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Optimal Trend Inflation[*]

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Abstract

We present a sticky-price model incorporating heterogeneous firms and systematic firm-level productivity trends. Aggregating the model in closed form, we show that it delivers radically different predictions for the optimal inflation rate than canonical sticky price models featuring homogenous firms: (1) the optimal steady-state inflation rate generically differs from zero and (2) inflation optimally responds to productivity disturbances. Using micro data from the US Census Bureau to estimate the inflation-relevant productivity trends at the firm level, we find that the optimal US inflation rate is positive. It was slightly above 2 percent in the year 1986, but continuously declined thereafter, reaching about 1 percent in the year 2013.

Keywords: optimal inflation rate, sticky prices, firm heterogeneity

JEL Class. No.: E52, E31, E32

1 Introduction

This paper introduces heterogeneous firms and empirically plausible firm-level productivity trends into an otherwise standard sticky-price economy. It shows that some of the most fundamental implications of canonical sticky-price models with homogeneous firms fail to survive within such a generalized sticky-price setup. The optimal steady-state inflation rate generically differs from zero and inflation optimally responds to productivity disturbances, unlike in settings with homogeneous firms. Moreover, the paper documents that the predictions of the homogeneous firm model turn out to be non-robust in the sense that they are discontinously affected by the presence of firm heterogeneity. We thus present an example in which microeconomic heterogeneity matters for macroeconomic policy prescriptions, an issue that has received renewed interest recently (Ahn et al. (2017), Kaplan and Violante (2014)).

Due to the technical difficulties associated with aggregating heterogeneous firm models, it is standard in the sticky-price literature to abstract from all firm-level heterogeneity beyond that generated by price adjustment frictions themselves. As is well known, price adjustment frictions then tightly anchor the optimal steady-state inflation rate at zero, e.g., Woodford (2003).[1] As we show, this rather robust but somewhat puzzling implication of standard sticky-price models arises precisely because of the homogeneity assumption. Homogeneity implies that the productivity of price-adjusting firms equals that of non-adjusting firms. With economic efficiency requiring relative prices to reflect relative productivities, it calls for price-adjusting firms to charge the same price as charged on average by non-adjusting firms, i.e., it calls for zero inflation.[2]

The present paper extends the basic sticky-price setup by introducing firm heterogeneity and systematic firm-level productivity trends. Such firm-level trends are clearly present in micro data, but are routinely abstracted from in the sticky price literature. New firms, for example, tend to be initially small, i.e., tend to be initially unproductive when compared to existing firms.[3] Some of the young firms become more productive over time and grow, others become unproductive and exit the economy. We show how such life-cycle related productivity dynamics cause the average productivity of price-adjusting firms to generally differ from the average productive of non-adjusting firms. Economic efficiency then requires that adjusting firms set on average different prices than existing firms, which causes inflation or deflation to be optimal in steady state. We show this by aggregating the non-linear sticky price model with heterogeneous firms in closed form and

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[1]Section 2 discusses a range of extensions of the basic framework considered in the literature and their implications for the optimal inflation rate.

[2]Yun (2005) shows, using a setting with homogeneous firms, that if initial prices do not reflect initial productivities, the optimal inflation rate can display deterministic transitory deviations from zero.

[3]This does not rule out that new firms are in age-adjusted terms more productive than old firms. Our setup will allow for this possibility.

by deriving analytical expressions for the optimal inflation rate.

The heterogeneous firm model that we present is formulated in abstract terms and allows for a variety of economic interpretations through which firm heterogeneity arises. One interpretation is - as alluded to above - that heterogeneity arises from firm entry and exit and the associated life-cycle dynamics of firm productivity. This is also the interpretation that we shall consider in our empirical analysis. Yet, as explained in the main text, the model can equally be interpreted as one in which heterogeneity arises from product substitution or product quality improvements.

To fix ideas, consider a sticky-price model with Calvo type or menu-cost type price adjustment frictions in which a measure $\delta \geq 0$ of randomly chosen firms becomes unproductive and exits the economy. Exiting firms are replaced by a measure δ of young new firms. Our setup then features three systematic productivity trends, each of which has different implications for the optimal inflation rate. First, there is a common trend in total factor productivity (TFP), which affects all firms equally. The common TFP trend captures general-purpose innovations that are adopted by all firms simultaneously. As in a standard homogeneous-firm model, it does not affect the optimal inflation rate. Second, there is an experience trend in firm TFP, which determines how firms accumulate experience with age. The experience trend may capture productivity gains from learning-by-doing or other forms of experience accumulation. As we show, this productivity trend generates a force towards positive inflation rates. Third, there is a cohort productivity trend, which determines the productivity level of newly entering firms. This trend captures the fact that new firms tend to bring new technologies into the economy that are not (yet) used by other firms.[4] The cohort trend will be a force towards making deflation optimal.

Taken together, the optimal steady-state inflation rate in our setting depends on the strength of the experience trend relative to the strength of the cohort trend, whenever there is some positive firm turnover ($\delta > 0$). The optimal steady-state inflation rate is itself independent of the firm turnover rate, as long as $\delta > 0$. Yet, in the absence of firm turnover ($\delta = 0$), the optimal steady-state inflation rate collapses to zero, i.e., to the optimal inflation rate of a homogeneous firm model. It is in this sense, that the inflation predictions of the homogeneous firm model turn out to be non-robust.

To provide economic intuition for these findings, consider two polar settings. The first setting abstracts from the presence of a cohort trend and considers a setting where the only trend is that firms accumulate experience over time.[5] If an old firm becomes unproductive and exits the economy, the new firm that replaces it will not have accumulated any

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[4]Newly entering firms are endowed with the cohort productivity level, in addition to the common TFP component, and then gradually accumulate experience over time.

[5]As mentioned before, we can abstract from the common TFP trend, as it does not affect the optimal inflation rate.

experience yet. The new firm will thus be less productive than the remaining set of old firms.[6] From a welfare standpoint, the optimal price of new firms should therefore exceed the average price of existing firms, so as to accurately reflect relative productivities. Achieving this requires either that new firms choose higher prices or that old firms reduce prices, or a combination thereof.

In the presence of sticky prices, price reductions by old firms are costly. In time-dependent adjustment models, they lead to inefficient price dispersion due to asynchronous price adjustment; in state-dependent pricing models, they require firms to pay adjustment costs. Therefore, it is optimal to implement the efficient relative price exclusively by having new firms charge higher prices, while all other firms hold their prices steady. Clearly, this implies that the aggregate inflation rate must be positive in the steady state.

Now consider the second polar setting, in which there is no experience effect and the only trend is a positive cohort trend. New firms are then more productive than the existing set of old firms, thus optimally charge lower prices than existing firms. This makes negative rates of inflation optimal.[7]

We also determine in closed form the dynamic response of the optimal inflation rate following shocks to experience and cohort productivity. We show that such shocks have fairly persistent effects on the optimal inflation rate, especially in settings in which δ is positive but close to zero. A low value for δ causes a persistent shock to the experience level of existing firms to give rise to a persistent change in relative productivity between existing firms and new entrants. Likewise, a persistent shock to the productivity level of new cohorts causes persistent relative productivity differences between existing and new firms. These persistent productivity differences require that inflation also moves persistently along the transition until the productivity distribution has again reached its steady state.

Importantly, one cannot infer the inflation-relevant cohort and experience trends by observing aggregate productivity. As we show, these trends are not identified because inflation-neutral TFP trends mask the underlying inflation-relevant cohort and experience trends at the aggregate level. In our empirical analysis we thus resort to micro evidence on firm dynamics.

To obtain a plausible framework for empirical analysis, we extend our baseline setting to a multi-sector economy. The multi-sector setup allows for sector-specific experience and cohort trends, sector-specific "common" TFP trends, as well as sector-specific firm turnover rates and degrees of price stickiness. We then show that the inflation rate that maximizes steady-state welfare is a weighted average of the inflation rates that would

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[6]The new firm will be in age-adjusted terms more productive than all old firms, once one allows for a positive cohort trend.

[7]Due to price-setting frictions, it is again not optimal that old firms adjust prices.

achieve efficient relative prices within each sector individually.[8] Based on this insight, we devise a model-based empirical strategy that allows us to estimate these sector-specific cohort and experience trends and thus the optimal inflation rate from firm-level data.

To estimate the relevant firm-level trends, we use the Longitudinal Business Database (LBD) of the US Census Bureau, which covers all private sector establishments in the United States, and estimate the relevant cohort and experience trends and their evolution over time. Our regression results show that the optimal inflation rate implied by our model is positive but approximately halved over the period 1986 to 2013. Depending on the precise value of the elasticity of product demand assumed, the level of the optimal inflation rate varies. For our preferred demand elasticity specification, the optimal inflation rate declined from around 2% in 1986 to approximately 1% in 2013.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents our heterogeneous firms model with sticky prices. Section 4 analytically aggregates the model, and section 5 shows that the flexible-price equilibrium is first best when a Pigouvian output subsidy corrects firms' monopoly power. The main results on the optimal rate of inflation for the non-linear model are presented in closed-form in section 6. Section 7 discusses the implications of the main results for the optimal steady-state inflation rate and steady-state welfare. It also shows how the optimal inflation rate jumps discontinuously when moving from a standard sticky-price economy ($\delta = 0$) to one including firm turnover ($\delta > 0$). Section 8 determines the utility costs of implementing suboptimal inflation and section 9 discusses the optimal response of the inflation rate to economic disturbances. Section 10 extends the baseline setup to a multi-sector economy, allowing for a considerable degree of sectoral heterogeneity. Using a model-consistent approach, section 11 estimates the optimal inflation rate for the US economy using the LBD data. Section 12 discusses the robustness of our findings towards various extensions. A conclusion briefly summarizes. Proofs and technical material are relegated to a series of appendices.

2 Related Literature

Only few papers discuss the relationship between the optimal inflation rate and productivity trends. All of these focus on aggregate or sectoral productivity trends and find that the optimal inflation rate is (slightly) negative in their calibrated models. Amano et al. (2009), for instance, consider an economy with aggregate productivity growth and sticky wages and prices. They show how monetary policy affects wage and price mark-ups and that this can make it optimal to implement deflation, so as to reduce wage mark-ups.

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[8]The inflation rate that achieves efficiency in a specific sector again depends on the sector-specific cohort and experience trends.

Wolman (2011) considers a two-sector sticky-price economy with sectoral productivity trends. He shows that - even in the absence of monetary frictions - the optimal inflation rate is either negative or close to zero, depending on the precise modeling of price adjustment frictions.

Golosov and Lucas (2007) and Nakamura and Steinsson (2010) consider sticky-price setups with heterogeneous firms and study monetary non-neutrality within these setups. They do not consider the issue of the optimal inflation rate. Firms in their settings are subject to random idiosyncratic productivity shocks. This differs from the present setup which features idiosyncratic shocks that give rise to systematic productivity adjustments (as implied by the cohort and experience trends). The idiosyncratic nature of productivity shocks in Golosov and Lucas (2007) and Nakamura and Steinsson (2010) causes firms with very positive or very negative idiosyncratic productivity shocks to adjust prices. The productivity of price-adjusting firms is thus on average similar to that of non-adjusting firms, suggesting zero inflation to be optimal.

The present paper is also related to a large literature studying the determinants of optimal inflation, most of which finds that the optimal inflation rate is either negative or close to zero. None of these papers makes a connection between the optimal inflation rate and firm-level productivity dynamics.

In classic work, Kahn, King and Wolman (2003) explore the trade-off between price adjustment frictions, which call for price stability, and monetary frictions, which call for a Friedman-type deflation. They demonstrate how a slight rate of deflation is optimal in such frameworks. In a comprehensive survey, Schmitt-Grohé and Uribe (2010) document the robustness of these findings to a large number of natural extensions. They show that taxation motives, including the presence of untaxed income, foreign demand for domestic currency (Schmitt-Grohé and Uribe (2012a)), as well as a potential quality bias in measured inflation rates (Schmitt-Grohé and Uribe (2012b)), are all unable to rationalize significantly positive rates of inflation.

