Fluctuations of Strangeness and Deconfinement Phase Transition in Nucleus–Nucleus Collisions

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Abstract

We suggest that the fluctuations of strange hadron multiplicity could be sensitive to the equation of state and microscopic structure of strongly interacting matter created at the early stage of high energy nucleus–nucleus collisions. They may serve as an important tool in the study of the deconfinement phase transition. We predict, within the statistical model of the early stage, that the ratio of properly filtered fluctuations of strange to non-strange hadron multiplicities should have a non–monotonic energy dependence with a minimum in the mixed phase region.

1 Introduction

Recently a new method was proposed [1] to study the equation of state (EoS) of the matter created at the early stage of nucleus–nucleus (A+A) collisions. It was suggested to analyze the collision energy dependence of the ratio of properly filtered multiplicity and energy fluctuations. It was shown that the fluctuation ratio measured at the final state may be directly dependent on the ratio of pressure to energy density at the early stage. In the present paper we make a next step within the same framework and consider strangeness fluctuations. A new observable connected with the fluctuations of the strange hadron yield is proposed. We derive the quantitative predictions of the Statistical Model of the Early Stage (SMES) [2] concerning the fluctuation of the number of strange hadrons. We find a non-monotonic dependence on the collision energy: the ratio of the relative multiplicity fluctuations of strange hadrons to those of negatively charged hadrons has a *minimum* in the domain where the onset of deconfinement occurs.

The paper is organized as follows. The basic relevant assumptions of the SMES are presented in Sect. 2. The role of dynamical fluctuations in the study of EoS at the early stage is stressed in Sect. 3. The energy dependence of the dynamical strangeness fluctuations is derived within SMES and discussed in Sect. 4. Summary and conclusions close the paper, Sect. 5.

2 Statistical Model of the Early Stage

Since we are going to discuss the collision energy dependence of the fluctuations within the SMES [2], let us present its basic assumptions. The volume, V, where the matter in confined, mixed or deconfined state is produced at the collision early stage, is given by the Lorentz contracted volume occupied by wounded nucleons. For the most central collisions the number of wounded nucleons is $N_W \approx 2 \cdot A$. The net baryon number of the *created* matter equals to zero. Even in the most central A+A collisions only a fraction $\eta < 1$ of the total collision energy is used for a particle production. The rest is carried away by the baryons which contribute to the baryon net number. The main postulates of the SMES [2] are the following:

- New particles are created at the early stage of A+A collision in a state of the global statistical equilibrium.
- The model assumes that the *created* matter is described in the grand canonical ensemble with all chemical potentials equal to zero. The EoS is chosen in the form of relativistic ideal gas with an additional bag–term contribution in the deconfined phase.
- The basic constituents of the deconfined phase are the light u and d (anti)quarks ($m_u \cong m_d \cong 0$), the strange (anti)quarks ($m_s \cong 175$ MeV) and gluons.
- The total entropy and strangeness created at the early stage are supposed to be approximately conserved during the expansion, hadronization and freeze-out.

We describe the system's EoS in terms of the pressure as a function of temperature: $p = p(T)$. For an ideal gas of particles 'j' with zero chemical potential one finds (m_j is a particle mass, g_j is a number of internal degrees of freedom):

$$p_j(T) \;=\; \frac{g_j}{6\pi^2}\int_0^\infty k^2dk\;\frac{k^2}{\sqrt{k^2+m_j^2}}\;\left[\exp\left(\frac{\sqrt{k^2+m_j^2}}{T}\right)\pm 1\right]^{-1}\;, \tag{1}$$

