Credit Scoring and Incentives for Loan Officers in a Principal Agent Model*

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Abstract

We analyze incentives for loan officers in a model with hidden action, limited liability and truth-telling constraints under the assumption that the principal has private information from an automatic scoring system. First we show that the truth-telling problem reduces the bank's expected profit whenever the loan officer cannot only conceal bad types, but can also falsely report bad types. Second, we investigate whether the bank should reveal her private information to the agent. We show that this depends on the percentage of good loans in the population and on the signal's informativeness. Though we had to define different regions for different parameters, we concluded that it might often be favorable to not reveal the signal. This contradicts current practice.

Keywords: delegated expertise, loan officers, loan origination, limited liability, informed principal.
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1 Introduction

As a reaction to increasing competition and bankruptcies,[1] banks all over the world are trying hard to improve the process of loan origination in corporate banking. Practitioners estimate that improvements in risk management can decrease credit losses by 20 to 40%.[2] Though effort has focused on technical scoring systems, based e.g. on an automatic analysis of the annual report or an automatic account analysis[3], automatic scoring systems cannot replace the loan officer's experience. Generally, the loan officer possesses the best information about the borrowers prospects to repay the loan. Especially in corporate banking the task of loan evaluation is often compared to corporate consulting or company evaluation which is too complex to be left up to an automatic scoring system.[4]

In view of the loan officer's central role in the process of loan origination, it is rather surprising that financial incentives are rare.[5] A fixed payment usually accounts for a large proportion of a loan officer's salary.[6] If there is a success related payment at all, it generally varies with some overall measure of success (e.g. company or division profit) but not with the loan repayment or failure. Though this may be related first to the dilution of responsibility (and thus to a team production problem), and second to incentives through career paths, monetary incentives for loan officers are clearly an interesting question.

When analyzing the process of loan origination, we focus on the agency problem between a bank and its loan officer. Our most important assumptions and results can be summarized as follows: first, we assume that the probability to detect bad loans depends on the loan officer's unobservable effort. Second, we introduce limited liability, meaning that the loan officer's wage has to be non-negative in each state of the world.[7] Limited liability

[1] For example in Germany, the number of company failures has tripled since 1990.

[2] See Wuffli and Hunt (1993).

[3] Contemporary creditworthiness tests still focus on the automatic evaluation of the annual report. In this context, a multivariate discriminant analysis (employed e.g by Bayerische Vereinsbank and Deutsche Bank), expert systems (e.g. Commerzbank), and neural networks (e.g. Chase Manhattan and American Express) are widely used.

[4] See Elsas and Krahnen (1998), and Ewert and Schenk (1998) for the significance of automatic credit scoring systems in corporate lending. Due to the lower complexity, scoring systems to evaluate retail customers generally provide better forecasts and often completely replace the lending officer. See e.g. Hyndman (1994) and Rusnak (1994)

[5] This is even more astonishing as anecdotical evidence suggests that loan officers tend to overestimate the creditworthiness of the borrowers. See Wuffli and Hunt (1993) and Kohls and Marciwiak (1987)

[6] See Scheepens (1995) and Siegel and Degener (1988)

[7] Alternatively, limited liability can be modeled by introducing a lower limit for the

aggravates the moral hazard problem and leads to an information rent and an effort below the first best level. Third, we assume that the loan officer truthfully reports the result of her check only if this is in her self-interest. We demonstrate that this truth-telling problem reduces the bank's expected revenue if and only if the loan officer can not only conceal that she has detected a bad loan, but also pretend that the loan is bad when its actually good.[8] Fourth and most importantly, we assume that the bank gets an additional signal from a scoring system, for instance by carrying out a multivariate discriminant analysis (MDA). The question that we are most interested in is whether the bank should commit itself to reveal the signal to the loan officer or not. On the one hand, informing the loan officer about the signal is favorable because it is then possible to implement different efforts for different signals. On the other hand, private information about the result of the scoring system allows the bank to reduce the loan officer's information rent. We demonstrate that it cannot be said generally which alternative is preferred by the bank, and we consider the comparative static with respect to the informativeness of the scoring system.

From a theoretical point of view, we consider a model with hidden action and limited liability where the risk neutral agent gets private information (about the prospective borrower's type) after contracting with the principal, and after having chosen her effort. Since the principal's decision depends on the agent's private information, the problem of truth-telling arises. Moreover, we assume that the principal has private information (resulting from the scoring system) and decides whether to reveal the information or not. Innes (1990) and Park (1995) also consider models with hidden action and limited liability, but they neither analyze the truth-telling problem nor private information on the principal's side. In the models by Demski, Sappington and Spiller (1988), and Lawarrée and van Audenrode (1996), the agent gets private information before choosing her effort, whereas in our model the agent is not better informed when choosing her effort.[9]

Our model is most closely related to the two models on delegated expertise and on the timing of information release by Demski and Sappington

agent's utility instead of the payments to her. This can lead to substantially different results, see e.g. Demougin and Garvie (1991).

[8]This is an important differerence to the literature on accountant's liability (see e.g. Chan and Pae (1996), and Ewert (1998)), where the agent can only conceal bad types. This is appropriate since an accountant cannot convey a mistake in a firm's statement that does not exist. In the context of loan origination, however, both types of false reports must be taken into account.

[9]See Sobel (1993) for a comparison of models with hidden information before and after the effort choice.

(1987) resp. Demski and Sappington (1986).[10] They consider a situation where a principal (for instance a patient) delegates a decision to an expert (for instance a doctor) whose private information depends on her unobservable effort. Since we assume that the bank follows the loan officer's recommendation, she actually delegates the decision.[11] However, our model differs from Demskis' and Sappingtons' (DS) at least in three respects: first, in their discrete models they do not characterize the optimal contract. Second, the agent is not liability constraint but risk averse. Third, though DS (1986) deals as well with the question of revealing or concealing the principal´s private information, none of their five "observations" shows the link between this question and the quality of the principal´s signal which is central in our paper. The last point relates our model to the literature on Informed Principals, for instance to Maskin and Tirole (1990), Maskin and Tirole (1992), and Beaudry (1994), since the result of the scoring system is private information to the principal. However, Maskin and Tirole consider an adverse selection problem, because the agent learns her type before contracting. Though Beaudry considers a moral hazard situation, the agent has no private information, and the principal cannot credibly reveal her information, whereas we analyze whether revelation is favorable or not. Finally, we are not aware of papers analyzing a loan officer's incentives in a comparable framework. Scheepens (1995) also considers an agency problem between a bank and its loan officer, but he focuses on the problem of collusion based on quite different assumptions.

Section 2 presents the model. In section 3, we consider the basic case where the bank has no private information. Section 4 introduces bank's private information and analyzes whether the information should be revealed or not. Section 5 concludes.