Adam and Billi (2006, 2007) and Coibion, Gorodnichenko and Wieland (2012) explicitly incorporate a lower bound on nominal interest rates into sticky-price economies. They find that fully optimal monetary policy is consistent with close to zero average rates of inflation. While zero lower bound episodes make it optimal to promise inflation in the future, these promises should only be made conditionally on being at the lower bound, which happens rather infrequently; see Eggertsson and Woodford (2003) for an early exposition.

A number of papers find positive inflation rates to be optimal on average when introducing downward nominal wage rigidities into the standard setup. Kim and Ruge-Murcia (2009) argue that such rigidities allow optimal inflation rates of approximately 0.35% on average to be justified when using a model with aggregate shocks only. Looking at a

setting with idiosyncratic shocks, Benigno and Ricci (2011) also find a positive steady-state inflation rate to be optimal.[9] Carlsson and Westermark (2016) consider a setting with nominal wage rigidities and search and matching frictions in the labor market. They show how a standard US calibration of the model implies failure of the Hosios condition and justifies an annual inflation rate of about 1.16%. Schmitt-Grohé and Uribe (2013) analyze the case for temporarily elevated inflation in the euro area due to the presence of downward rigidity of nominal wages.

Brunnermeier and Sannikov (2016) show that the optimal inflation rate can also be positive in a model without nominal rigidities. They present a model with undiversifiable idiosyncratic capital income risk in which the optimal inflation rate increases with the amount of idiosyncratic risk.

There is also a literature studying endogenous firm entry decisions in homogeneous firm economies, focusing on the effect of inflation on the firm entry margin, e.g., Bergin and Corsetti (2008), Bilbiie et al. (2008) and Bilbiie, Fujiwara and Ghironi (2014). Bilbiie et al. (2014) document - amongst other things - that the welfare optimal inflation rate is positive whenever the benefit of additional varieties to consumers falls short of the market incentives for creating these varieties. Inflation then reduces the value of creating varieties and brings firm entry closer to its efficient (lower) level. The present paper abstracts from endogenous firm entry decisions and thus from the implication of monetary policy for the entry margin, instead considers a setting with heterogeneous firms in which entry and exit is driven by exogenous productivity dynamics.

A set of empirical papers decompose the observed US inflation rate into a trend and cyclical component and shows that trend inflation displays substantial low-frequency variation over time, e.g., Cogley and Sargent (2001), Cogley, Primiceri and Sargent (2010). The sticky-price literature has reacted to these facts by incorporating trend inflation into their workhorse models; see Ascari and Sbordone (2014) and Cogley and Sbordone (2008). Trend inflation emerges in these setups because the central bank pursues an exogenous inflation target, which is non-zero and potentially time-varying. Primiceri (2006), Sargent (1999), and Sargent, Williams and Zha (2006) present settings in which policymakers learn about the Phillips curve trade-off and show how this can endogenously give rise to the observed low-frequency movements in US inflation. The present paper does not explore to what extent changes in firm-level productivity can contribute to explaining the observed US inflation history, as it focuses on the normative implications of these trends. Exploring the positive content of the theory presented in this paper appears to be an interesting avenue for further research.

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[9]Since positive inflation has no welfare costs in their setup, they do not quantify the optimal inflation rate.

3 Economic Model

We consider a cashless economy with nominal rigidities and monopolistically competitive firms. The model is entirely standard, except for the more detailed modeling of firm-level productivity and price adjustment dynamics. Specifically, we augment the standard sticky-price setup by idiosyncratic firm-level productivity adjustments that arrive in conjunction with a price adjustment opportunity. This gives rise to a setting with heterogeneous firm-level productivities in which the productivity of price-adjusting firms is not necessarily equal to that of non-adjusting firms.

For simplicity, we derive our results within a time-dependent price adjustment model à la Calvo (1983). As we argue in section 12.1, our main findings remain unaltered if we look instead at a setting where price adjustment frictions take the form of menu costs. The next section introduces our generalized firm setup in abstract terms. Section 3.2 provides alternative economic interpretations of the setup.

3.1 Technology, Prices and Price Adjustment Opportunities

Each period $t = 0, 1, \ldots$ there is a unit mass of monopolistically competitive firms indexed by $j \in [0, 1]$. Each firm j produces output Y_{jt}, which enters as an input into the production of an aggregate consumption/investment good Y_t according to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} \, \mathrm{dj} \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $1 < \theta < \infty$ denotes the price elasticity of product demand. Let P_{jt} denote the price charged by firm j in period t. Firms can adjust prices with probability $1 - \alpha$ each period ($0 < \alpha < 1$). The arrival of a Calvo price adjustment opportunity is thereby idiosyncratic and independent of all other exogenous random variables in the economy.

We augment this standard setting by a second price adjustment opportunity that arrives with probability $\delta \geq 0$ each period. This second adjustment opportunity is idiosyncratic across firms, but arrives in conjunction with a firm-level productivity change, as described in detail below. In particular, let $\delta_{jt} \in \{0, 1\}$ denote the idiosyncratic i.i.d. random variable governing this second price and productivity adjustment and let $\delta_{jt} = 1$ indicate the arrival of such an adjustment event for firm j in period t ($\Pr(\delta_{jt} = 1) = \delta$). We shall informally refer to the event $\delta_{jt} = 1$ as *the occurrence of a δ-shock*. We introduce such δ-shocks in abstract form below and discuss alternative economic interpretations in section 3.2.

Letting K_{jt} and L_{jt} denote the amount of capital and labor used by firm j, respectively, firm output Y_{jt} is given by

$$Y_{jt} = A_t Z_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right), \tag{2}$$

where A_t captures common productivity, Z_{jt} firm-specific productivity, and $F_t \geq 0$ the potential presence of fixed costs for operating the firm. To be consistent with balanced growth, we assume

$$F_t = f \cdot (\Gamma_t^e)^{1-\frac{1}{\phi}} \tag{3}$$

for some $f \geq 0$, where Γ_t^e captures the growth trend in the balanced growth path, as defined in equation (21) below.[10] Common productivity evolves according to

$$A_t = a_t A_{t-1},$$

firm-specific productivity according to

$$Z_{jt} = \begin{cases} g_t Z_{jt-1} & \text{if } \delta_{jt} = 0 \\ Q_t & \text{if } \delta_{jt} = 1, \end{cases} \tag{4}$$

where Q_t is given by

$$Q_t = q_t Q_{t-1}. \tag{5}$$

The productivity growth processes $a_t, g_t, q_t > 0$ are stationary and have unconditional mean $a, g, q > 0$, respectively.

Productivity dynamics in the previous setting feature three trends: (1) the common growth trend a_t; (2) the experience growth trend g_t, which applies in the absence of δ-shocks; and (3) the productivity growth trend q_t, which determines the effects of δ-shocks on technology. Each of these three growth trends has a different implication for the optimal inflation rate.

To understand the productivity dynamics implied by the previous setup, consider first the special case with $\delta = 0$. In the absence of idiosyncratic δ-shocks to firm technology, all firms experience the same productivity growth rate $a_t g_t$. Such a setting with homogeneous productivity growth across all firms is the one routinely considered in the sticky-price literature.[11]

Next, consider the case $\delta > 0$ and let s_{jt} denote the number of periods that have elapsed since firm j last experienced a δ-shock (i.e., $\delta_{j,t-s_{jt}} = 1$ and $\delta_{j\tilde{t}} = 0$ for $\tilde{t} = t - s_{jt} + 1, ..., t$). Firm-specific productivity Z_{jt} in equation (4) can then be written as

$$Z_{jt} = G_{jt} Q_{t-s_{jt}},$$

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[10]In the absent of aggregate technology growth, the formulation of fixed costs in equation (3) corresponds to that used in Melitz (2003).

[11]For the case $\delta = 0$, our setting still allows for a non-degenerate initial distribution of firm productivities. Typically, this initial distribution is also assumed to be degenerate in the sticky-price literature. As we show below, the additional assumption of a degenerate initial distribution is not key for the conclusion that zero inflation is optimal, as long as initial prices reflect initial productivities, see Yun (2005) for a discussion of this and related issues in a homogeneous firm setting.

where

$$G_{jt} = \begin{cases} 1 & \text{for } s_{jt} = 0 \\ g_t G_{jt-1} & \text{otherwise,} \end{cases}$$

and where Q_t follows equation (5). This alternative formulation illustrates that all firms hit by a δ-shock in t upgrade idiosyncratic productivity to $Z_{jt} = Q_t$, so that Q_t can be interpreted as capturing a "cohort effect" of productivity dynamics, where cohorts are determined by the arrival time of the last δ-shock. Following any δ-shock, the firm experiences productivity gains, as described by the process G_{jt}, as long as no further δ-shocks arrive. Since the productivity gains G_{jt} are lost with the arrival of the next δ-shock, one can interpret the process G_{jt} as capturing "experience" or "learning-by-doing effects" associated with the cohort production technology $Q_{t-s_{jt}}$. Following a δ-shock in period t, our specification thereby implies that firm productivity increases (temporarily decreases) if Q_t has been growing faster (slower) than G_{jt} since the time of arrival of the last δ-shock prior to period t. Note, however, that as long as Q_t displays a positive growth trend ($q > 0$), firms always become more productive over time in experience-adjusted terms, even if Q_t grows slower than G_{jt}. Indeed, in our setting the long-term growth rate of firms' productivity is determined by the process $a_t q_t$, as the experience growth rate g_t generates - due to the occasional reset - only temporary level effects for firm productivity.

As usual, we define the aggregate price level as

$$P_t \equiv \left(\int_0^1 P_{jt}^{1-\theta} \mathrm{d}j \right)^{\frac{1}{1-\theta}}. \tag{6}$$

Cost minimization in the production of final output Y_t implies

$$P_t = \int_0^1 \left(\frac{Y_{jt}}{Y_t} \right) P_{jt} \ \mathrm{d}j,$$

which shows that the price level is an expenditure-weighted average of the prices in the different expenditure categories, in line with the practice at statistical agencies. Following the approach of the Bureau of Labor Statistics, we furthermore assume that all current product versions enter the computation of the CPI and thus the inflation rate.[12] The inflation rate is defined as

$$\Pi_t \equiv P_t / P_{t-1}.$$

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[12] The section "Item replacement and quality adjustment" in chapter 17 of the BLS Handbook of Methods, BLS (2015), describes how the changeover of discontinued product versions is handled. If a data collector cannot find anymore a product version that was previously contained in the sample, she/he replaces it with a new version. The price of the old version enters the previous price index and the price of the new version enters the current price index. The BLS also seeks to adjust for quality differences across versions. Armknecht et al. (1996) shows that about 3% of products are discontinued each month and table 9.2 shows that more than 50% of the replacement versions fall into the category "direct comparisons", for which no quality adjustment is made; for the remaining replacements there is either a direct quality adjustment or quality adjustment is imputed via different methods. As will become clear in section 3.2, our setup will be consistent with statistical agencies making such quality adjustments.
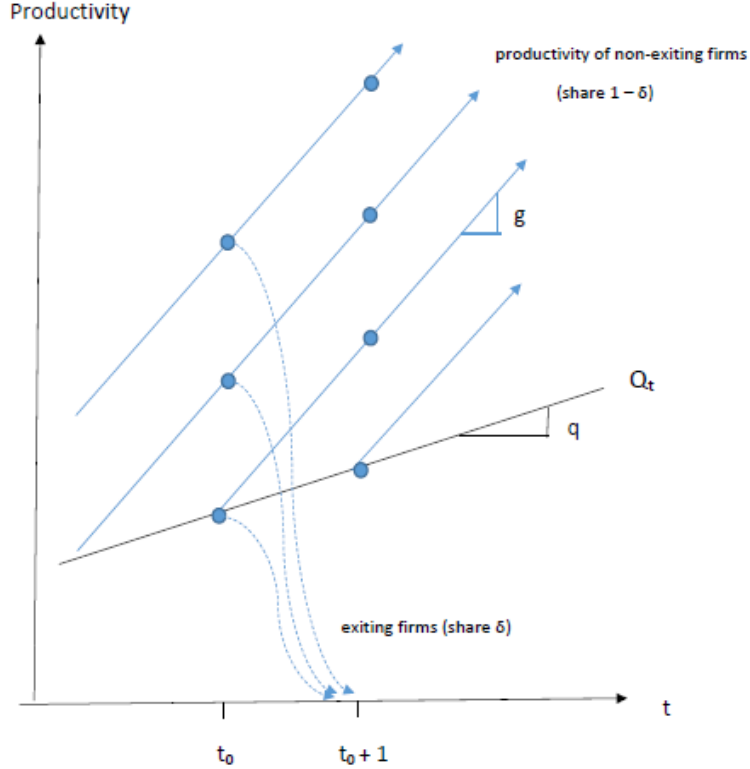
Figure 1: Productivity dynamics in a setting with firm entry and exit

