. I do the calculations with the benchmark calibration of the models S, SD and SDJ given in Section 5.

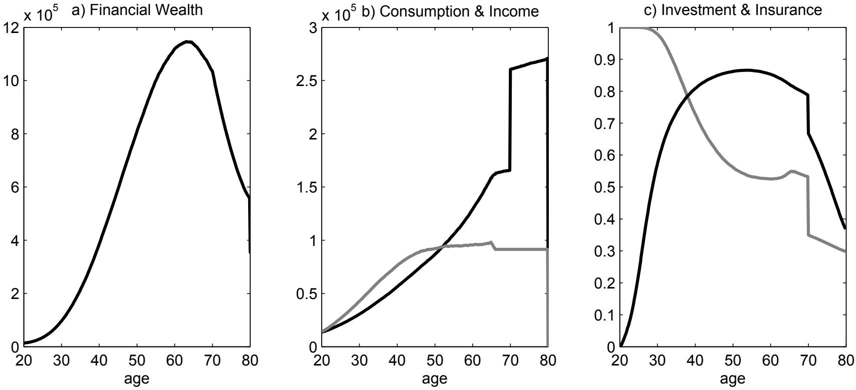

Figure 21: Sample Mortality Jump in the SDJ Model. The graphs depict the optimal wealth and controls in the SDJ model when agents face a jump in the hazard rate of death at the age of 70 and the death jump occurs at the age of 80. a) depicts the financial wealth evolution. b) shows the optimal consumption (dark line) and the income (grey line). c) presents the optimal fraction insured (dark line) and riskily invested (grey line). The model is calibrated with the SDJ parameters given in Section 5.

the importance of the stochastic hazard rate of death, I calculate a certainty equivalent defined by the inverse value function

$$CE(t,x,y,\pi) = \left[(1-\gamma)J(t,x,y,\pi)\right]^{\frac{1}{1-\gamma}}.$$

Table 2 provides the percentage gain of the SD and SDJ model compared to the S model calculated by the ratio of certainty equivalents. Adding a diffusion has only a minor effect with a gain smaller than 1.5%. However, the agent prefers the stochastic hazard rate of death. Adding the jump component has a significant impact over the life cycle. Middle-aged agents are more than 25% better off if they face stochastic mortality risk with jumps. This also highlights the importance of a jump component in the hazard rate of death. I extend the results of Huang, Milevsky, and Salisbury (2012). They state that in an analytically tractable model of retirement consumption, the stochastic mortality risk is relatively unimportant from an individual's perspective. Here, I approve this result for a realistically calibrated life cycle setup with unspanned labor income risk if the stochastic mortality risk is driven by a pure diffusive component. Moreover, I show that if the stochastic mortality risk is also driven by a jump component, the statement does not hold and the jump component becomes crucial from an individual's perspective. These results are valid in the models with and without the insurance.

In order to gain more intuition about the impact of a jump in the hazard rate of death, I consider graphs where all agents face a shock at the same time. Figure 21 depicts the results for agents with a health shock at the age of 70 and death occurs at the age of 80.

	age 20	age 50	age 80
S	0.14	8.15	13.43
SD	0.13	8.15	13.36
SDJ	0.17	8.33	12.96

Table 3: Welfare Impact of the Insurance. The table depicts the certainty equivalent increase in percentage when agents have access to the insurance market for differently aged agents. For the agent with age 20, I use the values $t = 0$, $x = 13\,912$, $y = 13\,912$, $\pi = 0.000074$. For the agent aged 50, the values are $t = 30$, $x = 750\,000$, $y = 92\,500$, $\pi = 0.0022$. For the agent at the age of 80, I have $t = 60$, $x = 690\,000$, $y = 91\,000$, $\pi = 0.0635$. I do the calculations with the benchmark calibration of the models S, SD and SDJ given in Section 5.

When the health shock occurs, mortality risk increases, the expected remaining lifetime decreases and the insurance premium increases. The optimal reaction to a health shock is an increase in consumption and a decrease in risky investment to capture the decreased expected remaining lifetime. The fraction insured is also effected in order to ensure optimal bequest. Due to the co-movement of expected bequest and optimal consumption, it is clear that the fraction insured decreases to reach a higher amount of desired bequest. Financial wealth growth reduces, i.e. the higher insurance premium due to the increased mortality risk does not offset the change in optimal controls. The reaction to a critical illness shock is more pronounced, the earlier the shock occurs. A second or third health shock further amplifies the effect.

The reason why the agent prefers a setup with jump risk in the hazard rate of death is that a jump gives an indication prior to a high death probability and the agent has the possibility to react in an optimal way. The agent likes extreme probabilities and dislikes probabilities in between. If the risk of death is very low, the agent faces the risk of a sudden death, whereas he especially faces longevity risk if mortality risk is very high. However, the agent can use the insurance to mitigate these extreme cases and can react in an optimal way to the predominant state. If there is a 50-50 chance of surviving the next few years, optimal decisions are mainly a tradeoff between both possible cases and semi-optimal whatever happens. In the S and SD model, the agent faces this situation especially in middle and older years. In the SDJ model, the agent faces a low probability of death as long as no jump has occurred and a high probability of death afterwards. This more extreme distribution explains the importance of the jump component for the individual agent.

A more accurate calibration, impact on the optimal controls and financial wealth evolution as well as a significant importance for the individual agent highlight that jumps in the hazard rate of death are important in life cycle consumption-investment problems with mortality risk. On the contrary, a diffusive component has only a minor effect.

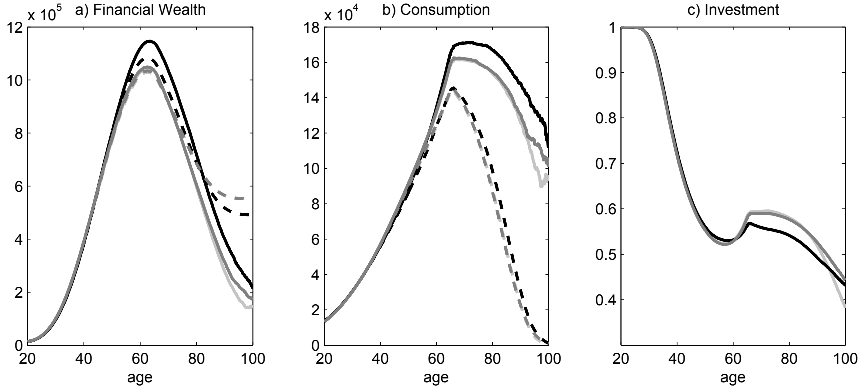

Figure 22: Model Comparison (S,SD,SDJ) without Insurance. The figure compares the deterministic hazard rate of death model S (light lines) with stochastic hazard rate of death models. The SD model (grey lines) has little uncertainty with respect to future mortality risk modeled with a diffusive component in the hazard rate. The SDJ model (dark lines) additionally has a jump component and more uncertainty with respect to future mortality risk. a) depicts the optimal average financial wealth over the life cycle. The solid lines include living agents only, whereas the dashed lines include all agents. b) shows the average optimal consumption for living agents only (solid lines) and for all agents (dashed lines). c) presents the optimal fraction riskily invested. The models S, SD and SDJ are calibrated as stated in Section 5.

The Impact of the Insurance In this paragraph, I consider the importance of the insurance in the three models and present results in the absence of the insurance. Table 3 gives the increase in the certainty equivalent of having access to the insurance market for the S, SD and SDJ model. At the age of 20, the agent is not significantly better off when having access to the insurance market. This is not surprising since the agent has little financial wealth to insurance and mortality risk is extremely low. At the age of 20, agents ideally invest nothing in the insurance at all. Due to the relatively high importance of the bequest motive and the low wealth early in the life cycle, the agent would even prefer to sell the insurance in order to get more bequest in the case of death. The middle-aged agent is significantly better off when having access to the insurance market in all models. Now, the agent has accumulated more financial wealth than he wants to hold for bequest. Therefore, he uses the insurance to ensure optimal bequest and he gets the insurance premium as additional source of income. For old agents, the importance of the insurance further increases although less financial wealth is available. Due to a higher mortality risk, the insurance becomes more important. Comparing the three models, a stochastic hazard rate of death has no significant impact on the importance of the insurance.

Figure 22 compares the three models in the absence of the insurance. Again, the S and SD model produce nearly identical results. As in Huang, Milevsky, and Salisbury (2012), the withdrawal rate at retirement, i.e. at the age of 65, is slightly higher in the model with a diffusive component. Comparing the models S and SD to the SDJ model, we see

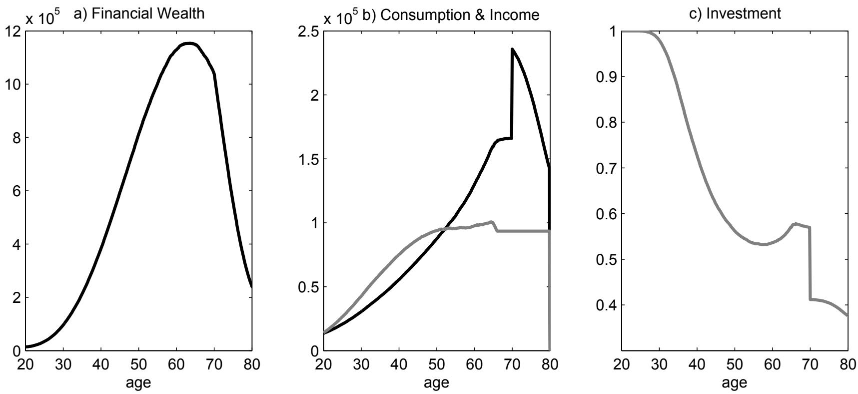

Figure 23: Sample Mortality Jump in the SDJ Model without Insurance. The figure shows the optimal wealth and controls in the SDJ model without insurance if agents face a mortality jump at the age of 70 and a death jump at the age of 80. a) shows the financial wealth evolution. b) presents the optimal consumption (dark line) and the income (grey line). c) depicts the optimal fraction riskily invested. The model is calibrated with the SDJ parameters given in Section 5.

more financial wealth and more consumption in the model with jump risk. However, the average bequest is lower although consumption and financial wealth are higher in the SDJ model. Hence, the SDJ model delivers significantly less accidental bequest. For an explanation, I consider the effect of mortality jumps in the model without insurance.

Figure 23 depicts the effect of a health shock at the age of 70 and a death shock at the age of 80 in the SDJ model without insurance. Consumption increases and financial wealth decreases when the health jump occurs. Since it is likely that death occurs a few years after a health jump, the agent has time to reduce financial wealth to prepare the bequest. This explains the lower fraction of bequest being accidental and also highlights why agents are better off in the SDJ model. Compared to the model with insurance, we see that financial wealth decreases even faster. This is necessary since the insurance is not available as a possibility to reduce financial wealth at death. Therefore, the financial wealth is reduced in advance. Consumption jumps after the mortality shock but decreases rapidly afterwards. Without an insurance premium, the agent cannot afford the high level of consumption over plenty of years without facing crucial longevity risk. Hence, consumption growth decreases after the jump. The risky investment reduces when the jump occurs as a reaction to the reduced expected remaining lifetime. Again, the earlier the shock occurs, the more pronounced is the effect and additional shocks further amplify the effect.

Without insurance, the inclusion of jumps in the hazard rate of death is important for the individual agent and has an impact on the aggregate optimal wealth and controls as well. Furthermore, the presence of jump risk reduces accidental bequest significantly. In contrast, a diffusive component in the hazard rate of death has again only little impact on the results.

9 Conclusion

Mortality risk with jumps in the hazard rate of death fits mortality data best and is important for the individual agents, especially in middle and older years. Further research can focus on stochastic mortality risk with jumps and combine this with a deeper analysis of either products that are relevant in the retirement phase or mortality-related and health-related assets or insurance products.

In Kraft, Schendel, and Steffensen (2014), we consider a suchlike model where a family faces the risk of a health shock or an early death and has the possibility to contract a term life insurance to partially mitigate the risk of losing the wage earner's income. Similar analyses can be done by considering critical illness insurance or disability insurance.

Jump risk in the hazard rate of death is also important with regard to retirement products like annuities. Since the jump risk is important for the individual agent and significantly effects consumption and investment decisions, further research can analyze the effect of mortality jumps on the retirement planning and annuity decisions.

Another idea for further research is to choose a different type of preferences in order to avoid the risk neutrality with respect to the time of death.[8] Bommier (2006b) points out that mortality risk implicitly defines risk neutrality with respect to the time of death if it can be added to the time preference rate. He develops non-additive preferences that produce a constant absolute risk aversion with respect to the time of death and that are still time-consistent when considering utility as an age-dependent function. With this utility, he shows that risk aversion with respect to the time of death increases consumption early in life. Examining portfolio holdings in this setup may lead to new insights how risk aversion with respect to the time of death affects the optimal asset allocation over the life cycle.

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[8] Bommier (2006a) analytically compares such a different preference type with the time-additive preferences (that I use here) in life cycle models and comments on similarities and differences.

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A Proof of Proposition 2

I start with the HJB (8) and set $\sigma_Y = 0$. Furthermore, π is redundant as a state variable. Therefore, I obtain the simpler HJB

$$\begin{aligned}\delta J = \sup_{c,\theta,\eta} \quad &\Bigg\{ \frac{c^{1-\gamma}}{1-\gamma} + J_t + J_x\left[x\left(r+\pi\eta+\theta\lambda\sigma_S\right)+y-c\right] + 0.5J_{xx}x^2\theta^2\sigma_S^2 + J_y y\mu_Y \\ &\quad + \pi\left[J(\tau,(1-\eta)x,0) - J(t,x,y)\right]\Bigg\}. \end{aligned} \tag{13}$$

Next, I use the following guess for the indirect utility:

$$J(t,x,y) = \mathbb{1}_{\{t<\tau\}}\left(\frac{1}{1-\gamma}(x+yf(t))^{1-\gamma}g(t)^\gamma\right) + \mathbb{1}_{\{t=\tau\}}\left(\epsilon(t)\frac{x^{1-\gamma}}{1-\gamma}\right).$$

I calculate the necessary values and derivatives using the guess and obtain

$$\begin{aligned} J(t\,|\,t<\tau,x,y) &= \frac{1}{1-\gamma}(x+yf(t))^{1-\gamma}g(t)^\gamma, \\ J_t(t\,|\,t<\tau,x,y) &= (x+yf)^{-\gamma}yf_tg^\gamma + \frac{\gamma}{1-\gamma}(x+yf)^{1-\gamma}g_tg^{\gamma-1}, \\ J_x(t\,|\,t<\tau,x,y) &= (x+yf)^{-\gamma}g^\gamma, \end{aligned}$$

$$
\begin{aligned}
J_{xx}(t \mid t<\tau, x, y) &= -\gamma (x+yf)^{-\gamma-1} g^{\gamma}, \\
J_y(t \mid t<\tau, x, y) &= (x+yf)^{-\gamma} f g^{\gamma}, \\
J(t \mid t=\tau, (1-\eta)x, 0) &= \epsilon(t) \frac{\big((1-\eta)x\big)^{1-\gamma}}{1-\gamma}.
\end{aligned}
$$

By substituting $\epsilon(t)\frac{((1-\eta)x)^{1-\gamma}}{1-\gamma}$ for $J(\tau,(1-\eta)x,0)$ the HJB (13) becomes

$$
\begin{aligned}
(\delta+\pi)J = \sup_{c,\theta,\eta} \quad & \Big\{ \frac{c^{1-\gamma}}{1-\gamma} + J_t + J_x \left[x\left(r+\pi\eta+\theta\lambda\sigma_S\right) + y - c \right] + 0.5 J_{xx} x^2 \theta^2 \sigma_S^2 + J_y y \mu_Y \\
& + \pi\epsilon \frac{\big((1-\eta)x\big)^{1-\gamma}}{1-\gamma} \Big\}.
\end{aligned}
$$

Next, taking the first order conditions for c, θ, η yields the optimal controls

$$
\begin{aligned}
c &= J_x^{-\frac{1}{\gamma}}, \\
\theta &= -\frac{J_x}{J_{xx}x}\frac{\lambda}{\sigma_S}, \\
\eta &= 1 - \frac{J_x^{-\frac{1}{\gamma}}}{x}\epsilon^{\frac{1}{\gamma}}.
\end{aligned}
$$

Substituting the calculated derivatives, I express the optimal controls as

$$
\begin{aligned}
c(t,x,y) &= \frac{x+yf(t)}{g(t)}, \\
\theta(t,x,y) &= \frac{\lambda}{\gamma x \sigma_S}\left(x+yf(t)\right), \\
\eta(t,x,y) &= 1 - \frac{x+yf(t)}{g(t)x}\epsilon(t)^{\frac{1}{\gamma}}.
\end{aligned}
$$

Next, I insert these optimal controls, the guess and the corresponding derivatives into the HJB (13). After doing a zero addition with $\pm(x+fy)^{-\gamma} g^{\gamma} y f (r+\pi)$, I sort terms with $(x+yf)^{1-\gamma}$ and $(x+yf)^{-\gamma}$ to get the rearranged expression:

$$
\begin{aligned}
0 = &(x+yf)^{-\gamma}\left[y g^{\gamma} f_t + g^{\gamma} y - g^{\gamma} y f (r+\pi) + f g^{\gamma} \mu_Y y \right] \\
& + (x+yf)^{1-\gamma} \Big[-\frac{\delta+\pi}{1-\gamma} g^{\gamma} + \frac{1}{1-\gamma} g^{\gamma-1}\left(1+\pi\epsilon^{\frac{1}{\gamma}}\right) + \frac{\gamma}{1-\gamma} g^{\gamma-1} g_t + g^{\gamma}(r+\pi) \\
& - g^{\gamma-1}\epsilon^{\frac{1}{\gamma}}\pi + g^{\gamma}\frac{1}{\gamma}\lambda^2 - g^{\gamma-1} - \frac{1}{2} g^{\gamma}\frac{1}{\gamma}\lambda^2 \Big].
\end{aligned}
$$

The equation is fulfilled, if both terms in the square brackets are zero. From the first square bracket, I get the following ordinary differential equation (ODE) for f:

$$f_t + f\left(\mu_Y(t) - r - \pi(t)\right) + 1 = 0$$

with the solution (can be verified using Leibniz rule)

$$f(t) = \int_t^{\infty} \mathrm{e}^{\int_t^s \mu_Y(u) - r - \pi(u)\, du}\, ds.$$

Considering the remaining terms, I obtain the following ODE for g:

$$g_t + g\left(\frac{1-\gamma}{\gamma} r - \frac{\delta}{\gamma} - \pi(t) + \frac{1}{2}\frac{1-\gamma}{\gamma^2}\lambda^2\right) + 1 + \pi(t)\epsilon(t)^{\frac{1}{\gamma}} = 0$$

with the solution (verifiable via Leibniz rule again)

$$g(t) = \int_t^{\infty} \mathrm{e}^{\int_t^s \frac{1-\gamma}{\gamma} r - \frac{1}{\gamma}\delta - \pi(u) + \frac{1}{2}\frac{1-\gamma}{\gamma^2}\lambda^2\, du} \left(1 + \pi(s)\epsilon(s)^{\frac{1}{\gamma}}\right) ds.$$

This proves the proposition. □

B The Numerical Solution Approach

It is not possible to find an analytical solution for the incomplete market case with unspanned labor income and with stochastic mortality risk. Therefore, I use a numerical approach which is similar to the approach of Munk and Sørensen (2010). First, I simplify the optimization problem by reducing the number of state variables from four to three.

Lemma 3. *The optimization problem (6) with mortality risk can be simplified by reducing the number of state variables. The indirect utility (7) is rewritten for $t < \tau$ as*

$$J(t, x, y, \pi) = y^{1-\gamma} F(t, z, \pi),$$

where $z = \frac{x}{y}$. The HJB (8) simplifies to

$$\begin{aligned}
0 = \sup_{\widehat{c},\theta,\eta} \Bigg\{ & \frac{\widehat{c}^{1-\gamma}}{1-\gamma} + \epsilon\pi \frac{\left((1-\eta)z\right)^{1-\gamma}}{1-\gamma} + F_t \\
& + F\left[-\delta - \pi - \kappa + (1-\gamma)\mu_Y - \frac{1}{2}\gamma(1-\gamma)\sigma_Y^2\right] \\
& + F_z\left[1 - \widehat{c} + z\left(r + \pi\eta + \theta\lambda\sigma_S - \mu_Y + \gamma\sigma_Y^2 - \gamma\sigma_S\sigma_Y\rho\theta\right)\right] \\
& + F_{zz} z^2 \left[\frac{1}{2}\theta^2\sigma_S^2 + \frac{1}{2}\sigma_Y^2 - \sigma_S\sigma_Y\rho\theta\right]
\end{aligned}$$

$$+F_\pi \pi \mu_\pi + \frac{1}{2} F_{\pi\pi} \pi^2 \sigma_\pi^2 + \kappa F(t,z,\pi+\beta) \Big\}$$

with $\widehat{c} = \frac{c}{y}$*. The optimal controls for* $t \in [0,\tau)$ *are*

$$\begin{aligned}
\widehat{c}(t,z,\pi) &= F_z^{-\frac{1}{\gamma}}, \\
\theta(t,z,\pi) &= \frac{\sigma_Y \rho}{\sigma_S} + \frac{F_z}{F_{zz} z} \frac{\gamma \sigma_Y \rho - \lambda}{\sigma_S}, \\
\eta(t,z,\pi) &= 1 - \frac{F_z^{-\frac{1}{\gamma}}}{z} \epsilon(t)^{\frac{1}{\gamma}}.
\end{aligned}$$

Proof. Due to the power utility setup as well as the linearity in wealth dynamics (5) and labor income dynamics (1), I can use the homogeneity property of the indirect utility. I reduce the number of state variables for $t < \tau$ as follows

$$\begin{aligned}
J(t,kx,ky,\pi) &= \sup_{\{c_s,\theta s,\eta_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,\pi}\left[\int_t^\tau e^{-\delta(s-t)} \frac{(kc_s)^{1-\gamma}}{1-\gamma} ds + \epsilon(t) \mathrm{e}^{-\delta(\tau-t)} \frac{(kX_\tau)^{1-\gamma}}{1-\gamma}\right] \\
&= k^{1-\gamma} \sup_{\{c_s,\theta s,\eta_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,\pi}\left[\int_t^\tau e^{-\delta(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} ds + \epsilon(t) \mathrm{e}^{-\delta(\tau-t)} \frac{X_\tau^{1-\gamma}}{1-\gamma}\right] \\
&= k^{1-\gamma} J(t,x,y,\pi).
\end{aligned}$$

Thus, the value function is expressed as

$$J(t,x,y,\pi) = k^{\gamma-1} J(t,kx,ky,\pi).$$

Plugging in $k = \frac{1}{y}$ reduces the number of state variables by one. Hence, I get for $t < \tau$

$$\begin{aligned}
J(t,x,y,\pi) &= y^{1-\gamma} J\left(t, \frac{x}{y}, 1, \pi\right) \\
&=: y^{1-\gamma} F(t,z,\pi),
\end{aligned}$$

The partial derivatives of J can be rewritten in terms of F as follows:

$$\begin{aligned}
J_t &= y^{1-\gamma} F_t, \\
J_x &= y^{1-\gamma} F_z \frac{1}{y}, \\
J_{xx} &= y^{1-\gamma} F_{zz} \frac{1}{y^2}, \\
J_y &= (1-\gamma) y^{-\gamma} F - y^{-\gamma} F_z \frac{x}{y}, \\
J_{yy} &= -\gamma(1-\gamma) y^{-\gamma-1} F + y^{-\gamma-1} F_{zz} \frac{x^2}{y^2} + 2\gamma y^{-\gamma-1} F_z \frac{x}{y},
\end{aligned}$$

$$\begin{aligned}
J_{xy} &= -y^{-\gamma-1}F_{zz}\frac{x}{y} - \gamma y^{-\gamma-1}F_z,\\
J_\pi &= y^{1-\gamma}F_\pi,\\
J_{\pi\pi} &= y^{1-\gamma}F_{\pi\pi}.
\end{aligned}$$

Furthermore, I notice for $t = \tau$:

$$J(\tau,(1-\eta)x,0,\pi) = \epsilon(t)\frac{\big((1-\eta)x\big)^{1-\gamma}}{1-\gamma}.$$

In order to get the new normalized HJB, I rewrite the HJB (8) in terms of F by substituting the calculated derivatives. I substitute z for $\frac{x}{y}$, divide by $y^{1-\gamma}$ and choose $\widehat{c} = \frac{c}{y}$ as new control variable instead of c. Then, I get rid off all x, y, c in the HJB such that the agent maximizes over portfolio holdings, normalized consumption and insurance holdings now. The only state variables are time, normalized wealth and the hazard rate of death. The resulting rearranged normalized HJB is shown in Lemma 3.