where $+1$ is used for fermions and -1 for bosons. The total pressure $p(T)$ is a sum of partial pressures $p_j(T)$ over all particle species 'j'. The energy density $\varepsilon(T)$ and entropy density $s(T)$ for the system with zero chemical potentials are calculated from the thermodynamical identities: $\varepsilon = T dp/dT - p$ and $s = dp/dT$. For massless particles one obtains: $p(T) = \sigma/3T^4$, $\varepsilon(T) = \sigma T^4$, $s(T) = 4\sigma/3T^3$, where $\sigma = (g^b + 7g^f/8) \cdot \pi^2/30$ with g^b and g^f being respectively the total number of degrees of freedom for bosons and fermions. In the quark gluon plasma (QGP) we have $g_Q^b = 2 \cdot 8 = 16$ and $g_Q^f = 2 \cdot 2 \cdot 2 \cdot 3 = 24$ massless degrees of freedom which correspond to the gluons and non-strange (anti)quarks, respectively. The pressure of strange quarks and anti-quarks is given by Eq. (1) with the degeneracy factor $g_Q^s = 2 \cdot 2 \cdot 3 = 12$ and quark mass $m_s = 175$ MeV. We use the bag–model EoS for the QGP [4]: $p_Q = p_{id} - B$, $\varepsilon_Q = \varepsilon_{id} + B$, i.e. the constant term $B > 0$ is subtracted from the total ideal gas pressure and is added to the total ideal gas energy density.

In the confined (hadron) phase we assume the effective degeneracy factor g_H^{ns} for massless non-strange degrees of freedom and g_H^s for strange degrees of freedom (the strange effective degrees of freedom are assumed to be massive with mass m_H close to the kaon mass). These parameters of the confined phase are obtained in Ref. [2] from fitting the data on multiplicities of strange and non-strange hadrons in Au+Au collisions at the AGS energies.

The temperature of the phase transition, T_c, is defined by the Gibbs criterion of equal pressures: $p_H(T_c) = p_Q(T_c)$. At $T = T_c$ the system stays in the mixed phase and its energy density equals to:

$$\varepsilon \; = \; (1-\xi) \cdot \varepsilon_H(T_c) \; + \; \xi \cdot \varepsilon_Q(T_c) \; , \tag{2}$$

where $0 \leq \xi \leq 1$ is the part of the system occupied by the Q-phase.

For the total strange particle number density in the H, Q and mixed phases we have, respectively:

$$n_H^s(T) \; = \; \frac{g_H^s}{2\pi^2} \int_0^\infty k^2 dk \; \exp\left(- \frac{\sqrt{k^2 + m_H^2}}{T} \right) \; = \; \frac{g_H^s}{2\pi^2} \cdot m_H^2 T \cdot K_2\left(\frac{m_H}{T}\right) \; , \tag{3}$$

$$n_Q^s(T) \; = \; \frac{g_Q^s}{2\pi^2} \int_0^\infty k^2 dk \left[\exp\left(\frac{\sqrt{k^2 + m_s^2}}{T} \right) \; + \; 1 \right]^{-1} \; , \tag{4}$$

$$n_{mix}^s(\varepsilon) \; = \; (1-\xi) \cdot n_H^s(T_c) + \xi \cdot n_Q^s(T_c) \; . \tag{5}$$

3 Statistical and Dynamical Fluctuations

Various values of the energy E and volume V of the initial equilibrium state lead to different, but uniquely determined, initial entropies S. When the collision energy is fixed, the energy, which is used for particle production still fluctuates. These fluctuations of the inelastic energy are caused by the fluctuations in the dynamical process which leads to the particle production. They are called the *dynamical* energy fluctuations [1]. Clearly the *dynamical* energy fluctuations lead to the *dynamical* fluctuations of any macroscopic parameter of the matter. In Ref. [1] the dynamical *entropy* fluctuations were considered and related to the dynamical energy fluctuations and EoS as:

$$R_e \; \equiv \; \frac{(\delta S)^2/S^2}{(\delta E)^2/E^2} \; = \; \left(1 + \frac{p}{\varepsilon}\right)^{-2} \; , \tag{6}$$

providing the volume fluctuations were absent $\delta V = 0$. The ratio p/ε reaches a minimum – the so called 'softest point' [5] of the EoS – at the boundary between the mixed phase and the QGP.

Thus, we expect a non-monotonic energy dependence of R_e with a maximum at this boundary point. The numerical results for the ratio R_e (6) calculated within the SMES in Ref. [1] are shown in Fig. 1.

