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Gradualism vs Cold Turkey –

How to establish credibility for the ECB

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How to establish credibility for the ECB

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Abstract

The paper analyzes the incentive for the ECB to establish reputation by pursuing a restrictive policy right at the start of its operation. The bank is modelled as risk averse with respect to deviations of both inflation and output from her target. The public, being imperfectly informed about the bank's preferences uses observed inflation as (imperfect) signal for the unknown preferences. Under linear learning rules - which are commonly used in the literature - a gradual build up of reputation is the optimal response. The paper shows that such a linear learning rule is not consistent with efficient signaling. It is shown that in a game with efficient signaling, a cold turkey approach - allowing for deflation - is optimal for a strong bank - accepting high current output losses at the beginning in order to demonstrate its toughness.

Zusammenfassung

Die Arbeit untersucht die Anreize der Europäischen Zentralbank, in der Startphase durch restriktive Politik Reputation aufzubauen. Die Öffentlichkeit kennt die Präferenzen der Zentralbank nicht; sie verwendet die beobachtete Inflationsrate als (imperfektes) Signal. Wird eine lineare Lernregel unterstellt - der Standardfall in der Literatur - erweist es sich als optimal, hohe Inflationserwartungen zumindest teilweise zu akkommodieren und so Reputation nur schrittweise aufzubauen. Die Arbeit zeigt aber, daß eine solche lineare Lernregel mit effizientem Signalverhalten nicht konsistent ist. Bei effizientem Signalisieren kann es für eine harte Zentralbank optimal sein, in der Startphase durch eine sehr restriktive, deflationäre Politik ihre Präferenzen zu offenbaren.

Keywords: central bank policy, credibility, signaling, time consistency

JEL classification: D 82, E 58

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(1) Introduction

When the European Central Bank starts its operations January 1999, a new era begins. Right from the start, the bank has to face many challenges. Careful and detailed preparations have been done by the EMI, the forerunner of the ECB, to provide a smooth transition of monetary policy from the national central banks to a European agency. But despite all this impressive work, the ECB has to travel in up to now unknown European territory. Since it has no track record on which it can build, there is considerable uncertainty about its policy.

Of course, the treaty of Maastricht tried to establish a number of devices to make sure that the ECB will care for price stability. In many ways, these rules follow the example of the success story of the Deutsche Bundesbank. Even though this design is meant to ease a difficult start by transferring at least part of the legendary reputation the Bundesbank has acquired during the past 50 years, the ECB certainly cannot inherit its reputation right from the beginning.

To overcome the difficulties caused by the lack of track record, some even propose the ECB should exactly copy the strategy of the Bundesbank. Otmar Issing for instance, the chief economist of the Bundesbank, suggested that the best way to gain reputation would be to adopt the Bundesbank's monetary targeting procedure rather than following the latest fashion of inflation targeting. Even if such a strategy turned out to be successful in the end, it is quite unlikely that a mere imitation of the Bundesbank model can be sufficient to persuade public opinion about the ECB's determination to fight inflation.[1]

Certainly, copying legal rules and procedures are not sufficient for success. They cannot credibly guarantee how actual monetary policy will be carried out. In Germany, for a long time, fear prevails in public that the hard German mark will be sacrificed for a much weaker Euro. Lacking the public support (the „stability culture") on which the Bundesbank could build on, so the argument, the European Central Bank is doomed to give in to pressures from a

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[1]In fact, it is quite doubtful that adopting monetary targeting would be of much help, given that in its actual monetary policy the Bundesbank behaved in a very pragmatic way. As several studies show (see *Bernanke/Mihov* 1996, *Clarida/Gertler* 1996), monetary aggregates play a rather limited role in its strategy; instead, the Bundesbank follows more a policy of disciplined discretion (see *Laubach/Posen* 1997). Whereas the Bundesbank with its immaculate reputation obviously can afford to miss the self imposed monetary targets whenever it seems appropriate, the ECB would have a much tougher time in justifying deviations from those targets, despite the fact that monetary aggregates on the European level are bound to be rather volatile right at the beginning.

coalition of weak countries - the infamous so called Latin countries France, Italy, Spain and Portugal) - gaining by a laxer policy. Public debates about strict enforcement of the Maastricht budget rules did not help much to dampen those fears, especially when the ambiguity of the statistics became evident and even the German government tried to manipulate the statistics in its rush for reevaluating gold reserves. The quarrel about who (and for how long) should be appointed as the first president of the ECB has been interpreted as evidence for such a tendency. Based on such fears, some economists even tried to go to court in order to prevent the start of a weak Euro.

On the other hand, as the start of the Euro comes closer, fears pointing in the opposite direction now dominate the public debate. More and more fears arise that, at least right at the beginning, the ECB may behave as an „inflation nutter,, in order to gain credibility. Since at the same time, the stability pact also severely limits the scope of fiscal policy for stabilizing the economy, massive unemployment as the result of an unfortunate mix between strict monetary and fiscal policy all over Europe appears to be a not unlikely scenario.

Of course, such a strict policy might hardly be sustainable, at least when the unemployed, united from all countries, begin to march to the Euro tower in Frankfurt. So the ECB is not only facing the complex task of designing a single monetary policy in dark and as yet not well known European territory, which should fit the diverging needs of all those countries joining. In addition, it has to avoid the Scylla of an overkill of the economy arising from the too ambitious zeal for price stability and the Charybdis of an accommodating, soft inflationary policy.

There has been a lot of interest in the problem whether a single monetary policy might be suitable for all prospective EMU members. Initially, the main focus has been on the question of whether those countries form some sort of common currency area, which would justify a common stabilization policy. Recently, attention shifted to the issue that even if all countries were hit by the same shocks, differences in the national transmission mechanism imply that a common policy may affect different countries in a quite different way (see *Dornbusch/ Favero/ Giavazzi* (1998), *Ramaswamy/Sloek* (1997)).

In the present paper, we focus on a quite different aspect of stabilization policy: What impact will the lack of track record have on the policy of the ECB? Since the public is uncertain about the policy stance of the ECB, additional risk is introduced in the markets. Obviously, the prior

credibility will be lower compared to some established institutions it is supposed to replace. So, there is a strong incentive for the ECB to establish a reputation for toughness by pursuing a restrictive policy right at the start. In particular, we are interested in the impact on stabilization policy. Could the zeal for gaining recognition result in an overkill of the economy or will the bank at least partly accommodate higher inflationary expectations and prefer to build up credibility at a smooth, gradual pace? When analyzing this issue, we will abstract from the problem of designing a common policy fit for all countries. That is, we neglect complications arising out of asymmetric demand and supply shocks.

Signaling models provide the natural starting point to analyze the issue of reputation. In the context of dynamic inconsistency, it has long been recognized that current monetary policy may be disciplined by reputational effects, when the public is uncertain about the central bank preferences. Based on the *Barro/Gordon* (1983) model, reputational effects have been studied - among others - by *Backus/Driffill* (1985), *Cukierman/Meltzer* (1986), *Vickers* (1986), *Mino/Tsutsui* (1990) and most recently by *Faust/Svensson* (1997).