I obtain the optimal (normalized) controls $\widehat{c}, \theta, \eta$ by taking the first order conditions with respect to $\widehat{c}, \theta, \eta$ from the normalized HJB. □

Basic Idea Using the results of Lemma 3, I solve the optimization problem (6) numerically. I use a backward iterative procedure. I discretize the setup and use finite difference approximations for the partial derivatives. Then, I guess the optimal normalized controls $\theta, \widehat{c}, \eta$ and use the guess to calculate the finite difference approximations for the partial derivatives. Afterwards, I calculate a new guess for the optimal normalized controls using the formulas from Lemma 3. I do this until the change in the normalized value function F between the iterations is very small. The solution technique is similar to the one used by Munk and Sørensen (2010).[9] The main difference is the infinite horizon setup of the model with uncertain time of death and the jump component. Due to the structure of the mortality risk, the survival probability of the agents to a specific high age is numerically not distinguishable from zero. I use this age as a start for the backward iterative procedure by assuming that agents die for sure if they would reach this age.

General Solution Technique I roughly present the solution technique here but I do not go into detail (e.g. boundary handling, numerical implementation of constraints) since this goes beyond the scope of the paper. First, I set up a grid about normalized wealth, hazard rate of death and time. The normalized indirect utility function in the grid point n, i, j is denoted by $F_{n,i,j}$ where i is the normalized wealth index, j the hazard rate of death index and n the time index. I denote the optimal controls on the grid by $\widehat{c}_{n,i,j}$, $\theta_{n,i,j}$,

[9] Munk and Sørensen (2010) provide details concerning the numerical approach in their appendix.

$\eta_{n,i,j}$ analogously. I start with the highest value of n and calculate backwards in time for all i and j simultaneously. First, I guess the optimal controls. A good guess is the previous value in the same grid point, e.g. $\eta_{n,i,j} = \eta_{n+1,i,j}$ since I do not expect optimal controls to vary much in a small time interval. After substituting the optimal controls into the normalized HJB of Lemma 3, the HJB becomes a partial differential equation (PDE) and can be expressed as

$$0 = K_1 + F_t + K_2 F + K_3 F_z + K_4 F_{zz} + K_5 F_\pi + K_6 F_{\pi\pi} + \kappa F(t,z,\pi+\beta)$$

with state-dependent coefficients K_i (I use K_i as a short form for $K_i(t,z,\pi)$). Next, I approximate the partial derivatives with finite differences. I get for the grid point (n,i,j)

$$\begin{aligned}
F_t:&\quad D_t^+ F_{n,i,j} = \frac{F_{n+1,i,j} - F_{n,i,j}}{\Delta_t},\\
F_z:&\quad D_z^+ F_{n,i,j} = \frac{F_{n,i+1,j} - F_{n,i,j}}{\Delta_z},\\
F_z:&\quad D_z^- F_{n,i,j} = \frac{F_{n,i,j} - F_{n,i-1,j}}{\Delta_z},\\
F_{zz}:&\quad D_z^2 F_{n,i,j} = \frac{F_{n,i+1,j} - 2F_{n,i,j} + F_{n,i-1,j}}{(\Delta_z)^2},\\
F_\pi:&\quad D_z^+ F_{n,i,j} = \frac{F_{n,i,j+1} - F_{n,i,j}}{\Delta_\pi},\\
F_\pi:&\quad D_z^- F_{n,i,j} = \frac{F_{n,i,j} - F_{n,i,j-1}}{\Delta_\pi},\\
F_{\pi\pi}:&\quad D_z^2 F_{n,i,j} = \frac{F_{n,i,j+1} - 2F_{n,i,j} + F_{n,i,j-1}}{(\Delta_\pi)^2},
\end{aligned}$$

where Δ denotes the difference between two grid points (e.g. $\Delta_t = t_{n+1} - t_n$). I use an implicit approach and thus, I consider the forward looking finite difference for the time derivative. For z and π, I use both forward and backward looking differences depending on the coefficients at each grid point in order to ensure the stability of the solution approach. I approximate the jump term via linear interpolation with the nearest grid points, i.e. $F(t,z,\pi+\beta) = f_1^\beta F_{n,i,j+\widetilde{\beta_1}} + f_2^\beta F_{n,i,j+\widetilde{\beta_2}}$ where the $\widetilde{\beta}$ are the nearest grid points of $\pi+\beta$ and the f^β $(= f^\beta(n,i,j))$ are the factors from the linear interpolation.

After inserting the approximations and sorting terms, the PDE is represented by the equation

$$\begin{aligned}
F_{n+1,i,j}\frac{1}{\Delta_t} + K_1 = F_{n,i,j}&\left(-K_2 + \frac{1}{\Delta_t} + \mathrm{abs}\left(\frac{K_3}{\Delta_z}\right) + \mathrm{abs}\left(\frac{K_5}{\Delta_\pi}\right) + 2\frac{K_4}{\Delta_z^2} + 2\frac{K_6}{\Delta_\pi^2}\right)\\
&+ F_{n,i-1,j}\left(\frac{\min\{K_3,0\}}{\Delta_z} - \frac{K_4}{\Delta_z^2}\right) + F_{n,i+1,j}\left(-\frac{\max\{K_3,0\}}{\Delta_z} - \frac{K_4}{\Delta_z^2}\right)
\end{aligned}$$

$$+F_{n,i,j-1}\left(\frac{\min\{K_5,0\}}{\Delta_\pi}-\frac{K_6}{\Delta_\pi^2}\right)+F_{n,i,j+1}\left(-\frac{\max\{K_5,0\}}{\Delta_\pi}-\frac{K_6}{\Delta_\pi^2}\right)$$
$$-F_{n,i,j+\widetilde{\beta_1}}\kappa f_1^\beta - F_{n,i,j+\widetilde{\beta_2}}\kappa f_2^\beta,$$

where K_i now depends on the specific grid point ($K_i = K_i(n,i,j)$). I use another shorthand notation where I suppress the n,i,j index again

$$F_{n+1,i,j}\frac{1}{\Delta_t}+C^0 = F_{n,i-1,j}C^1 + F_{n,i+1,j}C^2 + F_{n,i,j-1}C^3$$
$$+F_{n,i,j+1}C^4 + F_{n,i,j}C^5 + F_{n,i,j+\widetilde{\beta_1}}C^6 + F_{n,i,j+\widetilde{\beta_2}}C^7.$$

I vectorize the indirect utility for a fixed point in time such that I write

$$F_n = \begin{pmatrix} F_{n,1,1} & F_{n,1,2} & \dots & F_{n,1,N_\pi} & F_{n,2,1} & \dots & F_{n,2,N_\pi} & \dots & F_{n,N_z,N_\pi}\end{pmatrix}^T.$$

The ordering is mathematically unimportant but due to the jump component in π, this ordering results in an optimization problem with a smaller band width. Thus, this ordering increases the speed of solution algorithms dramatically. Then, I consider the equations above for all i,j for a fixed n. Resulting, I get a system of $N_\pi \times N_z$ equations that can be written in matrix form as follows

$$\begin{pmatrix} & \dots & \\ 0\dots 0\,C^1\,0\dots 0\,C^3\,C^5\,C^4\,0\dots 0\,C^6\,C^7\,0\dots 0\,C^2\,0\dots 0 \\ & \dots & \end{pmatrix} F_n = \begin{pmatrix} \vdots \\ F_{n+1,i,j}\frac{1}{\Delta_t}+C^0 \\ \vdots \end{pmatrix}.$$

The resulting system is characterized by a large, sparse, band matrix and can be solved relatively fast due to the fixed structure. With the solution F_n, I calculate a new guess using the finite difference approximations and the optimal control formulas of Lemma 3. A new iterative step starts using the new guess, or, if there is no significant change between old and new results, the algorithm continues with $n-1$ until the full system is solved.

With the solution, I simulate 100 000 life cycles for each model. I use the inverse transformation method to simulate the time of death and the mortality jumps.

Simulations of Section 6 and 7 In Section 6 and 7, π is redundant as a state variable. This simplifies the above approach significantly. The remaining matrix has a tridiagonal structure which allows a very fast solution. For the model D, I have specific boundary conditions for the last time period. In the model S, I start with $t = 110$ (agent aged 130). In more than 1 000 000 simulations no agent reached an age of 120 or more. An identical model with 200 as latest time of death yields the same results. I use 24 time steps for each year which yields overall 1441 time steps in the D model and 2641 time steps in the

S model. For the z grid, I choose $z \in (0,200]$ and use 10001 z-grid points in both models. The lower bound follows directly from the included liquidity constraint. I choose the upper bound in a way such that the normalized wealth process does most likely not reach the bound. In the results, the average value of z is below 20 throughout the lifetime.

Simulations of Section 8 The solution approach is more time-intensive with a stochastic hazard rate of death. I use the same algorithm for the S, SD and SDJ model to avoid a numerical impact in the comparison graphs. I start with $t = 110$ in all models. I use 6 time steps for each year, this yields overall 661 time steps. I choose $z \in (0,150]$ with 1001 z-grid points. For the hazard rate of death, I use $\pi \in (0,1]$ with 1001 π-grid points in all models.

Life Insurance Demand under Health Shock Risk

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Abstract: This paper studies the life cycle consumption-investment-insurance problem of a family. The wage earner faces the risk of a health shock that significantly increases his probability of dying. The family can buy term life insurance with realistic features. In particular, the available contracts are long term so that decisions are sticky and can only be revised at significant costs. Furthermore, a revision is only possible as long as the insured person is healthy. A second important and realistic feature of our model is that the labor income of the wage earner is unspanned. We document that the combination of unspanned labor income and the stickiness of insurance decisions reduces the insurance demand significantly. This is because an income shock induces the need to reduce the insurance coverage, since premia become less affordable. Since such a reduction is costly and families anticipate these potential costs, they buy less protection at all ages. In particular, young families stay away from life insurance markets altogether.

Keywords: Health shocks, Portfolio choice, Term life insurance, Mortality risk, Labor income risk

JEL-Classification: D14, D91, G11, G22

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Holger Kraft and Lorenz Schendel gratefully acknowledge research and financial support from the Center of Excellence SAFE, funded by the State of Hessen initiative for research LOEWE. Holger Kraft gratefully acknowledges financial support by Deutsche Forschungsgemeinschaft (DFG). The authors thank the Frankfurt Cloud for access to high-performance virtual servers. All remaining errors are of course our own.

1 Introduction

For most households, labor income is the essential source to finance lifetime consumption. Therefore, a potential income loss following an early death of the wage earner is a crucial risk. Consequently, a life insurance is of special importance to hedge future consumption of the remaining family members. Following Richard (1975) most studies simplify the insurance decision by including an instantaneous term insurance contract. However, in practice buying life insurance usually involves a long-term commitment and later changes are costly. Furthermore, health checks prevent agents from contracting an insurance if they already have a critical illness. Therefore, we study a life cycle problem where a family has access to a realistic term life insurance that is solely available as a long-term contract which can only be bought or sold at certain lump-sum costs.[1]

Another realistic feature of our model is that the family receives unspanned labor income earned by the head of the household. Since in practice agents cannot borrow against future income, we impose short-sale constraints that are binding, especially at young ages. In addition, we take a liquidity constraint into account so that the family's financial wealth has to stay non-negative at all points in time. Although papers studying life cycle problems without insurance decisions sometimes include unspanned labor income (see, e.g., Munk and Sørensen (2010)), this issue is usually disregarded in papers including insurance decisions. Additionally, we add another layer of incompleteness. In our model, the wage earner faces the risk of suffering from a health shock that we interpret as critical illness. After a health shock the family has no access to the insurance market any more, i.e. cannot buy new insurance or change existing contracts. Furthermore, the wage earner's probability of dying increases significantly. We calibrate the health shock and mortality process to cancer and mortality data.

The combination of these realistic features (long-term insurance contracts, transaction costs, unspanned labor income, short-sale and liquidity constraints, health shocks) distinguishes our model from the related literature discussed in Section 2. This combination generates interesting qualitative effects that are important for the optimal decisions of the family over the life cycle: The long-term nature of the insurance contract amplifies the effect of negative labor income shocks, since in the undesired case of a negative labor income shock a premature termination of the contract or a reduction of the insurance sum leads to additional losses. In an already bad state, the family might be worse off due to the stickiness of the insurance contract. Therefore, families with higher income uncertainty have significantly lower insurance demands. The amplifying effect also reduces the demands of families that are more risk averse and face labor income risk.

[1] Formally, we model the insurance decision as an impulse control problem.

Most importantly, we find that younger families (head of household less than 30 years old) optimally stay away from life insurance markets and do not buy term life insurance at all. Therefore, an unexpected death in younger years leads to severe problems for the family. Our finding is in line with the low participant rates that are observed empirically.[2] However, it differs significantly from the results in frameworks that model life insurance decisions via an instantaneous contract instead of a long-term contract as in our paper. In these frameworks, the theoretically optimal participation rates are typically much higher.

Furthermore, we find that it is optimal for families to increase insurance protection over the life cycle. This is because the long-term contract design becomes less relevant as agents get older, since the contract duration and the uncertainty about human wealth goes down. Therefore, if an older wage earner suddenly dies, the accumulated financial wealth and existing insurance contracts ensure that surviving family members can maintain their standard of living, although consumption growth must be reduced. To summarize, our results suggest that a high level of income, a high labor income volatility, large fees imposed by insurance companies and the presence of health shocks reduce the insurance demand of a family.

The remainder of the paper is organized as follows. Section 2 provides an overview of the related literature. Section 3 introduces the model setup. Section 4 presents the calibration. Section 5 discusses our benchmark results. Section 6 provides robustness checks. Section 7 concludes.

2 Related Literature

The modern portfolio optimization literature starts with Merton (1969) and Merton (1971). In discrete time, Cocco, Gomes, and Maenhout (2005) numerically solve a realistically calibrated life cycle model with deterministic mortality risk. One of their main objects is to study the effect of unspanned labor income that is calibrated to US data. Munk and Sørensen (2010) solve a realistically calibrated life cycle model in continuous time with unspanned labor income, but without mortality risk. Their main focus is on analyzing the effect of unspanned labor income and a stochastic riskfree rate on consumption-investment decisions. They adapt the labor income calibration results from Cocco, Gomes, and Maenhout (2005) to a continuous-time framework and find that *unspanned* labor income significantly effects consumption-investment decisions.

Merton (1975) points out the importance of the risk of dying as a source of risk. Campbell (1980) studies the corresponding optimization problem. He considers a two-period model and introduces an insurance market to allow a family to hedge the risk that the wage earner dies. Analytically, he derives the optimal demand for insurance. Yaari (1965)

[2] See, e.g., Hong and Ríos-Rull (2012) and the references therein.

analyzes the consumption decision of an agent when faced with longevity risk. Richard (1975) analytically solves a life cycle problem with deterministic labor income and a continuous instantaneous term insurance decision. He allows an uncertain time of death with deterministic distribution. Simple forms of an actuarially fair one-period life insurance contract are still widely used in the portfolio optimization literature. Pliska and Ye (2007) mathematically extend the model and analyze the effect of parameter choice on the life insurance demand. Closely related to our work is the paper Huang, Milevsky, and Wang (2008), who also consider the consumption-investment and life-insurance decision of a family with CRRA utility. They focus on the correlation between labor income and asset returns. Their main results are that life-insurance demand is insensitive to changing risk aversion and highly depends on labor income volatility. As the previous literature, they also model life insurance via a short-term contract and do not consider health shocks or stochastic mortality risk.

Recent papers building on the work of Richard (1975) use a continuous-time finite-state Markov chain approach to solve life cycle portfolio problems with insurance decisions analytically. Kraft and Steffensen (2008) focus on the consumption and insurance decision of a single person that faces the risk of dying and disability. Bruhn and Steffensen (2011) consider the consumption-investment and insurance decision of a two- and multi-person household. This strand of literature is able to provides analytical solutions of optimal insurance decisions, but relies on the assumptions that markets are complete and thus is not able to capture crucial realistic features such as unspanned labor income risk.

There are also recent papers studying life cycle problems of families that face mortality risk and can insure themselves via life insurance contracts. Love (2010) focuses on the effect of demographic shocks. In his model, the agent can exogenously get married, get divorced or have children. We adapt his calibration approach to capture the impact of the family size on the utility from consumption. His model involves a simple one-period term life insurance. Hubener, Maurer, and Rogalla (2013) analyze the optimal life insurance demand of retired couples. They model husband and wife via two separate mortality processes, but do not allow for changes in the family status (e.g. divorce). Since they consider only the retirement phase, they especially focus on annuities and disregard labor income. Hong and Ríos-Rull (2012) infer how individuals value consumption in different demographic stages using life insurance holdings by age, sex, and marital status. In particular, they estimate a consumption equivalence scale parameter.

There are also papers on life cycle problems with stochastic mortality risk. In a discrete-time setting, Cocco and Gomes (2012) include stochastic mortality risk in a realistically calibrated life cycle model which they solve numerically. They capture mortality risk by a Lee-Carter type model. The agent can invest in a riskless bond and in a longevity bond that is correlated with shocks in mortality rates. Furthermore, they allow the agent to choose the retirement date endogenously. Huang, Milevsky, and Salisbury (2012) analyze optimal consumption decisions analytically and compare results from a Yaari type model with a model allowing for stochastic mortality risk. In contrast to our paper, stochastic mortality risk is modeled as a geometric Brownian motion. Koijen, Van Nieuwerburgh, and Yogo (2013) consider a life cycle problem where the probability of dying can have unsystematic jumps. They develop risk measures for life and health insurance products that pool the effects of several insurance products. In a discrete-time setting, they calculate the corresponding optimal results for their risk measures. They also compare the model implied risk measures with empirically derived values where their focus on agents that are older than 50 years. Their model also involves critical illness jumps. In contrast to our paper, they focus on the insurance implications, but do not consider unspanned labor income or stock market risk.

3 Model Setup

In this section, we present the model setup and describe the optimization problem of the family.

Financial Assets The agent can invest into two financial assets, but faces short-sale constraints. The assets are a risky stock (index) S and a riskfree bond B. The riskfree rate is denoted by r. The dynamics are given by

$$\begin{aligned} dS_t &= S_t\left[(r+\sigma_S\lambda)\,dt+\sigma_S\,dW_t^S\right], \\ dB_t &= B_t r\,dt \end{aligned}$$

with a constant market price of risk λ and stock market volatility σ_S. The process $W^S = \left(W_t^S\right)$ is a standard Brownian motion.

Biometric Risk The sole wage earner faces the risk of a health shock (e.g. cancer) and of a death shock. The state variable A defined by

$$A_t = \begin{cases} 1 & \text{alive and healthy at } t, \\ 2 & \text{alive but unhealthy at } t, \\ 3 & \text{dead at } t, \end{cases}$$

captures the current status of the wage earner. The random age of death is denoted by τ^D and is modeled as doubly stochastic stopping time with intensity $\pi(t,A)$, the so-called hazard rate of death, where $\pi(t,3)=0$. Formally, τ^D is the time of the first jump of the jump process $N^D = (N_t^D)$. The health shock jump process $N^H = (N_t^H)$ has an only time-dependent intensity $\kappa(t)$ while $A_t = 1$. The health shock is permanent so that agents cannot recover again. Unhealthy agents cannot face another health shock.[3] The time of a health shock is denoted by τ^H. If the agent does not experience a health shock during his lifetime, then τ^H is infinity.

Unspanned Labor Income The family receives an uncertain income stream denoted by Y. Its dynamics are influenced by the health status and age of the wage earner and are given by

$$dY_t = \mathbb{1}_{\{A_t=1,2\}} Y_t \left(\mu_Y(t)\,dt + \sigma_Y(t)\left(\rho(t)\,dW_t^S + \sqrt{1-\rho(t)^2}\,dW_t^Y\right)\right) + \mathbb{1}_{\{A_{t^-}=1\}} Y_{t^-}(p^{1,2}(t)-1)dN_t^H$$
$$+ \mathbb{1}_{\{A_{t^-}=1\}} Y_{t^-}(p^{1,3}(t)-1)dN_t^D + \mathbb{1}_{\{A_{t^-}=2\}} Y_{t^-}(p^{2,3}(t)-1)dN_t^D, \tag{1}$$

where $W^Y = (W_t^Y)$ is a standard Brownian motion, independent of W^S. This income stream is unspanned for two reasons: First, the Brownian motion W^Y cannot be hedged in the financial market. Second, the health shock N^H cannot be fully insured.

Furthermore, $p^{i,j}$ is the fraction of income that remains after a jump from state i to state j. We assume that the agent retires at the prespecified date T_R. The income process has a drift of μ_Y, a volatility of σ_Y and is correlated with the stock via ρ. Before retirement, the family's income is interpreted as labor income, whereas it is a pension after the retirement date. After the death of the wage earner, the income stream can be interpreted as widow's pension indexed by the salary upon death. If a critical illness shock occurs, the decreased income can be interpreted in the sense that the wage earner is forced to reduce work effort or increased medical expenses reduce the net income. Alternatively, it can be interpreted as a transfer income that the family receives from the government.