We also assume that $a_t = a\epsilon_t^a$, $q_t = q\epsilon_t^q$, and $g_t = g\epsilon_t^g$ with $\epsilon_t^a, \epsilon_t^q, \epsilon_t^g > 0$ being stationary shocks with an arbitrary contemporaneous and intertemporal covariance structure, satisfying $E[\epsilon_t^a] = E[\epsilon_t^q] = E[\epsilon_t^g] = 1$. To obtain a well-defined steady state and to insure that relative prices in the flexible-price economy remain bounded, we assume throughout the paper

$$(1 - \delta)\,(g/q)^{\theta-1} < 1. \tag{7}$$

3.2 Alternative Interpretations of the Firm Setup

The previous section defined δ-shocks ($\delta_{jt} = 1$) as an idiosyncratic change in firm-level productivity that is associated with a price adjustment opportunity. This section presents three alternative economic interpretations of δ-shocks that highlight alternative economic sources of firm heterogeneity and that explain why productivity changes may plausibly be associated with price flexibility at the firm level.

Firm entry and exit. It is possible to interpret δ-shocks as a firm exit and entry event. Indeed, this is the interpretation that we will adopt in our empirical application of the model in section 11. Specifically, the event $\delta_{jt} = 1$ can be interpreted as an event in which firm j becomes permanently unproductive and thus exits the economy. Each exiting

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firms is then replaced by a newly entering firm to which we assign for simplicity the same firm index j. The variable Q_t then captures the productivity level of the cohort of firms that enters in period t, and G_{jt} captures the experience accumulated over the lifetime of the firm. The assumption that firm prices are flexible following a δ-shock should then be interpreted as newly entering firms being able to freely choose the price of their product. It is worth noting that firm entry and exit rates are high in the United States and range in the order of 8-12% per year, see figure 3 in Decker et al. (2014).

Figure 1 illustrates the firm level productivity dynamics for the empirically plausible setting in which the cohort trend is positive ($q > 0$), but less strong than the experience trend ($g > q$). To simplify the exposition, the figure depicts the deterministic dynamics and abstracts from the TFP trend a, which does not affect the distribution of relative productivities across firms. The line Q_t in the figure indicates the cohort trend and captures the productivity of newly entering firms at each point in time. The lines starting at the cohort trend line capture the productivity dynamics of the entering cohorts over time. Since $g > q$, the productivity of existing firms grows faster than that of new entrants, so that existing firms are initially more productive and thus larger than newly entering firms. In experience-adjusted terms, however, newly entering firms are the most productive firms in the economy. The downward-pointing dashed arrows indicate the productivity losses of exiting firms that have been hit by a δ-shock. For simplicity, the figure assumes that their productivity permanently drops to zero. As should be clear from the figure, the entry and exit dynamics imply an exponential distribution for firm age. Coad (2010) shows that such an age distribution is empirically plausible and how it generates, together with (productivity) growth shocks, a Pareto distribution for firm size, in line with the observed firm size distribution.

Product substitution. The event $\delta_{jt} = 1$ can also be interpreted as an event in which the product previously produced by firm j is no longer demanded by consumers. Firm j reacts to this by introducing a new product, which - for simplicity - is assigned the same product index j. The variable Q_t then captures the productivity level associated with products that are newly introduced in t and G_{jt} captures experience accumulation in producing the new product. Product substitutions, e.g., in the form of new product versions or models, take place rather frequently in the data and are also prevalent in the CPI baskets of statistical agencies (see section III.C in Nakamura and Steinsson (2008) for evidence on the rate of product substitution in the US CPI). Evidence provided in Moulton and Moses (1997), Bils (2009) and Melser and Syed (2016) furthermore shows that the prices of new products are typically higher than those that they replace, even after accounting for quality improvements.[13] It thus appears reasonable to assume price

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[13]Evidence provided in Bils (2009) shows that inflation for durables ex computers over the period 1988-2006 averaged 2.5% per year, but when including only matched items, the inflation rate was -3.7%

flexibility for new products.

Quality improvements. Let Q_{jt} denote the quality of the product produced by firm j in period t. Defining $Q_{jt} = Q_{t-s_{jt}}$, the event $\delta_{jt} = 1$ captures the situation in which firm j upgrades the quality of its product from level $Q_{t-1-s_{j,t-1}}$ to level Q_t. Let aggregate output produced with intermediate inputs of different quality be given by

$$Y_t = \left(\int_0^1 \left(Q_{jt} \widetilde{Y}_{jt} \right)^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}},$$

and let firm j's output of quality level Q_{jt} be given by

$$\widetilde{Y}_{jt} = A_t G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where G_{jt} now captures experience effects associated with producing quality Q_{jt}. Finally, let $\widetilde{P}_{jt}$ denote the price of a unit of good j of quality level Q_{jt}. Assuming that statistical agencies perfectly adjust the price level for quality changes over time, we have

$$P_t = \left(\int_0^1 \left(\frac{\widetilde{P}_{jt}}{Q_{jt}} \right)^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}.$$

As is easily verified, this setup with quality improvements is mathematically identical to the one with productivity changes spelled out in the previous section.[14] Again, as in the case with product substitution, it appears natural to assume that firms can flexibly price goods featuring improved quality features.

3.3 Optimal Price Setting

Firms choose prices, capital and hours worked to maximize profits. While price adjustment is subject to adjustment frictions, factor inputs can be chosen flexibly. Letting W_t denote the nominal wage and r_t the real rental rate of capital, firm j chooses the factor input mix so as to minimize production costs $K_{jt} P_t r_t + L_{jt} W_t$ subject to the constraints imposed by the production function (2). Let

$$I_{jt} \equiv F_t + Y_{jt}/(A_t Q_{t-s_{jt}} G_{jt})$$

denote the units of factor inputs $(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}})$ required to produce Y_{jt} units of output. As appendix A.1 shows, cost minimization implies that the marginal costs of I_{jt} are given by

$$MC_t = \left(\frac{W_t}{1/\phi} \right)^{\frac{1}{\phi}} \left(\frac{P_t r_t}{1 - 1/\phi} \right)^{1-\frac{1}{\phi}}. \tag{8}$$

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per year.

[14]The quality-adjusted price $\widetilde{P}_{jt}/Q_{jt}$ and the quality-adjusted quantity $\widetilde{Y}_{jt} Q_{jt}$ then correspond to the price P_{jt} and quantity Y_{jt}, respectively, in the previous section.

Now consider a firm in period t that can freely choose its price because it has experienced either a δ-shock or a Calvo adjustment shock. Letting α denote the probability implied by the Calvo process that the firm has to keep its price ($0 < \alpha < 1$), the firm will not be able to reoptimize its price with probability $\alpha(1-\delta)$ at any future date, i.e., whenever it undergoes neither a δ-shock nor a Calvo adjustment shock.[15] The price-setting problem of a firm that can optimize its price in period t is thus given by

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1+\tau)P_{jt+i}Y_{jt+i} - MC_{t+i}I_{jt+i}\right] \qquad (9)$$

$$\text{s.t.} \quad I_{jt+i} = F_{t+i} + Y_{jt+i}/A_{t+i}Q_{t-s_{jt}}G_{jt+i},$$

$$Y_{jt+i} = (P_{jt+i}/P_{t+i})^{-\theta} Y_{t+i},$$

$$P_{jt+i+1} = \Xi_{t+i,t+i+1}P_{jt+i}.$$

where τ denotes a sales tax/subsidy and $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$. The first constraint captures the firm's technology, the second constraint captures the demand function faced by the firm, as implied by equation (1), and the last constraint captures how the firm's price is indexed over time (if at all) in periods in which prices are not reset optimally. We consider general price indexation schemes and allow $\Xi_{t+i,t+i+1}$ to be a function of aggregate variables up to period $t+i$.[16] In the absence of indexation, we have $\Xi_{t+i,t+i+1} = 1$ for all $i \geq 0$.

Appendix A.2 shows that the optimal price $P_{jt}^{\star}$ can be expressed as

$$\frac{P_{jt}^{\star}}{P_t} \left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t}, \qquad (10)$$

where the variables N_t and D_t are functions of aggregate variables only and evolve recursively according to

$$N_t = \frac{MC_t}{P_t A_t Q_t} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \qquad (11)$$

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{1-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1}D_{t+1}\right]. \qquad (12)$$

Equation (10) shows that the optimal reset price of a firm depends only on how its own productivity ($A_t Q_{t-s_{jt}}G_{jt}$) relates to the productivity of a firm hit by a δ-shock in period

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[15]In any period, the firm can adjust its price with probability δ due to the occurrence of a δ-shock and with probability $(1-\alpha)(1-\delta)$ due to the occurrence of a Calvo price adjustment shock.

[16]We only require that price indexation is such that the price-setting problem remains well defined, that price indexation does not give rise to multiplicities of the optimal inflation rate and that indexation is such that $\Xi_{t,t+1} = 1$ in a steady state without inflation. For instance, when indexing occurs with respect to lagged inflation according to $\Xi_{t,t+1} = (\Pi_t)^{\kappa}$ with $\kappa \geq 0$, we rule out $\kappa > 1$ to avoid non-existence of optimal plans and rule out $\kappa = 1$ to avoid multiplicities of the steady-state inflation rate.

t $(A_t Q_t)$, as well as on aggregate variables. It is precisely this feature which permits aggregation of the model in closed form. Equation (10) furthermore shows that more productive firms optimally choose lower prices. For the special case with homogeneous firms, where relative productivity is always equal to one $(Q_{t-s_{jt}} G_{jt}/Q_t = 1)$, equations (10)-(12) reduce to those capturing price dynamics in a standard homogeneous-firm model.

3.4 Household Problem

There is a representative household with balanced growth consistent preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1 - \sigma} \right),$$

where C_t denotes private consumption of the aggregate good, L_t labor supply, ξ_t a preference shock with $E[\xi_t] = 1$ and $\beta \in (0,1)$ the discount factor. We assume $\sigma > 0$ and that $V(\cdot)$ is such that period utility is strictly concave in (C_t, L_t) and that Inada conditions are satisfied. The household faces the flow budget constraint

$$C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} \, \mathrm{dj} + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t,$$

where K_{t+1} denotes the capital stock, B_t nominal government bond holdings, i_{t-1} the nominal interest rate, W_t the nominal wage rate, r_t the real rental rate of capital, d the depreciation rate of capital, Θ_{jt} nominal profits from ownership of firm j, and T_t lump sum taxes. Household borrowing is subject to a no-Ponzi scheme constraint. The first-order conditions characterizing optimal household behavior are entirely standard and are derived in Appendix A.3. To insure existence of a well-defined balanced growth path, we assume throughout the paper that

$$\beta < (aq)^{\phi\sigma}.$$

3.5 Government

To close the model, we consider a government which faces the budget constraint

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + \tau \int_0^1 \left(\frac{P_{jt}}{P_t} \right) Y_{jt} \, \mathrm{dj} - T_t,$$

where τ denotes a sales subsidy, which will be used to correct for the monopolistic distortions in product markets. The government levies lump sum taxes T_t, so as to implement a bounded state-contingent path for government debt B_t/P_t.[17] Since we consider a cashless limit economy, there are no seigniorage revenues, even though the central bank controls the nominal interest rate. We furthermore assume that monetary policy is not constrained by a lower bound on nominal interest rates. The equilibrium concept is standard and defined in appendix A.5.