2 The model

There are two types of potential borrowers demanding bank loans of size I: "good" types with probability q, and "bad" types with probability $1-q$. The probability that good loans are repaid is p, with probability $1-p$ even good loans fail completely. Bad loans are lost with certainty. Since we consider

[10] See Lewis and Sappington (1997) for a similar model with focus on a cost reduction problem.

[11] To avoid misunderstandings, we wish to emphasize that our truth-telling problem is not related to the revelation principle, because the principal's utility depends on the agent's effort, and the agent learns the prospective borrower's type after having chosen her effort.

only the behavior of a single risk-neutral bank, we assume that the interest rate is exogenously given by r. We assume that

$$qp(1+r)I - I \geq 0 \tag{1}$$

so that granting a loan it is ex ante favorable. The bank can hire a risk-neutral loan officer, whose unobservable effort a depends on the probability π that a bad type is identified. We assume $a(\pi) = 0$ if $\pi = 0$, $\frac{\partial a}{\partial \pi} > 0$, $\frac{\partial^2 a}{\partial \pi^2} > 0$, and $\lim_{\pi \to 1}(a(\pi)) = \infty$.[12] Social welfare is

$$W(\pi) = qprI - (1-p)I - (1-q)(1-\pi)I - a(\pi) \tag{2}$$

yielding the FOC

$$(1-q)I = \frac{\partial a}{\partial \pi^f} \tag{3}$$

for the loan officer's optimal detection probability. However, the first best is not attainable through a simple franchising contract because the wage scheme is assumed to be restricted by the loan officer's limited liability. Particularly, we stipulate that the loan officer's wage must be non-negative in each state of the world.

Besides the unobservability of the loan officer's effort, there is an additional information problem, since the loan officer might lie. She cannot only conceal bad types, but can also pretend to have detected bad types.

The basic game can be summarized as follows:

1. The bank offers a contract to the loan officer specifying a wage for each state of the world.
2. The loan officer accepts or not.
3. The loan officer chooses her detection probability π (and thus her effort $a(\pi)$).
4. The loan officer reports a type.
5. The bank grants the loan if the loan officer reports the loan applicant to be good and rejects the loan if the loan officer's report classifies the prospective borrower as bad.[13]
6. The loan is paid back or fails, and the wage contract is enforced.

[12] Of course, we could use the inverse function $\pi(a)$ instead.

[13] Otherwise it would be senseless to employ a loan officer. However, this is not necessarily true if the bank gets an additional signal, see section 4 below.

3 The basic case

Assume first that the bank has no additional signal about the borrower's type. Since the bank grants a loan if and only if the loan officer reports a good type, the contract can consist of three different wages. Let L_b be the wage if the loan officer reports a bad type, L_g^0 the wage if she reports a good type and the project fails, and L_g^1 if she reports a good type and the project is successful. Due to the limited liability constraint, $L_g^0 \geq 0$, $L_g^1 \geq 0$, and $L_b \geq 0$. The loan officer's reservation level of utility is normalized to zero.

3.1 The pure moral hazard problem

Suppose for the moment that the loan officer is honest and reports his findings truthfully. Her objective function is then[14]

$$U(\pi^m) = qpL_g^1 + q(1-p)L_g^0 + (1-q)\left(1-\pi^m\right)L_g^0 + (1-q)\pi^m L_b - a(\pi^m) \quad (4)$$

yielding the FOC

$$(1-q)(L_b - L_g^0) = \frac{\partial a}{\partial \pi^m} \quad (5)$$

The bank's maximization problem is thus

$$\begin{aligned} R(\pi^m) \;=\; & qp(rI - L_g^1) - q(1-p)(I + L_g^0) \\ & -(1-q)\left(1-\pi\right)\left(I + L_g^0\right) - (1-q)\pi L_b \end{aligned} \quad (6)$$

subject to the incentive constraint given by Eqn. 5, the limited liability constraints and the loan officer's participation constraint (i.e. $U(\pi^m)$ must not be negative). Since the loan officer's effort is decreasing in L_g^0, and since L_g^1 has no influence on her effort, one can immediately conclude that $L_g^1 = L_g^0 = 0$. Substituting Eqn. 5 into the bank's objective function leads to[15]

$$R(\pi^m) = qprI - q(1-p)I - (1-q)\left(1-\pi^m\right)I - \pi^m \frac{\partial a}{\partial \pi^m} \rightarrow Max! \quad (7)$$

[14] "π^m" denotes the detection probability in the pure moral hazard case.

[15] Note that the loan officer's participation constraint is not binding, since the marginal effort is increasing in π, while the expected marginal wage is constant in π. Thus, total expected wages are always higher than total effort.

with the FOC

$$(1-q)I = \frac{\partial a}{\partial \pi^m} + \pi^m \frac{\partial^2 a}{(\partial \pi^m)^2} \tag{8}$$

Eqn. 8 shows that the probability of detecting bad loans in the moral hazard case is smaller than the first best probability ($\pi^m < \pi^f$), because $\pi \frac{\partial^2 a}{\partial \pi^2} > 0$. This is due to the limited liability constraint, since the first best would otherwise be achievable through a franchising contract as in Harris and Ravis (1979), and Holmström (1979), which grants the complete output to the agent except for a fixed fee. Since the principal equates the marginal information rent with the marginal efficiency loss, she prefers to implement an effort below the first best level.

3.2 The combined problem

Next we add the problem of truth-telling and assume that the loan officer reports his findings opportunistically. There are two constraints that have to be taken into account: The first constraint ensures that the loan officer honestly reveals a bad loan, and is given by

$$L_b \geq L_g^0 \tag{9}$$

Second, to ensure that the loan officer truthfully reports good types,[16]

$$\frac{q}{1-\pi^c(1-q)}\left(pL_g^1 + (1-p)L_g^0\right) + \left(1 - \frac{q}{1-\pi^c(1-q)}\right) L_g^0 \geq L_b \tag{10}$$

must hold. $\frac{q}{1-\pi^c(1-q)}$ is the ex post probability that a loan is good if the officer did not detect a bad type. $pL_g^1 + (1-p)L_g^0$ is her expected wage in this case, and if the type is bad she receives L_g^0. Summing up yields her expected wage if she truthfully reports a good type, which must not be smaller than L_b.