Insurance The family can buy a term life insurance to hedge the potential income loss resulting from the mortality risk of the wage earner. While the insurance contract is active, the insurance company pays the family a fixed payment I if the wage earner dies. The insurance offers a fixed set of contracts with specific payouts. The set of offered contracts is denoted by

$$\mathscr{I} = \{0, 50\,000, 100\,000, 150\,000, 200\,000, 300\,000, 500\,000, 750\,000, 1\,000\,000, 2\,000\,000\}. \tag{2}$$

The family must pay a constant insurance premium $\iota(I)$ as long as the contract is active. When the agent changes the insurance sum of the contract, a lump-sum payment η is due.

[3] Our results hardly change if we allow for more than one health shock.

This payment takes the previous insurance sum, the new insurance sum and the age of the agent that determines the mortality pattern into account. The lump-sum payment ensures an actuarially fair new contract, but it also involves a fee. The contract can be changed as long as the wage earner is healthy and younger than T_C. The insurance contract expires at $\min(\tau^D, T_I)$, i.e. at the death of the insured person or at the maturity of the contract, T_I. In the first case, the insurance pays the insurance sum I, in the second case the insurance pays nothing. We assume that $T_C < T_I$, i.e. after T_I there is no insurance available any more.

Technically, the insurance decision can be characterized by an impulse control problem. The family chooses the intervention times ζ_i, $i \in \mathbb{N}$, and the intervention actions ω_i, $i \in \mathbb{N}$. The intervention can take place at time ζ_i if the wage earner is alive ($\zeta_i < \tau^D$), healthy ($A_{\zeta_i} = 1$), and in the insurance market ($\zeta_i \le T_C$). A feasible intervention action requires that ω_i is chosen such that the new insurance sum is in the set of offered contracts, $I_{\zeta_i} \in \mathscr{I}$. We denote the set of possible interventions at ζ_i by

$$\mathscr{I}_{\zeta_i} = \{0 - I_{\zeta_i^-}, 50\,000 - I_{\zeta_i^-}, \ldots, 2\,000\,000 - I_{\zeta_i^-}\}.$$

Formally, the above statement can be expressed by the condition $\omega_i \in \mathscr{I}_{\zeta_i}$. At an intervention time, the family must pay the lump-sum payment $\eta(\zeta_i, \omega_i, I_{\zeta_i})$ that is a correction payment which makes the insurance contract actuarially fair and involves a fee. A detailed description is postponed to Section 4, see equation (4). Following the intervention, the family pays the new yearly premium $\iota(I_{\zeta_i})$ to maintain insurance protection until $\min(\tau^D, T_I)$.

Preferences The family has a fixed time horizon T and a power utility function given by

$$u(x, A) = \frac{\left(\frac{x}{\phi_A}\right)^{1-\gamma}}{1-\gamma}$$

with relative risk aversion γ. Here ϕ_A is a consumption scaling term that depends on the family size and for instance captures that two persons do not need twice as much consumption as a single person for the same utility level.[4] The family maximizes expected utility from intermediate consumption and terminal wealth given by

$$\mathrm{E}_{t,x,y,I,A}\left[\int_t^T \mathrm{e}^{-\delta(u-t)} \frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma} du + \varepsilon\, \mathrm{e}^{-\delta(T-t)} \frac{X_T{}^{1-\gamma}}{1-\gamma}\right]$$

[4] Preferences with a consumption scaling parameter are used by Love (2010) and Hubener, Maurer, and Rogalla (2013).

with time preference rate δ and financial wealth X. The constant ε specifies the importance of the bequest motive.

Financial Wealth Dynamics The family chooses consumption c and the fraction θ invested in the risky asset. As long as the wage earner is healthy and young, the family also optimizes the insurance sum decision via the impulse control strategy $(\zeta_i, \omega_i), i \in \mathbb{N}$. The wealth dynamics follow

$$
\begin{aligned}
dX_t &= X_t \left[(r + \theta_t \lambda \sigma_S)\, dt + \theta_t \sigma_S \, dW_t^S \right] + \left[Y_t - c_t - \mathbb{1}_{\{A_t = 1,2 \wedge t < T_I\}} \iota(I_t) \right] dt \\
&\quad + \mathbb{1}_{\{A_{t^-} = 1,2 \wedge t < T_I\}} I_{t^-} \, dN_t^D, \\
X_{\zeta_i} &= X_{\zeta_i^-} - \eta(\zeta_i, \omega_i, I_{\zeta_i}).
\end{aligned}
$$

Optimization Problem As stated above, the family optimizes expected utility from intermediate consumption and terminal wealth. The optimization problem is characterized by several state variables: financial wealth x, labor income y, the health status of the wage earner A and the current insurance choice I. The control variables are the consumption rate c, the proportion of wealth θ invested in risky assets, and the impulse control strategy for the insurance decision $(\zeta_i, \omega_i), i \in \mathbb{N}$. At time $t = 0$ the wage earner is assumed to be 20 years old. The optimization problem is then given by

$$
\begin{aligned}
&\max_{\{c_s, \theta_s\}_{s \in [0,T)}, \{(\zeta_i, \omega_i)\}_{i \in \mathbb{N}}} \mathrm{E}_{0,x,y,I,A} \left[\int_0^T \mathrm{e}^{-\delta u} \frac{\left(\frac{c_u}{\phi_{A_u}} \right)^{1-\gamma}}{1-\gamma} du + \varepsilon \, \mathrm{e}^{-\delta T} \frac{{X_T}^{1-\gamma}}{1-\gamma} \right] \\
&\text{s.t.} \qquad dX_t = X_t \left[(r + \theta_t \lambda \sigma_S)\, dt + \theta_t \sigma_S \, dW_t^S \right] + \left[Y_t - c_t - \mathbb{1}_{\{A_t = 1,2 \wedge t < T_I\}} \iota(I_t) \right] dt \\
&\qquad\qquad\quad + \mathbb{1}_{\{A_{t^-} = 1,2 \wedge t < T_I\}} I_{t^-} \, dN_t^D, \\
&\qquad\quad\; X_{\zeta_i} = X_{\zeta_i^-} - \eta(\zeta_i, \omega_i, I_{\zeta_i}),
\end{aligned}
$$

where we impose short-sale constraints, i.e. $\theta_t \in [0,1]$, and liquidity constraints, i.e. consumption has to be chosen in such a way that financial wealth stays positive, $X_t \geq 0$. The value function (indirect utility function) is defined by

$$
J(t,x,y,I,A) = \sup_{\{c_s, \theta_s\}_{s \in [t,T)}, \{(\zeta_i, \omega_i)\}_{i \in \mathbb{N}}} \mathrm{E}_{t,x,y,I,A} \left[\int_t^T \mathrm{e}^{-\delta(u-t)} \frac{\left(\frac{c_u}{\phi_{A_u}} \right)^{1-\gamma}}{1-\gamma} du + \varepsilon \, \mathrm{e}^{-\delta(T-t)} \frac{{X_T}^{1-\gamma}}{1-\gamma} \right].
$$

We split the problem into its impulse control and stochastic control part. Given no intervention at t, but optimal impulse control afterwards, the value function is denoted by

$$J^*(t,x,y,I,A) = \sup_{\{c_s,\theta_s\}_{s\in[t,T)},\{(\zeta_i\,|\,\zeta_i\neq t,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{t,x,y,I,A}\left[\int_t^T \mathrm{e}^{-\delta(u-t)}\frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma}du + \varepsilon\,\mathrm{e}^{-\delta(T-t)}\frac{{X_T}^{1-\gamma}}{1-\gamma}\right].$$

In this case, the optimization problem reduces to a stochastic control problem. The corresponding Hamilton-Jacobi-Bellman equation (HJB) is given by

$$\begin{aligned}
\delta J^* = \sup_{c,\theta} \quad & \Bigg\{\frac{\left(\frac{c}{\phi_A}\right)^{1-\gamma}}{1-\gamma} + J_t^* \\
& + J_x^*\left[x(r+\theta\lambda\sigma_S)+y-c-\mathbb{1}_{\{A=1,2\wedge t<T_I\}}\iota(I)\right] + \frac{1}{2}J_{xx}^* x^2\theta^2\sigma_S^2 \\
& + \mathbb{1}_{\{A=1,2\}}\left[J_y^* y\mu_Y(t) + \frac{1}{2}J_{yy}^* y^2\sigma_Y(t)^2 + J_{xy}^* xy\sigma_S\sigma_Y(t)\rho(t)\theta\right] \\
& + \mathbb{1}_{\{A=1\}}\kappa(t)\left[J^*(t,x,p^{1,2}(t)y,I,2) - J^*(t,x,y,I,1)\right] \\
& + \mathbb{1}_{\{A=1\}}\pi(t,A)\left[J^*(t,x+\mathbb{1}_{\{t\le T_I\}}I,p^{1,3}(t)y,0,3) - J^*(t,x,y,I,1)\right] \\
& + \mathbb{1}_{\{A=2\}}\pi(t,A)\left[J^*\left(t,x+\mathbb{1}_{\{t\le T_I\}}I,p^{2,3}(t)y,0,3\right) - J^*(t,x,y,I,2)\right]\Bigg\}
\end{aligned}$$

with terminal condition $J^*(T,x,y,I,A) = \varepsilon\frac{x^{1-\gamma}}{1-\gamma}$. Here subscripts on J indicate partial derivatives. Finally, we calculate the value function J by maximizing J^* over all possible interventions at $\zeta_i = t$:

$$J(t,x,y,I,A) = \sup_{\omega_i\in\mathscr{I}_{\zeta_i}} \left\{J^*\left(t,x-\eta(\zeta_i,\omega_i,I_{\zeta_i}),y,I+\omega_i,A\right)\right\}.$$

Note that in the case of $\omega_i = 0$ we have a continuation strategy, i.e. the family decides to keep its insurance decision. If this is optimal, then $J(t,x,y,I,A) = J^*(t,x,y,I,A)$. Consequently, there is no lump-sum payment, since we are in the no transaction region and $\eta(\zeta_i,0,I_{\zeta_i}) = 0$.

4 Calibration

This section describes the model calibration that is also summarized in Table 1.

Financial Assets We use standard values for the stock market drift ($\mu_S = 0.06$), the stock market volatility ($\sigma_S = 0.2$) and the riskfree rate ($r = 0.02$) that are similar to the values used by Cocco, Gomes, and Maenhout (2005) or Munk and Sørensen (2010), among others.

	Financial Market	
μ_S	Stock drift	0.06
σ_S	Stock volatility	0.2
r	Bond drift	0.02
	Preferences	
δ	Time preference rate	0.03
γ	Relative risk aversion	4
ε	Weight of the bequest motive	1
α_{Adult}	Number of adults in the household	2
α_{Child}	Number of children in the household	1
T	Time horizon of the family	80
X_0	Initial financial wealth	38 214
	Mortality Risk	
x	Age of the wage earner at $t = 0$	20
m	X-axis displacement	89.45
b	Steepness parameter	6.5
k_1	Constant impact of a health shock	0.048
k_2	Age-dependent impact of a health shock	0.0008
	Health Shock Risk	
a	Scaling parameter	0.02489
b	X-axis displacement	66.96
c	Steepness parameter	29.42
	Income	
ξ_0	Age and education independent wage increase	0.02
b	Education dependent wage increase	0.1682
c	Education and age dependent wage increase parameter	−0.00323
d	Education and age dependent wage increase parameter	0.00002
P	Replacement ratio	0.68212
T_R	Retirement time	45
Y_0	Initial income	19 107
σ_Y^w	Volatility while working	0.2
σ_Y^r	Volatility during retirement	0
ρ^w	Correlation with the stock while working	0
ρ^r	Correlation with the stock during retirement	0
$p^{1,2}(t < T_R)$	Income level after a health shock while working	0.8
$p^{1,2}(t \geq T_R)$	Income level after a health shock during retirement	1
$p^{1,3}$	Income level at death without previous health shock	0
$p^{2,3}$	Income level at death with previous health shock	0
	Insurance	
ψ_{ad}	Administrative fee	0.0299
ψ_{tr}	Transaction fee	0.0505
T_C	Latest time for changing the insurance contract	50
T_I	Contract maturity	55

Table 1: Benchmark Calibration Parameters. This table summarizes the calibration described in Section 4.

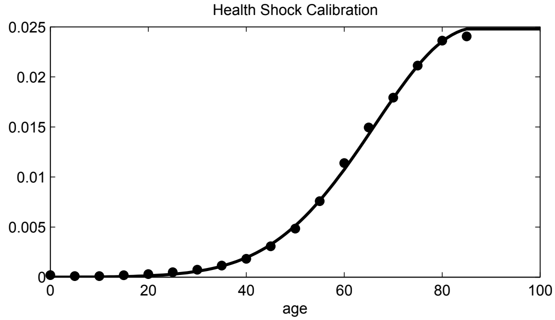

Figure 1: Health Shock Calibration. The figure shows empirical gender averaged 5-year cancer detection rates and our fitted curve κ. The empirical realizations (points) are gender averaged values from "Cancer in Germany 2007/2008, German Centre for Cancer Registry Data & Robert Koch Institute, 8th Edition 2012". The parameters for our fitted curve κ (solid line) are given in Section 4.

Biometric Risk Considering mortality risk, we use a Gompertz mortality model with constant parameters x, m, b for the healthy agent and increase the hazard rate of death by a constant term k_1 and an age-dependent term k_2 if the agent becomes unhealthy

$$\pi(t,A) = \begin{cases} \frac{1}{b}\,\mathrm{e}^{\left(\frac{x+t-m}{b}\right)} & \text{for } A_t = 1 \\ \frac{1}{b}\,\mathrm{e}^{\left(\frac{x+t-m}{b}\right)} + k_1 + k_2 t & \text{for } A_t = 2 \\ 0 & \text{for } A_t = 3 \end{cases}.$$

We calibrate the health shock using German cancer data.[5] We weight the gender specific data equally and do not distinguish between genders in the simulation. This yields an overall lifetime risk of getting cancer of 46.75%, a median age at diagnosis of 69 years and an absolute 5-year survival rate of 53.5%. Furthermore, the data provides age and genderspecific cancer incidence rates for 5-year intervals up to an age of 85. We calibrate the health shock rate κ using a gender averaged version of the age specific cancer incidence rates. We assume the following functional form for the health shock rate

$$\kappa(t) = a\,\mathrm{e}^{-\left(\frac{\min(t,65)-b}{c}\right)^2}$$

with constant parameters a, b, c. Since we do not have data for ages higher than 85, for simplicity we assume that cancer rates are constant for agents older than 85 years. We obtain the parametrization $a = 0.02489$, $b = 66.96$ and $c = 29.42$. Figure 1 illustrates the data points and our calibration of κ. For the calibration of the magnitude of the impact of the cancer shock on the mortality intensity (k_1, k_2) we use the absolute average 5-year

[5] We use German data taken from "Cancer in Germany 2007/2008, German Centre for Cancer Registry Data & Robert Koch Institute, 8th Edition 2012".

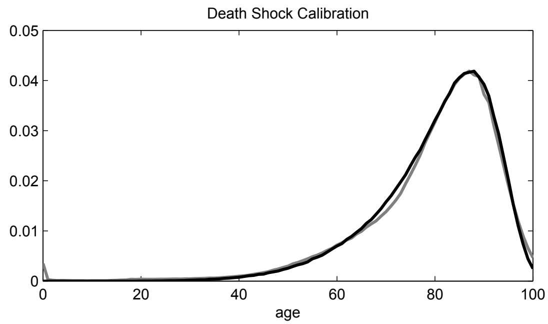

Figure 2: Death Shock Calibration. The figure depicts the number of yearly deaths for a normalized population of size 1. The mortality data (grey line) is gender averaged from a life table for Germany ("Sterbetafel 2009/11, Statistisches Bundesamt, 2013"). Our simulated values (dark line) are the averages from 100 000 death shock simulations using the biometric risk calibration of Section 4.

survival probability. Since there is an age dependency, we split the effect in a constant part k_1 and an age dependent part k_2. We calibrate k_1 and k_2 such that the simulated average 5-year survival probability and simulated death distribution match the empirical ones from the data. Here, we use German mortality data.[6] For the shock impact, the calibration yields $k_1 = 0.048$ and $k_2 = 0.0008$. Considering the Gompertz mortality risk parameters, we set the age at $t = 0$ to $x = 20$, the x-axis displacement to $m = 89.45$ and the growth rate (steepness parameter) to $b = 6.5$. Figure 2 compares our simulated yearly death rates with the empirical death rates in Germany. Our simulation fits the above-mentioned empirical means well: The average time of death is 80.4, the average time of cancer detection is 69.0, and over the lifetime 47.4% of the population face a health shock. The median survival rate at cancer detection is 5 years. Figure 3 shows the histogram of health shocks and death shocks and the corresponding health-state distribution of the wage earner in our simulation.

Labor Income For the income dynamics (see equation (1)), we use the labor income calibration from Munk and Sørensen (2010), which is a continuous-time version of the Cocco, Gomes, and Maenhout (2005) results. They estimate the labor income process using PSID data dependent on the agent's education. Its drift term is modeled as

$$\mu_Y(t) = \begin{cases} \xi_0 + b + 2ct + 3dt^2 & \text{for } t < T_R \\ -(1-P) & \text{for } T_R \le t \le T_R + 1 \\ 0 & \text{for } t > T_R + 1 \end{cases}.$$

[6] The German mortality data is taken from "Sterbetafel 2009/11, Statistisches Bundesamt, 2013".

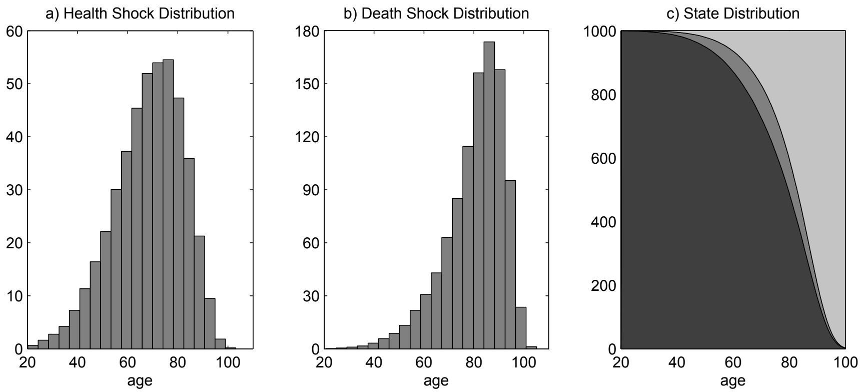

Figure 3: Simulated Biometric Risk Distribution. The graphs show the results of 100 000 simulated life cycles with the biometric risk parametrization of Section 4. a) depicts the histogram of the health shocks in our simulation. b) shows the corresponding histogram of death shocks. c) shows the state distribution of the families. The dark area corresponds to families with a healthy wage earner ($A = 1$), the middle grey area represents unhealthy agents ($A = 2$) and the light area marks families with a dead wage earner ($A = 3$).

In the benchmark calibration, we assume that the wage earner has a high school education and set the corresponding parameters according to Munk and Sørensen (2010). We use a retirement age of 65 ($T_R = 45$), an age- and education-independent real wage increase of $\xi_0 = 0.02$ and an initial income of $Y_0 = 19107$. The drift polynomial is given by $b = 0.1682, c = -0.00323, d = 0.000020$ and the retirement income reduction parameter is $P = 0.68212$. We also assume a different volatility parameter before and after retirement

$$\sigma_Y(t) = \begin{cases} \sigma_Y^w & \text{for } t < T_R \\ \sigma_Y^r & \text{for } t \geq T_R \end{cases},$$

where we use $\sigma_Y^w = 0.2$ and $\sigma_Y^r = 0$ in the benchmark calibration. In the same manner, we fix the correlation parameter

$$\rho(t) = \begin{cases} \rho^w & \text{for } t < T_R \\ \rho^r & \text{for } t \geq T_R \end{cases}.$$

Following Munk and Sørensen (2010), we assume zero correlation ($\rho^w = 0, \rho^r = 0$). If a jump occurs (critical illness or death), the income is reduced. In the critical illness case, we assume that the wage earner loses part of his income because he has to reduce his work effort.[7] We suppose that income decreases by 20%, i.e. $p^{1,2}(t < T_R) = 0.8$. If the critical

[7] In practice, an agent could buy disability insurance, but this usually does not cover all loses. At least potential future wage increases cannot be insured.

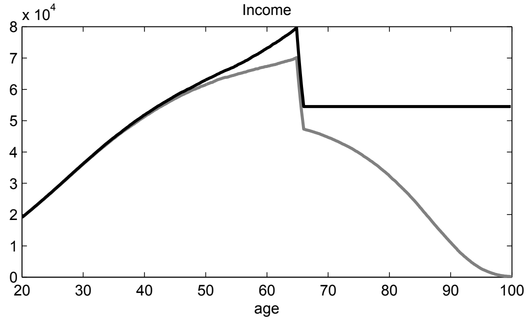

Figure 4: Expected Income Profile over the Life Cycle. The figure depicts the expected income of a wage earner conditional on survival (dark line) assuming that he has a high school education. The grey line gives the expected income of the family and incorporates the biometric risk of the wage earner. The labor income and biometric risk parameter values are stated in Section 4 and the lines depict the average values of 100 000 simulations.

illness occurs during retirement, the pension is however unaffected, i.e. $p^{1,2}(t \geq T_R) = 1$. In both cases, the income volatility remains unchanged. In the benchmark calibration, we do not include a social security system which is studied as a robustness check (see Section 6). Hence, we set $p^{1,3} = p^{2,3} = 0$ such that the family has no income if the wage earner dies. Figure 4 depicts the income profile of the wage earner and of the family over the life cycle.