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[17]The household's transversality condition will then automatically be satisfied in equilibrium.

4 Analytical Aggregation with Heterogeneous Firms

This section outlines the main steps that allow us to aggregate the model in closed form. In a first step, we derive a recursive representation describing the evolution of the aggregate price level P_t over time. In a second step, we derive a closed-form expression for the aggregate production function. In a last step, we show how to appropriately detrend aggregate variables, so as to render them stationary.

Evolution of the aggregate price level. Let $P^\star_{t-s,t-k}$ denote the optimal price of a firm that last experienced a δ-shock in $t-s$ and that has last reset its price in $t-k$ ($s \geq k \geq 0$). In period t, this firm's price is equal to $\Xi_{t-k,t}P^\star_{t-s,t-k}$, where $\Xi_{t-k,t} = \prod_{j=1}^{k} \Xi_{t-k+j-1,t-k+j}$ captures the cumulative effect of price indexation (with $\Xi_{t-k,t} \equiv 1$ in the absence of price indexation). Let $\Lambda_t(s)$ denote the weighted average price in period t of the cohort of firms that last experienced a δ-shock in period $t-s$, where all prices are raised to the power of $1 - \theta$, i.e.,

$$\Lambda_t(s) = (1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P^\star_{t-s,t-k})^{1-\theta} + \alpha^s(\Xi_{t-s,t}P^\star_{t-s,t-s})^{1-\theta}. \tag{13}$$

There are α^s firms that have not had a chance to optimally reset prices since receiving the δ-shock and $(1-\alpha)\alpha^k$ firms that have last adjusted $k < s$ periods ago. From equation (6) it follows that one can use the cohort average prices $\Lambda_t(s)$ to express the aggregate price level as

$$P_t^{1-\theta} = \sum_{s=0}^{\infty}(1-\delta)^s\delta\Lambda_t(s), \tag{14}$$

where δ is the mass of firms that experience a δ-shock each period and $(1-\delta)^s$ is the share of those firms that have not undergone another δ-shock for s periods.

To express the evolution of P_t in a recursive form, consider the optimal price $P^\star_{t-s,t}$ of a firm that sustained a δ-shock $s > 0$ periods ago, but can adjust the price in t due to the occurrence of a Calvo shock. Also, consider the price $P^\star_{t,t}$ of a firm where a δ-shock occurs in period t. The optimal price setting equation (10) then implies

$$P^\star_{t,t} = P^\star_{t-s,t}\left(\frac{g_t \times \cdots \times g_{t-s+1}}{q_t \times \cdots \times q_{t-s+1}}\right). \tag{15}$$

The previous equation shows that a stronger cohort productivity trend (higher values for q) causes the firm that experiences a δ-shock in period t to choose lower prices relative to firms that experienced δ-shocks further in the past, as a stronger cohort trend makes this firm relatively more productive. Conversely, the experience effect (higher values for g) increases the optimal relative price of the firm that underwent a δ-shock in t. The net effect depends on the relative strength of the cohort versus the experience effect.

Appendix A.4 shows how to combine equations (13), (14), and (15) to obtain a recursive representation for the evolution of the aggregate price level given by

$$P_t^{1-\theta} = \delta(P_{t,t}^\star)^{1-\theta} + (1-\alpha)(1-\delta)\frac{(p_t^e)^{\theta-1} - \delta}{1 - \delta}(P_{t,t}^\star)^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t}P_{t-1})^{1-\theta}, \quad (16)$$

where p_t^e summarizes the history of shocks to cohort and experience productivity and evolves recursively according to

$$(p_t^e)^{\theta-1} = \delta + (1-\delta)\left(p_{t-1}^e g_t/q_t\right)^{\theta-1}. \quad (17)$$

The last term on the r.h.s. of equation (16) captures the price-level effects from the share $\alpha(1-\delta)$ of firms that experienced neither a Calvo shock nor a δ-shock. These firms keep their old price (P_{t-1} on average), adjusted for possible effects of price indexation, as captured by the indexation term $\Xi_{t-1,t}$. The first term on the r.h.s. of equation (16) captures the price effects of the mass δ of firms that experienced a δ-shock in period t; these firms optimally charge price $P_{t,t}^\star$. The second term captures the average price of firms that experienced a Calvo shock in period t; their share is $(1-\alpha)(1-\delta)$ and they set a price that on average differs from the price charged by firms hit by a δ-shock, depending on the value of p_t^e. This latter aspect in equation (16) is the key difference relative to the standard model without firm heterogeneity in productivity. A stronger experience trend (a higher value for g_t), for instance, increases $(p_t^e)^{\theta-1}$, and - ceteris paribus - causes firms hit by a Calvo shock to choose a lower value for the optimal reset price. A stronger cohort trend (a higher value for q_t) has the opposite effect. Overall, the interesting new feature is that price dynamics now depend on the productivity trends.

In a setting where all firms have identical productivity, e.g., where the cohort effect is as strong as the experience effect ($q_t = g_t$ for all t), equation (17) implies that p_t^e converges to one, causing the price level to eventually evolve according to

$$P_t^{1-\theta} = [\delta + (1-\alpha)(1-\delta)]\,(P_{t,t}^\star)^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t}P_{t-1})^{1-\theta},$$

which is independent of productivity developments at the firm level. If in addition there are no δ-shocks ($\delta = 0$), the previous equation simplifies further to

$$P_t^{1-\theta} = (1-\alpha)(P_{t,t}^\star)^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1})^{1-\theta},$$

which describes the evolution of the aggregate price level in the standard Calvo model with homogeneous firms.

Aggregate production function. In appendix A.6 we show that aggregate output Y_t can be written as

$$Y_t = \frac{A_t Q_t}{\Delta_t}\left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t\right), \quad (18)$$

16

where K_t denotes the aggregate capital stock, L_t aggregate hours worked and

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right) \left(\frac{P_{jt}}{P_t} \right)^{-\theta} dj \tag{19}$$

evolves recursively according to

$$\Delta_t = \left[\delta + (1-\alpha)(1-\delta) \frac{(p_t^e)^{\theta-1} - \delta}{1-\delta} \right] \left(\frac{P_{t,t}^\star}{P_t} \right)^{-\theta} + \alpha(1-\delta) \left(\frac{q_t}{g_t} \right) \left(\frac{\Pi_t}{\Xi_{t-1,t}} \right)^\theta \Delta_{t-1}. \tag{20}$$

TFP in the aggregate production function (18) is a function of the TFP of the latest cohort hit by the δ-shock, $A_t Q_t$, and of the adjustment factor Δ_t. The latter is defined in equation (19) and captures a firm's productivity relative to that of the latest cohort, $Q_t / (Q_{t-s_{jt}} G_{jt})$, and weights this relative productivity with the firm's production share $(P_{jt}/P_t)^{-\theta}$. Equations (18) and (19) thus show how relative price distortions may lead to aggregate output losses by negatively affecting aggregate technology, e.g., by allocating more demand to relatively inefficient firms. The evolution of the adjustment factor over time is described by equation (20) and depends on firm-level productivity trends - amongst other ways - through the variable p_t^e. In the limit with homogeneous firm trends (i.e., $q_t = g_t$), p_t^e converges to one and the evolution of Δ_t becomes independent of productivity realizations. If - in addition - there are no δ-shocks ($\delta = 0$), then equation (20) simplifies further to

$$\Delta_t = (1-\alpha) \left(\frac{P_{t,t}^\star}{P_t} \right)^{-\theta} + \alpha \left(\frac{\Pi_t}{\Xi_{t-1,t}} \right)^\theta \Delta_{t-1},$$

the equation which captures the potential distortions from price dispersion within the standard homogeneous-firm model.

Balanced Growth Path. One can obtain stationary aggregate variables by rescaling them by the aggregate growth trend

$$\Gamma_t^e = (A_t Q_t / \Delta_t^e)^\phi, \tag{21}$$

where Δ_t^e denotes the efficient adjustment factor chosen by the planner, defined in equation (25) below. Specifically, the rescaled output $y_t = Y_t / \Gamma_t^e$ and the rescaled capital stock $k_t = K_t / \Gamma_t^e$ are now stationary and the aggregate production function (18) can be written as

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t} \right) \left(k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - f \right). \tag{22}$$

In the deterministic balanced growth path, the (gross) trend growth rate $\gamma_t^e = \Gamma_t^e / \Gamma_{t-1}^e$ is constant and equal to $(aq)^\phi$ and hours worked are constant whenever monetary policy implements a constant inflation rate. Appendices A.7 and A.8 write all model equations using stationary variables only and appendix A.9 determines the resulting deterministic steady state.

5 Efficiency of the Flexible-Price Equilibrium

This section derives the efficient allocation and provides conditions under which the flexible-price equilibrium is efficient. Appendix B shows that the efficient consumption, hours and capital allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ solves

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1 - \sigma} \right) \tag{23}$$

$$s.t. \quad C_t + K_{t+1} = (1 - d)K_t + \frac{A_t Q_t}{\Delta_t^e} \left((K_t)^{1-\frac{1}{\phi}} (L_t)^{\frac{1}{\phi}} - F_t \right), \tag{24}$$

where

$$\Delta_t^e \equiv \left(\int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right)^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}, \tag{25}$$

which evolves according to

$$(\Delta_t^e)^{1-\theta} = \delta + (1 - \delta) \left(\Delta_{t-1}^e q_t / g_t \right)^{1-\theta}. \tag{26}$$

Constraint (24) is the economy's resource constraint, when expressing aggregate output using the aggregate production function (18). The efficient productivity adjustment factor Δ_t^e showing up in the planner's production function is defined in equation (25); its recursive evolution is described by equation (26). The first-order conditions of problem (23)-(24) shown in appendix B are necessary and sufficient conditions characterizing the efficient allocation.

Decentralizing the efficient allocation requires that firms' prices, which enter Δ_t and thus in the aggregate production function (18), satisfy certain conditions. In particular, equation (19) implies that $\Delta_t = \Delta_t^e$ is achieved if prices satisfy

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}. \tag{27}$$

The previous equation requires relative prices to accurately reflect relative productivities. Furthermore, as in models without firm heterogeneity, one has to eliminate firms' monopoly power by a Pigouvian subsidy to obtain efficiency of market allocation. We thus impose the following condition:

Condition 1 *The sales subsidy corrects firms' market power, i.e., $\frac{\theta}{\theta-1} \frac{1}{1+\tau} = 1$.*

Appendix C then proves the following result:

Proposition 1 *The flexible-price equilibrium ($\alpha = 0$) is efficient if condition 1 holds.*

The proof of the proposition shows that condition (27) holds under flexible prices, so that one achieves $\Delta_t = \Delta_t^e$ and thereby productive efficiency. In the presence of the assumed sales subsidy, consumer decisions are also undistorted, which means the values of consumption, hours and capital in the flexible-price equilibrium are identical to the values that these variables assume in the efficient allocation.