Again, one can immediately conclude that $L_g^0 = 0$, because L_g^0 reduces the loan officer's incentive along Eqn. 5, whereas L_g^0 can be substituted by L_g^1 to fulfill the second truth-telling constraint (Ineq. 10). Thus, the first truth-telling constraint is not binding. Conversely, the second truth-telling constraint is binding, since L_g^1 does not influence the loan officer's effort, and is thus chosen as to guarantee that the loan officer reports truthfully. Setting $L_g^0 = 0$, and substituting the FOC (Eqn. 5) and the remaining truth-telling constraint (Ineq. 10) into the bank's objective function (Eqn. 6) leads to

[16] "π^c" denotes the detection probability in the combined problem.

$$R = qprI - \frac{\frac{\partial a}{\partial \pi^c}}{(1-q)} - q(1-p)I - (1-q)\left(1-\pi^c\right)I \tag{11}$$

with the FOC

$$(1-q)I = \frac{\frac{\partial^2 a}{(\partial \pi^c)^2}}{1-q} \tag{12}$$

Comparing π^c (Eqn. 12) to the detection probability π^m (Eqn. 8) shows that the loan officer's effort is smaller in the combined problem if and only if

$$\frac{\frac{\partial^2 a}{(\partial \pi^c)^2}}{1-q} > \frac{\partial a}{\partial \pi^m} + \pi^m \frac{\partial^2 a}{(\partial \pi^m)^2} \tag{13}$$

for $\pi^c = \pi^m$. Though it seems to be natural that the additional constraint reduces the effort, the result is ambiguous. For instance, for $a(\pi) = \frac{1}{(1-\pi)^n} - 1$, it can easily be shown that $\pi^c < \pi^m$ always holds. However, for $a(\pi) = \tan(\frac{\widehat{\pi}}{2}\pi)$,[17] it is possible that $\pi^c > \pi^m$ if q is sufficiently high. The reason for the ambiguousness is the following: The difference between the pure moral hazard problem and the combined problem is that $L_g^1 > 0$ in the combined problem. But $\frac{\partial L_g^1}{\partial L_b}$ is ambiguous in the combined problem, which can be seen from the truth-telling constraint (see Ineq. 10): for given π^c, the higher L_b, the higher the L_g^1 required to ensure truth-telling. But since π^c (and thus $\frac{q}{1-\pi^c(1-q)}$) is increasing in L_b , the overall effect is ambiguous. In other words: the higher L_b, the higher π^c, and the higher π^c, the higher is the ex post probability that the loan is good if the officer did not detect a bad type. This increases her incentive to reveal a good type.

Though the effect of the truth-telling constraint on π is ambiguous, the bank's expected profit is clearly decreasing due to the additional constraint.

Proposition 1: In the combined problem, the bank's expected revenue is strictly smaller than in the pure moral hazard case.

Proof: A feasible contract in the pure moral hazard problem is to set $L^b(c)$ and $L_g^1(c)$ as in the combined problem. Her expected profit would then be the same as in the combined problem. Denote this profit in the pure moral hazard problem as $R^m = R^c$. Now let $L^b(c)$ in the pure moral hazard problem again be the same as in the combined problem, but set $L_g^1 = 0$. Denote the bank's profit in this case as $\widetilde{R}^m$. Since L_g^1 does not influence the loan officer's effort, and since the expected profit is decreasing in L_g^1 for constant π, we have $\widetilde{R}^m > R^m = R^c$. And since π^m as given by Eqn.8 maximizes the profit in the pure moral hazard case, it follows that $R(\pi^m) \geq \widetilde{R}^m > R^m = R^c$.

[17] Since π is the loan officer's detection probability, we define $\widehat{\pi} \sim 3.14$.

4 The case with an additional signal

Now we assume that the bank gets an additional signal about the borrower's type. Let x be the probability to receive a good signal if the loan is good, and y if the loan is actually bad. Let $x > y$. Since q is the ex ante probability for good loans, the probability for a type-I-error is $(1-q)y$ and the probability for a type-II-error is $q(1-x)$ as shown by figure 1.

Insert figure 1 here.

Note that the ex ante probability to receive a good signal is $g = qx + (1-q)y$.

Because the bank's decision depends on ex post probabilities for good and bad loans if a signal is received, we define

$$q^g = \frac{qx}{qx + (1-q)y}$$

and

$$q^b = \frac{q(1-x)}{q(1-x) + (1-q)(1-x)}$$

as ex post probabilities for good loans if a good (bad) signal is received. Note that

$$gq^g + (1-g)q^b = q \tag{14}$$

because the probabilities to meet good types, weighed with the probabilities for good and bad signals, must add up to the original probability for good types. It follows $q^g > q > q^b$. The question we are interested in is whether the bank prefers to reveal her private information to the loan officer or not. Thus we have to consider two different submodels.

4.1 Strategy A: Informed loan officer

First we assume that the bank commits itself to revealing the signal when offering an incentive scheme to the loan officer (i.e. the loan officer knows the realization of the signal when choosing her effort). The advantage of revealing the signal is the possibility to offer different contracts in different states. Thus, we have two subgames (one for each signal) with the same structure as the basic game considered in section 3.2 - the only difference is that the probabilities for good types and bad types are different (q^g or q^b instead of q). To keep the analysis tractable, we add the following Assumption:

Assumption 1: It is always favorable to employ the loan officer.

Assumption 1 implies that q^g is not too close to 1 (otherwise it would be favorable to directly grant a loan) and that q^b is not too close to zero (otherwise it would be favorable to directly refuse).

Because $L_g^0 = 0$ in the optimal contract, the bank's objective function becomes[18]

$$\begin{aligned} R(\pi^g, \pi^b) &= gR(\pi^g) + (1-g)R(\pi^b) \\ &= A - gq^g pL_g^1(g) - g(1-q^g)\left(1-\pi^g\right)I \\ &\quad -g(1-q^g)\pi^g L_b(g) - (1-g)L_g^1(b) \\ &\quad -(1-g)(1-q^b)\left(1-\pi^b\right)I - g(1-q^b)\pi^b L_b(g) \end{aligned} \tag{15}$$

where

$$A = qI(p(1+r)-1) \tag{16}$$

cannot be influenced by the bank. Analogously to section 3.2, substituting the loan officer's FOC and the binding truth-telling constraint into the bank's objective function leads to

$$\begin{aligned} R(\pi^g, \pi^b) &= A - g\frac{\frac{\partial a}{\partial \pi^g}}{(1-\widetilde{q}^g)} - g(1-q^g)\left(1-\pi^g\right)I \\ &\quad -(1-g)\frac{\frac{\partial a}{\partial \pi^b}}{(1-q^b)} - (1-g)(1-q^b)\left(1-\pi^b\right)I \end{aligned} \tag{17}$$

with the FOC's

$$(1-q^g)I = \frac{\frac{\partial^2 a}{(\partial \pi^g)^2}}{1-q^g} \tag{18}$$

and

$$(1-q^b)I = \frac{\frac{\partial^2 a}{(\partial \pi^b)^2}}{1-q^b} \tag{19}$$

Since $1-q^g < 1-q < 1-q^g$, we obtain $\pi^g < \pi^c < \pi^b$, i.e. the effort is higher with a bad signal. Interestingly, it can generally not be said that the bank's expected profit is higher compared to the case without additional signal whenever one of the two signals is informative (i.e. if $q^g > q$ or $q^b < q$). More precisely, this is only the case if $\frac{\partial^2 R}{\partial q^2} > 0$. The deeper reason is

[18] π^g and π^b are the efforts with a good signal and a bad signal, respectively.

that an increase in the percentage of good types (i.e. an increasing q) does *not* necessarily increase the bank's expected profit (i.e. even $\frac{\partial R}{\partial q} < 0$ might hold)![19] This follows from the fact that an increasing q decreases the loan officer's probability of detecting bad types, thus decreasing her incentive to work, and this effect might outweigh the better chance to meet good types.