Insurance We assume a competitive insurance market, which leads to an actuarially fair insurance. First, we consider the continuation case where no intervention takes place. Then, the family only has to pay the insurance premium. The yearly insurance premium $\iota(I)$ is assumed to be constant over time and is defined by the actuarial fairness criterion at $t = 0$ so that the expected discounted value of the insurance premia, Σ_{agent}, is equal to the the expected discounted value of the payments of the insurance company to the family, Σ_{ins}, i.e.

$$\Sigma_{agent}(0,I) = \Sigma_{ins}(0,I)$$

with

$$\begin{aligned}\Sigma_{agent}(0,I) &= \mathrm{E}_0\left[\int_0^{T_I} \mathrm{e}^{-rs}\,\frac{\iota(I)}{1+\psi_{ad}+\psi_{tr}}\mathbb{1}_{\{s<\tau^D\}}\,ds\right]\\ &= \frac{\iota(I)}{1+\psi_{ad}+\psi_{tr}}\,\frac{1}{r}\left(1-\mathrm{E}_0\left[\mathrm{e}^{-r\min\{\tau^D,I\}}\right]\right),\end{aligned}$$

and

$$\Sigma_{ins}(0,I) = \mathrm{E}_0\left[\int_0^{T_I} \mathrm{e}^{-rs}\,I\,d\mathbb{1}_{\{s\geq\tau^D\}}\right] = \int_0^{T_I} \mathrm{e}^{-rs}\,I f_D(s)\,ds,$$

where f_D is the probability density function of τ^D. The constant ψ_{ad} captures yearly administrative costs and the constant ψ_{tr} captures deferred acquisition costs that are paid with the yearly premium. Hence, the yearly insurance premium can be expressed as

$$\iota(I) = \frac{Ir(1+\psi_{ad}+\psi_{tr})\int_0^{T_I} f_{\tau^D}(s)\mathrm{e}^{-rs}\,ds}{1-\mathrm{E}_0\left[\mathrm{e}^{-r\min(\tau^D,T_I)}\right]}. \tag{3}$$

Next, we consider the intervention case, i.e. a situation where the family increases or decreases the insurance sum I. If an intervention (ζ_i,ω_i) takes place, the family must pay a lump-sum payment $\eta(\zeta_i,\omega_i,I_{\zeta_i})$. This payment is necessary since the insurance premium ι is calculated based on survival patterns at $t=0$. Therefore, an insurance contract starting at $\zeta_i>0$ requires an adjusted premium. If the family reduces protection, $\omega_i<0$, no lump-sum payment is made.[8] If the family increases insurance protection, $\omega_i>0$, we assume that the insurance company takes previously paid premia into account so that the lump-sum payment is

$$\begin{aligned}\eta(\zeta_i,\omega_i,I_{\zeta_i}) = \left(1+\psi_{ad}+\psi_{tr}\right)\Big(&\Sigma_{ins}(\zeta_i,I_{\zeta_i})-\Sigma_{agent}(\zeta_i,I_{\zeta_i})\\ &-\Sigma_{ins}(\zeta_i,I_{\zeta_i}-\omega_i)+\Sigma_{agent}(\zeta_i,I_{\zeta_i}-\omega_i)\Big),\end{aligned} \tag{4}$$

where the two last terms depend on the previous insurance sum and capture the retrospective reserve. Note that they vanish in the special case where the family has no previous insurance protection. The variables $\Sigma_{ins}(\zeta_i,I_{\zeta_i})$ and $\Sigma_{agent}(\zeta_i,I_{\zeta_i})$ denote the conditional present values of the payments to the family and to the insurance company:

$$\begin{aligned}\Sigma_{ins}(\zeta_i,I_{\zeta_i}) &= \int_{\zeta_i}^{T_I} \mathrm{e}^{-r(s-\zeta_i)} I_{\zeta_i} f_D(s\,|\,\min(\tau^D,\tau^H)>\zeta_i)\,ds,\\ \Sigma_{agent}(\zeta_i,I_{\zeta_i}) &= \frac{\iota(I_{\zeta_i})}{1+\psi_{ad}+\psi_{tr}}\frac{1}{r}\left(1-\mathrm{E}_{\zeta_i}\left[\mathrm{e}^{-r(\min(\tau^D,T_I)-\zeta_i)}\,|\,\min(\tau^D,\tau^H)>\zeta_i\right]\right),\end{aligned}$$

where we condition on the agent being in the insurance market, i.e. being healthy and alive. The continuous premium ι is calculated according to (3). Notice that a valid intervention also requires $\zeta_i \le T_C$.

To summarize, there are two cases: If no intervention takes place, the family must pay the insurance premium $\iota(I)$. If an intervention (ζ_i,ω_i) takes place, an additional lump-sum payment becomes due

$$\eta(\zeta_i,\omega_i,I_{\zeta_i}) = \begin{cases} 0 & \text{if } \omega_i \le 0,\\ \left(1+\psi_{ad}+\psi_{tr}\right)\Big(\Sigma_{ins}(\zeta_i,I_{\zeta_i})-\Sigma_{agent}(\zeta_i,I_{\zeta_i}) & \\ \qquad -\Sigma_{ins}(\zeta_i,I_{\zeta_i}-\omega_i)+\Sigma_{agent}(\zeta_i,I_{\zeta_i}-\omega_i)\Big) & \text{else.}\end{cases}$$

[8] Note that in countries like Germany a term life insurance has usually no repurchase value.

We allow the family to choose among insurance contracts with payouts specified by $\mathscr{I}$ (see equation (2)). The family is able to change its insurance exposures until the wage earner is 70 ($T_C = 50$). The insurance contract expires at the age of 75 ($T_I = 55$). These ages are typical for German term life insurance contracts.

We set the administrative fee ($\psi_{ad} = 2.99\%$) and the transaction fee ($\psi_{tr} = 5.05\%$) to the average values of the German life insurance market.[9] We discretize the conditional expectations $\mathrm{E}_t\left[\mathrm{e}^{-r\min(\tau^D,T_I)} \mid \min(\tau^D,\tau^H) > t\right]$ and $\int_t^{T_I} \mathrm{e}^{-rs} f_D(s \mid \min(\tau^D,\tau^H) > t)ds$, for $t \in \{0,1,\ldots,T_C\}$ using German mortality data.[10] For intermediate values of t we use linear interpolation.

Preferences We choose standard values from the life cycle portfolio optimization literature for the risk aversion ($\gamma = 4$), the time preference rate ($\delta = 0.03$) and the bequest motive ($\varepsilon = 1$). In the benchmark calibration, the problem starts at time $t = 0$, when the wage earner is 20 years old, and ends at $t = T = 80$, when the wage earner is either 100 years old or dead. Following Love (2010), we calculate the consumption equivalence scaling parameter via

$$\phi = (\alpha_{Adult} + 0.7\alpha_{Child})^{0.7},$$

where α_{Adult} is the number of adults and α_{Child} is the number of children in the household. In the benchmark calibration, we study a family consisting of two adults and one child. Hence, we set the corresponding consumption scaling parameter to $\phi_{1,2} = 2.0043$ if the wage earner is alive and to $\phi_3 = 1.4498$ if the wage earner is dead. We assume that the family starts with a financial wealth level that is twice the initial annual labor income of the wage earner, $X_0 = 2Y_0$.

5 Benchmark Results

This section provides our main results for the model introduced in Section 3 with the calibration presented in Section 4.

Average Key Variables over the Life Cycle Figure 5 depicts the average optimal decisions as well as the average financial wealth and income over the life cycle. Similar as in the above mentioned papers, financial wealth is hump-shaped and the portfolio holdings are decreasing over the life cycle. The dark consumption line represents the consumption of

[9] Data about the transaction fees and administrative fees on the German insurance market are taken from "map-report no. 807-808".

[10] Mortality data is taken from Life table for Germany ("Sterbetafel 2009/11, Statistisches Bundesamt, 2013").

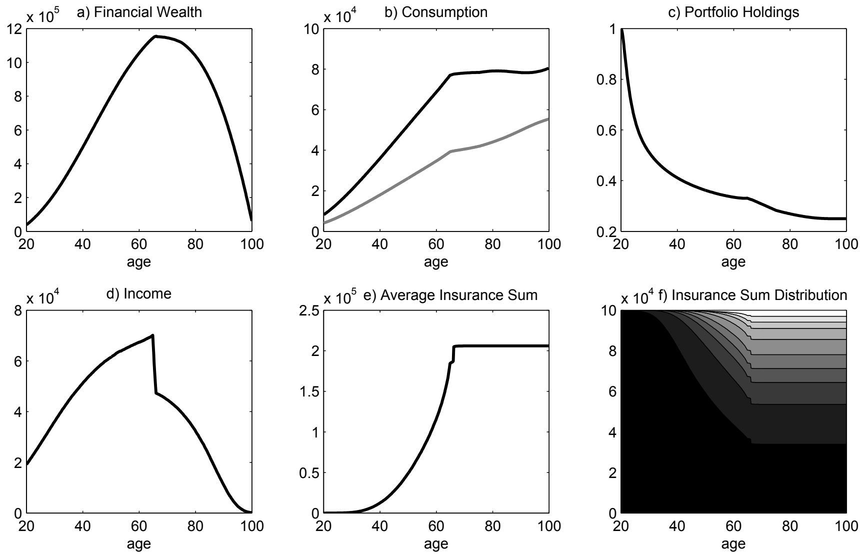

Figure 5: Average Key Variables over the Life Cycle. The graphs depict the average optimal control variables as well as the average financial wealth and income evolution over the life cycle based on 100 000 simulations with the benchmark calibration of Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The dark line corresponds to the consumption of the family, where the grey line represents the equivalent level of consumption for a one person household and is scaled with the consumption scaling parameter $\frac{c}{\phi}$. c) shows the optimal average portfolio holdings over the life cycle. d) depicts the average income of the family. e) shows the average insurance sum. f) depicts the distribution of the insurance sum over the life cycle. The darkest area marks families with no insurance contract, the white area families with the highest insurance sum (2 000 000) and the grey areas families with intermediate insurance sums.

the whole family. We see that consumption increases over the life cycle, although the slope decreases significantly after retirement. This is mainly due to two reasons. First, the labor income profile changes at retirement, especially the certainty of the retirement income leads to a flatter consumption path. Second, due to a higher mortality risk at older ages there are more families where the wage earner has already died. Then, the family has no more income and one person less to take care of, which both reduces consumption. The grey line represents the single person equivalent consumption.[11] This line increases almost linearly over the life cycle, which indicates that on average the death of the wage earner does not lead to a reduced utility from consumption for the remaining family members. This may be either due to a high amount of accumulated financial wealth, or a term life insurance contract. As the insurance sum distribution graph shows, it is a combination of both for most families. About 65% of the families buy a term life insurance over the life

[11] This is a comparable one-person consumption level that is calculated by weighting the consumption of the family with the consumption equivalence scaling parameter $\frac{c_t}{\phi_{A_t}}$.

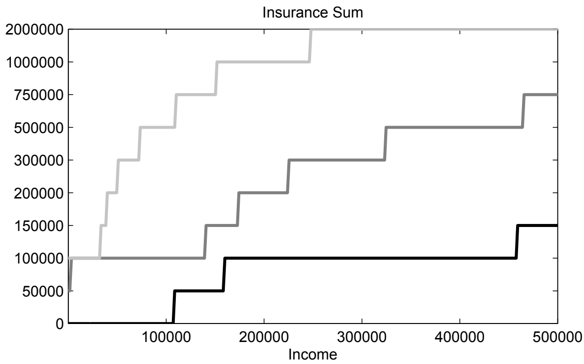

Figure 6: Insurance Demand Dependent on Income. This graph depicts the optimal insurance decision dependent on the income of a healthy agent ($A = 1$). The financial wealth is fixed to $x = 800\,000$, the previous insurance choice is fixed to $I = 100\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

cycle. At a young age, the families stay away from the insurance market, but start buying insurance at the age of about 30. Furthermore, the families increase the insurance sums over the life cycle and there is no age at which they systematically reduce the insurance exposures. Apparently, agents do not change the insurance contracts after the retirement age of 65, although changes would be possible until the age of 70. This can be explained by the certainty of the retirement income. Consequently, there is no uncertainty with respect to human wealth. Hence, there is no reason to change the insurance decision. The increasing insurance sum over the life cycle might be counterintuitive at first. Since an older agent has on average more financial wealth and more income to hedge the effect of a health shock, one could expect the insurance demand to be lower compared to a situation with less income and wealth. However, there are opposing effects. First, for an older agent the contract duration is shorter. Hence, reducing an insurance exposure is relatively more costly than keeping it. Second, uncertainty with respect to human wealth and financial wealth significantly reduces for an older agent. Therefore, it is less likely that he faces a wealth and income state in which the contract is not affordable or too expensive relatively to his financial situation. These two effects dominate and yield an overall increasing insurance demand over the life cycle.

Comparative Statics To analyze the effects of age, income, financial wealth, and the previous insurance contract, we consider the policy functions. Figure 6 depicts the insurance demand for different levels of the labor income. The lines represent different ages for a fixed level of financial wealth and a fixed previous insurance contract with insurance sum 100 000. For all ages, the insurance demand increases in the income level,

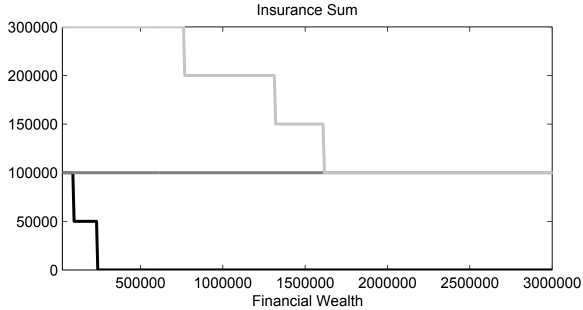

Figure 7: Insurance Demand Dependent on Financial Wealth. This graph depicts the optimal insurance decision dependent on the financial wealth of a healthy agent ($A = 1$). The income is fixed to $y = 50\,000$, the previous insurance choice is fixed to $I = 100\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

which has two reasons: First, if income is large, then a higher insurance sum becomes affordable. Second, since an insurance is a hedge against the income loss upon death of the wager earner, a higher income increases this hedging motive. Furthermore, the figure highlights a time-dependence. The black line shows the policy function for a young wage earner at the age of 25. For a reasonable labor income of below 100 000, it is always optimal for the family to ignore any previous insurance contract and to not pay the premium. This leads to a loss of insurance protection and all previous paid premiums are lost as well. We document a large continuation interval ranging from about 175 000 to 450 000, i.e. the family optimally sticks to the current insurance contract and no intervention takes place. The advantage is that the insurance protection maintains, past premiums are not lost and no expensive lump-sum costs must be paid for increasing the insurance sum. For a 50 year old middle-aged agent it is in general optimal to stick to the previous insurance decision, except for very high or low labor income. So the insurance choice at this age is also very robust. For an old retired agent at the age of 68 the optimal insurance sum crucially depends on the pension level. Furthermore, the insurance sum increases due to the certainty of the pension. This means that income volatility as a crucial source of uncertainty is no longer present. Overall, we document a strong dependence of the optimal insurance demand on labor income and time. The increasing insurance demand over time is in line with our previous findings.

Figure 7 depicts the corresponding policy functions dependent on financial wealth. Initially, we see that financial wealth has less impact on the insurance decision as income. However, this is not surprising, since the insurance is mainly used to hedge the loss of labor income in the case of death. Overall, the more financial wealth the agent has, the less insurance is optimal. This is intuitive, since with more wealth there is less need for a fixed payout in the case of death. For a middle-aged agent the continuation region includes

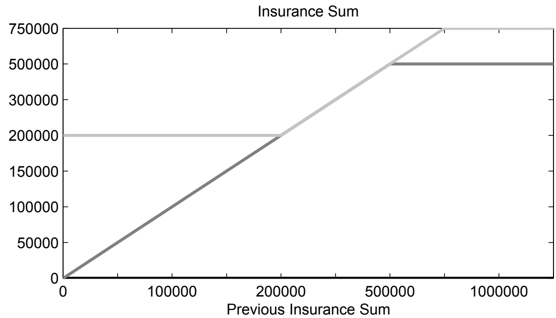

Figure 8: Insurance Demand Dependent on the Previous Insurance Sum. This graph depicts the optimal insurance decision dependent on the previous insurance sum of a healthy agent ($A = 1$). The financial wealth is fixed to $x = 800\,000$, the income is fixed to $y = 50\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

all reasonable financial wealth levels. The young agent cancels the insurance if he has a huge amount of financial wealth (that is above the average wealth level at this age), whereas the old agent raises the insurance contract if he has less than 1 600 000 wealth. To summarize, the insurance decision is rather insensitive towards the level of financial wealth.

The policy functions dependent on the previous insurance decision in Figure 8 document the stickiness of this decision. The diagonal line depicts a situation where the agent keeps his insurance decision, i.e. the previous insurance contract equals the current optimal decision. Below the diagonal, the agent reduces insurance protection, whereas he increases protection above the diagonal. A young agent would cancel any contract, independent of its insurance sum, and thus stays away from the insurance market in the first place. For him, the inflexibility of the contract is very severe, since he is tied to the contract for a potentially long time period or loses a significant amount of money if he cancels or reduces the contract prematurely. The uncertainty of human wealth at a young age amplifies this problem, as contracts are usually downward adjusted when human wealth has deteriorated and the current contract is not affordable any more. A middle-aged agent keeps a contract if the insurance sum is below 500 000. Contracts with higher insurance sum are reduced to this level. The old agent has a continuation region ranging from 200 000 to 750 000, where he sticks to his contract. To summarize, the decisions at every age are pretty stable: Young agents stay away from the insurance market, whereas middle-aged agents usually keep their current positions.

Impact of Critical Illness and Death Shocks Figure 9 depicts the effects of a critical illness shock at the age of 50 (black lines) and a death shock at the age of 60 (grey lines)

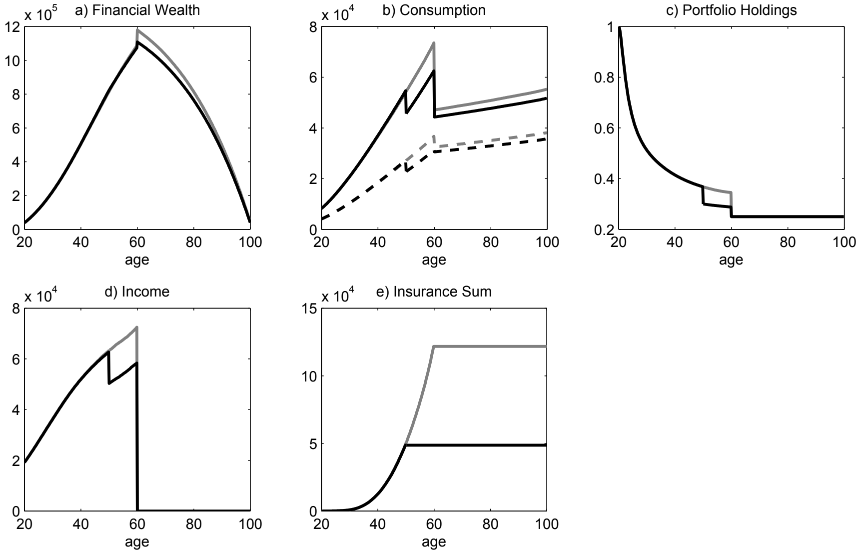

Figure 9: Optimal Reaction to Critical Illness and Death. The figure depicts the average financial wealth evolution and average optimal controls based on 100 000 simulations where all wage earners are assumed to die at the age of 60. The black lines correspond to the optimal behavior of a family where the wage earner gets a critical illness at the age of 50, whereas the grey lines represent a family whose wage earner dies without previous critical illness. The calibration equals the one of the benchmark results and is explained in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The solid lines are for the consumption of the family c and the dashed lines for the equivalent single person consumption level $\frac{c}{\phi}$. c) presents the average optimal portfolio holdings, d) gives the average income of the family and e) depicts the average optimal insurance sum over the life cycle.

on the optimal behavior and the financial wealth and income evolution. If the wage earner dies, financial wealth jumps upwards since the family receives an insurance payment. Furthermore, it leads to a negative jump in family consumption due to the reduced number of family members, although there is only a minor change in the level of single person equivalent consumption. However, the family reduces the slope of its consumption path to adjust to the new income situation. Besides, the portfolio holdings are reduced to the classical Merton demands, since there is no labor income any more.

If the agent is first exposed to a critical illness shock, he becomes aware of a high probability of an early death. Consumption is reduced since labor income decreases and the family tries to accumulate financial wealth for the remaining family members. Portfolio holdings are reduced as well. Unfortunately, the agent cannot increase his insurance protection any more. Therefore, at the time of death the average insurance sum is less than 50% of the sum of an agent without previous health shock. Finally, the single

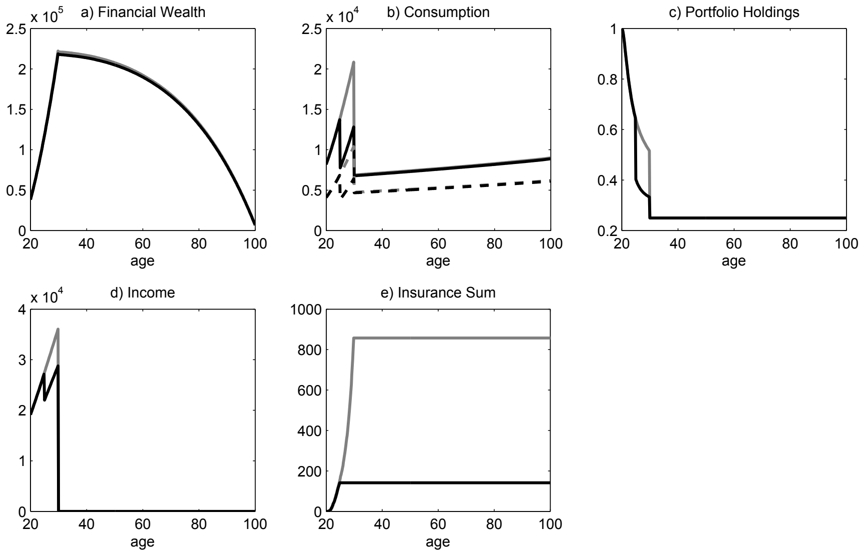

Figure 10: Optimal Reaction to Early Critical Illness and Death. The figure depicts the average financial wealth evolution and average optimal controls based on 100 000 simulations where all wage earners are assumed to die at the age of 30. The black lines correspond to the optimal behavior of a family where the wage earner gets a critical illness at the age of 25, whereas the grey lines represent a family whose wage earner dies without previous critical illness. The calibration equals the one of the benchmark results and is explained in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The solid lines are for the consumption of the family c and the dashed lines for the equivalent single person consumption level $\frac{c}{\phi}$. c) presents the average optimal portfolio holdings, d) gives the average income of the family and e) depicts the average optimal insurance sum over the life cycle.

person equivalent consumption is adjusted when the health shock occurs, and thus there is no significant decrease at the time of death.

Figure 10 illustrates a situation where the wage earner dies at the age of 30. In the first setting there is no previous health shock (grey lines), whereas in the second setting there is a health shock at the age of 25 (dark lines). In this case, the family has only little time to accumulate financial wealth. Furthermore, most families do not buy any insurance at this age, which explains the very low average insurance sum. These two facts together with the early death of the wage earner reduces consumption of the family and also the single person equivalent consumption significantly. Buying a term life insurance contract could easily double the available financial wealth.

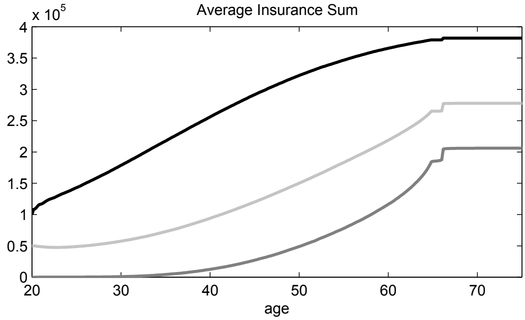

Figure 11: Insurance Demand for Different Levels of Labor Income Volatility. The dark line depicts the average insurance sum in a model where the labor income volatility before retirement is reduced to $\sigma_Y^w = 0.1$ and the light line depicts a reduction to $\sigma_Y^w = 0.15$. The grey line represents the benchmark results with a labor income volatility of $\sigma_Y^w = 0.2$. The remaining parameters are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

6 Robustness Checks

This section presents robustness checks for different labor income volatility, different insurance fees, a calibration without health shock and for a changed family size. We also discuss the main drivers that prevent families from increasing their insurance demands. Furthermore, we give results for a calibration with a social security system that pays a widow's pension after the death of the wage earner.