6 Optimal Inflation with Sticky Prices

This section determines the optimal inflation rate for an economy with sticky prices ($\alpha > 0$). It derives the optimal rate of inflation for the nonlinear stochastic economy with heterogeneous firms in closed form and shows how inflation optimally depends on the productivity growth rates a_t, q_t and g_t. As it turns out, the optimal inflation rate implements the efficient allocation (the flexible-price benchmark).

To establish our main result in the most straightforward manner, we impose an assumption on initial conditions, in particular on how firms' initial prices and initial productivities are related. Similar conditions are imposed in sticky-price models with homogeneous firms, where it is routinely assumed that initial dispersion of prices has reached its stationary outcome. We impose:

Condition 2 *Initial prices in $t = -1$ reflect firms' relative productivities, i.e.,*

$$P_{j,-1} \propto \frac{1}{Q_{-1-s_{j,-1}} G_{j,-1}} \qquad \text{for all } j \in [0,1].$$

We discuss the effects of relaxing this condition below. The following proposition states our main result:

Proposition 2 *Suppose conditions 1 and 2 hold. The equilibrium allocation in the sticky-price economy is efficient if monetary policy implements the gross inflation rate*

$$\Pi_t^\star = \Xi_{t-1,t}^\star \left(\frac{1 - \delta \left(\Delta_t^e \right)^{\theta-1}}{1-\delta} \right)^{\frac{1}{\theta-1}} \qquad \text{for all } t \geq 0, \tag{28}$$

where $\Xi_{t-1,t}^\star$ captures price indexation between periods $t-1$ and t ($\Xi_{t-1,t}^\star \equiv 1$ in the absence of indexation) and Δ_t^e is defined in equation (25) and evolves according to equation (26).

In the absence of price indexation ($\Xi_{t-1,t}^\star \equiv 1$), the optimal inflation rate $\Pi_t^\star$ is only a function of the variable Δ_t^e, which captures the distribution of relative productivities between all firms firms and those with a δ-shock; see equation (25). Since these relative productivities are independent of the common TFP growth rate a_t, it follows that the optimal inflation rate does not depend on the realizations of a_t. In contrast, the cohort productivity growth rate q_t and the experience growth rate g_t do affect Δ_t^e, see equation (26). Yet, these trends affect the optimal inflation rate in opposite directions: a stronger cohort productivity growth rate q_t decreases the optimal inflation rate, while a stronger experience growth rate g_t increases the optimal inflation rate.

For the special case in which all firms have identical productivity trends ($\delta = 0$ or $g_t = q_t$) or even identical productivities ($\Delta_t^e = 1$), the optimal gross inflation rate is equal

to one in the absence of price indexation, as in a standard homogeneous-firm model. Perfect price stability is then optimal at all times.

Price indexation by non-adjusting firms ($\Xi^{\star}_{t-1,t} \neq 1$), say because of indexation to the lagged inflation rate, introduces additional components into the optimal aggregate inflation rate. In particular, it requires that price-adjusting firms, i.e., firms hit by either a δ-shock or a Calvo shock, also adjust their price by the indexation component. This way prices continue to accurately reflect relative productivities at all times. This explains why indexation affects the optimal inflation rate one-for-one.

Although proposition 2 assumes that firms' initial prices accurately reflect the initial relative productivities, the initial productivity distribution itself is unrestricted. We conjecture that for a setting where condition 2 fails to hold, one would obtain additional transitory and deterministic components to the optimal inflation rate, as in the homogeneous firm setting studied by Yun (2005). The inflation rate stated in proposition 2 would then become optimal only asymptotically.

The proof of proposition 2, which is contained in appendix D, establishes that with the optimal inflation rate firms choose relative prices as in the flexible-price equilibrium. This result is established by showing that (1) firms hit by a δ-shock choose the same optimal relative price as in the flexible price economy, and that (2) firms hit by a Calvo shock optimally choose not to adjust their price, which avoids the emergence of price dispersion between otherwise identical firms. This, together with the fact that (3) initial prices reflect initial productivities, ensures that all relative prices are identical to those in the flexible-price equilibrium. Under the assumed output subsidy, it then follows that household allocations are also identical to the flexible-price equilibrium, which has been shown to be efficient; see proposition 1.

Interestingly, it follows from the proof of proposition 2 that the inflation rate (28) continues to ensure productive efficiency (but not full efficiency) in settings where condition 1 fails to hold. From the theory of optimal taxation it then follows that it remains optimal to implement the inflation rate (28), as it is suboptimal to distort intermediate production as long as (distortionary) taxes on final goods are available.

7   The Optimal Steady-State Inflation Rate

This section discusses the optimal steady-state inflation rate implied by the model. To simplify the discussion, we abstract from price indexation, unless otherwise stated.

Proposition 2 makes it clear that in the case in which the productivity of all firms grows at the same rate ($\delta = 0$), which includes as a special case a setting with homogeneous firms, we obtain that the optimal inflation rate $\Pi^{\star}_t = 1$, independently of all shock processes. For $\delta = 0$, the optimal (gross) steady-state inflation rate is thus trivially equal to one.

For the case $\delta > 0$, the optimal steady-state inflation rate jumps discontinuously away from $\Pi_t^\star = 1$, but turns out to be itself independent of the value of δ.[18] The following lemma summarizes this result:

Lemma 1 *Suppose conditions 1 and 2 hold, there are no economic disturbances, there is no price indexation ($\Xi_{t-1,t}^\star \equiv 1$) and $\delta > 0$. The optimal inflation rate then satisfies*

$$\lim_{t\to\infty} \Pi_t^\star = g/q. \tag{29}$$

Proof. From equations (7) and (26) it follows that $(\Delta_t^e)^{\theta-1} \to [1 - (1-\delta)\,(g/q)^{\theta-1}]/\delta$. It then follows from proposition 2 that $\lim_{t\to\infty} \Pi_t^\star = g/q$. ∎

Since we allow for arbitrary initial productivity distributions, the absence of shocks does not necessarily imply that the optimal inflation rate is constant from the beginning. This only happens asymptotically, once the productivity distribution converges to its stationary distribution (in detrended terms).[19] The lemma provides the inflation rate that is asymptotically optimal as this stationary distribution is reached.[20]

Interestingly, the optimal long-run inflation rate is completely independent of the intensity of δ-shocks, which may appear surprising. To understand the source of this invariance, consider a setting where δ-shocks capture firm exit and entry events and where $g > q$, so that entering firms are smaller and less productive than the set of non-exiting firms. A higher value for δ implies that more young and relatively unproductive firms are amongst the set of price-setting firms. This calls - ceteris paribus - for a higher inflation rate. Yet, the productivity distribution of non-exiting firms is not invariant to changes in δ: a higher δ also implies more firm turnover and thus less experience accumulation. Non-exiting firms thus tend to be less productive relative to new entrants, which calls for lower inflation rates. In net terms, these two effects exactly cancel each other.

On empirical grounds, it appears plausible to assume $g > q$, so that according to lemma 1 the optimal steady-state inflation rate is positive. For the case with firm turnover, the fact that young firms are small relative to old firms requires $g > q$. Likewise, interpreting δ-shocks as representing product substitution shocks, the case $g > q$ implies that new products are relatively more expensive than old products and that their relative price is falling over the life cycle of the product. Both of these facts are in line with evidence

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[18]Note that the efficient allocation also discontinuously jumps when moving from $\delta = 0$ to $\delta > 0$, as in the former case efficient aggregate growth is equal to $(ag)^\phi$ and in the latter case it is equal to $(aq)^\phi$ in steady state. Appendix E shows that the discontinuity of the optimal steady-state inflation rate is not due to the discontinuity of the efficient real allocation.

[19]This is not an issue when $\delta = 0$: the initial distribution then remains unchanged (in detrended terms).

[20]The transitional dynamics can easily be derived from proposition 2 using the initial productivity distribution and equation (26).

provided by Melser and Syed (2016) but would not be obtained if we assumed $g < q$. Thus, while the setup allows the optimal steady-state inflation rate to be potentially negative, these considerations suggest positive inflation to be optimal in steady state.

Interestingly, aggregate productivity dynamics turn out not to be informative about the optimal inflation rate. The aggregate steady-state growth is equal to $(aq)^\phi$ and is driven by a factor that affects the optimal inflation rate (q) and a factor that does not affect it (a). Moreover, the experience effect (g) has no aggregate growth rate implications, but affects the optimal inflation rate. Determining the optimal inflation rate thus requires studying the firm-level productivity trends g and q, as aggregate productivity fails to identify the inflation-relevant productivity trends. We shall come back to this issue in our empirical section 11.

Finally, we discuss the effects of price indexation. For $\delta > 0$ the optimal long-run inflation rate is then given by $\Xi^\star_{t-1,t}(g/q)$. For the case where prices are indexed to lagged inflation according to $\Xi^\star_{t-1,t} = \left(\Pi^\star_{t-1}\right)^\kappa$ for some $\kappa \in [0,1)$, we obtain

$$\lim_{t \to \infty} \Pi^\star_t = (g/q)^{\frac{1}{1-\kappa}}.$$

Standard forms of price indexation thus amplify the divergence of the optimal gross inflation rate from one.

8 The Welfare Costs of Strict Price Stability

This section shows that suboptimally implementing strict price stability, as suggested by sticky-price models with homogeneous firms, gives rise to strictly positive welfare costs whenever $g \neq q$. We derive this fact first analytically for a special case. The analytic result allows us to consider also the limit $\delta \to 0$. In a second step, we use numerical simulations to highlight the source of the welfare losses and their magnitude.

The following proposition shows that - as long as $g \neq q$ - there is a strictly positive welfare loss that is bounded away from zero when implementing strict price stability; this holds true even for the limit $\delta \to 0$.[21]

Proposition 3 *Suppose conditions 1 and 2 hold, there are no economic disturbances, $\delta > 0$, fixed costs of production are zero ($f = 0$), there is no price indexation ($\Xi^\star_{t-1,t} \equiv 1$), and the disutility of work is given by*

$$V(L) = 1 - \psi L^\nu,$$

with $\nu > 1$ and $\psi > 0$. Assume $g/q > \alpha(1 - \delta)$, so that a well-defined steady state with strict price stability exists.

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[21]The proof of the proposition is contained in appendix F.

A. Relative cohort price:
cohort mean, different inflation rates

B. Relative cohort price: mean and 2 std.dev. bands
0% inflation rate.
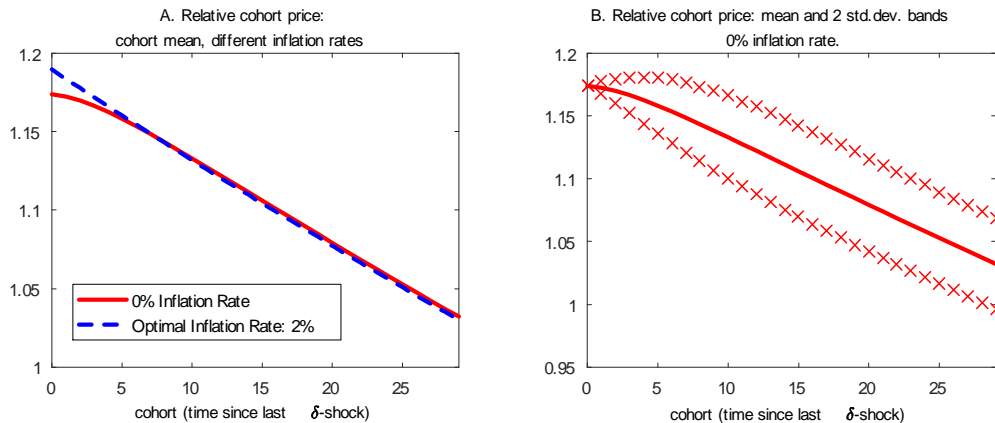
Figure 2: Relative prices and inflation

Consider the limit $\beta(\gamma^e)^{1-\sigma} \to 1$ and a policy implementing the optimal inflation rate $\Pi_t^\star$ from proposition 2, which satisfies $\lim_{t\to\infty} \Pi_t^\star = \Pi^\star = g/q$. Let $c(\Pi^\star)$ and $L(\Pi^\star)$ denote the limit outcomes for $t \to \infty$ for consumption and hours, respectively, under this policy. Similarly, let $c(1)$ and $L(1)$ denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

$$L(1) = L(\Pi^\star)$$

and

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{1 - \alpha(1-\delta)(g/q)^{\theta-1}}{1 - \alpha(1-\delta)} \right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1 - \alpha(1-\delta)(g/q)^{-1}}{1 - \alpha(1-\delta)(g/q)^{\theta-1}} \right)^{\phi} \leq 1. \qquad (30)$$

For $g \neq q$ the previous inequality is strict and $\lim_{\delta\to 0} c(1)/c(\Pi^\star) < 1$.