4.2 Strategy B: Uninformed loan officer

Now we assume that the bank decides to not reveal the signal to the loan officer. Thus it is impossible to implement different efforts for different signals, which is an unwarranted consequence. On the other hand, the bank has private information allowing to reduce the loan officer's information rent. Analogously to section 4.1, we add an assumption allowing to restrict our attention to the basic case:

Assumption 2: It is favorable to the bank to grant a loan if and only if the loan officer reports "good".

Assumption 2 excludes a situation where the quality of a bad signal is extremely high (i.e. q^b close to zero) so that the bank would prefer to rely on the signal and not on the loan officer's report.[20]

The bank's objective function is then

$$\begin{aligned} R(\pi^i) \;=\; & A - gq^g pL_g^1(g) - gq^g(1-p)L_g^0(g) \\ & -g(1-q^g)\left(1-\pi^i\right)L_g^0(g) - g(1-q^g)\pi^i L_b(g) \\ & -(1-g)q^b pL_g^1(b) - (1-g)q^b(1-p)L_g^0(b) \\ & -(1-g)(1-q^b)\left(1-\pi^i\right)L_g^0(b) - (1-g)(1-q^b)\pi^i L_b(b) \end{aligned} \tag{20}$$

where A is defined as in section 4.1, π^i is the detection probability when the signal is unobservable to the loan officer, and the wage scheme L depends on the signal.

4.2.1 Truth-telling constraints

Since the loan officer's wage depends on the signal, she must calculate the ex post probabilities for good and bad signals as a function of the result of her test. Let

[19] Proof available on request.

[20] Clearly the bank will always relie on the loan officer if she claims to have detected a bad applicant as in our model the type-II-error is excluded.

$$\overline{g} = \frac{g(1-q^g)\pi^i}{g(1-q^g)\pi^i + (1-g)(1-q^b)\pi^i} = \frac{g(1-q^g)}{1-q} \tag{21}$$

be the ex post probability for a good signal if she detected a bad type, and let $1-\overline{g}$ be the ex post probability for a bad signal if she found a mistake. As to guarantee that the loan officer truthfully reports bad types,

$$\overline{g}L_b(g) + (1-\overline{g})L_b(b) \geq \overline{g}L_g^0(g) + (1-\overline{g})L_g^0(b) \tag{22}$$

must be fulfilled, where the wage depends not only on whether the loan is paid back, but also on the result of the scoring system.

To calculate the condition required that the loan officer truthfully reports a good test, let[21]

$$\overline{d} = q + (1-q)(1-\pi^i) \tag{23}$$

the ex ante probability that no mistake is found. If the loan officer honestly reports, there are six different probabilities that she has to take into account:

1. The signal was good, the type is good and the loan is paid back: $\overline{g}_g^1 = gq^g p/\overline{d}$
2. The signal was good, the type is good, but the project fails: $\overline{g}_g^0 = gq^g(1-p)/\overline{d}$.
3. The signal was good, but the type is bad: $\overline{g}_b = g(1-q^g)(1-\pi^i)/\overline{d}$.
4. The signal was bad, the type is good and the loan is paid back: $\overline{b}_g^1 = (1-g)q^b p/\overline{d}$.
5. The signal was bad, the type is good, but the project fails: $\overline{b}_g^0 = (1-g)q^b(1-p)/\overline{d}$.
6. The signal was bad and the type is bad: $\overline{b}_b = (1-g)(1-q^b)(1-\pi^i)/\overline{d}$

If she always claims to having detected a bad type, there are only two cases that she has to consider:

1. The signal was good: $\widehat{g} = \frac{gq^g + g(1-q^g)(1-\pi^i)}{\overline{d}}$.

[21]Note that the probability can be expressed by q (and thus q^g and q^b can be neglected), because the signal must be unbiased.

2. The signal was bad: $\widehat{b} = \frac{(1-g)q^b + (1-g)(1-q^b)(1-\pi^i)}{\overline{d}}$.

Thus, the second truth-telling constraint can be written as

$$\begin{aligned} & L_g^1(g)\overline{g}_g^1 + L_g^0(g)\overline{g}_g^0 + L_g^0(g)\overline{g}_b + L_g^1(b)\overline{b}_g^1 + L_g^0(b)\overline{b}_g^0 + L_g^0(b)\overline{b}_b \quad (24) \\ \geq \; & L_b(g)\widehat{g} + L_b(b)\widehat{b} \end{aligned}$$

4.2.2 Effort constraint

Whereas the truth-telling constraints depend on ex post probabilities for good and bad signals given the result of the loan officer's test, the effort constraint depends on ex ante probabilities. The loan officer's objective function yields the FOC

$$g(1-q^g)\left(L_b(g) - L_g^0(g)\right) + (1-g)(1-q^b)\left(L_b(b) - L_g^0(b)\right) = \frac{\partial a}{\partial \pi^i} \quad (25)$$

4.2.3 Analysis

Again, the loan officer's effort depends on the difference between L_b and L_g^0, now weighed with the ex ante probabilities for good and bad signals, and the probabilities for bad types given the result of the scoring system. Again, $L_g^0 = 0$ in the optimal contract. Thus, the first truth-telling constraint (Ineq. 22) can be neglected, and the effort constraint turns out to be

$$g(1-q^g)L_b(g) + (1-g)(1-q^b)L_b(b) = \frac{\partial a}{\partial \pi^i} \quad (26)$$

Moreover, the second truth-telling constraint (Ineq. 24) becomes

$$L_g^1(g)\overline{g}_g^1 + L_g^1(b)\overline{b}_g^1 \geq L_b(g)\widehat{g} + L_b(b)\widehat{b} \quad (27)$$

To deduce the optimal contract, Lemma 1 is required.

Lemma 1: In the optimal contract, the bank sets $L_b(g) = 0$.

Proof: see Appendix.