Labor Income Volatility A term insurance allows a family to (partially) hedge the risk resulting from an early death of the wage owner. Since the death of the wage earner predominately leads to a loss of labor income, the optimal insurance choice crucially depends on the labor income process. A negative feature of an insurance contract is the stickiness of its premia. Consequently, an insurance contract amplifies the effect of negative labor income shock. For instance, if a family is optimally insured and a negative labor income shock occurs, then the family has too much insurance protection given the actual income situation and must cut down on consumption. Alternatively, the family can reduce or terminate the insurance contract yielding to a loss, since term life insurance has no surrender value. In both cases, the effect of a negative labor income shock is stronger if the family has a higher insurance exposure. Therefore, hedging mortality risk comes at the cost of amplifying the effect of a negative labor income shock. In line with these findings, Figure 11 shows that the insurance demand is significantly higher for families with lower income volatility. Besides, these families also buy insurance earlier so that young families are insured as well.

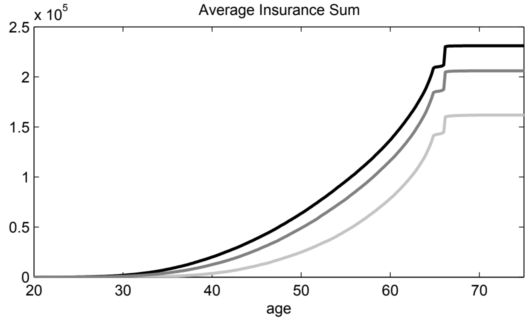

Figure 12: Insurance Demand for Different Fees. The light line depicts the average insurance sum in a model where the fees of the insurance company are increased such that the administrative fee is ψ_{ad} = 12.68% and the transaction fee equals ψ_{tr} = 13.62%. The values are taken from "map-report no. 807-808" and represent the highest fees in the German life insurance market. The dark line is for an actuarially fair insurance without fees ($\psi_{ad} = 0, \psi_{tr} = 0$). The grey line represents the benchmark results with fees of ψ_{ad} = 2.99% and ψ_{tr} = 5.05% that correspond to average fees in the German life insurance market. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

Insurance Structure The structure of a term life insurance contract varies among insurance companies. Especially the fees ψ_{tr}, ψ_{ad}, the date T_C until the insurance decision can be revised and the expiration date T_I can be different. Figure 12 depicts the effect of a change of the fees. We compare the average fees of the benchmark result (grey line) with a regime with high fees (light line) and without fees (dark line). Clearly, the insurance demand decreases if fees are raised. However, although the fees are increased significantly, the decrease in insurance demand is not dramatic. This indicates that the insurance profit, captured by the fees, has a rather small impact on the insurance demand.

Health Shocks A health shock prevents the family from increasing the insurance protection or buying a new contract. Intuitively, one might expect that the family anticipates this restriction and buys more protection at an earlier age. Figure 13 compares the benchmark model with an alternative calibration without health shocks. Surprisingly, our findings document a higher insurance demand in the case without health shocks. The reason is that a health shock can be interpreted as a warning that death becomes more likely. If the wage earner faces a health shock, the family knows that he will die with a high probability in the next few years. Although an early death is clearly negative for the family, the health shock partially resolves uncertainty about the timing of dying, which itself is beneficial. With this new information the family is better able to plan consumption and investment decisions. Therefore, the family reduces consumption in order to accumulate more wealth, which can be seen in Figure 9 and 10. Due to the additional savings, the insurance demand goes down. Notice that in our benchmark calibration labor income is

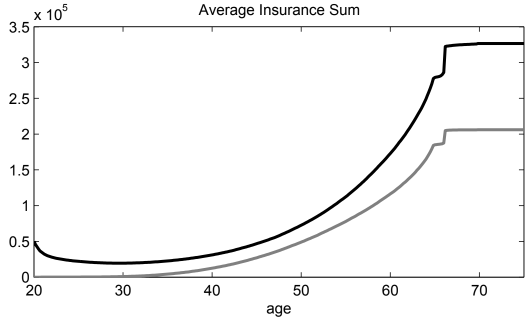

Figure 13: Insurance Demand Compared to a Model without Health Shocks. The dark line depicts the average insurance sum for a different biometric risk calibration without health shocks. Thus, the wage earner can only be in two health states ($A = 1,3$) and the family can contract and increase term life insurance contracts as long as the wage earner is alive and $t < T_C$. The health shock rate and magnitude are set to zero ($\kappa = 0$, $k_1 = 0$, $k_2 = 0$) and the mortality model has a standard Gompertz structure. The parameter calibration is changed to $b = 8.9$, $m = 85.47$ to get a similar death shock distribution as in the benchmark model with critical illness shocks. The grey line represents the benchmark results including health shocks and with the original mortality parameters. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

reduced after a health shock, which triggers a decrease in consumption. This is however also true in a calibration where the labor income is not reduced in the critical illness state. Furthermore, one might argue that the increased insurance demand without health shock results from the fact that in this setup there is no state in which the insurance acquisition is forbidden. However, the results also hold when we only consider families that do not face a health shock.

Family Size Figure 14 confirms the intuition that a larger family buys more insurance protection. In our model, this is captured by the consumption scaling parameter ϕ_A. The relative difference in the consumption scaling parameter in state $A = 1,2$ and $A = 3$ is smaller, the larger the family. Consequently, for a large family more consumption is needed to obtain the same single-person equivalent utility level. If the wage earner dies, the whole income is lost, but the bigger part of consumption remains if the family size is large. This increases the insurance demand.

Risk Aversion Figure 15 shows the impact of the relative risk aversion on the average insurance sum. Apparently, risk aversion has little impact before the age of 30 and after retirement. In between, more risk averse agents demand less insurance. Hence, a more risk averse agent perceives the insurance contract as more risky compared to financial investments (stocks, bonds). However, overall the degree of relative risk aversion has only little impact on the insurance decision.

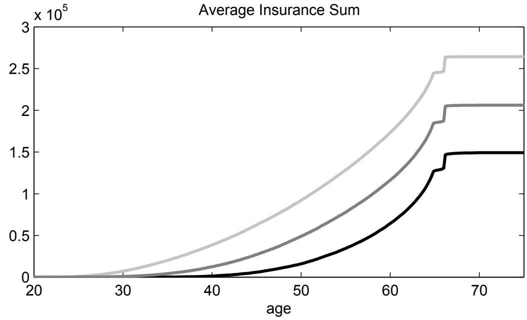

Figure 14: Insurance Demand for Different Family Sizes. The dark line depicts the average insurance sum over the life cycle for a family without children. The consumption scaling parameter are changed to $\phi_{1,2} = 1.6245$ and $\phi_3 = 1$. The light line gives the insurance demand with three children and the corresponding parameters are $\phi_{1,2} = 2.6850$, $\phi_3 = 2.2078$. The grey line represents the benchmark results for a family with one child and consumption scaling parameters are $\phi_{1,2} = 2.0043$ and $\phi_3 = 1.4498$. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

Social Security System In this paragraph, we add a social security system to the model that pays the family an income after the death of the wage earner. This can be interpreted as a widow's pension and the corresponding income stream is calibrated using

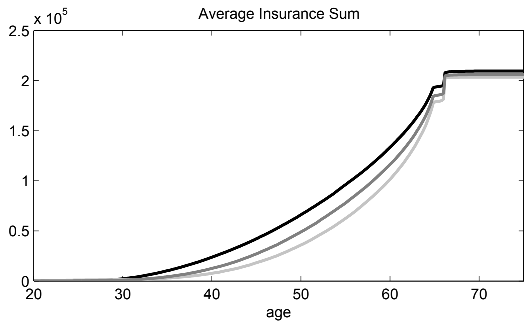

Figure 15: Insurance Demand for Different Risk Aversions. The figure depicts the average insurance sum over the life cycle for different values of the relative risk aversion. The dark line shows results for a low level of relative risk aversion ($\gamma = 3$), the grey line corresponds to the benchmark case with $\gamma = 4$ and the light line presents a more risk averse agent with $\gamma = 5$. The remaining parameters are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

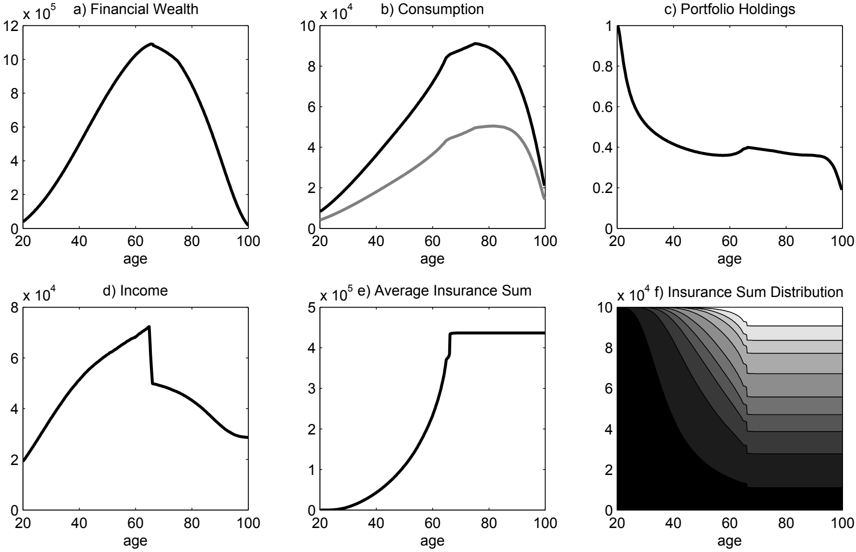

Figure 16: Average Key Variables over the Life Cycle with Social Security. The graphs depict the average optimal control variables as well as the average financial wealth and income evolution over the life cycle based on 100 000 simulations with a social security system as given in (5). The remaining calibration is described in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The dark line corresponds to the consumption of the family, where the grey line represents the equivalent level of consumption for a one person household and is scaled with the consumption scaling parameter $\frac{c}{\phi}$. c) shows the optimal average portfolio holdings over the life cycle. d) depicts the average income of the family. e) shows the average insurance sum. f) depicts the distribution of the insurance sum over the life cycle. The darkest area marks families with no insurance contract, the white area families with the highest insurance sum (2 000 000) and the grey areas families with intermediate insurance sums.

data of the German social security system. Therefore, we recalibrate the effects of the wage earner's death on the income process as follows:[12]

$$p^{1,3}(t) = p^{2,3}(t) = \begin{cases} 0.55 \cdot 0.68212(1-0.108) & \text{for } t < 40, \\ 0.55 \cdot 0.68212(1-(43-t)0.036) & \text{for } 40 \leq t < 43, \\ 0.55 \cdot 0.68212 & \text{for } 43 \leq t < 45, \\ 0.55 & \text{for } t \geq 45. \end{cases} \tag{5}$$

Notice that at most a widow receives about 55% of the pension. Besides, before retirement ($t < 45$) there is also an adjustment for for the replacement rate of 0.68212 and deductions. The income jump if the wage earner gets unhealthy $p^{1,2}$ remains unchanged.

[12] Details can be found in the German Social Security Code (SGB).

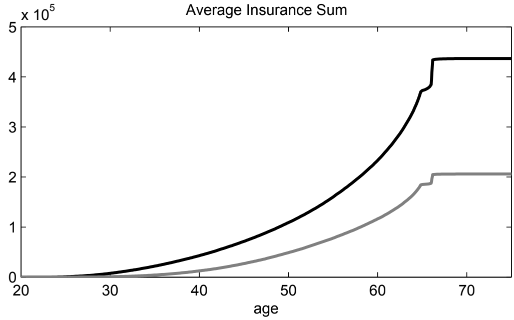

Figure 17: Insurance Demand with Social Security System. The figure depicts the average insurance sum over the life cycle for a different income reduction at death $p^{1,3}, p^{2,3}$. The dark line presents results of a model with social security system as given in (5). The grey line corresponds to the benchmark results without social security system $p^{1,3} = p^{2,3} = 0$. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

Figure 16 depicts the average key variables over the life cycle. Compared to an economy without social security system (see Figure 5), the average income is higher and does not approach zero. Furthermore, the optimal consumption now shows a hump-shaped pattern. The financial wealth and the portfolio holdings are only little affected. The insurance sum distribution highlights that about 90% of the population buy term life insurance over the lifetime, which is a significant increase. However, one of our main results stands: Young families do not participate in term life insurance markets. Figure 17 compares the average insurance demand to the benchmark results without social security system. One might conjecture that a social security system crowds out most of the insurance demand and, consequently, the insurance demand is significantly reduced. Our results however point in a different direction: With a social security system, the insurance demand significantly increases for all ages. This increase can be explained by the changes in characteristics of the income process. First, the human wealth is higher due to the additional payment. Second, human wealth uncertainty reduces since the income loss at death is less pronounced and the widow's pension is deterministic.

7 Conclusion

This paper studies the insurance demand of a family that is exposed to health shocks and mortality risk. The wage earner receives an unspanned income stream and can buy term life insurance up to the age of 70 as long as he is healthy. We model the available insurance contracts in a realistic way by assuming that they are of long-term nature and that decisions can only be revised at certain costs. The combination of unspanned income as well as the stickiness of the insurance contracts reduces the insurance demand significantly. In particular, young families do not participate in the insurance market at all.

Our results have potentially important policy implications. From a welfare perspective, one might argue that it is beneficial that people buy insurance contracts to avoid poverty. Our results document that long-term contracts prevent families from doing so, since they get locked into these contracts and have difficulties to pay premia if they are in financially dire situations. This finding suggests that families should have access to more flexible insurance contracts. For instance, a contract with a variable insurance sum that is linked to the actual labor income evolution (similar to occupational pensions) could avoid the negative amplifying effect of a labor income shock.

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Critical Illness Insurance in Life Cycle Portfolio Problems

Lorenz S. Schendel*

Abstract: I analyze a critical illness insurance in a consumption-investment model over the life cycle. I solve a model with stochastic mortality risk and health shock risk numerically. These shocks are interpreted as critical illness and can negatively affect the expected remaining lifetime, the health expenses, and the income. In order to hedge the health expense effect of a shock, the agent has the possibility to contract a critical illness insurance. My results highlight that the critical illness insurance is strongly desired by the agents. With an insurance profit of 20%, nearly all agents contract the insurance in the working stage of the life cycle and more than 50% of the agents contract the insurance during retirement. With an insurance profit of 200%, still nearly all working agents contract the insurance, whereas there is little demand in the retirement stage.

Keywords: Health shocks, Health expenses, Labor income risk, Stochastic mortality risk, Portfolio choice

JEL-Classification: D91, G11, I13

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I gratefully acknowledge research and financial support from the Center of Excellence SAFE, funded by the State of Hessen initiative for research LOEWE. I thank the Frankfurt Cloud for access to high-performance virtual servers. All remaining errors are of course my own.

1 Introduction

A critical illness (CI) insurance delivers a fixed payment if the insured person is diagnosed a critical illness from the list of insured illnesses. Although the CI insurance is growing in popularity, there is to the best of my knowledge no life cycle model explicitly considering such an insurance.

I consider a life cycle consumption-investment-insurance problem in continuous time. The agent has to pay exogenously determined health expenses that can jump due to a critical illness of the agent. In order to avoid the excess health expenses, the agent can contract a CI insurance. The agent receives unspanned labor income and decides about the optimal consumption, investment, and insurance strategy. The financial market consists of a riskless bond and a stock. The time of death is random. The hazard rate of death can jump due to a mortality shock. A critical illness may lead to an increased mortality risk but this is not necessarily the case. In this work, I analyze whether the agent wants to contract the CI insurance or not. Moreover, I investigate the driving factors of the resulting CI insurance demand.

The health expense effect of shocks has a crucial impact both for the aggregate results and for the individual results. Middle-aged agents (age 45) are more than 40% better off if they do not face jumps in the health expenses. Consequently, there is a huge demand for the CI insurance. Until the age of 50, nearly all agents contract the CI insurance, even if the insurance profit is set to 200%. With human wealth becoming less uncertain when approaching retirement, the CI insurance demand decreases since a more certain income can be better used to counter the effects of a health expense jump. However, still more than 50% of the agents contract the insurance during retirement with an insurance profit set to 20%. Middle-aged agents are about 18% better off when having access to this insurance. Before retirement when income is uncertain, low actual health expenses and a high actual income support the insurance decision. During retirement with certain income, either a low income or low health expenses can prevent the agent from contracting the insurance. A low income volatility and a low level of risk aversion decrease the demand for the CI insurance. The insurance demand also reduces significantly if agents underestimate the health expense effect of jumps or the health jump intensity.

The importance of the health status for investment and consumption decisions is empirically analyzed in the literature with mixed results. The studies of Rosen and Wu (2004), Berkowitz and Qiu (2006) as well as Fan and Zhao (2009) find a strong relation between health status and portfolio choice. However, Love and Smith (2010) disentangle the relation between health status and portfolio choice by analyzing which part is causal and which is due to unobserved heterogeneity. They argue that health has no significant impact on portfolio choice. Of course, one expects a strong relation between health expenses and health status. Since uncertain health expenses affect the agents optimization problem in the same way as uncertain labor income does, it is a reasonable assumption that health expenses have a significant impact on consumption and investment decisions. The relevance of unspanned labor income for portfolio choice is without doubt and among others highlighted by Viceira (2001) as well as Cocco, Gomes, and Maenhout (2005).

Some recent papers include health expenses in a portfolio choice framework. Edwards (2008, 2010) analyzes a retired investor with a Cobb-Douglas utility that depends on consumption and health. In his model, agents that are unhealthy have to purchase health. In this setup, he argues that health risk can partially explain the decrease in risky investment for older people. Davidoff (2009) considers the annuity and the long-term care insurance demand as well as the consumption decision for a retired house owner with uncertain health status in a two-period model. The paper of Pang and Warshawsky (2010) is most related to my work. They model stochastic health expenses for a retired agent in a discrete-time model and analyze the impact on the optimal stock, bond, and annuity portfolio. They show that health risk leads to less risky investment and increases the annuity demand. Yogo (2012) focuses on the retirement state as well. He allows for health expenditures as a choice variable besides consumption and investment. In a discrete-time model, the agent optimizes utility from consumption, housing, and health. In contrast to the papers mentioned in this paragraph, I do not restrict my analysis to the retirement state. Furthermore, I analyze a CI insurance as a possibility to avoid excess health expenditures.

The remaining paper is organized as follows. Section 2 introduces the model setup. In Section 3, I calibrate the health expense process and present the calibrated parameters that I use in the simulations. Section 4 analyses the health expense impact of shocks and motivates the existence of a CI insurance. The CI insurance is calibrated in Section 5. Furthermore, I present results for different values of the insurance profit. Section 6 gives several sensitivity analyses with a special focus on the difference between the real-world and model-based CI insurance demand. Finally, Section 7 concludes and presents ideas for further research.

2 Model Setup

Financial Assets and Investment Decision There are two assets in the financial market. The first one is a bond B that yields the constant risk-free rate r and the second one is a stock S with constant market price of risk λ and constant volatility σ_S. The corresponding dynamics are given by

$$dB_t = B_t r\,dt,$$
$$dS_t = S_t(r + \sigma_S \lambda)\,dt + S_t \sigma_S\,dW_t^S,$$

where $W^S = (W_t^S)$ is a standard Brownian motion. The agent continuously chooses θ_t which is the fraction of financial wealth X_t that he invests into the risky asset. The remaining part, $(1-\theta_t)X_t$, is invested into the bond. I impose short-sale constraints such that the agent is restricted to $\theta_t \in [0,1]$.

Mortality Risk The time of death, denoted by τ, is uncertain and is given by the first jump of a jump process $N^D = (N_t^D)$ with intensity (hazard rate of death) $\pi(t)$. The intensity is increasing with age and can jump due to a mortality shock that permanently increases the hazard rate of death. I interpret a mortality shock as a critical illness that highly influences mortality risk, e.g. cancer. The time-dependent part of the mortality risk is modeled with a Gompertz structure. The hazard rate of death is given by

$$\pi(t) = \frac{1}{b} e^{\left(\frac{t-m}{b}\right)} + \beta_\pi(t) N_t^\pi,$$

where $N^\pi = (N_t^\pi)$ is a jump process with intensity $\kappa_\pi(t)$ and is independent of all other sources of risk. In the model, N^π is allowed to jump only once. Hence, the intensity $\kappa_\pi(t)$ is set to zero after the first jump. I denote the time-dependent jump size by $\beta_\pi(t)$, whereas b and m are constant parameters that capture the increasing mortality risk over the life cycle.

Health Expenses and Insurance Decision The agent faces health expenses that are exogenously given and modeled by a geometric Brownian motion with time-dependent drift $\mu_H(t)$ and volatility $\sigma_H(t)$. The drift captures that average health expenses increase in age. The diffusive part accounts for small deviations in health expenses, e.g. induced by a common cold. Furthermore, the agent faces additional health expenses if a mortality shock or a health shock occurs. The health shock increases the health expenses significantly without increasing the mortality risk and can be interpreted as a psychical illness or a physical disability. It is modeled by the jump process $N^H = (N_t^H)$ with intensity $\kappa_H(t)$ and independently of all other sources of risk. In the model, the health expenses can jump only once. In order to hedge the health expense jump risk, the agent can contract an insurance. The insurance decision is denoted by $\iota \in \{0,1\}$. If insured, $\iota = 1$, the insurance company pays all excess health expenses due to a jump. The insurance premium depends on the actual health expense level and is denoted by $\eta(t)H_t$. The insurance decision takes place continuously and is only contracted for an interval of length dt. Thus, the health expense jump term becomes relevant only if the agent is uninsured, $\iota = 0$. The health expense dynamics are then summarized as

$$dH_t = H_t\mu_H(t)\,dt + H_t\sigma_H(t)\,dW_t^H + \mathbb{1}_{\{N_{t-}^\pi + N_{t-}^H = 0 \,\wedge\, \iota_{t-}=0\}} H_{t-}\beta_H(t)\left(dN_t^\pi + dN_t^H\right), \tag{1}$$

where $W^H = (W_t^H)$ is a standard Brownian motion that is independent of all other sources of risk. Here, $\beta_H(t)$ denotes the time-dependent health expense jump size.