Following standard game theoretic literature, most of the initial work confined the analysis to the case of a limited number of central bank types with linear preferences w.r.t. output, choosing among perfectly reliable signals in a mixed strategy equilibrium. This set up gives a far too simplified picture of monetary policy, disregarding the continuous nature of uncertainty about central bank behavior and the problem of imperfect control. A noticeable exception is the highly original paper by *Cukierman/Meltzer* (1986). Even in that paper, however, central bank preferences are assumed to be linear in output for computational reasons. Such preferences imply risk neutrality w.r.t. output fluctuations, so all shocks can be absorbed by output fluctuations without any welfare losses. Given this assumption, the issue of stabilization policy becomes trivial and rather uninteresting.

Faust/Svensson (1997) modify the set up of *Cukierman/Meltzer* (1986) by allowing for quadratic preferences for output in order to study the impact of risk aversion for output fluctuations on stabilization policy. Following *Cukierman/Meltzer* (1986), they use a stationary, infinite horizon scenario in which central banks preferences are private information and change continuously across time. As in *Cukierman/Meltzer*, the public uses a linear learning rule to update its beliefs after observing the central bank's policy (the inflation rate). Noise, however, makes the central banks´s actions not completely transparent to the public, so the signals can reveal current preferences only partially.

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Because of the complexity arising from the quadratic loss function, Faust/Svensson do not solve directly the learning process and the bank's optimization problem. Instead, exploiting the stationary structure of their model, they use Kalman filter and dynamic programming technique to find a stationary solution. They show that reputation will be build up gradually: A central bank with low credibility will partly accommodate higher inflationary expectations. So the lower its credibility, the more expansionary the policy of a bank with given preferences will be. But since expectations will be accommodated only partly, output will, on average, be less than expected. Even though inflation is below the expected rate, the gradual build up of reputation will result both in less employment and higher inflation. In contrast, low credibility does not affect the bank's flexibility to respond optimally to supply shocks.

The intuition behind *Faust/Svenssons* result, at first sight, appears to be fairly straightforward: The lower the initial reputation, the higher inflationary expectations. Taking these into account, the bank will accommodate expectations to some extent, since an overly restrictive policy would involve even higher output losses. So it seems not to be optimal to gain credibility at a faster pace - implying that a gradual build up of reputation should be the optimal response.

This intuition, however, turns out not to be robust. As the current paper shows, the fact that gradualism is the optimal strategy is a consequence of the implausible linear learning rule imposed on private agents. We argue that linear learning rules do not capture the strategic nature of the asymmetric information game at issue: Using standard results from the signaling literature, it is shown that such rules are not robust against strategic experimentation. In a game with efficient signaling, a strong central bank may very well have an incentive to signal its intentions in a drastic way, rather than to accommodate high inflationary expectations at least partially.

The model is closely related to Faust/Svensson except that we use a two period model.[23] This set up captures the non stationarity nature of problem faced by introducing the Euro. The start of the ECB (implying a drastic change of the rules of the game) can hardly be interpreted as the outcome of a stationary process. The main advantage of the current set up, however, is that it allows to derive the explicit solution of the credibility game even in a model with quadratic

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[2] *Mino/Tutsui* (1991) analyzed reputational constraints in a two period version of the Barro/Gordon model. Again, however, they limit their analysis to the case of preferences being linear w.r.t. to output and to linear learning rules.

[3] In contrast to *Backus/Driffill* (1985), the reputation effects of the model are robust to an extension to infinite horizon, they are not driven by some strange effects of an end game scenario.

preferences. Characterizing this solution may be of interest in its own. But the main economic payoff comes from testing the robustness of the results derived in Faust/Svensson.

The next section presents the two period model outlining the signaling problem: Inflation today serves as a (noisy) signal about the bank's preferences. Then, section 3 characterizes equilibrium under linear learning rules. Both the Bayesian learning problem the private agents are facing and the optimal central bank strategy are analyzed in detail. The setup allows to derive central results in a straightforward way: If the public is able to observe output shocks when updating estimates about the bank's preferences, the bank's stabilization response will be independent of her credibility. Concern for reputation dampens her temptation to pursue a discretionary policy. Reputational concerns are stronger the higher the discount factor and the more transparent the policy (the more reliable the signal is for the public). Under linear learning rules, partial accommodation of inflationary expectations (implying a gradual response) is always optimal.

Linear learning rules, however, even though very popular in macroeconomics, are not robust against strategic experimentation. Section 4 analyzes the efficient signaling strategy for the case of a completely revealing signal. We show that their results do not hold in an efficient signaling game. It is shown that in that case, a strong bank is willing to incur high current output losses in order to demonstrate its toughness to the public.

(2) The signaling problem

(2).1 Reputation under imperfect control

We analyze the following two period model. The central bank has the quadratic loss function:

$$\mathbf{1} \qquad E\, L = E\, L_t(\boldsymbol{p}_t, \boldsymbol{p}_t - \boldsymbol{p}_t^e) + \boldsymbol{d}\, E\, L_{t+1}(\boldsymbol{p}_{t+1}, \boldsymbol{p}_{t+1} - \boldsymbol{p}_{t+1}^e)$$

with $\dfrac{\P L_t}{\P \boldsymbol{p}_t} > 0$, and $\left.\dfrac{\P L_t}{\P (\boldsymbol{p}_t - \boldsymbol{p}_t^e)}\right|_{\boldsymbol{p}_t = \boldsymbol{p}_t^e} < 0$

Throughout the paper, we assume the quadratic loss function:

$$\boldsymbol{p}_t^2 + (y_t - \Delta)^2 + \boldsymbol{d}\left\{\boldsymbol{p}_{t+1}^2 + (y_{t+1} - \Delta)^2\right\}$$

The short run aggregate supply curve is given by:

2 $y_t = (p_t - p^e_{t|t-1}) + e_t$

e_t is a supply shock with $E(e_t) = 0$; $Var(e_t) = s^2_e$. Since the central bank can observe e_t, it can, in principle, stabilize output fluctuations. Private agents are uncertain about central bank's true preferences. As in *Faust/Svensson* (1997), this uncertainty is modeled by incomplete information about the output target Δ: The optimal policy depends on the output target, but Δ is assumed to be private information of the central bank. For the public, Δ is a random variable with $E(\Delta) > 0$; $Var(\Delta) = s^2_\Delta$. At the beginning of period t, the public knows only $E(\Delta)$. $E(\Delta)$ characterizes initial reputation (the higher $E(\Delta)$, the lower the reputation).

The conduct of monetary policy provides a signal about the underlying preferences: The lower Δ, the more restrictive policy will be. The central bank may be willing to incur current output losses in order to signal a low output target Δ and so gain higher reputation in the next period. In general, however, the signal will be noisy – the actions of the central bank cannot reveal completely the underlying preferences.