We now illustrate the nature of the relative price distortions that are generated by suboptimal rate of inflation and how they give rise to welfare losses. Panel A in figure 2 reports the mean cohort price (relative to the price of all firms), depicted on the y-axis, as a function of the cohort age in quarters (x-axis). It does so once for a setting where monetary policy implements a 2% inflation rate annually (in net terms) and once when monetary policy pursues strict price stability. The assumed optimal inflation rate is thereby 2%.[22] Panel A shows that young cohorts charge a higher (relative) price and that this price decreases over the lifetime of the cohort. Under the optimal inflation rate (2%) the decline happens at a constant rate.[23] Under strict price stability, firms anticipate that their relative prices will not necessarily fall, due to Calvo price stickiness. This causes them to initially "front load" prices, i.e., in an environment with strict price stability

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[22]Figure 2 is computed using $g = 1.02^{0.25}, q = 1, \alpha = 0.75, \delta = 0.035, \theta = 3.8$ and assumes that the initial productivity distribution is equal to the stationary distribution (in detrended terms).

[23]The figure assumes that no shocks hit the economy.

young cohorts charge initially lower prices than under the optimal inflation rate. Over time, some firms in the cohort will get the opportunity to lower their prices in response to Calvo shocks, but the average relative price of the cohort will eventually be slightly higher than under the optimal inflation rate. Beyond these distortions in average cohort prices, the suboptimal inflation rate also generates prices distortions within a cohort of firms. This is illustrated in panel B of figure 2. Panel B reports the mean cohort price and the +/-2 standard deviation bands of the cross-sectional price distribution within the cohort, assuming monetary policy pursues strict price stability. It shows that suboptimal inflation not only gives rise to distortion in mean prices but also to substantial amounts of price dispersion within the cohort. Under the optimal inflation rate, price dispersion is zero at the cohort level.

Figure 3 reports the steady-state value for the ratio Δ_t^e/Δ_t (y-axis) as a function of the implemented steady-state inflation rate (x-axis), when the optimal inflation rate is 2% per year.[24] The aggregate production function (18) shows that one can interpret Δ_t^e/Δ_t as a measure of the aggregate productivity distortion that is implied by the relative price distortions associated with suboptimal inflation rates.[25] The figure shows that a 10% shortfall of the inflation rate below its optimal value of 2% is associated with an aggregate productivity loss equal to about 1%. In the process, the productivity losses arise rather nonlinearly: a shortfall of inflation of 2% below its optimal value is associated with an aggregate productivity loss of just 0.05%. Furthermore, inflation losses are asymmetric, with above-optimal inflation leading to relatively larger losses. For instance, increasing inflation 8% above its optimal value generates a productivity loss of 0.94%, while decreasing inflation by the same amount below its optimal value leads to a productivity losses of only 0.37%.

9   The Variance of the Optimal Inflation Rate

This section discusses the optimal dynamic response of inflation to productivity disturbances. In the absence of δ-shocks, we obtain from proposition 2

$$\Pi_t^\star = 1 \text{ for all } t,$$

i.e., the optimal inflation rate is then independent of productivity shocks at all times. For $\delta > 0$, it follows from equations (26) and (28) that the optimal nonlinear inflation response to productivity disturbances is given by

$$\frac{1}{1 - (1 - \delta)\left(\frac{\Pi_t^\star}{\Xi_{t-1,t}}\right)^{\theta-1}} = 1 + \frac{(1 - \delta)\left(\frac{g_t}{q_t}\right)^{\theta-1}}{1 - (1 - \delta)\left(\frac{\Pi_{t-1}^\star}{\Xi_{t-2,t-1}}\right)^{\theta-1}}. \tag{31}$$

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[24] The figure is based on the same parameterization as figure 2.

[25] Appendix F shows that $c(1)/c(\Pi^\star) = (\Delta^e/\Delta)^\phi$.
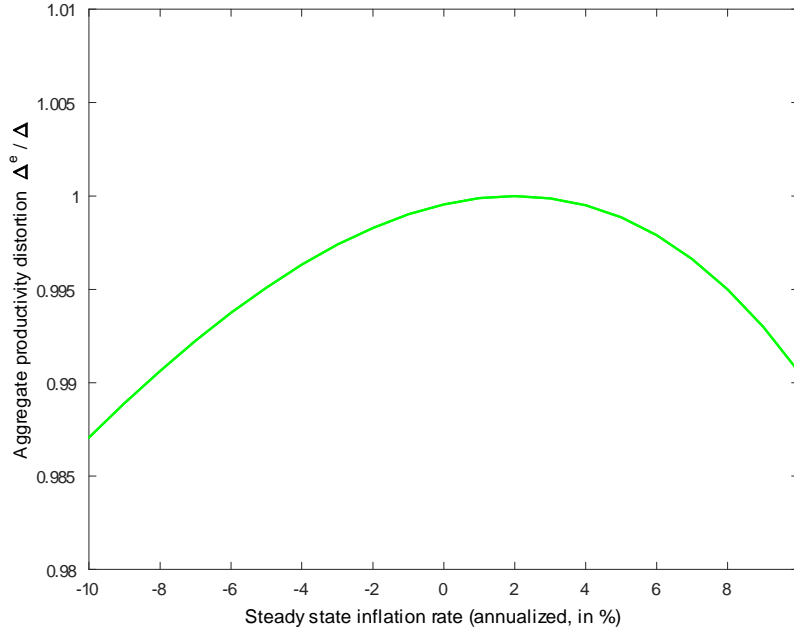
Figure 3: Aggregate productivity as a function of steady state inflation (optimal inflation rate is 2%)

Abstracting from price indexation ($\Xi_{t-1,t} \equiv 1$), a linearization of equation (31) delivers[26]

$$\pi_t^\star = (1 - \delta)\pi_{t-1}^\star + \delta \left(\frac{g_t}{q_t} - 1 \right). \tag{32}$$

As long as $\delta < 1$, the optimal inflation rate will thus display persistent responses to any deviation of g_t/q_t from its average value. A positive surprise to experience productivity growth g_t, for example, shifts up permanently the experience level of old cohorts. This requires persistently higher inflation rates, as new cohorts (firms hit by a δ-shock) now have to keep raising their prices continuously until the productivity distribution returns to its stationary distribution (in detrended terms). The speed with which the productivity distribution returns to its stationary distribution depends on δ. For $\delta = 1$, the return is immediate and the optimal inflation rate inherits the persistence properties of the exogenous driving process g_t/q_t. For values of δ close to zero, the optimal inflation rate approximately behaves like a random walk whenever g_t/q_t is an i.i.d. process, but the unconditional variance of inflation decreases with δ and approaches zero as $\delta \to 0$.

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[26]We log-linearize with respect to the variables $\Pi_t^\star$ and g_t/q_t at the point $(\Pi_t^\star, g_t/q_t) = (1, 1)$.

10 Extension to a Multi-Sector Economy

We now extend the basic sticky-price setup to a multi-sector setting that allows (inter alia) for sector-specific productivity trends and sector-specific price stickiness. Such an extension is relevant when seeking to bring the model to the data, as we do in the next section, because productivity trends and price stickiness tend to be different across the manufacturing and the service sectors. We show below that the optimal steady-state inflation rate in a multi-sector economy is a weighted average of the inflation rates that would achieve efficiency in the respective sectors individually.

Consider an economy with $z = 1, \ldots, Z$ sectors in which aggregate output Y_t is

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z},$$

with Y_{zt} denoting output in sector z and $\psi_z \geq 0$ being the sector's expenditure share, with the expenditure shares satisfying $\sum_{z=1}^{Z} \psi_z = 1$. Sectoral output Y_{zt} itself is a Dixit-Stiglitz aggregate of the output of a unit mass of firms j in sector z, in close analogy to the one-sector setup.

Let A_{zt} denote the TFP component, Q_{zt} the cohort-specific component and G_{jzt} the experience component to firm productivity in sector $z = 1, \ldots, Z$. Output of the firm producing product $j \in [0,1]$ in sector z is then given by

$$Y_{jzt} = A_{zt} Q_{t-s_{jzt}} G_{jzt} \left(K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}} - F_{zt} \right),$$

where s_{jzt} denotes the number of periods since the last δ-shock, K_{jzt} employed capital, L_{jzt} employed labor, and $F_{zt} \geq 0$ a sector-specific fixed cost of production. The sector-specific growth rates of A_{zt}, Q_{zt} and G_{jzt} are given by $a_{zt} = a_z \epsilon_{zt}^a$, $q_{zt} = q_z \epsilon_{zt}^q$ and $g_{zt} = g_z \epsilon_{zt}^g$, respectively, where a_z, q_z and g_z denote the steady-state growth trends. We also allow for sector-specific degrees of price stickiness $\alpha_z \in (0,1)$ and for sector-specific δ-shock intensities $\delta_z \in (0,1)$. We do not introduce additional sectors specific frictions, i.e., we assume the existence of a common capital and labor market.

The aggregate price level is defined as

$$P_t = \prod_{z=1}^{Z} \left(\frac{P_{zt}}{\psi_z} \right)^{\psi_z},$$

where the sectoral price level P_{zt} depends on the prices charged by firms in sector z in the same way as in the one-sector economy; see equation (6). Further details of the multi-sector economy are provided in a separate technical appendix, jointly with the proof of the following proposition.

Proposition 4 *Suppose condition 1 holds, there are no economic disturbances, there is no price indexation ($\Xi^\star_{t-1,t} \equiv 1$), there is positive δ-shock intensity in all sectors ($\delta_z > 0$ for all $z = 1, \ldots Z$), and the discount factor approaches $\beta(\gamma^e)^{1-\sigma} \to 1$, where $\gamma^e = \prod_{z=1}^{Z} (a_z q_z)^{\psi_z \phi}$ denotes the growth trend of the aggregate economy. Suppose monetary policy implements the inflation rate $\Pi_t = \Pi$ for all t. The inflation rate Π^* that maximizes utility in the steady state of the multi-sector economy is*

$$\Pi^\star = \sum_{z=1}^{Z} \omega_z \left(\frac{g_z}{q_z} \frac{\gamma^e_z}{\gamma^e} \right), \tag{33}$$

where

$$\frac{\gamma^e_z}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^{Z} (a_z q_z)^{\psi_z}}$$

denotes the growth trend of sector z relative to the growth trend of the aggregate economy. The sectoral weights $\omega_z \geq 0$ are given by

$$\omega_z = \frac{\tilde{\omega}_z}{\sum_{z=1}^{Z} \tilde{\omega}_z},$$

with

$$\tilde{\omega}_z = \frac{\psi_z \theta \alpha_z (1 - \delta_z)(\gamma^e/\gamma^e_z)^\theta (\Pi^\star)^\theta (q_z/g_z)}{[1 - \alpha_z(1-\delta_z)(\gamma^e/\gamma^e_z)^\theta (\Pi^\star)^\theta (q_z/g_z)] \, [1 - \alpha_z(1-\delta_z)(\gamma^e/\gamma^e_z)^{\theta-1}(\Pi^\star)^{\theta-1}]}.$$

The proposition shows that the result from the one-sector economy naturally extends to a multi-sector setup. The main new element consists of the fact that the sector-specific optimal inflation rates g_z/q_z need to be rescaled by the sectors' relative growth trends γ^e_z/γ^e. This implies that the sector-specific optimal inflation rate g_z/q_z is scaled upwards for faster growing sectors ($\gamma^e_z/\gamma^e > 1$) and scaled downwards for sectors that grow slower than the aggregate economy.