Labor Income The agent receives a continuous income stream Y as long as he is alive. The income can be interpreted as labor income before retirement and pension payments after retirement. Additional to labor income uncertainty, the agent faces the risk of having to reduce work effort permanently or getting disabled due to a mortality or health shock. In this case, the income permanently reduces. The agent has no possibility to hedge the income reduction. The labor income process is allowed to jump only once. The income dynamics are given by

$$dY_t = Y_t\mu_Y(t)\,dt + Y_t\sigma_Y(t)\,dW_t^Y + \mathbb{1}_{\{N_{t-}^\pi + N_{t-}^H = 0\}} Y_{t-}\beta_Y(t)\left(dN_t^\pi + dN_t^H\right) \tag{2}$$

with another standard Brownian motion $W^Y = (W_t^Y)$ that is independent of all other sources of risk. The income drift $\mu_Y(t)$, volatility $\sigma_Y(t)$, and jump magnitude $\beta_Y(t)$ are allowed to be time-dependent.

Preferences The agent gains utility from intermediate consumption c and terminal wealth X_τ. The utility has a constant relative risk aversion with risk aversion parameter γ. The time preference rate is given by δ and the weight of the bequest motive is denoted by ε. Hence, lifetime utility at time t is given by

$$\mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}\,du + \varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right],$$

where A is a state variable that captures the current health status of the agent. A is determined by the jump processes and defined as follows:

$$A_t = \begin{cases} 1 & \text{(healthy)} & \text{if } N_t^H = 0 \wedge N_t^\pi = 0 \wedge N_t^D = 0, \\ 2 & \text{(health shock)} & \text{if } N_t^H = 1 \wedge N_t^\pi = 0 \wedge N_t^D = 0, \\ 3 & \text{(mortality shock)} & \text{if } N_t^\pi = 1 \wedge N_t^D = 0, \\ 4 & \text{(dead)} & \text{if } N_t^D = 1. \end{cases}$$

A healthy agent, $A=1$, faces health shock risk, mortality shock risk and death shock risk. Agents that only faced a health shock, $A=2$, have mortality shock risk and death shock risk, further health shocks cannot occur. If an agent suffered a mortality shock, $A=3$, he only faces risk of dying since further health or mortality shocks are not possible.

Financial Wealth and the Optimization Problem The financial wealth of the agent is denoted by X. The following wealth dynamics arise from the above model setup

$$dX_t = \left[X_t(r+\lambda\sigma_S\theta_t)+y_t-c_t-h_t-\mathbb{1}_{\{N_t^\pi+N_t^H=0\wedge\iota_t=1\}}h_t\eta(t)\right]dt+X_t\sigma_S\theta_t\,dW_t^S. \tag{3}$$

The agent maximizes lifetime utility from consumption and terminal wealth. The optimization problem is characterized by the control variables consumption c, portfolio holdings θ, and the insurance decision ι. The state variables are time t, financial wealth x, income y, health expenses h, and the health state of the agent A. The optimization problem is expressed as

$$\max_{\{c_u,\theta_u,\iota_u\}_{u\in[0,\tau)}} \quad \mathrm{E}_{0,x,y,h,A}\left[\int_0^\tau \mathrm{e}^{-\delta u}\frac{c_u^{1-\gamma}}{1-\gamma}du+\varepsilon\,\mathrm{e}^{-\delta\tau}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]$$

$$\text{s.t.}\quad dX_t = \left[X_t(r+\lambda\sigma_S\theta_t)+y_t-c_t-h_t-\mathbb{1}_{\{A_t=1\wedge\iota_t=1\}}h_t\eta(t)\right]dt+X_t\sigma_S\theta_t\,dW_t^S, \tag{4}$$

and also includes short-sale constraints, $\theta_t\in[0,1]$, and liquidity constraints, i.e. the optimal choice variables have to ensure $X_t>0$. I denote the corresponding value function by J:

$$J(t,x,y,h,A) = \sup_{\{c_u,\theta_u,\iota_u\}_{u\in[t,\tau)}} \quad \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau \mathrm{e}^{-\delta(u-t)}\frac{c_u^{1-\gamma}}{1-\gamma}du+\varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right]. \tag{5}$$

The Hamilton-Jacobi-Bellman (HJB) equation of the problem is given by

$$\begin{aligned}
\delta J = \sup_{c,\theta,\iota} \quad &\left\{\frac{c^{1-\gamma}}{1-\gamma}+J_t+J_x\left[(r+\lambda\sigma_S\theta)+y-c-h-\mathbb{1}_{\{A=1\wedge\iota=1\}}h\eta\right]+\frac{1}{2}J_{xx}x^2\sigma_S^2\theta^2\right.\\
&+J_y y\mu_Y+\frac{1}{2}J_{yy}y^2\sigma_Y^2+J_h h\mu_H+\frac{1}{2}J_{hh}h^2\sigma_H^2\\
&+\mathbb{1}_{\{A=1\}}\kappa_H\left[J(t,x,(1+\beta_Y)y,(1+\mathbb{1}_{\{\iota=0\}}\beta_H)h,2)-J(t,x,y,h,A)\right]\\
&+\mathbb{1}_{\{A=1\vee A=2\}}\kappa_\pi\left[J(t,x,(1+\mathbb{1}_{\{A=1\}}\beta_Y)y,(1+\mathbb{1}_{\{A=1\wedge\iota=0\}}\beta_H)h,3)-J(t,x,y,h,A)\right]\\
&\left.+\pi\left[J(\tau,x,y,h,4)-J(t,x,y,h,A)\right]\right\},
\end{aligned} \tag{6}$$

for $A=\{1,2,3\}$. Subscripts of J denote partial derivatives, for example $J_t=\frac{\partial J}{\partial t}$. I solve the optimization problem numerically. An outline of the solution method is described in Appendix A.

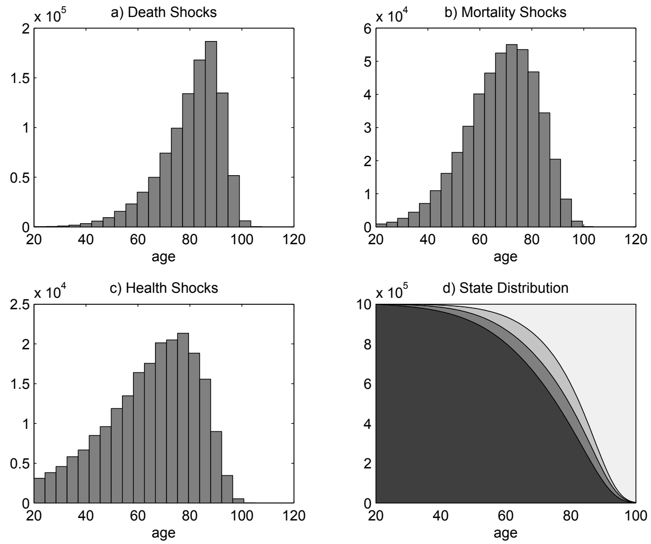

Figure 1: Shock Distribution. The figure depicts the histogram of the shocks after 1 000 000 simulations and the corresponding state distribution. a) shows the death shock distribution. b) depicts the histogram of the mortality shocks. c) presents the health shock distribution. d) depicts the resulting state distribution over the lifetime. The areas from bottom left to top right are explained as follows: The dark area corresponds to healthy agents ($A = 1$), the dark grey area to agents that faced a health shock ($A = 2$), the light grey area represents agents that faced a mortality shock ($A = 3$), and the light area indicates dead agents ($A = 4$). The processes are calibrated as stated in Section 3.

3 Calibration

Financial Assets My financial market calibration is based on Munk and Sørensen (2010). I set the risk-free rate to $r = 0.02$. I calibrate the stock with a market price of risk of $\lambda = 0.2$ and I set the volatility parameter to $\sigma_S = 0.2$.

Mortality Risk I use the mortality process and mortality shock calibration of Kraft, Schendel, and Steffensen (2014). They interpret the mortality shock as critical illness as well and calibrate it with cancer data for Germany. The intensity and magnitude of the mortality shock are given by

$$\kappa_\pi(t) = 0.02489\,\mathrm{e}^{\left(\frac{\min(t,65)+66.96}{29.42}\right)^2},$$
$$\beta_\pi(t) = 0.048 + 0.0008t.$$

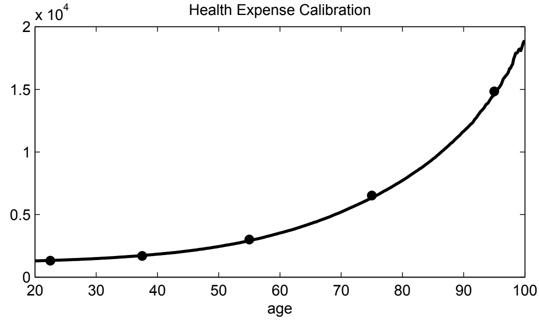

Figure 2: Health Expense Calibration. The figure compares the average health expenses after 1 000 000 simulations (line) with the data for Germany (points). The calibration for the simulated results is stated in Section 3. The data points represent average health expenses in Germany provided by Statistisches Bundesamt. The data yields average costs for the age intervals 15-29,30-44,45-64,65-84, and 85+. I draw the data points in the middle of the corresponding interval and assume a length of 20 years for the last interval. The insurance is not used here ($\iota_t = 0\,\forall t$).

The mortality process is calibrated with mortality data for Germany. The results for the parameters are $b = 6.5$ and $m = 69.45$. Figure 1 depicts the resulting death shock and mortality shock distribution after 1 000 000 simulations.

Health Expenses I calibrate the health expenses using data for Germany.[1] The data provides the average medical expenses in 2008 for six age groups. I calibrate the health jump together with the health expense drift and diffusion such that the resulting health expenses match the data. Since I do not have data for very old agents, I assume that the health expense pattern remains unchanged for agents that are older than 100. In order to simplify notation, I set: $\tilde{t} = \min(t, 80)$. I calibrate the health jump intensity and magnitude according to

$$\kappa_H(t) = \frac{1}{20.5} \mathrm{e}^{\left(\frac{\tilde{t}-88.45}{20.5}\right)},$$
$$\beta_H(t) = 6\left(\mathrm{e}^{0.01\tilde{t}}\right)^{-1} + 0.08\tilde{t} - 0.0008\tilde{t}^2.$$

The drift and volatility of the health expense process are calibrated with

$$\mu_H(t) = 0.0055 + 0.0004\tilde{t} - 0.0000137\tilde{t}^2 + 0.000000187\tilde{t}^3,$$
$$\sigma_H(t) = 0.03.$$

[1] The German health expense data is taken from “Statistisches Bundesamt, Wirtschaft und Statistik Juli 2011, p. 666”, available online at: `https://www.destatis.de/DE/Publikationen/WirtschaftStatistik/Monatsausgaben/WistaJuli11.pdf`, last access: January 21, 2014.

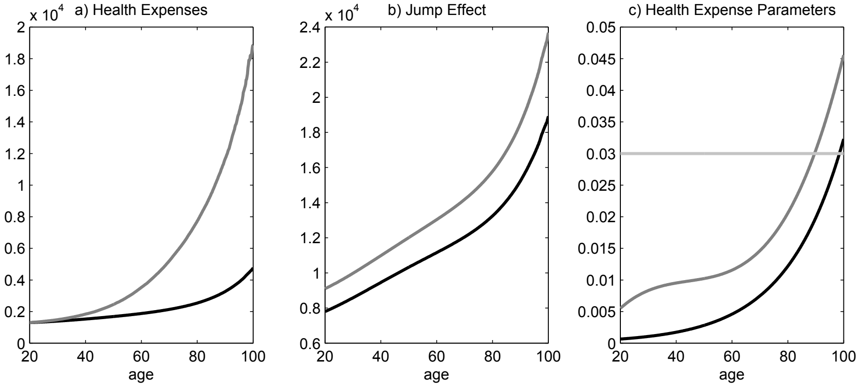

Figure 3: Health Expenses, Parameters and the Jump Effect. a) depicts the average health expenses after 1000000 simulations if shocks have no impact on the health expenses (i.e. $\beta_H = 0$) $\tilde{H}$ (dark line) and compares it to the benchmark case including the health expense impact of the shocks H (grey line). b) presents the effect of a health jump. The dark line is the average jump size $\tilde{H}\beta_H$ after 1000000 simulations and the grey line gives the corresponding average health expenses immediately after a jump $\tilde{H}(1+\beta_H)$. c) depicts the health expense parameter calibration: the jump intensity κ_H (dark line), the drift parameter μ_H (grey line), and the volatility σ_H (light line). The corresponding calibration used in the three graphs is given in Section 3. The insurance is not used here ($\iota_t = 0\,\forall t$).

I set the initial value to $H_0 = 1300$ which is a EUR 2008 value. Figure 2 compares the simulated health expenses with the above calibration to the German health expense data. The figure highlights that the simulated health expenses fit the data well. Figure 3 a) compares the average health expenses with and without the health expense effects of the shocks if the agents are not insured. The huge difference stresses the importance of the health expense jump for the agents. The difference between those lines would be captured by the insurance, if contracted. b) depicts the average jump size and the average health expenses after a jump. Intuitively, they are increasing with age. c) depicts the health jump intensity and health expense parameters over the lifetime. Figure 1 c) shows the resulting health shock distribution after 1000000 simulations. In the sample, 21.1% of the agents face a health shock. Furthermore, 45.6% of the population suffer a mortality shock that also increases health expenses for agents that had no health shock before. Figure 1 d) depicts the resulting state distribution over the life cycle. We see that most agents are either healthy or dead and only few are at the states with high health expenses at the same time.

Labor Income I calibrate the drift of the income process as in Munk and Sørensen (2010). They use PSID (Panel Study of Income Dynamics) data that yields the income dependent on the education level. The drift polynomial was originally estimated by Cocco, Gomes, and Maenhout (2005) for a discrete-time setup. They assume that the agent

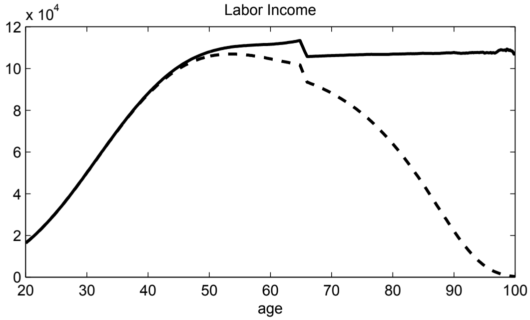

Figure 4: Expected Labor Income over the Life Cycle. The figure depicts the expected labor income profile over the life cycle after 1 000 000 simulations with the calibration given in Section 3. The solid line represents the earnings profile of all living agents, whereas the dashed line denotes the expected earnings of all agents (i.e. dead agents are included with zero income).

retires at the age of 65 and the drift is set to zero afterwards. The retirement income is a fraction of the last income before retirement. I use the continuous-time version of Munk and Sørensen (2010) which smooths the income reduction at retirement. The drift for the college education level is then given by

$$\mu_Y(t) = \mathbb{1}_{\{t<45\}}\left(0.3394 - 0.01154t + 0.000099t^2\right) - \mathbb{1}_{\{45\leq t\leq 46\}}0.06113.$$

For the diffusive component, I assume that retirement income is riskless in contrast to labor income before retirement. The diffusive component is calibrated by

$$\sigma_Y(t) = \mathbb{1}_{\{t<45\}}0.15.$$

In the case that a health jump or a mortality jump occurs, the labor income is reduced since the agent has to reduce work effort or gets disabled. I calibrate the jump size as

$$\beta_Y(t) = -\mathbb{1}_{\{t\leq 45\}}0.2.$$

Hence, the labor income reduces by 20% if the agent is still working. In contrast, the income remains unaffected if the agent is already retired. Last, I calibrate the initial value Y_0. Munk and Sørensen (2010) give a starting value for the college calibration of 13 912 USD in 2002. To be consistent with the health expense calibration, I translate the value to a 2008 value in EUR. In order to do this, I use the average EUR-USD closing mid exchange rate (source: WM/Reuters via Datastream) in 2002. Afterwards, I assume that the average income change follows the German consumer price index for the corresponding years (source: Statistisches Bundesamt)[2]. This results in $Y_0 = 16\,369$. Figure 4 depicts the average earnings profile over the life cycle.

Preferences I use standard values for the relative risk aversion $\gamma = 4$, the time preference rate $\delta = 0.03$, and the weight of the bequest motive $\varepsilon = 1$. The agent starts at the age of 20 with a financial wealth equal to one year of labor income $X_0 = Y_0$.

4 Why a Critical Illness Insurance?

In this section, I justify the existence of a CI insurance in my model and comment on the situation in the real-world insurance market. The calibration highlights a huge difference in average health expenses depending on whether shocks have an impact on the health expenses ($\beta_H \neq 0$) or not ($\beta_H = 0$) . Now, I analyze the impact of the health expense effect of the shocks on the optimal controls for the aggregate results. Furthermore, I analyze the effects for an individual agent who suffers from a health shock and/or from a mortality shock.

Aggregate Results Figure 5 compares the financial wealth evolution and optimal controls in a model with and without health expense effects of jumps. We see that the existence of the health expense jump effect increases average financial wealth. The fraction riskily invested is on average smaller and the consumption is reduced in early years. The agents are afraid of the health expense effect of the shocks. Therefore, they save more, consume less, and invest less riskily. Consequently, in later years, consumption is higher due to a high amount of accumulated wealth. This explains that the average bequest is higher as well. The differences between both models diminish for old ages and vanish almost completely at the age of 100 since shocks become less important with decreasing expected remaining lifetime.

Overall, we see that the existence of the health expense impact of the health and mortality shock has a significant effect on the aggregate results. The inclusion of the health expense effect of the jumps has qualitatively the same effect as an increase in risk aversion.

Individual Results Figure 6 depicts the effects of the shocks for an individual agent without insurance. The direct effect of a health shock at the age of 50 are a decreased income and increased health expenses. The optimal reaction is less consumption and less risky investment compared to agents without shock. Both effects are due to the reduced

[2] Data available at: `https://www-genesis.destatis.de`, table code: 61111-0001, last access: January 21, 2014.

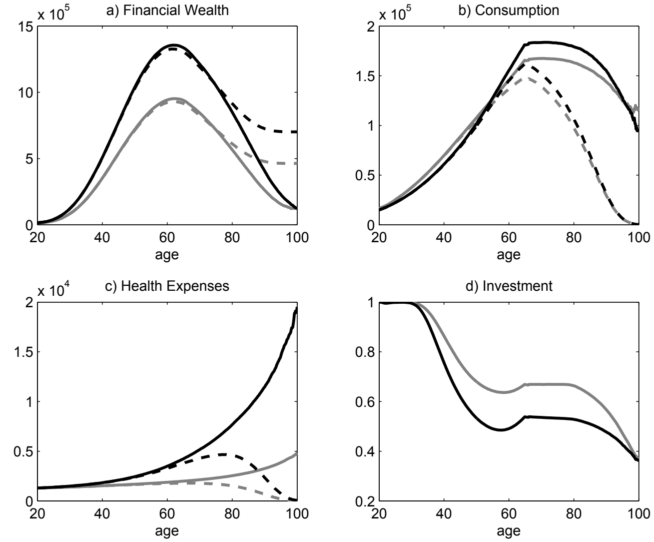

Figure 5: Aggregate Effects of Health Expense Jumps without Insurance. The figure compares the optimal controls and financial wealth evolution in two models without insurance ($\iota_t = 0\,\forall t$). The dark lines represent the benchmark model with health expense jumps, the grey lines are for a model in which shocks have no impact on health expenses, i.e. $\beta_H = 0$. The models are calibrated as stated in Section 3 and the results are averaged after 100 000 simulations. a) depicts the financial wealth evolution, b) shows consumption, c) plots the average health expenses, and d) gives the fractions of wealth invested into the risky asset. Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents where dead agents are included with zero consumption, zero health expenses and financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$).

human wealth. The financial wealth evolution shows only a slight reduction in growth. A mortality shock at the age of 70 does not decrease income since the agent is already retired. However, it increases health expenses if there was no previous health shock. With a previous health shock, income and health expenses remain unchanged. Additionally, the mortality shock increases the hazard rate of death, which reduces the expected remaining lifetime. The optimal reaction to the mortality shock is also a decrease in the fraction of wealth that is riskily invested. The lower expected lifetime further reduces the share riskily invested as agents prefer a less risky investment for a shorter time horizon. With a previous health shock, the reduction is small and only due to the increased mortality risk. Without a previous shock, the reduction is larger and due to the reduced human wealth and the increased mortality risk. The optimal consumption increases as a result of two opposing effects. On the one hand, the increased health expenses reduce human wealth which leads to a decrease in consumption. On the other hand, the increased mortality risk increases consumption as the agent wants to spend excess wealth before his death. In this case, the mortality effect outweighs the human wealth effect. However, if a mortality

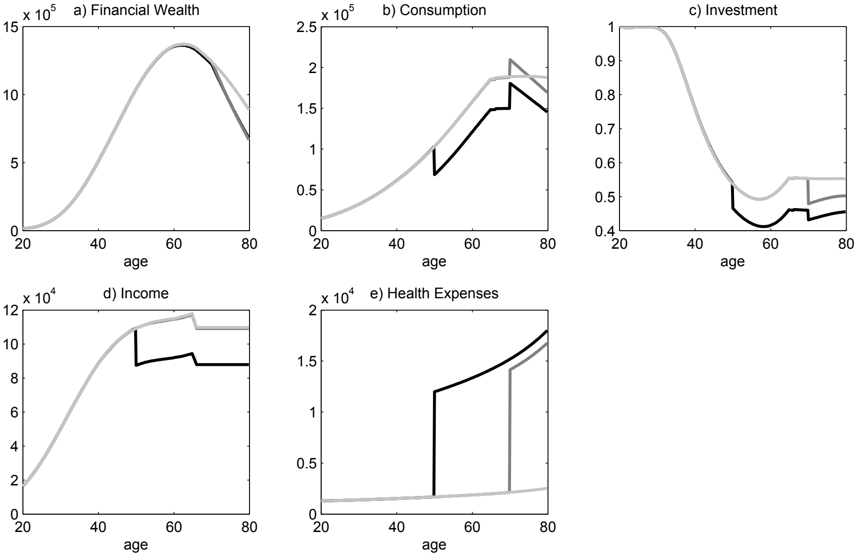

Figure 6: Sample Shocks without Insurance. The figure depicts the effects of health and mortality jumps in the lifetime, averaged from 100 000 simulations with the calibration of Section 3 without insurance ($\iota_t = 0 \,\forall t$). The death shock occurs at the age of 80. The light lines show the results for no previous shocks, the grey lines are for agents with a mortality shock at the age of 70, and the agents depicted by the black lines additionally have a health shock at the age of 50. a) depicts the optimal financial wealth evolution, b) the optimal consumption, c) the optimal fraction riskily invested, d) the income over the lifetime, and e) the health expenses.

shock occurs earlier in lifetime, e.g. at the age of 50, then the human wealth effect has a larger impact and outweighs the mortality risk effect. As a result, optimal consumption would decrease as a reaction to an early mortality shock. Independent of the age, if the agent is already unhealthy and a mortality shock occurs, then consumption always increases since there is no human wealth effect and only the mortality risk effect remains. Consumption growth always decreases after a mortality shock as a result of the reduced expected lifetime. Financial wealth growth also reduces after a mortality shock because agents dissave and want to reduce accidental bequest, independent of a previous health shock. At the age of 80, the agent dies.