Following *Cukierman/Meltzer* (1986), this noise is modeled as imperfect control of the central bank about the inflation rate captured – control errors introduce additive noise in the signal:

3 $p_t = i_t + h_t$

i_t are the (unobservable) intentions of the central bank, h_t is a control error with $E(h) = 0$; $Var(h) = s^2_h$. The public cannot observe the intentions i_t (it certainly has more or less detailed information about the actual monetary policy instruments (the interest rate set by the central bank is directly observable) , but it does not have sufficient knowledge about information motivated a change (resp. no change) in the monetary policy instruments. The more transparent the policy, the more precise the central bank's intentions can be deduced by the public. So higher transparency (such as publishing detailed reports revealing intentions behind the policy actions) is equivalent to reducing the variance s^2_h. This has the somewhat implausible implication that higher transparency at the same time also reduces the control error of monetary policy. As in *Faust/Svensson* (1997), we could distinguish between transparency and control error with somewhat more complex notation, but no additional insights will be gained by doing so.

After observing $\boldsymbol{p}_t$, agents update their beliefs about central bank and revise inflationary expectations. The higher the inflation rate $\boldsymbol{p}_t$, the more damaged, in general, the reputation will be. There are, however, two caveats. First, high inflation may be the result of a negative supply shock. If the central bank tries to stabilize the economy and if private agents can observe that (ex post, once monetary policy has been carried out, but before next periods inflationary expectations have been formed), then the public will take that information into account and may absolve the central bank for its action. Furthermore, the high inflation may simply be due to control errors. When updating reputation, the public will take into account this noise.

Of course, stronger types will try to signal their policy stance via pursuing a strict policy. So observing a low inflation rate (having adjusted for the effects of supply side shocks), raises the posterior for a strict central bank. The probability distribution will be shifted. The smaller $\boldsymbol{s}_h^2$ and/or the larger $\boldsymbol{s}_\Delta^2$, the more valuable information about Δ the signal provides and thus the stronger expectations will be updated. If there were no noise ($\boldsymbol{s}_h^2=0$), a completely separating equilibrium will occur (in that case, actions reveal the true type). The concern for reputation works as a disciplinary device, limiting the temptation for surprise inflation. Therefore, first period inflation will be lower than in the discretionary equilibrium - except for the case that future payoffs are irrelevant ($\boldsymbol{d}=0$) or that there is no uncertainty about the preferences ($\boldsymbol{s}_\Delta^2=0$).

(2).2 The central bank's strategy

In a two period setting, obviously, reputational effects are of no concern in the last period. So the optimal policy at t+1 simply depends on preferences and expected inflation: $i_{t+1}=f(\boldsymbol{p}_{t+1}^e,\Delta)$. In contrast, when formulating its policy in period t, the central bank will take into account that an increase in the inflation target (a higher i_t) will affect next period's reputation ($E(\boldsymbol{p}_{t+1}|\boldsymbol{p}_t(i_t))$). Policy today is disciplined by the impact on reputation next period, as long as $\boldsymbol{p}_t$ serves as a (noisy) signal about Δ. In general terms, the central bank's objective is:

4   Min $EL(i_t)=EL_t(i_t,i_t-\boldsymbol{p}_t^e)+\boldsymbol{d}\ EL_{t+1}(i_{t+1},i_{t+1}-\boldsymbol{p}_{t+1}^e(\boldsymbol{p}_t))$

Using 2, the central bank's quadratic loss function 1 can be written as:

5 $EL(\boldsymbol{p}_t)=\boldsymbol{p}_t^2+(\boldsymbol{p}_t-\boldsymbol{p}_t^e-\Delta+\boldsymbol{e}_t)^2+\boldsymbol{d}\{\boldsymbol{p}_{t+1}^2+(\boldsymbol{p}_{t+1}-\boldsymbol{p}_{t+1}^e(\boldsymbol{p}_t)-\Delta+\boldsymbol{e}_{t+1})^2\}$

In terms of the instrument, the objective is:

$$EL(i_t) = E(i_t)^2 + E(i_t + e_t - p_t^e - \Delta)^2 + d\left\{ E(i_{t+1})^2 + E(i_{t+1} + e_{t+1} - p_{t+1}^e(p_t) - \Delta)^2 \right\} +$$

$$2s_{h_t}^2 + d2s_{h_{t+1}}^2$$

The first order condition for the optimal strategy is:

6 $$\frac{\partial\, E\, L}{\partial\, i_t} = i_t + (i_t - p_t^e - \Delta + e_t) + d\,\frac{\partial\, EL_{t+1}(p_{t+1}^e)}{\partial\, p_{t+1}^e}\,\frac{\partial\, p_{t+1}^e}{\partial\, i_t} = \mathbf{0}$$

The first term characterizes the marginal loss in period t arising out of higher expected inflation, whereas the second term captures the benefit from inflation at t (via increased output). The final term represents the discounted future loss arising from a laxer policy at t. For $d > 0$, inflation will, in general, be lower than in the absence of reputational effects since $\frac{\partial\, L_{t+1}(p_{t+1}^e)}{\partial\, p_{t+1}^e} > 0;\ \frac{\partial\, p_t}{\partial\, i_t} = 1$ and $\frac{\partial\, p_{t+1}^e}{\partial\, p_t} > 0$ (except for the limit case that the signal has no informational content).

Following backward induction, the optimal policy in t+1 can be solved for given p_{t+1}^e as:

7 $$i_{t+1} = \tfrac{1}{2}\,(p_{t+1}^e + \Delta - e_{t+1}).$$

From 7 follows that the rational forecast for inflation is just the forecast for the unknown parameter Δ:

8 $$p_{t+1}^e(p_t) = E(\Delta | p_t)$$

This is because $E(p_{t+1}) = E(i_{t+1})$ and, according to **7**, $E(i_{t+1}) = \tfrac{1}{2}p_{t+1}^e + \tfrac{1}{2}E(\Delta)$. Thus, with rational expectations $p_{t+1}^e = E(p_{t+1})$**,** we get $p_{t+1}^e(p_t) = E(\Delta | p_t)$**.**

Equation 6 characterizes the optimal policy. Obviously, the bank's strategy depends on how the public interprets the outcome p_t. In the following section, we discuss the public's learning problem. We consider the special case that the updating can be described by a linear learning rule. In this case, it turns out that the rational expectations equilibrium is indeed characterized by a linear rule.

(3) **Linear learning rules**

(3).1 *The learning problem*

At the end of period t, private agents observe the inflation rate p_t and the supply shock e_t. Suppose that private agents believe the bank's policy rule 6 can be described by the following linear equation:

$$9 \qquad\qquad i_t = a + b\,\Delta + g\,e_t$$

Because of 3 and 9, then $p_t = i_t + h_t = a + b\,\Delta + g\,e_t + h_t$. Using all available information, the public tries to infer the central bank's preferences Δ (resp. i_t) in order to minimize expectational errors about next periods inflation rate: $p^e_{t+1} = F(p_t)$.