Lemma 1 states that the loan officer gets nothing if she states a bad loan, whereas the signal indicates a good one. The intuition is that a good signal increases the probability of a good type, so that it is more likely that the loan officer lies if she reported a bad type. Without loss of generality, we stipulate that the loan officer also gets nothing if she announces a good type, whereas

the signal is bad ($L_g^1(b) = 0$).[22] Thus, the bank's maximization problem becomes

$$\begin{aligned} R(\pi^i) &= A - gq^g pL_g^1(g) - g(1-q^g)\left(1-\pi^i\right)I \\ &\quad -(1-g)(1-q^b)(1-\pi^i)I \\ &\quad -(1-g)(1-q^b)\pi^i L_b(b) \end{aligned} \tag{28}$$

s.t.

$$(1-g)(1-q^b)L_b(b) = \frac{\partial a}{\partial \pi^i} \tag{29}$$

$$L_g^1(g)\overline{g}_g^1 = L_b(b)\widehat{b} \tag{30}$$

Substituting Eqn. 29 and Eqn. 30 into Eqn. 28, taking into account that $g = \frac{q-q^b}{q^g-q^b}$, $1-g = \frac{q^g-q}{q^g-q^b}$, and simplifying leads to

$$R(\pi^i) = A - \frac{\frac{\partial a}{\partial \pi^i}}{1-q^b} - \left(1-\pi^i\right)(1-q)I \tag{31}$$

with the FOC

$$(1-q)I = \frac{\frac{\partial^2 a}{(\partial \pi^i)^2}}{1-q^b} \tag{32}$$

Conversely to strategy A, the detection probability implemented by the wage contract is independent of q^g, *i.e. independent of the signal's quality if it indicates a good type*. There are two reasons for this result: First, q^g does not enter into the loan officer's first order condition (and thus has no influence on π^i) because the bank sets $L_b(g) = 0$. Second, the *expected* wage required to ensure that the truth-telling constraint is fulfilled is also independent of q^g: Though $L_g^1(g)$ can be reduced if the signal's quality increases, this effect is equalized by the fact that $L_g^1(g)$ must be paid more often if q^g increases. In other words: for a given right hand in the loan officer's truth-telling constraint, the left hand $gpq^g L_g^1(g)$ must remain constant as to guarantee a truthful report, so that q^g and $L_g^1(g)$ are substitutes with respect to the loan officer's truth-telling constraint.

[22] It can easily be shown that $L_g^1(b)$ and $L_g^1(g)$ are perfect substitutes, so that $L_g^1(b) = 0$ is not restrictive.

4.3 Comparative Statics

Next we analyze whether it is favorable to the bank to reveal the signal. Comparing $R(\pi^i)$ and $R(\pi^g, \pi^b)$, it follows immediately that $R(\pi^i) > R(\pi^g, \pi^b)$ iff[23]

$$\begin{aligned} & (1-q)\pi^i I - \frac{\frac{\partial a}{\partial \pi^i}}{1-q^b} \qquad (33) \\ > \quad & g(1-q^g)\pi^g I - g\frac{\frac{\partial a}{\partial \pi^g}}{1-q^g} + (1-g)(1-q^b)\pi^b I - (1-g)\frac{\frac{\partial a}{\partial \pi^b}}{1-q^b} \end{aligned}$$

Whether Ineq. 33 holds depends on q, q^b and q^g. Thus we have to consider different regions depending on q, q^b and q^g. These different regions are expressed by Propositions 2-4.

Proposition 2: Strategy B is superior if $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$.

Proof: see Appendix.

Proposition 2 means that, independently of the percentage of good loans, and independently of the signal's informativeness, strategy B is superior if the marginal disutility of effort increases steeply enough. This follows from the fact that the optimal detection probability decreases if the marginal disutility of effort increases. Thus, the advantage of implementing two different efforts in strategy A becomes less important. Simultaneously, the marginal information rent increases due to the higher disutility, so that the reduction in information rent caused by strategy B becomes more valuable. Thus, concealing the signal pays off if the disutility of effort rises steeply.

Since $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$ is sufficient for the superiority of strategy B, we now assume $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} < 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$.

Proposition 3: There is always a critical $\widehat{q}$ as to guarantee that strategy B is superior.

[23]For the special case $a(\pi) = \frac{1}{(1-\pi)^n} - 1$ it can be shown that Ineq. 33 can simply be written as $(1-q)\pi^i > g(1-\widetilde{q}^g)\pi^g + (1-g)(1-\widetilde{q}^b)\pi^b$. This means that it is favorable to conceal the result of the scoring system if the average detection probability implemented through the optimal contract is higher (i.e. the costs $\frac{\frac{\partial a}{\partial \pi^i}}{1-\widetilde{q}^b}$ can be neglected), since π^g and π^b are weighted with the probabilities of bad loans in case of good and bad signals. However, this is only true iff $\frac{\partial^2\left(\pi - \frac{\frac{\partial a}{\partial \pi}}{\frac{\partial^2 a}{\partial \pi^2}}\right)}{\partial \pi^2} = 0$, which holds for $a(\pi) = \frac{1}{(1-\pi)^n} - 1$. If $\frac{\partial^2\left(\pi - \frac{\frac{\partial a}{\partial \pi}}{\frac{\partial^2 a}{\partial \pi^2}}\right)}{\partial \pi^2} < 0$ (this holds for $a(\pi) = \tan(\frac{\widehat{\pi}}{2}\pi)$), then $R(\pi^i)$ can be higher even if $(1-q)\pi^i < g(1-\widetilde{q}^g)\pi^g + (1-g)(1-\widetilde{q}^b)\pi^b$
(proof available on request).

Proof: see Appendix.

Proposition 3 expresses that, independently of the signal's quality (i.e. independently of q^g and q^b), strategy B is always superior if the percentage of good loans exceeds a critical value. This is so because the optimal effort decreases in the percentage of good loans, so that it becomes less important to implement two different efforts in strategy A. Clearly, the critical value of q depends on the other parameters of the model and can become zero, so that there are constellations where strategy B is always superior even if $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} < 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$.

Proposition 4: Suppose $q < \widehat{q}$. Then, depending on q^g and q^b, either strategy A or strategy B can be favorable.

Proof: see Appendix.

Table 1 shows how the superiority of A or B depends on q^g and q^b if neither $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$ nor $q > \widehat{q}$ holds. First, independently of q^b, strategy B is always superior if q^g is sufficiently small. If q^g is small, then that the optimal effort is relatively high even if the bank receives a good signal, so that the information rent is more important compared to the possibility to implement different efforts. However, if q^g increases, strategy A can be superior. This is the more likely the higher q^b is, since the bank's profit in strategy B is independent of q^b as explained in subsection 4.2.3. Only if q^g is high enough, strategy A is superior if neither $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$ nor $q > \widehat{q}$ holds.

Considering Propositions 2-4 together, our results doubt the current practice of always revealling the signal to the loan officer. Strategy A can only be superior if the disutility of effort does not increase to fast, if the percentage of good loans is relatively small, and if the informativeness of the good signal is high.

5 Discussion

We analyzed wage contracts for loan officers in a model with hidden action, limited liability and truth-telling constraints under the assumption that the principal has private information. We demonstrated that the truth-telling problem reduces the bank's expected profit because the loan officer cannot only conceal bad types, but can also falsely report bad types.