Figure 7 depicts the corresponding graphs without health expense effect of the shocks, i.e. $\beta_H = 0$. The health shock at the age of 50 has a less pronounced effect since the human wealth is less reduced in the absence of the health expense effect. However, consumption and risky investment is also decreased due to the income reduction. The less pronounced consumption reduction explains that financial wealth growth is slightly more reduced. The mortality shock at the age of 70 now has no human wealth effect independent of a previous

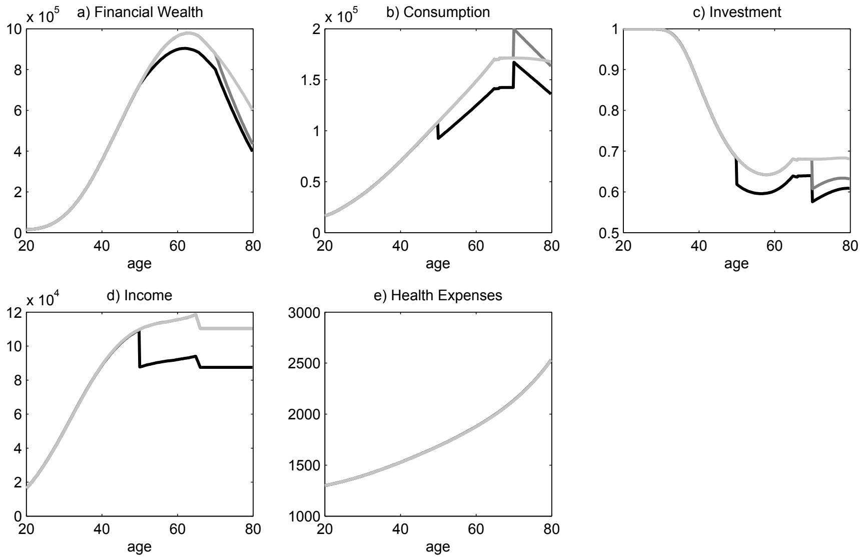

Figure 7: Sample Shocks without Health Expense Effect. The figure depicts the effects of health and mortality jumps in the lifetime, averaged from 100 000 simulations with the calibration of Section 3 without health expense effect of the shocks ($\beta_H = 0$). The death shock occurs at the age of 80. The light lines show the results for no previous shocks, the grey lines are for agents with a mortality shock at the age of 70, and the agents depicted by the black lines additionally have a health shock at the age of 50. a) depicts the optimal financial wealth evolution, b) the optimal consumption, c) the optimal fraction riskily invested, d) the income over the lifetime, and e) the health expenses.

health shock. Hence, all effects occur as an optimal reaction to the increased mortality risk and the reduced expected remaining lifetime. The shorter expected lifetime leads to less risky investment, an increase in consumption but a decrease in consumption growth and a decrease in financial wealth growth. If a mortality shock would occur before retirement, the human wealth effect is present due to the reduced income. Then, it depends on the age whether consumption increases or decreases.

Comparing Figure 6 and 7, we see that the health expense effect of the shocks is also important for the individual agent as the optimal controls differ significantly. Without health expense effect of the shocks, the human wealth effect is less pronounced which reduces the impact of the shocks for the agents.

Hence, the health expense effect of the shocks is important both for the aggregate results and for the reaction of the individual agents to the shocks. This raises the questions whether, and at which costs, agents are willing to hedge the health expense jump risk using the CI insurance.

	age 25	age 45	age 75
$CE(t,x,y,h,A)$	24.26	44.11	2.02
$CE(t,\frac{1}{2}x,y,h,A)$	24.45	46.16	2.05
$CE(t,2x,y,h,A)$	23.59	39.71	1.87
$CE(t,x,\frac{1}{2}y,h,A)$	25.83	57.71	4.71
$CE(t,x,2y,h,A)$	17.62	24.78	0.93
$CE(t,x,y,\frac{1}{2}h,A)$	17.48	23.08	0.92
$CE(t,x,y,2h,A)$	26.38	60.76	5.21

Table 1: Gain of Having no Health Expense Jumps. The table gives the percentage gain in the certainty equivalent (7) of having no health expense effect of the shocks ($\beta_H = 0$) compared to the model with health expense impact of shocks and without insurance. The percentage gain is given for a young ($t = 5$), middle-aged ($t = 25$), and old ($t = 55$) healthy ($A = 1$) agent. The other state variables are set to $x = 500\,000$, $y = 100\,000$, $h = 1\,700$. The model calibration is given in Section 3.

Welfare Impact In order to quantify the impact of the health expense effect of the shocks, I calculate a certainty equivalent which is given by

$$CE(t,x,y,h,A) = \left[\left(1-\gamma\right) J(t,x,y,h,A)\right]^{\frac{1}{1-\gamma}}. \tag{7}$$

Table 1 gives the percentage gain of having no health expense effect of the shocks for healthy agents ($A = 1$), which equals having a CI insurance for free. As expected, the gain is always positive. Considering the state variables, the age crucially influences the gain. The young agent strongly profits from having no health expense effect, whereas the middle-aged agent profits even more, but the old aged agent is only a little better off without health expense effect of the shocks. The old agent has a short remaining lifetime which damps the effect of a health expense jump. In contrast, the young agent would suffer from a health expense shock due to the long remaining lifetime but the probability of a shock in younger years is low. The middle-aged agent has a non-negligible probability for a health and mortality jump and a long enough expected remaining lifetime such that the shock has a crucial impact. The financial wealth has only a little impact on the certainty equivalent gain. For all ages, the gain increases for less financial wealth but the weak effect highlights that financial wealth is not a main driving factor. Due to the permanent effect of the health expense jump, financial wealth cannot compensate the effect, especially in early years. Variations in income have a larger impact. The more income the agent has, the less he profits from having no health expense effect of the shocks, independent of the age. A high income directly compensates high health expenses since both provide a continuous cash-flow. The older the agent is, the lower is the uncertainty with respect to future income and the better can a high income counter high health expenses. The actual health expenses have the most pronounced effect on the gain. The higher the health expenses are, the more important is the absence of the health expense effect of the shocks for the agent. The actual health expenses are particularly crucial as they directly determine the jump size. Besides, the health expense effect is similar to the income effect in the opposite direction since it provides a continuous cash-flow as well.

Thus, the health expense effect of the shocks is crucial from a qualitative and a quantitative point of view. Particularly, the young and middle-aged agents have a huge benefit from having no health expense effect of the shocks.

Actual Situation The first CI insurance (also known as dread disease insurance) was developed in South Africa in 1983.[3] The insurance pays a previously fixed lump sum if the insured person is diagnosed with a critical illness from a list of insured illnesses. The insurance is typically offered as a long-term contract. Hence, the CI insurance in my model differs from real contracts in the way that it offers a perfect hedge against the excess health expenses and is only contracted for an interval of length dt.

Although the CI insurance is becoming more popular,[4] it is still rarely used in Germany compared with disability insurance or other health-related insurance products. The relatively low demand for CI insurance is surprising due to the possible benefits. To give an example, blindness or deafness are critical illnesses that are often covered by a CI insurance. These illnesses might or might not trigger a disability insurance and lead to large costs that are not fully covered by a health insurance. A handicapped-accessible house, books for blind persons, or a special computer produce large costs. Since the expected lifetime is usually not reduced, this messes up the financial planning. I model such illnesses with the health shocks in the model. Cancer or a heart attack are examples for insurable critical illnesses that reduce the expected remaining lifetime and might or might not trigger a disability insurance as well. These illnesses also produce large costs, e.g. for medicine and health care. The mortality shocks in my model capture such illnesses.

The seemingly huge benefits of the CI insurance raise the question why there is little demand for this type of insurance. Therefore, I analyze the driving factors of the CI insurance demand.

5 Insurance Demand

In this section, I add the CI insurance to the model. I calibrate the insurance premium, present results with insurance and comment on the effects and importance of the insurance premium level.

[3] Information on real-world CI insurance contracts in this paragraph is based on: CoverTen (Incisive Financial Publishing), October 2007, available online at: `http://db.riskwaters.com/data/cover/pdf/cover_supp_1007.pdf`, last access: January 24, 2014.

[4] Estimate as of 2007, more than 20 million contracts are yearly sold worldwide (source: CoverTen, p. 13-14).

Insurance Calibration To calibrate the insurance premium, I consider 1 000 000 agents and assume that every agent is always insured. The agents are denoted with a superscript i. I calculate the average costs that occur for the insurance company corresponding to every age. I compare these costs with the income of the insurance such that the CI contract is fair for every age. For each time t, I consider all agents that are in the insurance market, i.e. agents that are healthy ($A_t^i = 1$) or face a health or mortality shock at t and were healthy before ($A_{t^-}^i = 1$). The healthy ones pay the insurance premium which yields the average insurance income (9). The agents that face a health or mortality shock at t receive a payment from the insurance. This payment is determined by the discounted difference of health expenses with and without jump effect until the time of death τ^i. This gives the average insurance outgoings (8).

$$Ins_{out}(t) = \frac{1}{\left|\{i \mid A_{t^-}^i = 1\}\right|} \sum_{i \in \{i \mid A_{t^-}^i = 1, A_t^i \neq 1\}} \int_t^{\tau^i} \mathrm{e}^{-r(u-t)} \left(H^i(u \mid \iota_{t^-}^i = 0) - H^i(u \mid \iota_{t^-}^i = 1) \right) du, \quad (8)$$

$$Ins_{in}(t) = \frac{1}{\left|\{i \mid A_{t^-}^i = 1\}\right|} \sum_{i \in \{i \mid A_t^i = 1\}} \eta(t) H^i(t). \quad (9)$$

In order to get smooth and reliable results, I consider the average income and outgoings on a yearly basis. Hence, all health, mortality, and death shocks are rounded to a full year. Now, I calibrate the insurance premium η such that the average income and outgoings are approximately identical for all ages. Resulting, I set

$$\eta(t) = \tilde{\eta} \left(0.1 + 1.227 \, \mathrm{e}^{-\left(\frac{\tilde{t} - 58.47}{17.35}\right)^2} + 0.936 \, \mathrm{e}^{-\left(\frac{\tilde{t} - 44.81}{28.05}\right)^2} \right), \quad (10)$$

where $\tilde{\eta}$ is a scaling parameter that determines the level of the insurance premium and thus, the insurance profit. For $\tilde{\eta} = 1$, the income approximately equals the outgoings such that the CI insurance is approximately actuarially fair given the insurance company faces no administrative or transaction costs. Figure 8 depicts the income and outgoings for $\tilde{\eta} = 1$.

Unfortunately, I do not have any data considering the fees and profits of CI insurance contracts. I use the average administrative fee (2.99%) and transaction fee (5.05%) given in Kraft, Schendel, and Steffensen (2014). These are the average values for German life insurance companies in 2011. Furthermore, I add the average equity return in 2011 (11.9%) to account for the insurance profit.[5] Resulting, I set $\tilde{\eta} = 1.2$ in the benchmark calibration such that the insurance company has an average profit of 20% (excluding fees). Additionally, I present results for a more expensive insurance with an average profit of 100% ($\tilde{\eta} = 2.0$) and 200% ($\tilde{\eta} = 3.0$).

[5] The average equity return is taken from the 20 biggest international insurance companies. Source: `http://www.presseportal.de/pm/39565/2367580`, last access: January 25, 2014.

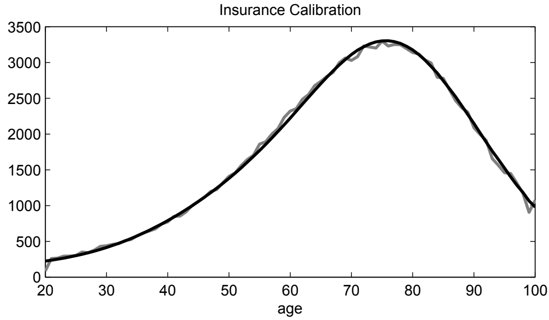

Figure 8: Insurance Calibration. The figure compares the average earnings and expenditures of the insurance company given that all agents are always insured. The grey line represents the average expenditures of the insurance calculated by (8). The dark line depicts the average earnings of the insurance calculated by (9) where η is calibrated according to (10) with $\tilde{\eta} = 1$. The model calibration is given in Section 3. The calibration is based on 1 000 000 simulations.

Benchmark Results Figure 9 depicts the results in the benchmark calibration and for more expensive CI insurance contracts. I compare the figure with Figure 5, which can be interpreted as comparing an insurance for free ($\tilde{\eta} = 0$, grey lines) and an infinitely costly insurance ($\tilde{\eta} = \infty$, dark lines). In the first case, there is no health expense effect since it is always optimal for all agents to contract the CI insurance. In the latter case, no agent can afford contracting the insurance. Hence, the setup is equal to the absence of the insurance. Consequently, I expect the results for $\tilde{\eta} \in \{1.2, 2.0, 3.0\}$ being in between those for $\tilde{\eta} \in \{0, \infty\}$. The financial wealth, consumption, and investment graphs in Figure 9 show that this is the case. The higher the insurance premium is, the more risk averse the agent behaves. Thus, he has more financial wealth, less risky investment, less consumption early in the life cycle, and more consumption later in lifetime.

Comparing the insurance decisions for the different premiums, we intuitively see that the higher the insurance premium is, the less agents contract the insurance. Considering the insurance decision in detail, we notice that nearly all agents are insured until the age of 50 independent of the insurance premium level. Then, the demand for a CI insurance decreases rapidly which is due to less uncertainty in human wealth when approaching retirement. In the benchmark calibration, the median agent is insured after retirement, whereas the median agent for the more expensive insurance is not insured anymore. For very old ages, there are only few agents in the insurance market such that the CI insurance demand is not that accurate any more. We observe that the insurance demand approaches similar levels for the different insurance premiums. Hence, the demand is less dependent on the premium level for very old ages.

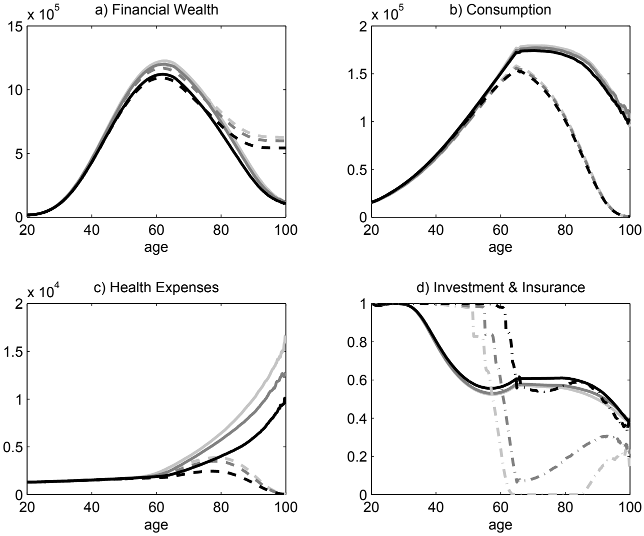

Figure 9: Results for Different Insurance Premiums. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution for different insurance premiums. The insurance premium is determined by (10). The dark lines show results for $\tilde{\eta} = 1.2$, the grey lines for $\tilde{\eta} = 2.0$ and the light lines for $\tilde{\eta} = 3.0$. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are based on 100000 simulations with the model calibration given in Section 3.

Policy Functions Next, I consider policy functions for the CI insurance demand in the benchmark case with $\tilde{\eta} = 1.2$ to analyze the impact of the state variables on the insurance decision. Figure 10 depicts the corresponding graphs. The dark grey area indicates that the agent optimally contracts the insurance, whereas he optimally has no insurance protection in the light grey area. We see that the policy functions of the young and middle-aged agent look similar, whereas the policy functions of the old agent show a completely different pattern. Considering the young and middle-aged agents in detail, the graphs for fixed health expenses depict that both income and financial wealth have no crucial impact on the CI insurance decision. The graphs for fixed income highlight that unreasonable high health expenses would be necessary such that it would be optimal not to contract the insurance and little financial wealth further supports this. In this case, the insurance comes at too high costs for the agent and is therefore not optimal. Since the contract is based on the actual health expenses, the agent cannot or does not want to afford the contract as it becomes to expensive. The graphs for fixed financial wealth

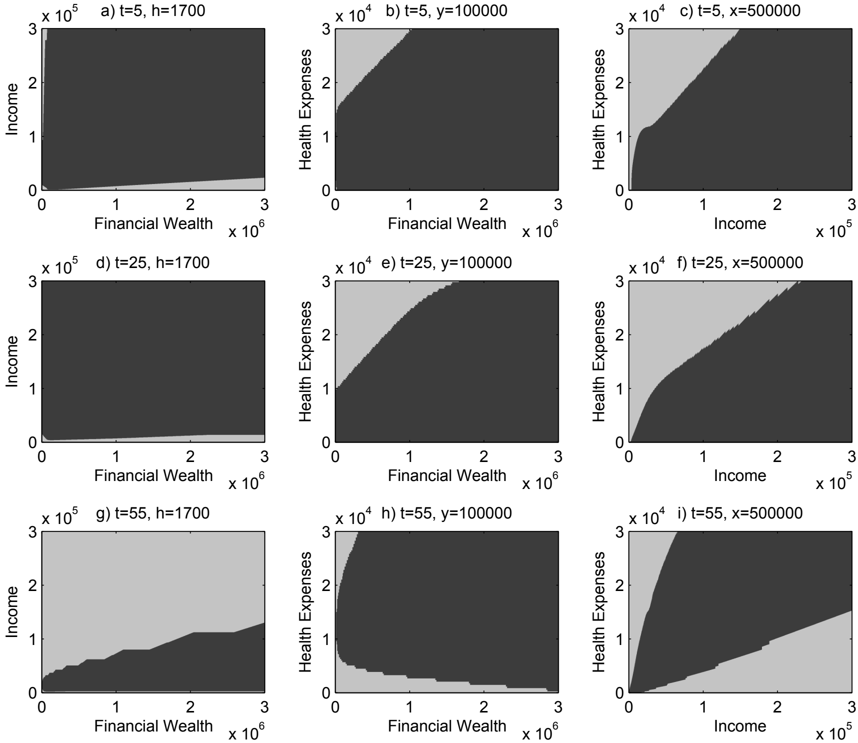

Figure 10: Policy Functions for CI Insurance Demand. The figure depicts policy functions for the CI insurance demand with the benchmark calibration in Section 3 and the insurance premium given in (10) with $\tilde{\eta} = 1.2$. The first row (a,b,c) gives policy functions for a young agent ($t = 5$), the second row (d,e,f) for a middle-aged agent ($t = 25$), and the last row (g,h,i) for an old agent ($t = 55$). The agents are healthy ($A = 1$). The first column (a,d,g) shows the policy functions for fixed health expenses with $h = 1700$, the second column (b,e,h) for fixed income with $y = 100000$, and the last column (c,f,i) for a fixed financial wealth of $x = 500000$. The dark grey area indicates that the agent contracts the CI insurance, whereas the light grey area indicates that the agent does not contract the insurance.

deliver the same pattern. If health expenses are extremely high and income is low, the agent does not contract the insurance since a contract becomes to expensive. Comparing the young and middle-aged agent, we see that the no-contract area increases as the age increases.

For the old agent, income has an increased importance since it is certain now. The graph with fixed health expenses highlights the importance of the income, whereas the actual financial wealth is still of little importance. With a lot certain income, the agent does not contract the CI insurance since he is able to pay the increased health expenses in the case of a jump. More financial wealth supports the decision to contract the insurance since

	age 25	age 45	age 75
$CE(t,x,y,h,A \mid \tilde{\eta} = 0.0)$	24.26	44.11	2.02
$CE(t,x,y,h,A \mid \tilde{\eta} = 1.2)$	10.33	18.30	0.00
$CE(t,x,y,h,A \mid \tilde{\eta} = 2.0)$	6.03	7.31	0.00
$CE(t,x,y,h,A \mid \tilde{\eta} = 3.0)$	3.68	3.02	0.00

Table 2: Gain of Having Access to the CI Insurance. The table gives the percentage gain in the certainty equivalent (7) of having access to the CI insurance for different insurance calibrations ($\tilde{\eta} \in \{0.0, 1.2, 2.0, 3.0\}$) compared to a model without CI insurance ($\iota = 0$). The table shows the percentage gain for a young ($t = 5$), a middle-aged ($t = 25$), and an old ($t = 55$) healthy ($A = 1$) agent. The other state variables are set to $x = 500\,000$, $y = 100\,000$, and $h = 1\,700$. The model calibration is given in Section 3 and the insurance calibration in (10).

it becomes affordable even with less income. Financial wealth cannot substitute income here, which is due to mortality risk. Particularly, income delivers a certain cash-flow until the time of death, in contrast to financial wealth. Hence, a high income can hedge a potential health expense jump that would also have an effect until the time of death. In contrast, the agent possibly outlives his financial wealth if he wants to counter increased health expenses using financial wealth and faces a late time of death. Considering the graph with a fixed income, health expenses either have to be low or extremely high such that no CI insurance contract is optimal. In the first case, the contract is not necessary, whereas the contract is too expensive in the second case. In between, contracting the CI insurance is the optimal decision. Again, more financial wealth supports an insurance contract. The graph for fixed financial wealth highlights the interaction of health expenses and income. On the one hand, a high income and low health expenses result in a rejection of the insurance. Since the agent is able to pay the increased expenses after a jump without problems, he avoids the costly insurance. On the other hand, high health expenses combined with a low income lead to no insurance as well. In this case, the agent cannot or does not want to afford the CI insurance since it is too expensive.

Altogether, health expenses have a crucial impact on the CI insurance decision over the lifetime. Income is also important for the insurance decision and the importance increases in age as the uncertainty of human wealth decreases. Financial wealth is comparably unimportant for the insurance decision since it cannot reliably hedge the effects of a health expense jump.

Welfare Analysis To evaluate the influence of the insurance premium on the importance of the insurance, I consider the percentage gain in the certainty equivalent when having access to the insurance as calculated in (7). Table 2 yields the corresponding results. It is intuitive that agents are less better off as the insurance premium increases. If the insurance is not for free, the old agent is not significantly better off when having access to the insurance, independent of the premium level. This is not surprising since the insurance demand is low for old agents due to no uncertainty in human wealth. For the middle-aged agent, the level of the premium matters most. In the benchmark calibration, the agent crucially benefits from having access to the CI insurance, whereas the gain is much lower with a high insurance premium. The young agent prefers to contract the insurance due to huge uncertainty with respect to human wealth. In contrast, the middle-aged agent has already less uncertainty with respect to human wealth, which decreases the need for a CI insurance. Therefore, the level of the premium becomes more important for the middle-aged agent.

Altogether, young and middle-aged agents are significantly better off when having access to the CI insurance. This statement holds for all premium levels, even if the insurance profit is set to 200%. This indicates a strong desire to hedge the health expense jump risk as long as there is uncertainty with respect to human wealth.