The early stage energy and entropy are not directly measurable. In an experiment only momenta and energies of final state particles in a limited acceptance are reconstructed. Thus, the question arises whether the early stage fluctuations can be reconstructed from the experimentally accessible information. In Ref. [1] we argued that this may be possible in the case of entropy, provide that the distortions caused by final state correlations and statistical noise are removed. The distortions can be minimized by a proper choice of the studied observables. For instance in the case of study of dynamical entropy fluctuations it is better to extract them from the fluctuations of negatively charged hadrons than from all charged hadrons, as the latter is influenced by correlations due to resonance decays and global charge conservation. As a method to extract the dynamical fluctuations from the statistical noise the so-called sub-event method [3] can be used. In this method the fluctuations are measured in two different non-overlapping but dynamically equivalent regions of the phase space (see Ref. [1] for further details).

The idea to study the EoS of the matter created at the early stage of collisions by an analysis of the dynamical fluctuations, proposed originally for entropy [1], can be extended for strangeness. This subject is discussed in the next section. We note that among different macroscopic parameters of the system the entropy and strangeness content are of special importance [2]: they are created at the early stage of A+A collisions, they are sensitive to the EoS of the matter and they are approximately conserved during the system expansion, hadronisation and freeze-out.

4 Fluctuations of Strangeness

In this central section of the paper we study within the SMES the energy dependence of the dynamical *strangeness* fluctuations caused by the dynamical energy fluctuations. We define strangeness N_s as a total number of all strange and anti-strange particles and consider the fluctuation ratio defined as:

$$R_s \;=\; \frac{(\delta \overline{N}_s)^2/\overline{N}_s^2}{(\delta E)^2/E^2}~. \tag{7}$$

As in the case of entropy fluctuations we assume that experimental procedure allows to eliminate the event–by–event fluctuations of the initial volume V. Since $\overline{N}_s = n^s \cdot V$, the dynamical fluctuations of $\overline{N}_s$ are defined by that of the strangeness density: $\delta\overline{N}_s/\delta E = \delta n^s/\delta\varepsilon$ and consequently $R_s = (\delta n^s/\delta\varepsilon)^2 \cdot (\varepsilon/n^s)^2$.

The strangeness density depends on T in the pure confined and deconfined phases according to Eqs. (3) and (4), respectively. One has therefore to calculate the fluctuations of n^s due to the fluctuations of temperature and then the fluctuations of T due to those of ε:

$$\frac{\delta n^s_{(H,Q)}}{\delta\varepsilon} \;=\; \frac{dn^s_{(H,Q)}}{dT}\cdot\frac{dT}{d\varepsilon}~. \tag{8}$$

In the mixed phase region, $\varepsilon_H(T_c) < \varepsilon < \varepsilon_Q(T_c)$, the strangeness dynamical fluctuations can be found from Eqs. (2) and (5):

$$\frac{\delta n^s_{mix}}{\delta\varepsilon} \;=\; \frac{n^s_Q(T_c) - n^s_H(T_c)}{\varepsilon_Q(T_c) - \varepsilon_H(T_c)}~. \tag{9}$$

We note that the fluctuation ratio (9) does not depend on the energy density ε.

Let us first find the asymptotic behavior of R_s at high and small T. When $T \to \infty$ the system is in the QGP phase. The strange (anti)quarks can be considered as massless and the bag constant can be neglected. Then $\varepsilon \propto T^4$ and $n^s \propto T^3$ and consequently $d\varepsilon/\varepsilon = 4 \cdot dT/T$ and $dn^s/n^s = 3 \cdot dT/T$, which result in $R_s = (3/4)^2 \cong 0.56$. In the confined phase, $T < T_c$, the energy density is still approximately given by $\varepsilon_H \propto T^4$ due to the dominant contributions of non-strange hadron constituents. However, the dependence of strangeness density on T is dominated in this case by the exponential factor, $n^s_H \propto \exp(-m_H/T)$, as $T << m_H$. Therefore, at small T one finds $d\varepsilon_H/\varepsilon_H \propto 4 \cdot dT/T$ and $dn^s_H/n^s_H \propto m_H \cdot dT/T^2$, so that the ratio $R_s \cong m_H/(4T)$ decreases with increasing temperature (energy density). The strangeness density n^s_H is small and goes to zero at $T \to 0$, but the fluctuation ratio R_s (7) is large and goes to infinity at zero temperature limit.