The question we were most interested in was whether the bank should reveal her private information to the agent. We showed that this depends on the percentage of good loans in the population and on the signal's informativeness. Though we had to define different regions for different parameters, we concluded that it might often be favorable to not reveal the signal. This

contradicts current practice. In this context it should be interesting to see how the results change if assumptions 1 and 2 are relaxed. In this case it could be favorable to the bank make the employment of the loan officer contingent on the signal, i.e. to employ the loan officer only if the signal is good or to employ her only if the signal is bad. Concerning strategy B, the bank might better rely on the signal than rely on the agent if the signal is bad.

One might wish to extend our model in many respects. First, one could add a type-II-error on the loan officer's side (and not only for the scoring system). Second, one could distinguish among more than two types of borrowers, maybe combined with different rates of interest when granting a loan. Third, one could compare a scoring system to other kinds of supervision, namely to a senior loan officer. Fourth, and perhaps most interesting, one might wish to incorporate the process of loan review in the analysis. If the borrower's type changes after the loan is granted, the bank must set incentives to not prolong the loan. However, there arises a trade-off between ex ante and ex post incentives - the higher the payment for the detection of bad loans during time, the higher the ex ante incentive to grant bad loans.

We would like to mention that we also analyzed the situation where it is not possible to make the loan officer's wage contingent on whether the loan is repaid or not (unobservable resp. incontractible outcome). This is an important case from a practitioner's point of view because there are often many persons involved in the process of loan origination, and the loan officer might already work for an other firm when the project fails.[24] In the pure moral hazard case (see section 3.1), it does not matter whether the outcome is observable or not, since the wage is always zero if the loan is granted. If the truth-telling constraint is taken into account and the outcome is unobservable, then it is no longer possible to employ a loan officer, because she will always report a good type (a bad type) if L_b is higher (lower) than L_g. Even with an additional signal, it is not possible to employ a loan officer if the signal is conveyed - depending on the signal, we have two subgames where the loan officer again reports "bad" as long as $L_b > L_g$. Thus, an unobservable outcome strengthens the argument that it might often be better to not inform the loan officer about the signal, since the signal partially substitutes the output as a variable in the optimal wage scheme. Along the lines of the proof to Proposition 1, it is easy to show that the bank's profit is lower compared to the case where the contract is contingent on the result of the project. However, again in accordance with Proposition 1, the effort decreases if the outcome is unobservable if and only if $\frac{\frac{\partial^2 a}{\partial \pi^2}}{1-q^b} > \frac{\partial a}{\partial \pi} + \pi^m \frac{\partial^2 a}{\partial \pi^2}$.

Appendix

[24] See Venohr (1996)

Proof of Lemma 1:

1. Suppose the bank wishes to implement a specific detection probability $\widehat{\pi}$. Since $L_b(g)$ and $L_b(b)$ are substitutes in the loan officer's objective function, one can calculate the marginal rate of substitution between $L_b(g)$ and $L_b(b)$ from the partial derivatives of the loan officer's objective function:[25]

$$\frac{\frac{\partial U}{\partial L_b(g)}}{\frac{\partial U}{\partial L_b(b)}} = \left|\frac{dL_b(b)}{dL_b(g)}\right|^{EC} = \frac{g}{1-g}\frac{1-q^g}{1-q^b} \tag{34}$$

The marginal rate of substitution between $L_b(g)$ and $L_b(b)$ expresses to which degree $L_b(b)$ can be decreased if $L_b(g)$ is increased by one unit, and if the same probability (effort) shall be implemented.

2. Now suppose that the bank increases $L_b(g)$ by one unit and consider the effect on the right hand of the loan officer's truth-telling constraint (see Ineq. 25). The marginal rate of substitution between $L_b(g)$ and $L_b(b)$ is[26]

$$\frac{\frac{\partial R}{\partial L_b(g)}}{\frac{\partial R}{\partial L_b(b)}} = \left|\frac{dL_b(b)}{dL_b(g)}\right|^{TC} = \frac{g}{1-g}\frac{1-(1-q^g)\pi^i}{1-(1-q^b)\pi^i} \tag{35}$$

3. Comparing the marginal rates of substitution shows that $\left|\frac{dL_b(b)}{dL_b(g)}\right|^{EC} < \left|\frac{dL_b(b)}{dL_b(g)}\right|^{TC}$ if and only if $q^b < q^g$, which is always fulfilled. $\left|\frac{dL_b(b)}{dL_b(g)}\right|^{EC} < \left|\frac{dL_b(b)}{dL_b(g)}\right|^{TC}$ means that, if the bank wishes to implement some effort $\widetilde{\pi}$, it is always cheaper for her to decrease $L_b(g)$ and to increase $L_b(b)$ according to the marginal rate of substitution in the effort constraint, since this alleviates the truth-telling constraint without altering the effort or the bank's cost.[27] Thus, the bank prefers to set $L_b(g) = 0$.

The proofs of Propositions 2-4 are tedious, because $\frac{\partial\left[R(\pi^i)-R(\pi^g,\pi^b)\right]}{\partial q}$ as well as $\frac{\partial\left[R(\pi^i)-R(\pi^g,\pi^b)\right]}{\partial q^b}$ are ambiguous, so that we have to proceed indirectly. Thus, some parts are only sketched. More detailed calculations are available on request.

Proof of Proposition 2: Strategy B is always superior if $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$.

Let $R(\pi^g, \pi^b)$ be the bank's profit under strategy A, and $R(\pi^i)$ the profit under strategy B. We proceed in two steps:

[25] "EC" denotes "effort constraint".

[26] "TC" denotes "truth-telling constraint.

[27] Recall that the marginal rate of substitution between $L_b(g)$ and $L_b(b)$ is the same in the objective functions of the manager and the loan officer.

First we show that $R(\pi^i)$ is decreasing in q^b, so that $R(\pi^i)$ is minimum if $q^b = q$. Second we demonstrate that $R(\pi^g, \pi^b)$ is increasing in q^b if $\frac{\partial^3 a}{\partial \pi^3}\frac{\partial a}{\partial \pi} \geq 2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$. Because $R(\pi^g, \pi^b) = R(\pi^i)$ if $q^b = q$, this proves Proposition 2.