Since proposition 4 provides a result specifying the inflation rate that maximizes utility in the limiting steady state, rather than a result about the limit of the optimal inflation rate itself, we do not have to impose condition 2, unlike in proposition 2. Furthermore, unlike in proposition 1, the optimal inflation rate fails to implement the first-best allocation, which would generally require different inflation rates for different sectors. The limiting condition $\beta(\gamma^e)^{1-\sigma} \to 1$ is required in proposition 4 to ensure that the utility losses due to aggregate markup distortions and those due to relative price distortions are minimized by the same inflation rate, namely the one given in the proposition. Absent this condition, minimizing these distortions individually would call for different inflation rates. One could then use sector-specific output subsidies to undo the sectoral markup distortions. The inflation rate $\Pi^\star$ stated in proposition 4 is then optimal even if $\beta(\gamma^e)^{1-\sigma}$ is strictly smaller than unity.

For the special case with $a_z q_z = aq$, which implies that $\gamma_z^e = \gamma^e$, and $g_z/q_z = g/q$, we obtain from equation (33) that $\Pi^\star = g/q$, which is the result for the one-sector economy stated in lemma 1. In this special case, the central bank does not face a trade-off between different sector-specific optimal inflation rates and can achieve the first-best allocation in the resulting steady state of the multi-sector economy, despite the presence of sector-specific degrees of price stickiness and sector-specific δ-shock intensities.

Since the closed-form expressions for the sector weights ω_z in proposition 4 are difficult to interpret and also depend on the optimal inflation rate, the subsequent lemma shows that these weights are - to a first order approximation - equal to the sector's expenditure weights ψ_z:

Lemma 2 *The optimal steady-state inflation rate in the multi-sector economy is equal to*

$$\Pi^* = \sum_{z=1}^{Z} \psi_z \left(\frac{g_z \gamma_z^e}{q_z \gamma^e} \right) + O(2), \tag{34}$$

where $O(2)$ denotes a second order approximation error and where the approximation to equation (33) has been taken around a point, in which $\frac{g_z}{q_z} \frac{\gamma_z^e}{\gamma^e}$ and $\alpha_z (1 - \delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$ are constant across sectors $z = 1, \ldots Z$.

Interestingly, the optimal steady-state inflation rate turns out to be (to first order) independent of the sector-specific degree of price stickiness (α_z), unlike in Benigno (2004). This happens because the point of approximation is chosen such that sector-specific productivity trends and the effects of sector-specific price stickiness cancel each other. Our result then shows that to a first order approximation, the optimal inflation rate remains independent of α_z in the neighborhood of this point. This contrasts with the effects of sector-specific firm-level productivity trends (q_z/g_z), which do have first order implications for the optimal steady-state inflation rate.

11   The Optimal Inflation Rate for the US Economy

This section quantifies the optimal inflation rate for the US economy using the multi-sector setup presented in the previous section. The next section explains our empirical approach and the subsequent section presents the estimation results.

11.1   Empirical Strategy

To quantify the optimal inflation rate implied by microeconomic productivity trends, one would ideally estimate these trends directly at the firm or establishment level. Yet, it is

generally difficult to measure physical productivity at the firm or establishment level because output prices are not widely observed at this level of observation.[27] Given this, we proceed by using establishment-level employment trends to estimate the establishment-level productivity trends. Employment and productivity are related to each other via the elasticity of product demand ($\theta - 1$), which maps any firm-level productivity (and associated product price) difference into an employment difference.[28] Clearly, to the extent that firms face additional constraints for expanding production beyond being insufficiently productive/competitive (e.g., financial constraints, adjustment costs, regulatory constraints), there may be biases in the productivity trends that are estimated from employment trends. To deal with this concern, we shall mainly look at changes in the estimated trends over time, which should remove any fixed effects that arise from other frictions affecting firm-level employment.

We estimate the employment trends using information provided by the Longitudinal Business Database (LBD) of the US Census Bureau. The LBD reports establishment-level employment data and covers all US establishments at an annual frequency. Coverage starts in the year 1976 and we use data up the year 2013. For this period, there are a total of 176 million employment observations at the establishment level. Working with this data source, we shall interpret δ-shocks as an event in which the establishment is closed down, in line with the "firm entry and exit" interpretation spelled out in section 3.2. The variable s_{jzt} can then be interpreted as the establishment age in sector z. Using the multi-sector setup from the previous section, we then can derive a model-implied relationship between establishment-level employment, establishment age, both of which are observed in the LBD, and the productivity trends of interest:[29]

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[27] As explained in Foster, Haltiwanger and Syverson (2008), the productivity literature usually measures revenue productivity instead of physical productivity at the firm level, which deflates firm-level output with some industry-level price index. In our setting, firms' revenue productivity is completely unrelated to their physical productivity in the absence of fixed costs of production. For the few industries for which physical and revenue productivities can both be observed, the two productivity measures can be rather different; see Foster, Haltiwanger and Syverson (2008).

[28] An alternative approach would consist in considering product-level price data, e.g., the price information entering into the construction of the CPI, as used for example in Nakamura and Steinsson (2008). The results documented in Bils (2009) and Moulton and Moses (1997) show that the inflation rate of so-called "forced substitution" items, i.e., items which become permanently unavailable and are replaced by other "new" items, is significantly larger than that of so-called matched items, which are products that continue to be available. Our model implies that one can infer the inflation-relevant trend g/q from the inflation difference in these two-item categories. However, this would require making accurate quality adjustments in the computation of the inflation rate for forced substitution items, which is a task that is difficult to achieve.

[29] The proof of the following proposition can be found in a separate technical appendix which also spells out the details of the multi-sector setup.

Proposition 5 *Suppose that $\sum_{i=0}^{t} \ln(\epsilon_{zi}^q/\epsilon_{zi}^g)$ is a stationary process in t and that fixed costs are equal to zero ($f_z = 0$). Employment L_{jzt} of the firm producing product j in sector z at time t in the flexible-price equilibrium is then equal to*

$$\ln(L_{jzt}) = d_{zt} + \eta_z \cdot s_{jzt} + \epsilon_{jzt}, \qquad (35)$$

where d_{zt} denotes a sector-specific time dummy, s_{jzt} the age of the firm and ϵ_{jzt} a stationary residual term. The regression coefficient η_z is given by

$$\eta_z = (\theta - 1) \ln(g_z/q_z).$$

The requirement that $\sum_{i=0}^{t} \ln(\epsilon_{zi}^q/\epsilon_{zi}^g)$ is stationary is essential for obtaining stationarity of the regression residuals ϵ_{jzt}. It is satisfied, for instance, if $\ln Q_{zt}$ and $\ln G_{jzt}$ are both trend stationary or non-stationary but co-integrated processes. The coefficient of interest, η_z, captures the inflation-relevant productivity trends, multiplied by elasticity of product demand $(\theta - 1)$.

Note that proposition 5 derives a property about firm-level employment in the flexible-price equilibrium. In the one-sector economy, the optimal inflation rate replicates the flexible-price equilibrium exactly, indicating that the flexible-price employment would indeed be observed in equilibrium under optimal monetary policy. This fails to be exactly true in the multi-sector economy or when policy is conducted in a suboptimal way. Relative price distortions associated with sticky prices then generally affect the equilibrium distribution of firm-level employment. Since these distortions do not tend to systematically vary with firm age, they will likely get absorbed by the firm-level residuals in equation (35). Moreover, since we identify trends by looking at yearly observations, the more short-lived effects of price stickiness at the firm level can plausibly be expected to be averaged out.

Note also that equation (35) holds whenever fixed costs of production are zero. From the proof of proposition 5 it follows that further nonlinear terms in age can show up on the right-hand side of equation (35), for strictly positive fixed costs. In our empirical analysis, we therefore explore the robustness of the estimates η_z when including also age squared as a regressor on the right-hand side.

Equation (35) is of interest because it allows us to identify, when combined with information about the demand elasticity θ, the sector-specific relative productivity trends g_z/q_z from establishment-level data. The values for g_z/q_z for all sectors together then determine - jointly with the sector specific relative growth trends γ_z^e/γ^e and information about sector size ψ_z - the optimal aggregate inflation rate, as implied by lemma 2.

To avoid censoring of the age variable when estimating η_z, we consider a restricted LBD sample that excludes the firms that are already present in the initial year (1976), for which age information is not available. Furthermore, we start estimating η_z from

the year 1986 onwards, so as to minimize any effects from having only young establishments in the sample during the initial years of the database. This leaves us with 147 million establishment-age observations. The mean age for this sample is 8.16 years, with a standard deviation of 7.05 years, which means that a wide range of age observations is covered. We then estimate equation (35) for 65 private BEA industries, which is the level of disaggregation at which sectoral GDP information is available. The sectoral GDP information allows us to compute the sector-specific relative growth trends γ_z^e/γ^e showing up in equation (34). To this end, we map the NAICS industries in the LBD database into the 65 private BEA industries (for the early part of the sample we map SIC codes). Finally, the GDP weights ψ_z are computed using sectoral GDP information for the year 2013. As another robustness exercise, we use time-varying weights ψ_{zt} using sectoral GDP information for each of the considered years $t = 1986, \ldots 2013$. Details of our approach are provided in appendix G.

11.2   Empirical Results

For the years 1986 to 2013, we report the time series

$$\Phi_t \equiv \sum_{z=1}^{65} \left(\psi_z \frac{\gamma_z^e}{\gamma^e} \right) \exp(\widehat{\eta}_{zt}), \tag{36}$$

where $\widehat{\eta}_{zt}$ is the time t OLS estimate for η_z, for sector $z = 1, \ldots 65$. The time series Φ_t is of interest, because it is proportional to the optimal steady-state inflation rate[30]

$$\Pi_t^\star - 1 = \frac{1}{\theta - 1}(\Phi_t - 1) + O(2), \tag{37}$$

where $\Pi_t^\star$ is the time t estimate of the steady-state inflation rate from lemma 2. Transforming Φ_t into an implied inflation rate thus requires us to take a stand on the value of the demand elasticity θ. For statements about the relative evolution of the optimal inflation rate, the value of θ does not matter. For the year 2007, which is the last year before the start of the financial crisis, we report in appendix G.3 detailed information on the cross-sectional estimates $\widehat{\eta}_z$, descriptive statistics for the various sectors, and the outcome of a robustness exercise. For the year 2007, we can also compute the estimation uncertainty about Φ_t, which turns out to be very small: the standard error is below one basis point.[31]

Figure 4 presents our baseline estimate for Φ_t. It shows that the optimal inflation rate is positive, in line with the empirical observation that older establishments tend to

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[30] This follows from $\exp((\theta-1)\ln(g_z/q_z)) = 1 + (\theta-1)(g_z/q_z - 1) + O(2)$ and $\sum_z \left(\psi_z \frac{\gamma_z^e}{\gamma^e} \right) = 1 + O(2)$.