Assume $\Delta \sim N(E(\Delta); s^2_\Delta)$ and $h \sim N(0; s^2_h)$. Furthermore, Δ and h are assumed to be uncorrelated. Then

$$p_t \sim N(a + b\,E(\Delta) + g\,e_t;\ b^2\,s^2_\Delta + s^2_h)$$

When the shock e_t can be observed perfectly ex post, private agents can deduce the impact of e_t on p_t, as long as the stabilization response is independent of the type Δ. This will be true if the policy indeed can be characterized by a linear rule such as in 9. Knowing the optimal reaction $g\,e_t$, private agents can sort out[4] the effect of e_t on p_t.

Using Bayesian updating, the best estimates for Δ and i_t, based on the signals p_t and e_t and the central bank's strategy 9, are:

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[4] This argument implies that stabilization policy will be independent of credibility- that is the optimal response to shocks e_t will be $g_t = -\frac{1}{2}$ (see equation 19 below). The independence between stabilization impact and reputation holds as long as we can separate the effect of e_t on p_t when making inference about the type Δ.

Obviously, this result does no longer hold when there is uncertainty w. r. t. to the weight the central bank attaches to output stabilization (as is well known, the lower this weight, the less dampened will be output fluctuations). Depending on the central banks weight relative to the society's weight, bias in stabilization policy may be positive or negative (see *Beetsma/Jensen* 1998). This, however, does not imply that a given type will not be able to implement the policy which is efficient from her point of view. The present set up allows to work out the impact of reputation on the stabilization response in the clearest way.

10 $\quad E(\Delta | \boldsymbol{p}_t, \boldsymbol{e}_t) = E(\Delta) \; \boldsymbol{r} - \dfrac{\boldsymbol{a}}{\boldsymbol{b}} (1 - \boldsymbol{r}) + \dfrac{1 - \boldsymbol{r}}{\boldsymbol{b}} (\boldsymbol{p}_t - \boldsymbol{g} \, \boldsymbol{e}_t)$

and correspondingly:

11 $\quad E(i_t | \boldsymbol{p}_t, \boldsymbol{e}_t) = E(i_t) + \dfrac{Cov(i_t, \boldsymbol{p}_t)}{Var(\boldsymbol{p}_t)} \left[\boldsymbol{p}_t - E(\boldsymbol{p}_t) \right] = (\boldsymbol{a} + \boldsymbol{b} \, E(\Delta) + \boldsymbol{g} \, \boldsymbol{e}_t) \, \boldsymbol{r} + \boldsymbol{p}_t \, (1 - \boldsymbol{r})$

12 $\quad$ with $\boldsymbol{r} = \dfrac{\boldsymbol{s}_h^2}{\boldsymbol{b}^2 \, \boldsymbol{s}_\Delta^2 + \boldsymbol{s}_h^2}; 1 - \boldsymbol{r} = \dfrac{\boldsymbol{b}^2 \, \boldsymbol{s}_\Delta^2}{\boldsymbol{b}^2 \, \boldsymbol{s}_\Delta^2 + \boldsymbol{s}_h^2} = \dfrac{Cov(i_t, \boldsymbol{p}_t)}{Var(\boldsymbol{p}_t)}$

Given the belief 9, private agents make a linear forecast:

13 $\quad \boldsymbol{p}_{t+1}^e = \boldsymbol{l}_0 + \boldsymbol{l}_1 \, \boldsymbol{p}_t + \boldsymbol{l}_2 \boldsymbol{e}_t$

The impact of a change in the rate of inflation at t on next period's expectation is $\dfrac{\partial \boldsymbol{p}_{t+1}^e}{\partial \boldsymbol{p}_t} = \boldsymbol{l}_1$.

Obviously, $\boldsymbol{l}_0$ will depend on $\boldsymbol{p}_t^e$ (resp. on the prior knowledge $E(\Delta)$). To solve for the equilibrium, the central bank takes $\boldsymbol{l}_0; \boldsymbol{l}_1; \boldsymbol{l}_2$ as given. This defines her optimal rule (specifying equation 9), given the assumed updating behavior of private agents. In a REE, the estimate $\boldsymbol{p}_{t+1}^e = \boldsymbol{l}_0 + \boldsymbol{l}_1 \, \boldsymbol{p}_t + \boldsymbol{l}_2 \, \boldsymbol{e}_t$ must correspond to the actual optimal behavior of the central bank. Thus, in REE, updating must correspond to the actual behavior –this determines the solution.

(3).2 Equilibrium under linear learning rules

The optimal forecast for $E(\Delta | \boldsymbol{p}_t)$ is defined by equation 10 and the inflation forecast $\boldsymbol{p}_{t+1}^e(\boldsymbol{p}_t)$ by equation 13. Because $\boldsymbol{p}_{t+1}^e(\boldsymbol{p}_t) = E(\Delta | \boldsymbol{p}_t)$, in REE the parameters in 10 and 13 must be identical:

14 $\quad \boldsymbol{l}_0 = E(\Delta) \, \boldsymbol{r} - \dfrac{\boldsymbol{a}}{\boldsymbol{b}} (1 - \boldsymbol{r})$

15 $\quad \boldsymbol{l}_1 = \dfrac{1 - \boldsymbol{r}}{\boldsymbol{b}}$

16 $\quad \boldsymbol{l}_2 = -\boldsymbol{g} \, \boldsymbol{l}_1$

As shown in the appendix A, using the restrictions above the optimal linear strategy of the central bank is characterized by:

17 $$a = E(\Delta)\left[\frac{1-d\,l_1}{2+d\,l_1^2} - d\,l_1\,r\right] = E(\Delta)[b - d\,l_1\,r]$$

18 $$b = \frac{1-d\,l_1}{2+d\,l_1^2}$$

19 $$g = -\frac{1}{2}$$

20 $$i_t = E(\Delta)\left[\frac{1-d\,l_1}{2+d\,l_1^2} - d\,l_1\,r\right] + \Delta\frac{1-d\,l_1}{2+d\,l_1^2} - \tfrac{1}{2}e_t = p_t^e + \frac{1-d\,l_1}{2+d\,l_1^2}\,[\Delta - E(\Delta)] - \tfrac{1}{2}e_t$$

with $p_t^e = E(\Delta)\left[2\frac{1-d\,l_1}{2+d\,l_1^2} - dl_1\,r\right]$

According to 19, the central bank responds efficiently to output shocks, independent of its credibility. The reason is similar to the standard Barro/Gordon model: There, lack of credibility results just in an inflation bias, leaving the stabilization response and thus real output unchanged.

Here, however, the impact on stabilization policy is more subtle due to the lack of information about central bank's preferences. Since a strong bank (with $\Delta < E(\Delta)$) pursues a policy resulting on average in less inflation than expected, it will cause a recession on average in order to gain reputation for the future. The opposite is true for a weak bank ($\Delta > E(\Delta)$). A stronger concern for reputation (due to a higher d) will cause a more stable policy for all types and thus reduce inflationary expectations: $\frac{\partial p_t^e}{\partial d} < 0$ with $p_t^e < E(\Delta)$ for $d > 0$ and $r < 1$.