Step 1:

$$\frac{\partial R(\pi^i)}{\partial q^b} = \frac{\partial\left[I\pi^i(1-q) - \frac{\frac{\partial a}{\partial \pi^i}}{1-q^b}\right]}{\partial q^b} = \frac{-\frac{\partial a}{\partial \pi^i}}{(1-q^b)^2} < 0 \tag{36}$$

Step 2:

$$\begin{aligned}\frac{\partial R(\pi^g, \pi^b)}{\partial q^b} &= \frac{\partial g}{\partial q^b}R(\pi^g) + g\frac{\partial R(\pi^g)}{\partial q^b} + \frac{\partial(1-g)}{\partial q^b}R(\pi^b) &(37)\\ &\quad +(1-g)\frac{\partial R(\pi^b)}{\partial q^b} &(38)\\ &= \frac{q-q^g}{(q^g-q^b)^2}\left(R(\pi^g) - R(\pi^b)\right) + \frac{q^g-q}{q^g-q^b}\frac{\partial R(\pi^b)}{\partial q^b}\end{aligned}$$

This is positive iff $\frac{\partial R(\pi^b)}{\partial q^b} > \frac{R(\pi^g)-R(\pi^b)}{q^g-q^b}$, i.e. iff $\frac{\partial^2 R(\pi)}{\partial q^2} < 0$.[28] Thus we have to calculate $\frac{\partial^2 R(\pi)}{\partial q^2}$. It can be shown that

$$\frac{\partial R(\pi)}{\partial q} = (pr + p - 1)I + \frac{(1-\pi)\frac{\partial^2 a}{\partial \pi^2} - \frac{\partial a}{\partial \pi}}{(1-q)^2} \tag{39}$$

and

$$\frac{\partial^2 R(\pi)}{\partial q^2} = \frac{1}{\frac{\partial^3 a}{\partial \pi^3}(1-q)^3}\left(4\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2 - 2\frac{\partial a}{\partial \pi}\frac{\partial^3 a}{\partial \pi^3}\right)$$

which is negative iff $2\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2 < \frac{\partial a}{\partial \pi}\frac{\partial^3 a}{\partial \pi^3}$.

Proof of Proposition 3: There is always a critical $\widehat{q}$ guaranteeing that strategy B is superior.

Define $R(\pi^i) - R(\pi^g, \pi^b) = \Delta$. In step 1, we show that $\Delta(q^b)$ has no maximum. It follows that strategy B is always (i.e. for each q^g and q^b) superior to strategy A if $\frac{\partial \Delta}{\partial q^b} \leq 0$ at $q^b = q$ when q^g is chosen as to maximize $R(\pi^g, \pi^b)$. Thus, in step 2, we calculate the critical $\widehat{q}$ where $\frac{\partial \Delta}{\partial q^b} = 0$ at $q^b = q$. It remains to be shown in step 3 that $\Delta > 0$ for each $q > \widehat{q}$.

Step 1: Let V denote the variable part of R, i.e.$V(\pi^i) = I\pi^i(1-q) - \frac{\frac{\partial a}{\partial \pi^i}}{1-q^b}$, $V(\pi^g, \pi^b) = gV(\pi^g) + (1-g)V(\pi^b) = g\left(I\pi^g(1-q^g) - \frac{\frac{\partial a}{\partial \pi^g}}{1-q^g}\right)$

[28] Clearly, π is the optimal detection probability for a given q.

$+(1-g)\left(I\pi^b(1-q^b)-\frac{\frac{\partial a}{\partial \pi^b}}{1-q^b}\right)$. Then one can show that

$$\begin{aligned}\frac{\partial^2\Delta}{(\partial q^b)^2} &= \frac{2\left(\frac{\partial^2 a}{\partial(\pi^i)^2}\right)^2}{\frac{\partial^3 a}{\partial(\pi^i)^3}(1-q^b)^3}+\frac{2}{1-q^b}\frac{\partial V(\pi^i)}{\partial q^b}\\ &\quad -\frac{2}{q^g-q^b}\frac{\partial V(\pi^g,\pi^b)}{\partial q^b}-(1-g)\frac{\partial V(\pi^b)^2}{\partial (q^b)^2}\end{aligned}$$

If there is an extremum ($\frac{\partial\Delta}{\partial q^b}=0$) then $\frac{\partial V(\pi^i)}{\partial q^b}$ can be replaced by $\frac{\partial V(\pi^g,\pi^b)}{\partial q^b}$. Thus

$$\begin{aligned}\frac{\partial^2\Delta}{\partial\left(q^b\right)^2} &= \frac{2\left(\frac{\partial^2 a}{\partial(\pi^i)^2}\right)^2}{\frac{\partial^3 a}{\partial(\pi^i)^3}(1-q^b)^3}-\frac{q^g-q}{q^g-q^b}\frac{\partial V(\pi^b)}{\partial q^b}\\ -2\frac{1-q^g}{(1-q^b)(q^g-q^b)}&\left(\frac{q-q^g}{(q^g-q^b)^2}\left[V(\pi^g)-V(\pi^b)\right]+\frac{q^g-q}{q^g-q^b}\frac{\partial V(\pi^b)}{\partial q^b}\right)\end{aligned}$$

which can be shown to be positive whenever $\frac{\partial^2 V(\pi)}{\partial q^2}=\frac{\partial^2 R(\pi)}{\partial q^2}>0$, which is identical to $\frac{\partial^3 a}{\partial\pi^3}\frac{\partial a}{\partial\pi^3}<2\left(\frac{\partial^2 a}{\partial\pi^2}\right)^2$. [29]

Step 2: Now we are looking for the critical $\widehat{q}$ where $\frac{\partial\Delta}{\partial q^b}=0$ at $q=q^b$ and $q^g=\widetilde{q}^g$, where $\widetilde{q}^g$ is the probability where the bank is indifferent between employing a loan officer or not. Tedious calculations yield

$$\frac{\partial\Delta}{\partial q^b}\mid_{q=q^b,q^g=\widetilde{q}^g}=\frac{1}{(1-q)^2(\widetilde{q}^g-q)}\left((1-q)\frac{\partial a}{\partial\pi}-(1-\widetilde{q}^g)\frac{\partial^2 a}{\partial\pi^2}\pi\right)$$

Since we know from step 1 that there is no maximum, it follows that $R(\pi^i)>R(\pi^g,\pi^b)$ for all q^g and q^b if $\frac{\partial\Delta}{\partial q^b}=0$ at $q^b=q$ and $q^g=\widetilde{q}^g$. Thus, $\widehat{q}$ is implicitly defined by

$$\frac{(1-q)\frac{\partial a}{\partial\pi}-(1-\widetilde{q}^g)\frac{\partial^2 a}{\partial\pi^2}\pi}{1-q}=\frac{\partial a}{\partial\pi}-(1-\widetilde{q}^g)I(1-q)\pi=0$$

For step 3, we define $\frac{\partial a}{\partial\pi}-(1-\widetilde{q}^g)I(1-q)\pi\equiv F$

Step 3: First note that $F(q)$ has a root for $q=\widetilde{q}^g$, because then $\frac{\partial a}{\partial\pi}=\frac{\partial^2 a}{\partial\pi}\pi$. We show that $F(q)$ has no maximum to prove that $F(q)$ has not

[29] Recall that strategy B is always superior if $\frac{\partial^3 a}{\partial\pi^3}\frac{\partial a}{\partial\pi^3}<2\left(\frac{\partial^2 a}{\partial\pi^2}\right)^2$ is violated.