The results of numerical calculations for the collision energy dependence of R_s (7) within SMES are presented in Fig. 2. As expected the fluctuation ratio R_s is independent of energy and equals approximately 0.56 at high collision energies when the initial state corresponds to the hot QGP. R_s rapidly decreases at small collision energies. A pronounced minimum-structure is observed in the dependence of R_s on the collision energy. It is located in the collision energy region $30 \div 60$ A·GeV, where the mixed phase is created at the early stage of A+A collision.

The ratio R_s (7) is defined above for the infinitesimal energy fluctuations. From Fig. 2 one observes the 'discontinuities' of R_s at the boundaries of the pure hadron and QGP phases with the mixed phase. It is easy to understand this result, as we have used different equations for n^s and ε in each separate phase: the function n^s does not have discontinuities, but its derivative $\delta n^s/\delta\varepsilon$ does. In fact, the dynamical energy fluctuations are not infinitesimal. Even more, the dynamical fluctuations should be not too small to separate them safely from the statistical fluctuations. The quantitative analysis presented below demonstrates that by introducing the finite size of the energy fluctuations we avoid the artificial discontinuities in the R_s ratio.

We denote by E an *average* energy of the created matter at fixed collision energy. We further assume that event-by-event values of energy E' vary around E according to the Gauss distribution:

$$P(E',E) \;=\; C\;\exp\left[\;-\;\frac{(E-E')^2}{2\sigma^2}\right]\;, \tag{10}$$

where $\sigma = aE$ with $a = const$, and $C \cong (2\pi\sigma^2)^{-1/2}$ is defined by the normalization condition $\int_0^\infty dE' P(E',E) = 1$. The dynamical averaging of any observable $f(E')$ can be defined now as:

$$\langle f\rangle \;=\; \int_0^\infty dE'\;P(E',E)\;f(E')\;. \tag{11}$$

The ratio R_s for the finite energy fluctuations (10) is given by

$$R_s \;\equiv\; \frac{\langle(\delta N_s)^2\rangle/\langle N_s\rangle^2}{\langle(\delta E)^2\rangle/\langle E\rangle^2}\;, \tag{12}$$

where $(\delta X)^2 \equiv (X - \langle X\rangle)^2$. Providing $E >> \sigma$ one gets $\langle E\rangle \equiv \int_0^\infty dE' P(E',E)\; E' \cong E$ and $\langle(\delta E)^2\rangle = \langle E^2\rangle - \langle E\rangle^2 \cong (\sigma)^2$, and consequently $\langle(\delta E)^2\rangle/\langle E\rangle^2 = (\sigma/E)^2 = a^2$, i.e. for distribution (10) the relative fluctuations are independent of the collision energy.

The function $N_s(E')$ needed to calculate $\langle N_s\rangle$ and $\langle N_s^2\rangle$ is defined by Eqs. (3-5) with $\varepsilon \equiv E'/V$. The results of numerical calculations of R_s (12) with $\sigma/E = a = 0.1$ are shown in Fig. 2. The qualitative features of the R_s dependence on the collision energy are not changed by the introduction of finite energy fluctuations (very similar plots are obtained for $a = 0.05$ and $a = 0.2$). The main difference is the disappearance of the sharp edges characteristic for infinitesimal fluctuations.

We apply now our procedure of averaging over finite energy fluctuations to the ratio R_s (6). The calculations are done according to Eq. (11) with $S(E') = V(\varepsilon + p)/T$:

$$\langle S^n \rangle = \int_0^{\infty} dE'\ P(E', E)\ S^n(E')\ , \quad n = 1, 2; \qquad \langle (\delta S)^2 \rangle \ = \ \langle S^2 \rangle - \langle S \rangle^2\ . \tag{13}$$

The dependence of R_e on the collision energy for distribution (10) with $\sigma/E = a = 0.1$ is shown in Fig. 1 by dashed line. Again, as in the case of R_s, the averaging procedure does not change the basic features of the dependence.