The independence between stabilization response and reputation holds as long as the effect of e_t on p_t can be separated when private agents make inference about the type Δ. This is also true for nonlinear (e.g. multiplicative) control errors. Take the case of multiplicative uncertainty: $p_t = h\,i_t$ with $E(h) = \bar{h} = 1$ and $s_h^2 = E(h)^2 - (E(h))^2$ Then, the FOC has to be modified to

$$2\,E(\boldsymbol{h})^2 i_t - E(\boldsymbol{h})(\boldsymbol{p}_t^{\,e} + \Delta - \boldsymbol{e}_t) + \boldsymbol{d}\,\frac{\partial\,EL_{t+1}(\boldsymbol{p}_{t+1}^e)}{\partial\,\boldsymbol{p}_{t+1}^e}\,\frac{\partial\,\boldsymbol{p}_{t+1}^e}{\partial\,i_t} \quad \text{or}$$

$$i_t = \frac{1}{2\,(1+\boldsymbol{s}_{\boldsymbol{h}}^2)}\left[(\boldsymbol{p}_t^{\,e} + \Delta - \boldsymbol{e}_t) - \boldsymbol{d}\,\frac{\partial\,EL_{t+1}(\boldsymbol{p}_{t+1}^e)}{\partial\,\boldsymbol{p}_{t+1}^e}\,\frac{\partial\,\boldsymbol{p}_{t+1}^e}{\partial\,i_t}\right]$$

Now, the bank will act more cautiously in general, and so – as is well known from *Brainard* (1967) – it will stabilize shocks to a lesser extent. But as long as the impact of $\boldsymbol{e}_t$ on i_t can be inferred precisely, the stabilization policy does not affect updating by private agents, and so credibility has no impact on the stabilization response.[5]

What is the impact of damaged reputation as a result of changing from a well established institution as the Bundesbank to an institution lacking any track record? In order to evaluate the impact of reputation, assume that the new institution has identical preferences (the same Δ). The public, however, being unsure about Δ, raises its expectation $E(\Delta)$. That is, the loss of reputation for the new institution is captured by an increase in $E(\Delta)$ (of course, in general this goes hand in hand with an increase in variance $\boldsymbol{s}_{\Delta}^2$, but the effects are similar). Obviously, type Δ will also partly accommodate the higher inflationary expectations, but only to some extent: According to equation 20, the response will be less than ½ for $\boldsymbol{d} > 0$ and $\boldsymbol{r} < 1$. Thus, given the linear learning rule, a gradual response is optimal when reputation deteriorates. As a result, on average output and employment will be lower and inflation higher when reputation deteriorates.

The endogeneity of the signal to noise ratio $1 - \boldsymbol{r}$ complicates calculation of the explicit solution: The stronger the central bank responds to its own preferences (the higher $\boldsymbol{b}$), the stronger the public will update its priors when observing a high rate of inflation, thus dampening the bank's incentive to raise $\boldsymbol{b}$. The two period set up allows to calculate explicitly the central bank's reaction as a function of time preference and the variance of the shocks. As shown in appendix B, comparative static results show that, in general, the central bank responds more cautiously to its own preferences, the more valuable future reputation (the higher the discount factor $\boldsymbol{d}$) and the more reliable the informational content of the signal (the more precise the information $\boldsymbol{p}_t$, that is, the smaller the ratio $\boldsymbol{s}_{\boldsymbol{h}}^2/\boldsymbol{s}_{\Delta}^2$):

---

[5] This argument no longer holds if instead $\boldsymbol{e}_t$ can only be observed with some noise. Then, when they update their priors, private agents can no longer separate the impact of supply shocks on the inflation rate, and this may affect the central bank's incentives to stabilize efficiently. The impact of noisy observation is left for future research.

$$\frac{\partial\, b}{\partial\, d} < 0 \ \text{ and } \ \frac{\partial\, b}{\partial\, s_h^2 / s_\Delta^2} > 0$$

(4) Efficient Signaling Equilibrium

Up to now, following standard procedure in the macro literature (*Cukierman/Meltzer* (1986), *Mino/Tsutsui* (1990) and *Faust/Svensson* (1997)), a linear learning rule was imposed on the updating behavior of the public. Such a rule preserves the linear structure of the model and makes calculation of expected values fairly straightforward. As is well known from signaling models, there can be an infinite number of equilibria depending on the beliefs imposed on the uninformed players. Most of these beliefs, however, are rather implausible. For the discrete type analysis, game theoretic tools provides a menu of refinements to narrow down the set of equilibria. Refinements are needed because, in the discrete type case, in equilibrium most of the signals are not used, and so arbitrary restrictions can be imposed on out of equilibrium behavior. In a continuous type setting, a continuous set of signals will be sent in equilibrium, and so the problem of indeterminacy becomes less serious. As shown below, imposing a linear learning curve is not justified in this context.

Following the standard signaling literature (see *Riley* 1979), the efficient signaling equilibrium will be analyzed. It turns out that under efficient signaling, it is quite likely that a tough bank has incentives to enforce a very restrictive cold turkey policy (even risking an „overkill„) in order to prove its toughness. Since the equilibrium outcome is highly nonlinear, updating rules under noisy signals become fairly messy and unsolvable. We illustrate the argument for the case of perfect transparency (a completely revealing signal $r = 0$) and a finite support of the random variable Δ. The finite support allows to characterize the strategy of the worst central banker in a straightforward way. To what extent the argument below can be extended to the case of distributions with infinite support is left for future research. So from now on assume that

21 $p_t = i_t$

Furthermore, Δ is assumed to be distributed continuously with finite support $\Delta \in \left[\Delta_{\min}, \Delta_{\max} \right]$.

First, we show that strategies implied by a linear learning rule cannot be the optimal strategy for the weakest type. Let $\hat{\Delta} = \Delta_{\max}$ be the type with the highest preference for output

stimulation. Under a linear learning rule as imposed in 13, the public assumes that she will always choose some $\boldsymbol{p}_t(\hat{\Delta}) = \boldsymbol{q}\,\hat{\Delta} < \hat{\Delta}$. Obviously, this choice will reveal her type in the second period anyway $\boldsymbol{p}_{t+1}^e(\boldsymbol{p}_t(\hat{\Delta})) = \hat{\Delta}$. But then, what should prevent type $\hat{\Delta}$ from pursuing the short run optimal policy $\boldsymbol{p}_t(\hat{\Delta}) = \hat{\Delta}$ which clearly would give her a higher payoff?