more than two roots, and thus that $\widehat{q}$ is unique if it exists. It follows that $\frac{\partial \Delta}{\partial q^b} \mid_{q=q^b, q^g=\widetilde{q}^g} < 0$ for each $q > \widehat{q}$ and therefore $\Delta > 0$ for each $q > \widehat{q}$. From

$$\frac{\partial F}{\partial q} = (q - \widetilde{q}^g) I \frac{2\frac{\partial^2 a}{\partial \pi^2}}{\frac{\partial^3 a}{\partial \pi^3}} + (1 - \widetilde{q}^g) I \pi$$

it follows

$$(1 - \widetilde{q}^g) = \frac{(1-q) 2\frac{\partial^2 a}{\partial \pi^2}}{2\frac{\partial^2 a}{\partial \pi^2} + \pi \frac{\partial^3 a}{\partial \pi^3}}$$

for $\frac{\partial F}{\partial q} = 0$. Calculating $\frac{\partial^2 F}{\partial q^2}$, and substituting $(1 - \widetilde{q}^g) = \frac{(1-q) 2\frac{\partial^2 a}{\partial \pi^2}}{2\frac{\partial^2 a}{\partial \pi^2} + \pi \frac{\partial^3 a}{\partial \pi^3}}$ into $\frac{\partial^2 F}{\partial q^2}$, leads to the result that a sufficient condition for $\frac{\partial^2 F}{\partial q^2} > 0$ to hold (and thus that the extremum of $F(q)$ is a minimum if it exists) is $\frac{\partial^4 a}{\partial \pi^4} \frac{\partial^2 a}{\partial \pi^2} < \frac{3}{2} \left(\frac{\partial^3 a}{\partial \pi^3}\right)^2$. This is implied by $\frac{\partial^3 a}{\partial \pi^3} \frac{\partial a}{\partial \pi} < 2 \left(\frac{\partial^2 a}{\partial \pi^2}\right)^2$(otherwise, strategy B is always superior, see Proposition 2) if $\frac{\frac{\partial^3 a}{\partial \pi^3} \frac{\partial a}{\partial \pi}}{\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2}$ is constant or decreasing in π.[30] Thus, $\widehat{q}$ is unique if it exists. $\widehat{q}$ exists iff $F(q) \geq 0$ at $q = 0$, i.e. iff $\frac{\partial a}{\partial \pi} - (1 - \widetilde{q}^g) I \pi \geq 0$. However, if $\frac{\partial a}{\partial \pi} - (1 - \widetilde{q}^g) I \pi < 0$, then $\Delta > 0$ always holds (and thus $\widehat{q} = 0$).

Proof of Proposition 4: Suppose $2 \left(\frac{\partial^2 a}{\partial \pi^2}\right)^2 > \frac{\partial a}{\partial \pi} \frac{\partial^3 a}{\partial \pi^3}$ and $q < \widehat{q}$. Then, depending on q^g and q^b, either strategy A or strategy B can be superior.

First we show that there exists a $\widehat{q}^g < \widetilde{q}^g$ guaranteeing that, independently of q^b, strategy B is superior (row 1 in table 1) if $q^g < \widehat{q}^g$.

We have to search for a pair $(q, \widehat{q}^g)$ where $\frac{\partial \Delta}{\partial q^b} = 0$ for $q^b = q$. Recalling that $\frac{\partial \Delta}{\partial q^b}$ has no maximum, and taking into account that $\frac{\partial R(\pi^g, \pi^b)}{\partial q^g} > 0$, it follows that strategy B is then superior for $q^g < \widehat{q}^g$.

Define $\frac{\partial \Delta}{\partial q^b} \mid_{q^b=q} \equiv \frac{\partial \widetilde{\Delta}}{\partial q^b}$. It can be shown that $\frac{\partial \widetilde{\Delta}}{\partial q^b} = 0$ if

$$(1 - q) \left(\pi^g \frac{\partial^2 a}{\partial \left(\pi^g\right)^2} - \frac{\partial a}{\partial \pi^g} \right) + (1 - q^g) \left(\frac{\partial a}{\partial \pi} - I(1 - q)(1 - q^g) \pi \right) \equiv G = 0$$

Obviously, for the minimum q^g (i.e. for $q^g = q$), we have $G = 0$. For the maximum q^g (i.e. for $q^g = \widetilde{q}^g$) it follows $G > 0$ when taking into account that $q < \widehat{q}$. Moreover, we get

[30] This must be the case as otherwise $\frac{\frac{\partial^3 a}{\partial \pi^3} \frac{\partial a}{\partial \pi}}{\left(\frac{\partial^2 a}{\partial \pi^2}\right)^2}$ would at some point become > 2.

$$\frac{\partial G}{\partial q^g} = -\frac{\partial a}{\partial \pi} + 2I(1-q)(1-q^g)(\pi - \pi^g)$$

Thus $\frac{\partial G}{\partial q^g} < 0$ if $q = q^g$. Concerning the slope of $G(q^g)$, one can now conclude that G has a root at $q^g = q$, and is decreasing at this point. As $G > 0$ for $q^g = \widetilde{q}^g$, it follows that G has a second root at $q^g = \widehat{q}^g < \widetilde{q}^g$. Because $\frac{\partial \Delta}{\partial q^g} < 0$, this solution must be unique.

Now assume that $q^g > \widehat{q}^g$. For $q^b = q - \varepsilon$, we have $\frac{\partial \Delta}{\partial q^b} < 0$ because $\frac{\partial \Delta}{\partial q^b} = 0$ for $q^b = q$ and $q^g = \widehat{q}^g$ and because $\frac{\partial^2 \Delta}{\partial q^g \partial q^b} > 0$.[31] Thus, $\Delta < 0$ in this region.

However, if $q^b < \widehat{q}^b$, then strategy B remains superior, and $\widehat{q}^b$ is implicitly defined by

$$\begin{aligned} & I\pi^i(1-q) - \frac{\frac{\partial a}{\partial \pi^i}}{1-\widehat{q}^b} \\ = \; & g\left(I\pi^g(1-q^g) - \frac{\frac{\partial a}{\partial \pi^g}}{1-q^g}\right) + (1-g)\left(I\pi^b(1-\widehat{q}^b) - \frac{\frac{\partial a}{\partial \pi^b}}{1-\widehat{q}^b}\right) \end{aligned}$$

If q^g exceeds a critical value $\overline{q}^g$, strategy A becomes superior independently of q^b (third row in table 1). This is so because $\frac{\partial^2 \Delta}{\partial q^g \partial q^b} > 0$, and thus, increasing q^g makes strategy A superior even for smaller q^b. The critical $\overline{q}^g$ is defined by equating $R(\pi^i)$ and $R(\pi^g, \pi^b)$ at $q^b = 0$.

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