Both the entropy and strangeness fluctuation measures, R_e and R_s, show 'anomalous' behavior in the transition region: the maximum is observed for R_e and the minimum for R_s. Consequently, even stronger anomaly is observed in the ratio:

$$R_{s/e} \ \equiv \ \frac{R_s}{R_e} \ = \ \frac{\langle (\delta N_s)^2 \rangle / \langle N_s \rangle^2}{\langle (\delta S)^2 \rangle / \langle S \rangle^2} \ \cong \ \frac{\langle (\delta N_s)^2 \rangle / \langle N_s \rangle^2}{\langle (\delta N_-)^2 \rangle / \langle N_- \rangle^2}\ . \tag{14}$$

Its dependence on the collision energy is shown in Fig. 3 for infinitely small energy fluctuations and for $\sigma/E = a = 0.1$. Experimental measurements of $R_{s/e}$ may be easier than the measurements of R_e or R_s because the ratio $R_{s/e}$ requires measurements of particle multiplicities only, whereas both R_e and R_s involve also measurements of particle energies. Finally we note that the relative dynamical fluctuations of strangeness and entropy are approximatelly equal (see [1]) to the corresponding fluctuations of K^+ meson and negatively charged hadron multiplicities, respectively. Thus the $R_{s/e}$ fluctuation ratio can be expressed by the measurable quantities as:

$$R_{s/e} \cong \frac{\langle (\delta N_{K^+})^2 \rangle / \langle N_{K^+} \rangle^2}{\langle (\delta N_-)^2 \rangle / \langle N_- \rangle^2}\ . \tag{15}$$

5 Conclusions

We have considered strangeness fluctuations as a potential probe of the equation of state and microscopic content of the strongly interacting matter created at the early stage of high energy nucleus–nucleus collisions. In order to quantify the fluctuations we have introduced a new measure $R_{s/e}$ (14) constructed from fluctuations of strange and non–strange hadron multiplicities. We have predicted, within statistical model of the early stage [2], the energy dependence of the $R_{s/e}$ measure and we have shown that it is strongly sensitive to the form of matter created at the early stage of nucleus–nucleus collisions. In particular, a "tooth" structure (see Fig. 3) is expected in the collision energy domain $30 \div 60$ A·GeV in which deconfinement phase transition is located.

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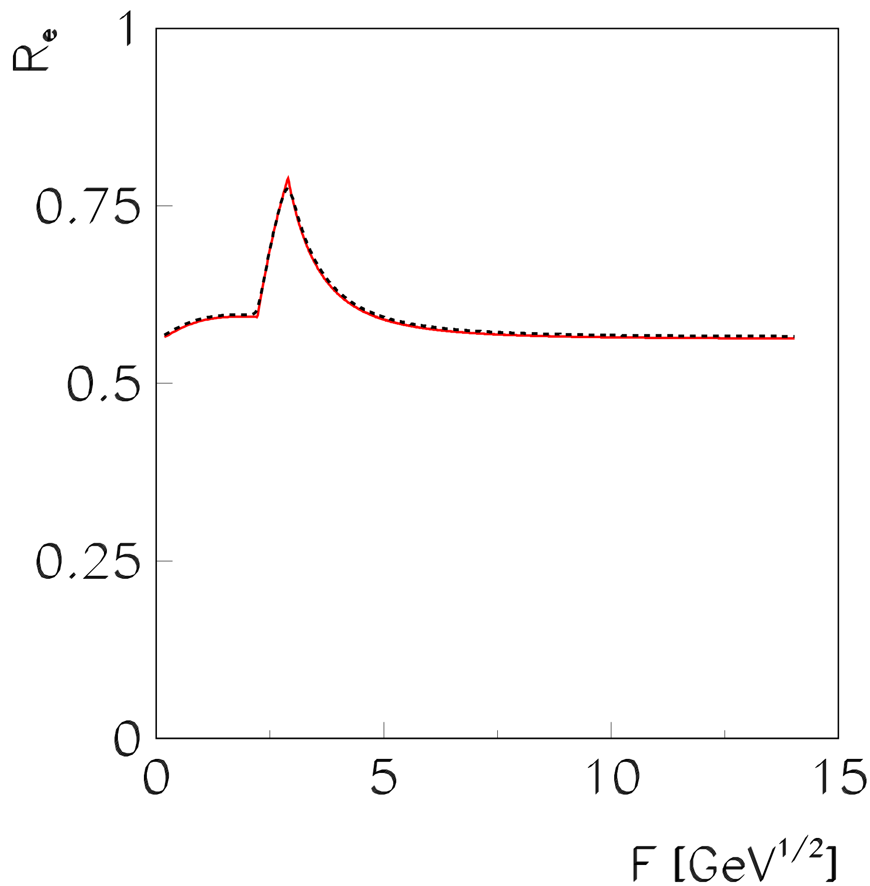