The problem with the linear rule is that it implies for any $\tilde{\boldsymbol{p}}_t > \boldsymbol{p}_t(\hat{\Delta})$ inflationary expectations will be even higher ($\boldsymbol{p}_{t+1}^e(\tilde{\boldsymbol{p}}_t) > \hat{\Delta}$), because $\dfrac{\partial \boldsymbol{p}_{t+1}^e}{\partial \boldsymbol{p}_t(\hat{\Delta})} = \boldsymbol{l}_1 > 0$ (see *figure 1*). This, however, does not make sense, since in a revealing equilibrium, the highest possible rate at t+1 is $\boldsymbol{p}_{t+1} = \hat{\Delta}$. So, the learning curve should become flat for $\boldsymbol{p}_t > \boldsymbol{p}_t(\hat{\Delta})$ (all other learning strategies are strictly dominated). Given a flat updating, however, the best type $\hat{\Delta}$ can do is to choose $\boldsymbol{p}_t(\hat{\Delta}) = \hat{\Delta}$. The weakest type has no reason to signal at all. That means that any refined learning curve must be flat at $\hat{\Delta}$ (that is $\boldsymbol{p}_{t+1}^e(\hat{\Delta}) = \hat{\Delta}$ with $\dfrac{\partial \boldsymbol{p}_{t+1}^e}{\partial \hat{\Delta}} = 0$ - see *figure 2*).
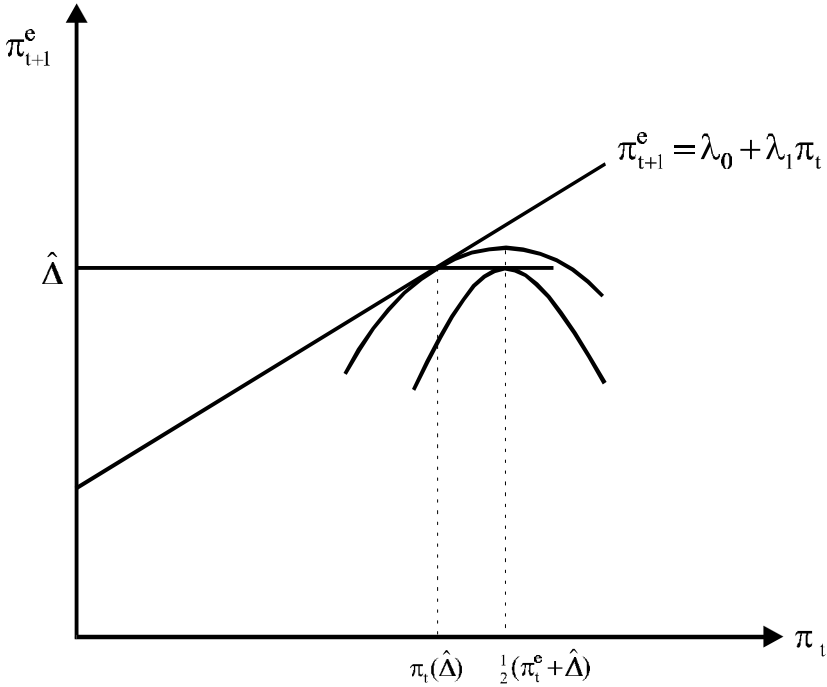

Figure 1

In contrast, all lower types have an incentive to signal. The efficient signaling is pinned down by incentive conditions: It must not be in the interest of high Δ types to disguise as low Δ types (weak types must not be able to gain by reducing inflation and so establishing a

reputation as being strong). Given some learning curve $p_{t+1}^e(\Delta)$ of the public, each type Δ will choose her optimal strategy according to the FOC. In a rational expectations equilibrium, the optimal response of type Δ must confirm to the belief held by the public.

The central banks problem is to minimize the loss:

22 $\quad EL(p_t) = p_t^2 + (p_t - p_t^e - \Delta + e_t)^2 + d\left[\frac{1}{2}(p_{t+1}^e)^2 + p_{t+1}^e(\Delta) + \frac{1}{2}(\Delta)^2\right]$

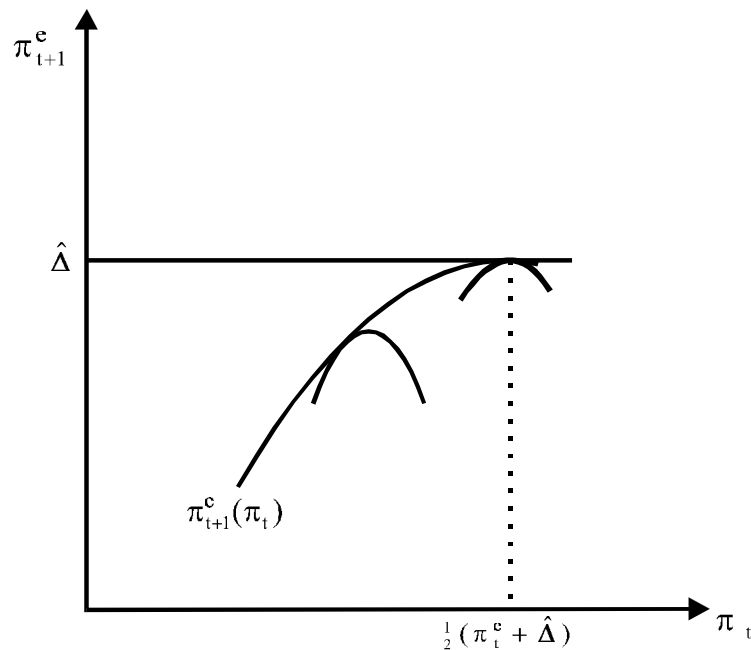with $p_{t+1}^e(p_t)$. Indifference curves in the (p_t, p_{t+1}^e) space are sloped according to:

Figure 2

$(4p_t - 2(p_t^e + \Delta))\, d\, p_t + d\, (p_{t+1}^e + \Delta)\, d\, p_{t+1}^e = 0$

So given any learning rule $p_{t+1}^e(p_t)$, the optimal choice for each Δ is characterized by the condition (compare equation **4** and figure 2):

23 $\qquad\qquad \dfrac{\partial\, p_{t+1}^e}{\partial\, p_t} = \dfrac{2(p_t^e + \Delta) - 4p_t}{d\, (p_{t+1}^e + \Delta)}$

$\dfrac{\partial\, p_{t+1}^e}{\partial\, p_t} \geq 0$ for $p_t \leq \frac{1}{2}(p_t^e + \Delta)$ (which always holds)

23 pins down the signaling choice for any type Δ. If the learning rule $p^e_{t+1}(p_t)$ is continuous, it should correspond for each type Δ to her actual behavior, which is characterized by this condition. Integrating 23 gives the optimal learning rule:

24
$$\tfrac{1}{2}\boldsymbol{d}\,(p^e_{t+1})^2 + \boldsymbol{d}\,\Delta p^e_{t+1} - 2(p^e_t + \Delta)p_t + 2p^2_t = A$$

In a revealing equilibrium, the following additional conditions must hold:

1) Given that p_t is a revealing signal for Δ, in equilibrium, expectations must conform to actual behavior.