6 Sensitivity Analyses

In this section, I analyze how the insurance demand is influenced by important market features and characteristics of the agent. Especially, I consider the impact of the income parameters, the risk aversion, and underestimating the probability or magnitude of a jump in health expenses. I am seeking for explanations considering the difference between the high CI insurance demand in the benchmark model, and the low CI insurance demand in the real world.

Impact of the Income Volatility Since income is the major source of wealth for the agent, the income parameters are of special importance. In the previous section, the certain retirement income was considered to be an explanation for the lower insurance demand during and shortly before the retirement state. Figure 11 compares the benchmark model with certain retirement income to a model in which retirement income is also uncertain. Considering the CI insurance demand, the previous explanations get justified. With uncertain retirement income, the insurance demand is higher throughout the lifetime, in particular during the retirement period. Due to a higher uncertainty of human wealth, the income is less suitable as a buffer against a health expense jump. This increases the CI insurance demand. As a direct reaction to the higher insurance demand, the average health expenses reduce. The consumption and wealth graphs reflect the increased uncertainty with respect to the major source of wealth as well. The consumption is reduced in early years but is higher later in the lifetime. Hence, the agent saves more as a protection against a possibly low future income and spends the excess wealth when the

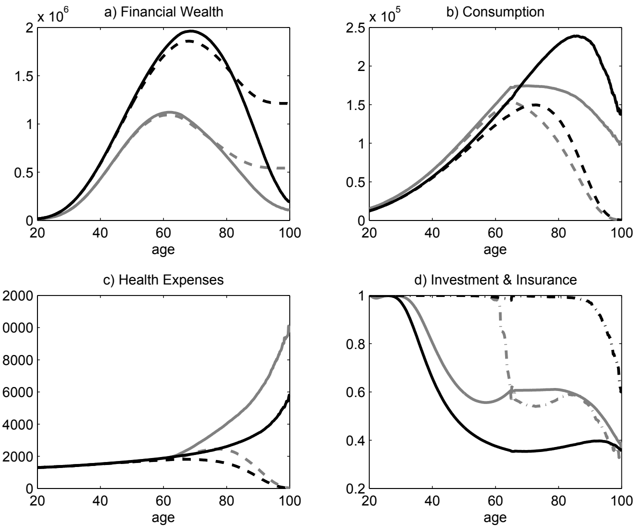

Figure 11: Impact of an Uncertain Retirement Income. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution. It compares the results with uncertain retirement income, $\sigma_Y(t) = 0.15\,\forall t$ (dark lines), to the benchmark results with certain retirement income (grey lines). a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

mortality risk increases. As a result, the agents have more financial wealth on average and leave more than twice as much bequest which is mainly accidental.

Having these results in mind, the effects of a change in the income volatility before retirement on the CI insurance demand are predictable. An increase in the volatility would further increase the insurance demand but the effect would be negligible since already nearly all agents contract the insurance early in lifetime. A reduction of the income volatility decreases the CI insurance demand before retirement. However, the income volatility is already low compared to other studies[6] such that a further reduction in income volatility is difficult to justify economically. Therefore, the income volatility cannot explain the empirically low insurance demand generally. However, it can partially explain a low insurance demand for people that have a less volatile income.

It is intuitive in the model that a reduced income volatility also reduces the CI insurance demand. In contrast, this effect is not obvious in the real world. In this paper, I model

[6] For example, Munk and Sørensen (2010) use $\sigma_Y = 0.2$ and Cocco, Gomes, and Maenhout (2005) have an overall income volatility of $\sigma_Y \approx 0.37$.

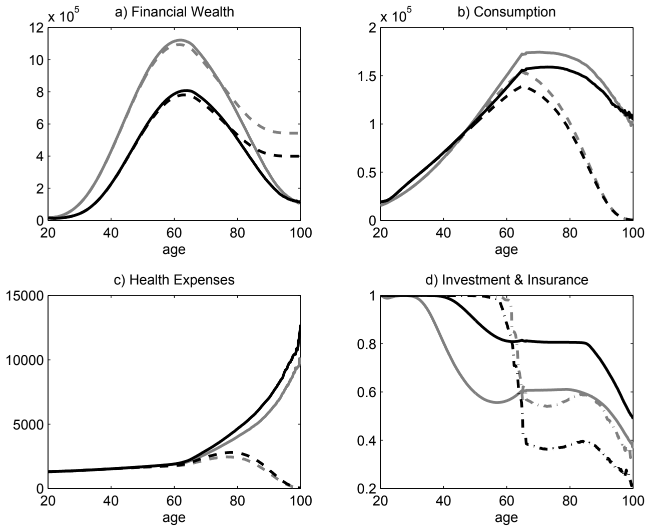

Figure 12: Impact of the Risk Aversion. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution if the agent is less risk averse, $\gamma = 3$ (dark lines), compared to the benchmark results, $\gamma = 4$ (grey lines). a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

the CI insurance as an instantaneous term insurance with a contract duration of dt. However, the real-world contracts typically have a longer duration of several years. Kraft, Schendel, and Steffensen (2014) consider a model with a term life insurance and account for the typically long contract duration. In their model, the demand for term life insurance increases if the income volatility is reduced. They argue that contracting the insurance enhances the impact of a negative income evolution since a belated change of the insurance contract is costly. In an already bad state, the agent has either an undesired insurance contract or faces additional costs for changing the contract. If I modeled long-term CI insurance contracts, I would expect the same effect to take place. Then, it is unclear whether it is the fear of having an undesired contract in a bad income state or the fear of suffering a health expense jump while having a low income that dominates. However, modeling long-term CI insurance contracts would decrease the overall insurance demand. Therefore, the instantaneous term insurance modeling can serve as one explanation for the too high insurance demand in the model compared to the real world.

Impact of the Risk Aversion In previous sections, I argue that the absence of the health expense jump effect or the effect of the premium level is qualitatively equivalent to a change in risk aversion. The more expensive the CI insurance is, the more risk averse the agent behaves. In order to verify these statements, I consider the effect of a change in risk aversion in Figure 12. As noted in the previous sections, the direct effects of a reduced risk aversion are less financial wealth, less bequest, and more risky investment. The consumption is increased in early years and decreased in later years. These effects are verified in the figure. Furthermore, the figure shows that a reduced risk aversion leads to less insurance demand and therefore to higher health expenses. However, the decrease in insurance demand is low before retirement and the value of risk aversion used in the benchmark calibration is not unreasonably high. Hence, a too high risk aversion in the model compared to reality is unlikely as an explanation for the different insurance demand in the model and the real world.

Effects of Underestimating the Health Expense Jumps Another possible explanation for the low insurance demand in the real world might be that agents underestimate the probability that a shock occurs or underestimate the financial impact of a shock. To analyze this hypothesis, I consider the impact of a different belief about the health shock intensity κ_H and the health expense effect of the shocks β_H in the model. A different belief about the mortality shock intensity κ_π would also lead to a different belief about the expected remaining lifetime and is therefore not studied here.

Figure 13 depicts the effects that arise from an underestimation of the health shock intensity. The agent thinks that health shocks occur less often and thus, he expects lower average health expenses and a higher average income. Furthermore, he thinks that the insurance has a worse cost-benefit relation. Compared to the benchmark case, there is no significant effect regarding financial wealth, consumption and portfolio holdings. However, the insurance demand is reduced, especially shortly before and during retirement which yields higher health expenses.

Figure 14 depicts the effects of underestimating the health expense effect of the health and mortality shocks. The agent underestimates average health expenses and thinks that the insurance has a worse cost-benefit relation but he has a correct belief about average income. The results are similar compared to the underestimation of the intensity. There is only a little impact on financial wealth, consumption, and investment. The CI insurance demand is reduced. In particular, there is only little demand in the retirement state. However, the demand in early years is still very high. Health expenses increase again due to the reduced insurance demand.

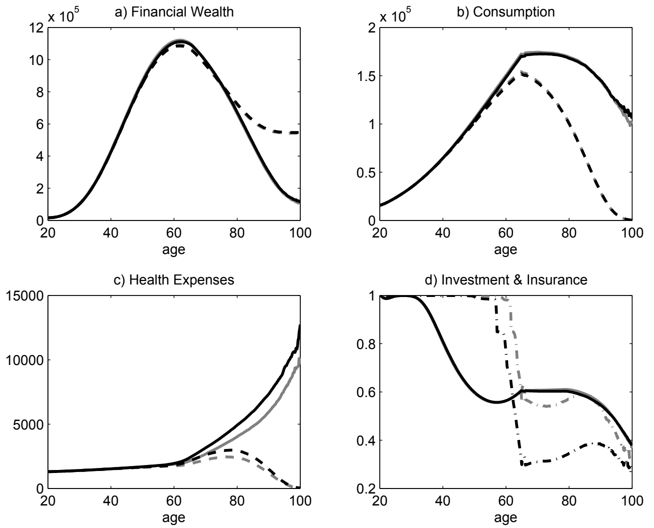

Figure 13: Impact of Underestimation of the Health Shock Intensity. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution in the benchmark model (grey lines) and in a model in which the agent underestimates the true health shock intensity (black lines). The agent thinks that health shocks occur only half as often compared to the true probability. Hence, the agent uses $\tilde{\kappa}_H(t) = 0.5\kappa_H(t)$ for his optimization, whereas $\kappa_H(t)$ is the true health jump intensity as used in the benchmark model. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

Agents that underestimate the intensity or impact of shocks are likely a partial explanation for the different insurance demand in the model and the real world. However, the CI insurance demand is mainly affected shortly before and during the retirement period.

Further Sensitivity Analyses Further sensitivity analyses cannot explain the difference between the model-based and the real-world CI insurance demand. I briefly give the results, the corresponding figures are available upon request. First, I change the preference parameters. The time preference rate δ influences the insurance demand such that for a higher time preference rate the CI insurance demand decreases. The intuition is that present cash-flows increase in value compared to future cash-flows. The CI insurance premium has to be paid when contracted, whereas the monetary benefits last throughout the lifetime if a shock occurs. Hence, the benefits decrease in value compared to the premium. For the agent, the subjective cost-benefit relation of the insurance is worse,

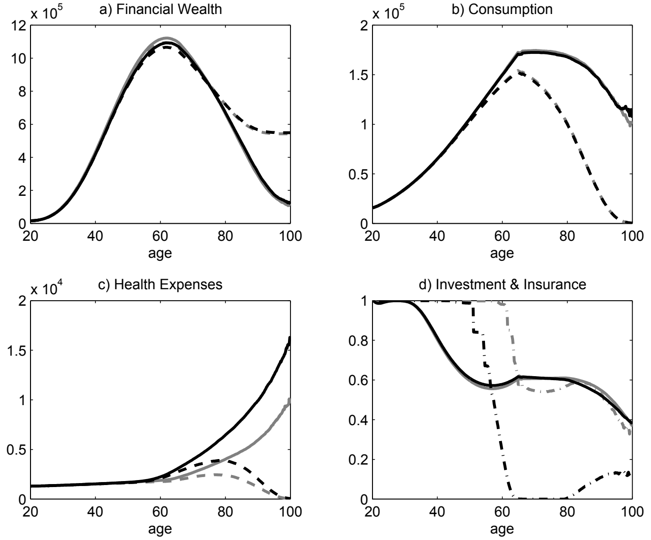

Figure 14: Impact of Underestimation of the Health Expense Jump Effect. The figure depicts the average optimal controls as well as the average financial wealth and health expense evolution in the benchmark model (grey lines) and in a model in which the agent underestimates the true health expense jump effect (black lines). The agent thinks that the health expense impact of the shocks is only half as high compared to the true impact. Hence, the agent uses $\tilde{\beta}_H(t) = 0.5\beta_H(t)$ for his optimization, whereas $\beta_H(t)$ is the true health expense effect of shocks as used in the benchmark model. a) presents the financial wealth evolution, b) gives the corresponding optimal consumption, and c) depicts the health expenses. d) shows the optimal fraction of risky investment as well as the fraction of agents that is in the insurance market ($A = 1$) and contracts a CI insurance (dash-dotted lines). Solid lines represent results for living agents ($A \neq 4$) only and dashed lines include all agents, whereas dead agents are included with zero consumption, zero health expenses, and a financial wealth equal to their bequest ($X_t = X_\tau$ if $A_t = 4$). The results are averaged based on 100 000 simulations with the model calibration of Section 3 and insurance calibration (10) with $\tilde{\eta} = 1.2$.

which yields a decreased insurance demand. Next, I consider the bequest motive and find that a change in the weight of the bequest motive ε has no significant effect on the insurance demand. In order to vary the investment opportunity set, I consider the impact of changing the stock volatility σ_S. An increase in the volatility leads to an increase in the insurance demand. The more volatile stock makes the financial wealth more volatile as well. Thus, the financial wealth is less suitable as a protection against shocks, which leads to the increased CI insurance demand. Next, I analyze the labor income drift μ_Y and the starting value Y_0. Using the high school or no high school calibration given in Munk and Sørensen (2010) instead of the college calibration, the CI insurance demand increases. The increase can be explained by the fact that the college calibration leads to an overall higher income and the retirement income is less reduced compared to the other two calibrations. Hence, the less educated agents have less human wealth which can serve as a protection against shocks. Consequently, the CI insurance demand increases. Finally, I consider a different labor income reduction if a shock occurs before retirement β_Y. An increase in β_Y means that the agent keeps more income if a shock occurs. This goes along with a decrease in the CI insurance demand before retirement since the additional income can be used to partially counter the negative effect on the health expenses. Since there is no income reduction after retirement, the effect vanishes then.

However, all of the above effects have either only a little impact or would require an unreasonable parametrization to be eligible as an explanation for the different CI insurance demand in the model and the real world.

7 Conclusion

In my model, the agents have a very high CI insurance demand, especially early in lifetime almost all agents contract the insurance. However, the CI insurance demand is significantly lower in the real world. On the one hand, the real-world CI insurance demand might be too low since it is a rather new type of insurance and it is still not very popular in most countries. On the other hand, the model-based CI insurance demand might be too high. One explanation is, that I model instantaneous term contracts which differ from real-world contracts. A long term contract, as insurance companies offer in reality, is less flexible and would therefore reduce the insurance demand. Furthermore, the insurance is modeled such that it is a perfect hedge against excess health expenses, which is also not true in reality where a fixed payout is delivered. Another explanation is that agents systematically underestimate the probability of health shocks or the magnitude of the health expense impact of the shocks. Furthermore, I do not have data on the health expense impact of the critical illnesses. Therefore, my health expense calibration might overestimate the importance of the jump part. This could also result in a too high insurance demand in the model. Despite these points, the high insurance demand, even if insurance profits are set unreasonably high, indicates that a CI insurance is worth further studies.

Further research can focus on the question why the real-world CI insurance demand is so low, despite the benefits of the insurance in the model. Modeling long-term insurance contracts would be an interesting extension to analyze the impact of the contract design in detail. Another promising extension is adding a disability insurance to analyze which type of insurance is preferred. Furthermore, a high disability insurance demand might be one explanation for the relatively low CI insurance demand in the real world.

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A The Numerical Solution Method

I solve the optimization problem (4) presented in Section 2 numerically. The numerical approach is similar to the one used in Schendel (2014a) which is based on the procedure in Munk and Sørensen (2010). Details considering the numerical method are given in the papers mentioned above. For the numerical solution, I simplify the problem by reducing the number of state variables by one.

Lemma 1. *The number of state variables in the optimization problem (4) can be reduced by one. The value function (5) can be expressed as*

$$J(t,x,y,h,A) = y^{1-\gamma}F(t,\hat{x},\hat{h},A)$$

with $\hat{x} = \frac{x}{y}$ *and* $\hat{h} = \frac{h}{y}$*. The corresponding HJB of the simplified problem is given by*

$$\begin{aligned}
0 = \sup_{\hat{c},\theta,\iota} \quad & \Bigg\{ \frac{\hat{c}^{1-\gamma}}{1-\gamma} + F\left[-\delta - \pi - \mathbb{1}_{\{A=1\}}\kappa_H - \mathbb{1}_{\{A=1\vee A=2\}}\kappa_\pi + (1-\gamma)\mu_Y - 0.5\gamma(1-\gamma)\sigma_Y^2\right] + F_t \\
& + F_{\hat{x}}\left[\hat{x}\left(r + \lambda\sigma_S\theta - \mu_Y + \gamma\sigma_Y^2\right) + 1 - \hat{c} - \hat{h} - \mathbb{1}_{\{A=1\wedge\iota=1\}}\hat{h}\eta\right] + 0.5F_{\hat{x}\hat{x}}\hat{x}^2\left[\sigma_S^2\theta^2 + \sigma_Y^2\right] \\
& + F_{\hat{h}}\hat{h}\left[\mu_H - \mu_Y + \gamma\sigma_Y^2\right] + 0.5F_{\hat{h}\hat{h}}\hat{h}^2\left[\sigma_H^2 + \sigma_Y^2\right] \\
& + \mathbb{1}_{\{A=1\}}\kappa_H F\left(t, \left(\frac{1}{1+\beta_Y}\right)\hat{x}, \left(\frac{1+\mathbb{1}_{\{\iota=0\}}\beta_H}{1+\beta_Y}\right)\hat{h}, A=2\right) \\
& + \mathbb{1}_{\{A=1\vee A=2\}}\kappa_\pi F\left(t, \left(\frac{1}{1+\mathbb{1}_{\{A=1\}}\beta_Y}\right)\hat{x}, \left(\frac{1+\mathbb{1}_{\{A=1\wedge\iota=0\}}\beta_H}{1+\mathbb{1}_{\{A=1\}}\beta_Y}\right)\hat{h}, A=3\right) \\
& + \pi F\left(\tau,\hat{x},\hat{h},A=4\right)\Bigg\}, \qquad (11)
\end{aligned}$$

for $A \in \{1,2,3\}$ *where* $\hat{c} = \frac{c}{y}$*. The simplified optimization problem has only four state variables* $t,\hat{x},\hat{h},A$ *and three control variables* $\hat{c},\theta,\iota$*, whereat hat-variables are normalized by the income level. The optimal normalized consumption and optimal portfolio holdings for a given insurance decision* $\iota \in \{0,1\}$ *can be calculated according to*

$$\hat{c} = F_{\hat{x}}^{-\frac{1}{\gamma}},$$
$$\theta = -\frac{F_{\hat{x}}\lambda}{F_{\hat{x}\hat{x}}\hat{x}\sigma_S}.$$

With the optimal controls $\hat{c}$ *and* θ *for both possible insurance decisions,* ι *is calculated by*

$$\iota = \underset{\iota\in\{0,1\}}{\arg\max} \quad F(t,\hat{x},\hat{y},A).$$

Proof. First, I reduce the number of state variables. Due to the linearity of the financial wealth (3), income (2), and health expense (1) dynamics and the power utility setup, I can calculate for $k > 0, k \in \mathbb{R}$

$$
\begin{aligned}
J(t,kx,ky,kh,A) &= \sup_{\{c_s,\theta_s,\iota_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau e^{-\delta(s-t)}\frac{(kc_s)^{1-\gamma}}{1-\gamma}ds + \varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{(kX_\tau)^{1-\gamma}}{1-\gamma}\right] \\
&= k^{1-\gamma}\sup_{\{c_s,\theta_s,\iota_s\}_{s\in[t,\tau)}} \mathrm{E}_{t,x,y,h,A}\left[\int_t^\tau e^{-\delta(s-t)}\frac{c_s^{1-\gamma}}{1-\gamma}ds + \varepsilon\,\mathrm{e}^{-\delta(\tau-t)}\frac{X_\tau^{1-\gamma}}{1-\gamma}\right] \\
&= k^{1-\gamma}J(t,x,y,h,A).
\end{aligned}
$$

Thus, I can reduce the number of state variables by one via expressing the indirect utility as

$$
\begin{aligned}
J(t,x,y,h,A) &= y^{1-\gamma}J\left(t,\frac{x}{y},\frac{y}{y},\frac{h}{y},A\right) \\
&=: y^{1-\gamma}F(t,\hat{x},\hat{h},A)
\end{aligned}
$$

with the new introduced normalized state variables $\hat{x} = \frac{x}{y}$ and $\hat{h} = \frac{h}{y}$. I express the partial derivatives of the HJB (6) in terms of F, which yields

$$
\begin{aligned}
J_t &= y^{1-\gamma}F_t, \\
J_x &= y^{1-\gamma}\frac{1}{y}F_{\hat{x}}, \\
J_{xx} &= y^{1-\gamma}\frac{1}{y^2}F_{\hat{x}\hat{x}}, \\
J_h &= y^{1-\gamma}\frac{1}{y}F_{\hat{h}}, \\
J_{hh} &= y^{1-\gamma}\frac{1}{y^2}F_{\hat{h}\hat{h}}, \\
J_y &= (1-\gamma)y^{-\gamma}F - y^{-\gamma}\frac{x}{y}F_{\hat{x}} - y^{-\gamma}\frac{h}{y}F_{\hat{h}}, \\
J_{yy} &= -\gamma(1-\gamma)y^{-\gamma-1}F + 2\gamma\frac{x}{y}y^{-\gamma-1}F_{\hat{x}} + 2\gamma\frac{h}{y}y^{-\gamma-1}F_{\hat{h}} + \frac{x^2}{y^2}y^{-\gamma-1}F_{\hat{x}\hat{x}} + \frac{h^2}{y^2}y^{-\gamma-1}F_{\hat{h}\hat{h}}.
\end{aligned}
$$

Inserting the new value function and the above derivatives in the HJB (6) and defining the new normalized control variable $\hat{c} = \frac{c}{y}$ leads to the new simplified HJB (11). For solving the simplified optimization problem, I first consider the HJB (11) for a fixed $\iota \in \{0,1\}$. Then, I can calculate the optimal normalized consumption and the optimal portfolio holdings, conditional on ι by taking the first order conditions of the HJB (11). Next, I substitute the calculated optimal controls into the HJB. The optimal insurance decision ι is then defined as the argument that maximizes the value function. □

I solve the new simplified optimization problem numerically with an implicit finite difference backward iterative approach. I set up a grid of normalized wealth $\hat{x} \in (0, 150]$ with 1000 grid points, of normalized health expenses $\hat{h} \in (0, 3]$ with 500 grid points, and of time $t \in [0, 110]$ with 661 grid points. I start with the solution for the case $A = 4$, which is trivial as there is no decision. Afterwards, I calculate the solution for $A = 3$, followed by $A = 2$, and lastly $A = 1$. In the state $A = 1$, I first calculate optimal normalized consumption and portfolio holdings for both possible insurance decisions. Then, I take the insurance decision that maximizes the value function calculated with the other optimal controls in each grid point. With the optimal insurance decision, I choose the corresponding optimal normalized consumption and portfolio holdings. After having calculated the optimal controls, I simulate 100 000 life cycles.

Part III

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Bibliographie

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Part IV

Anhang

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LIFE INSURANCE DEMAND UNDER HEALTH SHOCK RISK
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