Figure 1: The dependence of the entropy to energy fluctuations, R_e, calculated within SMES [2] on Fermi's collision energy measure F ($F \equiv (\sqrt{s_{NN}} - 2m_N)^{3/4}/\sqrt{s_{NN}}^{1/4}$, where $\sqrt{s_{NN}}$ is the c.m.s. energy per nucleon–nucleon pair and m_N the rest mass of the nucleon). The non–monotonic behavior, the "shark fin" structure, is caused by the modification of fluctuations expected in the vicinity of the mixed phase region. The solid and dashed lines indicate results for infinitely small and finite ($\sigma/E = 0.1$) energy fluctuations, respectively.

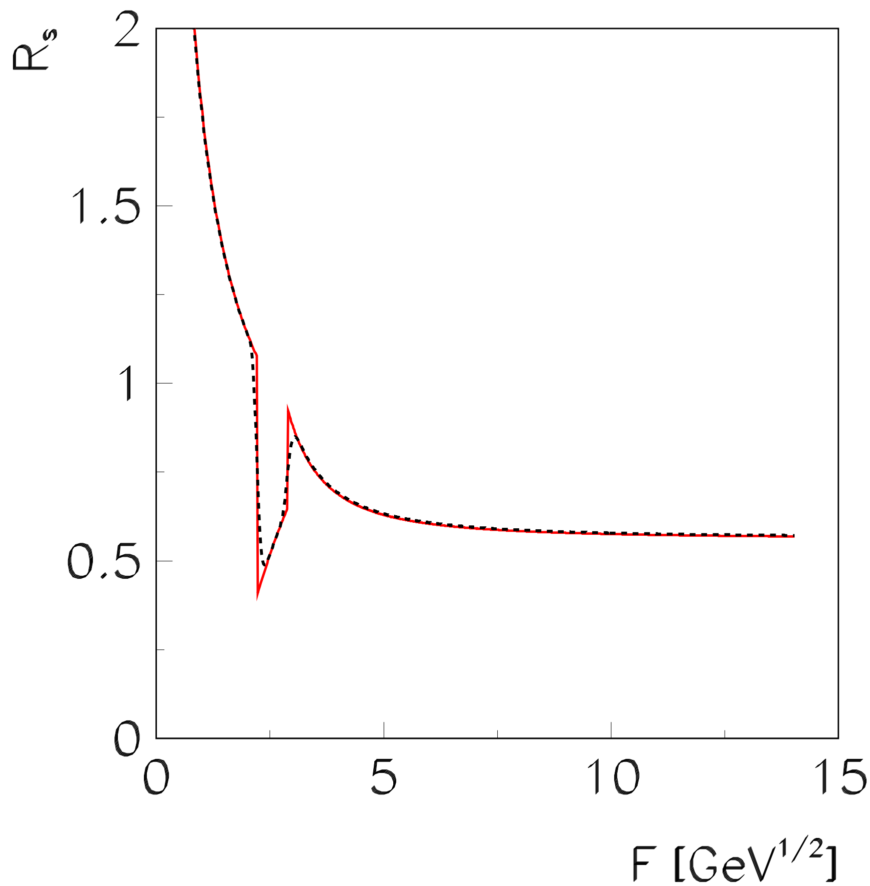

Figure 2: The dependence of the strangeness to energy fluctuations, R_s, calculated within SMES [2] on Fermi's collision energy measure F ($F \equiv (\sqrt{s_{NN}} - 2m_N)^{3/4}/\sqrt{s_{NN}}^{1/4}$, where $\sqrt{s_{NN}}$ is the c.m.s. energy per nucleon–nucleon pair and m_N the rest