25
$$p^e_{t+1}(p_t(\Delta), \Delta) = \Delta$$

2) The weakest type cannot gain by distorting his short run optimal choice:

26
$$p_t(\overline{\Delta}) = \tfrac{1}{2}(p^e_t + \overline{\Delta}) \quad \text{for } \overline{\Delta} = \Delta_{max}$$

Using 25 and 26, the constant A is determined by:

27
$$A = \tfrac{3}{2}\overline{\boldsymbol{d}}\Delta^2 - \tfrac{1}{2}(p^e_t + \overline{\Delta})^2$$

The optimal policy p_t for type Δ is characterized by the equation:

$$p^2_t - (p^e_t + \Delta)p_t - \tfrac{3}{4}\boldsymbol{d}(\overline{\Delta}^2 - \Delta^2) + \tfrac{1}{4}(p^e_t + \overline{\Delta})^2 = 0$$

which yields the solution:

28
$$p_t(\Delta) = \tfrac{1}{2}(p^e_t + \Delta) - \tfrac{1}{2}\sqrt{(\overline{\Delta} - \Delta)\left[(3\boldsymbol{d} - 1)(\overline{\Delta} + \Delta) - 2p^e_t\right]}$$

According to 28, p_t is increasing in Δ. The higher the discount factor $\boldsymbol{d}$, the more current policy is disciplined by the impact on future reputation. Reputation works as disciplinary device only for $\boldsymbol{d} > \hat{\boldsymbol{d}} = \tfrac{1}{3} + \tfrac{1}{3}\dfrac{2p^e_t}{\overline{\Delta} + \Delta}$. $\hat{\boldsymbol{d}}$ is decreasing in Δ. For $\boldsymbol{d} \le \hat{\boldsymbol{d}}$, reputational effects do not constrain monetary policy, so we get the unconstrained optimum as a corner solution:
$$p_t(\Delta) = \tfrac{1}{2}(p^e_t + \Delta)$$

Compared to the equilibrium under linear learning rules (equation 20), obviously $\boldsymbol{p}_t$ is higher for weak types (high Δ). In contrast to 20, however, a policy of gradualism (accommodating high inflationary expectations to some extent), is not optimal for strong types. For Δ low enough, it pays to cause deflation in the first period in order to demonstrate toughness. Equation 28 shows that $\boldsymbol{p}_t < 0$ for.

$$\Delta < \sqrt{\overline{\Delta}^2 - \tfrac{1}{3d}(\boldsymbol{p}_t^e + \overline{\Delta})^2}$$

Thus, in contrast to the outcome under linear learning rule, under efficient signaling a cold turkey approach is quite likely for strong central bank types.

(5) Conclusions and further research

The paper analyzes the signaling problem a central bank is facing in a two-period model when the bank is risk averse with respect to deviations of both inflation and output from her target and the public is imperfectly informed about the bank's preferences (represented by a continuous random variable). Since the public updates the priors after observing the policy outcome, concern for reputation dampens the bank's temptation to pursue a discretionary policy.

When the public uses a linear learning rule, the rational expectations equilibrium can be characterized explicitly. As long as the public observes both $\boldsymbol{e}_t$ and $\boldsymbol{p}_t$ before updating the prior and so is able to separate the impact of supply shocks on the inflation rate, the central bank will respond efficiently to output shocks, independent of her credibility. Under a linear learning rule, inflationary expectations will always be partly accommodated - it will never be optimal to pursue a deflationary policy in order to prove her toughness.

This result, however, is not robust against more sophisticated learning rules. Under efficient signaling, it is shown for the case of perfect transparency, that a tough bank has strong incentives to enforce a very restrictive cold turkey policy, in order to prove its toughness. Thus imposing linear learning rules, as is standard in the macroeconomic literature, gives misleading results.

The paper can be extended in several ways. First, it seems rather implausible to assume that the public is able to observe supply shocks perfectly. If these shocks can be observed only with some noise, private agents can no longer separate the impact of supply shocks on the inflation rate, when updating their priors. This may affect the central bank's incentives to stabilize efficiently, and so the separation between efficient stabilization response and credibility may no longer hold. The impact of noisy observation is left for future research.

Efficient signaling equilibria result in highly nonlinear outcomes. Consequently, updating rules under noisy signals become fairly messy and unsolvable. For that reason, the analysis in that part was restricted to the case of perfect transparency. Future work will show whether the results can be extended to signaling equilibria even in the case of noisy signals.

(6) Appendix

(6).1 Appendix A

According to 7, $EL_{t+1}(\boldsymbol{p}_{t+1}^e) = \frac{1}{2}(\boldsymbol{p}_{t+1}^e)^2 + \boldsymbol{p}_{t+1}^e \Delta + \frac{1}{2}(\Delta)^2$ and thus

29
$$\frac{\partial \, EL_{t+1}(\boldsymbol{p}_{t+1}^e)}{\partial \, \boldsymbol{p}_{t+1}^e} = \boldsymbol{p}_{t+1}^e + \Delta \, .$$

Given that private agents make a linear forecast $\boldsymbol{p}_{t+1}^e = \boldsymbol{l}_0 + \boldsymbol{l}_1 \, \boldsymbol{p}_t + \boldsymbol{l}_2 \boldsymbol{e}_t$, (implying $\dfrac{\partial \, \boldsymbol{p}_{t+1}^e}{\partial \, \boldsymbol{p}_t} = \boldsymbol{l}_1$), we can reformulate the FOC condition 4 as:

$$2 \, i_t = (\boldsymbol{p}_t^e + \Delta - \boldsymbol{e}_t) - \boldsymbol{d} \, \boldsymbol{l}_1 \, (\, \boldsymbol{p}_{t+1}^e + \Delta) \text{ or:}$$

30 $i_t = \dfrac{1}{2 + \boldsymbol{d} \, \boldsymbol{l}_1^2}(\boldsymbol{p}_t^e + \Delta \, (1 - \boldsymbol{d} \, \boldsymbol{l}_1) - \boldsymbol{d} \, \boldsymbol{l}_1 \boldsymbol{l}_0 - (1 + \boldsymbol{d} \, \boldsymbol{l}_1 \boldsymbol{l}_2) \, \boldsymbol{e}_t)$

30 defines the optimal strategy depending on private forecasts and thus determines the coefficients in $i_t = \boldsymbol{a} + \boldsymbol{b} \, \Delta + \boldsymbol{g} \, \boldsymbol{e}_t$ Substituting $\boldsymbol{l}_0$ and $\boldsymbol{l}_2$ gives 30 as a function of $\boldsymbol{a}, \boldsymbol{b} \,_, \boldsymbol{g}$ and $\boldsymbol{l}_1$ and thus characterizes together with 15 and 12 the explicit solution. First, solve for $\boldsymbol{p}_t^e$:

$$\boldsymbol{p}_t^e = \frac{1}{1 + \boldsymbol{d}\boldsymbol{l}_1^2}\Big[E(\Delta) \, (1 - \boldsymbol{d} \, \boldsymbol{l}_1) - \boldsymbol{d} \, \boldsymbol{l}_0 \, \boldsymbol{l}_1\Big] = \boldsymbol{p}_t^e = \frac{1}{1 + \boldsymbol{d}\boldsymbol{l}_1^2}\Big[E(\Delta) \, (1 - \boldsymbol{d} \, \boldsymbol{l}_1(1 + \boldsymbol{r})) + \boldsymbol{d} \, \boldsymbol{a} \, \boldsymbol{l}_1^2\Big]$$