mass of the nucleon). The non–monotonic behavior is caused by the modification of fluctuations expected in the vicinity of the mixed phase region. The solid and dashed lines indicate results for infinitely small and finite ($\sigma/E = 0.1$) energy fluctuations, respectively.

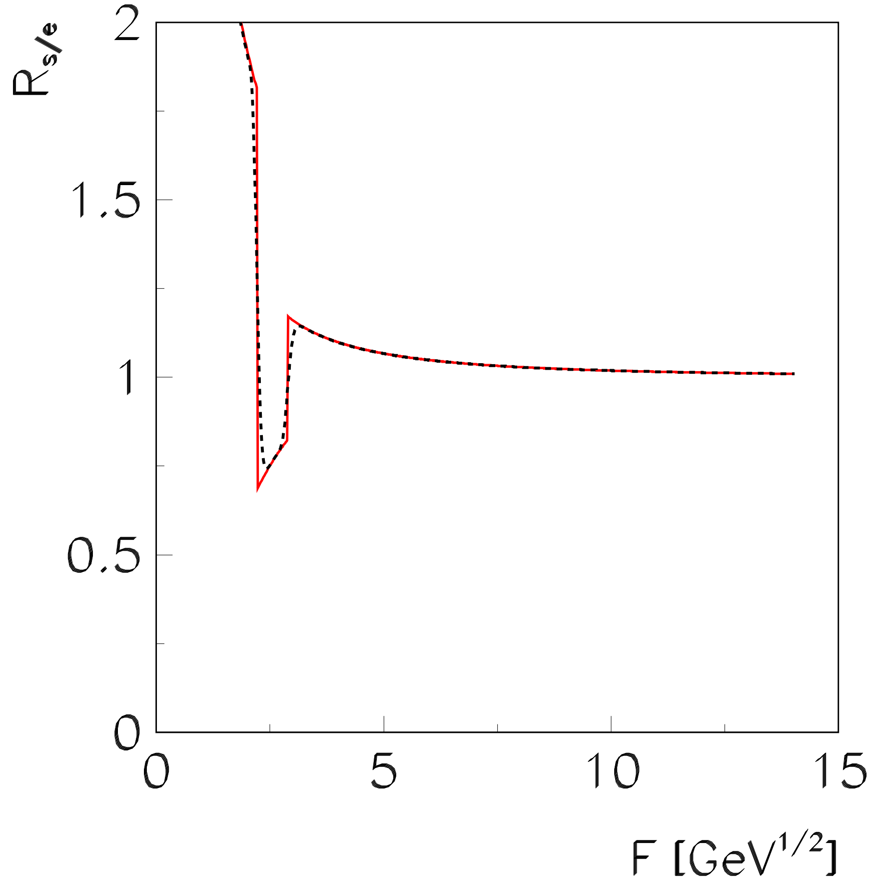

Figure 3: The dependence of the strangeness to entropy fluctuations, $R_{s/e}$, calculated within SMES [2] on Fermi's collision energy measure F ($F \equiv (\sqrt{s_{NN}} - 2m_N)^{3/4} / \sqrt{s_{NN}}^{1/4}$, where $\sqrt{s_{NN}}$ is the c.m.s. energy per nucleon–nucleon pair and m_N the rest mass of the nucleon). The non–monotonic behavior, the "tooth" structure, is caused by the modification of fluctuations expected in the vicinity of the mixed phase region. The solid and dashed lines indicate results for infinitely small and finite ($\sigma/E = 0.1$) energy fluctuations, respectively.