This gives:

31 $i_t = \dfrac{1}{1 + \boldsymbol{d} \, \boldsymbol{l}_1^2}\Big[E(\Delta) \, (1 - \boldsymbol{d} \, \boldsymbol{l}_1(1 + \boldsymbol{r})) + \boldsymbol{a} \, \boldsymbol{d} \, \boldsymbol{l}_1^2)\Big] + \dfrac{1 - \boldsymbol{d} \, \boldsymbol{l}_1}{2 + \boldsymbol{d} \, \boldsymbol{l}_1^2}\Big[\Delta - E(\Delta)\Big] + \dfrac{\boldsymbol{g} \, \boldsymbol{d} \, \boldsymbol{l}_1^2 - 1}{2 + \boldsymbol{d} \, \boldsymbol{l}_1^2} \boldsymbol{e}$

So $\boldsymbol{a} = E(\Delta)\left[\dfrac{1 - \boldsymbol{d} \, \boldsymbol{l}_1(1 + \boldsymbol{r})}{1 + \boldsymbol{d} \, \boldsymbol{l}_1^2} - \dfrac{1 - \boldsymbol{d} \, \boldsymbol{l}_1}{2 + \boldsymbol{d} \, \boldsymbol{l}_1^2}\right] + \dfrac{1}{1 + \boldsymbol{d} \, \boldsymbol{l}_1^2}\boldsymbol{a} \, \boldsymbol{d} \, \boldsymbol{l}_1^2$. From this equation, we can immediately solve for $\boldsymbol{a}$ to get 17.

(6).2 Appendix B

From 18, $\boldsymbol{b}$ is defined implicitly (using 15 and 12) as:

$$g(b) := 2\,b + d\,\frac{1}{b}\left[\frac{b^4\,s_\Delta^4}{(b^2\,s_\Delta^2 + s_h^2)^2} + \frac{b^2\,s_\Delta^2}{b^2\,s_\Delta^2 + s_h^2}\right] - 1 = 0$$

$g(b)$ characterizes the solution for b as a function of d and of the variances of the shocks. For $s_h^2 > 0$ $g(b=0) = -1 < 0$ and $g(b = \frac{1}{2}) > 0$. Since g is continuous in b, there exists (at least) one solution $b \in (0, \frac{1}{2})$. Uniquenesss of the solution can be established when g is monotone increasing in b, that is if:

32

$$\frac{dg}{db} = 2 + d\,\frac{(b^2 s_\Delta^2 + s_h^2)(2b^2 s_\Delta^4 + s_\Delta^2 s_h^2) - 4b^4 s_\Delta^6}{(b^2 s_\Delta^2 + s_h^2)^3} > 0$$

$$\Leftrightarrow \quad 2(b^2 - d)b^4 s_\Delta^6 + (6b^2 + 3d)b^2 s_\Delta^4 s_h^2 + (3b^2 + d)s_\Delta^2 s_h^4 + 2s_h^6 > 0$$

$$\Leftrightarrow \quad 2b^6 + 2b^4(3s_h^2/s_\Delta^2 - d) + 3b^2(ds_h^2/s_\Delta^2 + s_h^4/s_\Delta^4) + ds_h^4/s_\Delta^4 + 2s_h^6/s_\Delta^6 > 0 \text{ Sufficient}$$

conditions for 32 to hold are

33: **either:** $b \geq \sqrt{d}$ **or** $d < 3s_h^2/s_\Delta^2$.

If 33 is fulfilled, the central bank responds more cautiously to its own preferences, the more valuable future reputation (the higher the discount factor d) and the more reliable the informational content of the signal (the more precise the information p_t, that is, the smaller the ratio s_h^2/s_Δ^2).

$$\frac{\partial b}{\partial d} < 0 \text{ (since } \frac{\partial g}{\partial d} > 0 \text{ for } b > 0, \ \frac{\partial b}{\partial d} < 0 \text{ whenever } \frac{\partial g}{\partial b} > 0$$

For the same reason, $\dfrac{\partial b}{\partial s_h^2/s_\Delta^2} > 0$ since $\dfrac{\partial g}{\partial s_h^2/s_\Delta^2} < 0$

If 33 holds, we also get both $\dfrac{\partial a}{\partial d} < 0$ and $\dfrac{\partial p_t^e}{\partial d} < 0$, since $a = E(\Delta)\left[b - \dfrac{d\,(1-r)\,r}{b}\right]$ and

$p_t^e = E(\Delta)\left[2b - \dfrac{d\,(1-r)\,r}{b}\right]$. Thus, according to 20, expected inflation decreases with

increasing discount factor. In contrast, the impact of the quality of the signal on expected

inflation is indetermined: $\dfrac{\partial a}{\partial r} \lessgtr 0$

When $s_h^2 = 0$, $b = 1/4(1 \pm \sqrt{1 - 16d})$

We can rule out the negative root for the following reason: for the limit case $d = 0$ and/or
$r = 1$ (because $s_\Delta^2 = 0$ or $s_h^2 \to \infty$), the two solutions for b are $b = 0$ and $b = \frac{1}{2}$. Since
only $b = \frac{1}{2}$ makes sense in this case, we might exclude the solution $b = 0$. If we require the
solution to be continuous for small changes in d, only the positive solution for the square root
makes economic sense:

(6).3 Special cases:

a) $r = 1$ or $s_\Delta^2 = 0$ (no uncertainty about Δ). Then $b = \dfrac{1}{2}$ $l_0 = \Delta; l_1 = 0$. Obviously, there

 cannot be any reputational effects; so $i_t = \frac{1}{2}(p_t^e + \Delta)$ with $p_t^e = \Delta$

b) $d = 0$ (no reputational effect) Then again $b = \frac{1}{2}$; $i_t = \frac{1}{2} E(\Delta) + \frac{1}{2}\Delta$ Updating gives:

 $l_0 = (2r - 1) E(\Delta); l_1 = 2(1 - r)$,   so

 $E(\Delta | p_t) = l_0 + l_1 p_t = E(\Delta)(2r - 1) + (1 - r)(E(\Delta) + \Delta) : = \overline{\Delta} r + \Delta(1 - r)$

c) $r = 0$ or $s_h^2 = 0$ Limit regime without noise. When p_t is a perfect signal, the a priori

 information E(Δ) will not be used to estimate Δ. That is, $l_0 = -E(\Delta); l_1 = \dfrac{1}{b}$. Since

 now $a = b$ and   so $p_t = i_t = bE(\Delta) + b\Delta$,   we   get

 $E(\Delta | p_t) = l_0 + l_1 p_t = -E(\Delta) + (E(\Delta) + \Delta) = \Delta$

 $p_t^e = E(\Delta)\, 2b$ with $b = \dfrac{1 + \sqrt{1 - 16d}}{4}$ decreasing in d for $d < \frac{1}{16}$.

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