[31] We use the delta method to compute the uncertainty about Φ_t assuming that the individual estimates $\widehat{\eta}_z$ are independent across z. The latter assumption is needed as we do not have information about the full covariance matrix for the vector consisting of the elements η_z, $z = 1, ..., 65$.
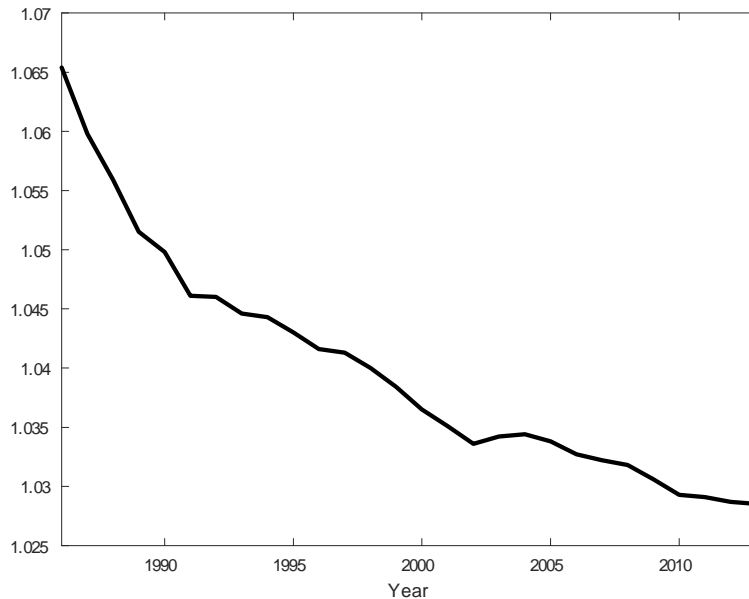
Figure 4: Baseline estimate of Φ_t (fixed 2013 sector weights, linear specification in age)

employ more workers on average ($g_z/q_z > 1$). It also shows that the optimal inflation rate dropped by approximately fifty percent over the period 1986 to 2013. The decline is rather steady over time and there are only weak indications for cyclical fluctuations. The drop in the estimate implies that either the experience trend in productivity weakened over the considered time period or the cohort productivity trend affecting new entrants accelerated.[32] While we cannot disclose the cross-sectional estimates for the year 2002, comparing 2002 estimates of η_z to the 2007 estimates reported in appendix G, we find that the decline in $\widehat{\eta}_z$ is widespread in most economic sectors and not driven by a small set of sectors experiencing very large declines.

Figure 5 investigates the robustness of the baseline estimates to alternative estimation approaches. As a first alternative, we consider weights ψ_{zt} that reflect the sectoral GDP share of each economic sector in period t rather than using fixed weights from the year 2013. This is motivated by the observation that the GDP shares of sectors have shifted considerably over the considered period. For instance, the share of manufacturing in private GDP dropped from 21.1 percent in 1986 to 14.0 percent in 2013. As figure 5 shows, using these time-varying weights leads to only negligible changes in the estimates. The sectoral rebalancing taking place in the US economy thus does not co-vary significantly with the changes in the $\widehat{\eta}_z$ (and thus g_z/q_z). This is consistent with the previous observation that the drop in η_z is present in almost all sectors of the US economy.

Figure 5 also presents estimates for Φ_t when additionally including a term in age

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[32]It is impossible to identify from the η_z estimates, which of the two effects actually drives the decline.
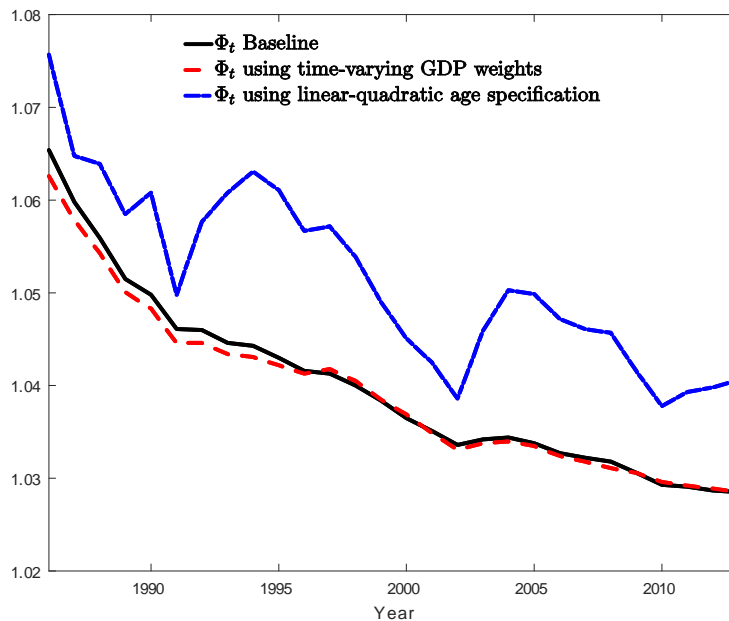
Figure 5: Robustness of Φ_t estimate to alternative estimation approaches

squared on the right-hand side of the regression equation (35).[33] The Φ_t estimates then become slightly larger in magnitude and also considerably more cyclical, displaying drops around the recession years 1991, 2001, and 2009. The overall message, however, remains unchanged: The optimal inflation rate is positive and approximately halved over the considered time period.

Translating the estimates presented in figures 4 and 5 into optimal inflation rates requires us to take a stand on the value of the demand elasticity parameter θ. As our baseline, we follow Bilbiie et al. (2012) and Bernard et al. (2003) who use $\theta = 3.8$ based on a calibration that fits US plant and macro trade data. As a robustness exercise, we also consider $\theta = 5$, based on a calibration in Eusepi et al. (2011) that fits the revenue labor share.[34]

Table 1 reports the outcomes for the optimal inflation rate over the sample period. It shows that, for most of the specifications, the optimal inflation rate dropped from a value of close to two percent in 1986 to approximately one percent in 2013. The optimal steady-state inflation rate thus appears to have dropped significantly over these 27 years.

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[33]The definition of Φ_t is still given by equation (36).

[34]Eusepi et al. (2011) also show that the average wholesale markup implies a demand elasticity of 5.1 for the industries in the 1997 Census of Wholesale Trade that Bils and Klenow (2004) could match to consumer goods in the CPI.

Table 1: Optimal Inflation Rate (Net)

	Baseline	TV Weights	LQ Specification	Baseline	TV Weights	LQ Specification
	$\theta = 3.8$			$\theta = 5$		
$\Pi^{\star}_{1986}$	2.34%	2.24%	2.70%	1.64%	1.57%	1.89%
$\Pi^{\star}_{2013}$	1.02%	1.02%	1.45%	0.71%	0.71%	1.01%

Notes: "Baseline" refers to the baseline estimate of Φ_t with fixed GDP weights and age as single regressor. "TV Weights" refers to the estimate of Φ_t that is based on time-varying GDP weights. "LQ Specification" refers to the estimate of Φ_t that is based on a specification with both age and age squared as regressors. The parameter θ denotes the product demand elasticity.

12 Extensions and Robustness of Results

This section considers various extensions and alternative model setups. Section 12.1 shows that our main finding about the optimal inflation rate (proposition 2) continues to apply in a setting where price adjustment frictions take the form of menu costs. Section 12.2 discusses the effects of introducing additional firm-specific productivity components.

12.1 Menu Cost Frictions

While our results are illustrated using time-dependent Calvo price-setting frictions, our main theoretical finding from proposition 2 extends to a setting in which firms have to pay a fixed cost to adjust their price. Since the optimal inflation rate in proposition 2 replicates the flexible-price allocation, firms that do not experience a δ-shock have - independently of the nature of their price setting frictions - no incentives to adjust their prices, whenever monetary policy implements the optimal inflation rate. Since the flexible price allocation is efficient, see proposition 1, monetary policy also has no incentive to deviate from the flexible-price allocation. Both observations together imply that the optimal inflation rate does not depend on whether price-setting frictions are state or time dependent.[35]

The previous logic does not fully extend to our results for a multi-sector economy in section 10. The optimal inflation rate there fails to exactly implement the efficient flexible-price allocation, because generically each sector has its own sector-specific optimal inflation rate. The precise form of the postulated price setting frictions can then have an influence on the optimal rate of inflation, as it determines the details of how monetary policy can get allocations closer to the efficient flexible-price benchmark. Determining

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[35]Obviously, this requires that, in a setting with menu cost frictions, δ-shocks lead either to these menu cost not having to be paid or always having to be paid, independently of whether or not prices are adjusted. The latter situation appears plausible when interpreting δ-shocks as being associated with new products, new product qualities or firm entry and exit; see section 3.2.

the optimal inflation rate for menu cost type models, in which monetary policy cannot replicate the efficient allocation, e.g., the setting considered in Golosov and Lucas (2007), is of interest for further research but beyond the scope of the present paper.

12.2   Idiosyncratic Firm Productivity

The model that we present allows for a considerably richer firm-specific productivity process than many other sticky-price models. At the same time, however, it abstracts from a number of potentially interesting additional dimensions of firm-level heterogeneity. In particular, one simplifying assumption entertained throughout the paper is that there are no firm-specific productivity components: firms receiving a δ-shock, for example, are homogeneous and - absent further δ-shocks - their productivity grows according to common trends defined by (a_t, g_t).

This said, some of our results continue to hold even in the presence of additional idiosyncratic elements to firm productivity. Adding to the setup, for instance, a multiplicative firm fixed effect to productivity, i.e., letting firm-specific productivity be given by

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_{j,t-s_{jt}},$$

where $\widetilde{Z}_{j,t-s_{jt}}$ is chosen in the period in which a δ-shock hits, independently across firms and from a time-invariant distribution with mean one, our main results in proposition 1 and 2 continue to apply. The same holds true if we incorporated instead a time-invariant firm-fixed effect $\widetilde{Z}_j$ for productivity, i.e.,

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_j.$$

For these extended settings, proposition 5, which derives our model implied empirical specification for estimating the relevant sectoral productivity trends, also holds in unchanged form because firm fixed effects get absorbed by the firm specific error term.

The situation is different if firm-specific components are time-varying, e.g., take the form

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_{jt}$$

for some idiosyncratic productivity shock process $\widetilde{Z}_{jt}$ with unconditional mean one. While such idiosyncratic shocks may be present in the data, it is hard to know to what extent they represent measurement noise. In any case, the presence of such shocks prevents full replication of the flexible-price equilibrium, as adjustment frictions then become strictly binding.[36] While this makes it difficult to obtain closed-form solutions for the optimal inflation rate, studying the implications of such time-varying idiosyncratic shocks for the optimal inflation rate appears to be worth exploring further in future work.

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[36]This holds true for the case with time-dependent pricing frictions and the case with menu cost type frictions.

13 Conclusions

This paper shows how firm-level productivity trends affect the inflation rate that is optimal for the aggregate economy. Since the inflation-relevant firm-level productivity trends cannot be inferred from aggregate productivity dynamics, we analyze data from the Longitudinal Business Database, which reports establishment-level employment for all US establishments. We find that the US employment trends imply establishment-level productivity trends that can rationalize significantly positive rates of inflation as being optimal. At the same time, our estimates show that important changes in firm-level productivity trends have been taking place over the period 1986 to 2013 in the US economy. These changes caused the optimal inflation rate to fall by approximately 50 percent over the considered 27 years. The economic forces behind these changes in establishment-level productivity trends are certainly worth exploring further. It also appears interesting to explore to what extent similar changes are present in other advanced economies.

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Appendix

A   Derivation of the Sticky-Price Economy

A.1   Cost Minimization Problem of Firms

The cost minimization problem of firm j,

$$\min_{K_{jt}, L_{jt}} K_{jt} r_t + L_{jt} W_t / P_t \quad s.t. \quad Y_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

yields the first-order conditions

$$0 = r_t + \left(1 - \frac{1}{\phi} \right) \lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}} \right)^{\frac{1}{\phi}}$$

$$0 = W_t / P_t + \frac{1}{\phi} \lambda_t A_t Q_{t-s_{jt}} G_{jt} \left(\frac{L_{jt}}{K_{jt}} \right)^{\frac{1}{\phi}-1},$$

where λ_t denotes the Lagrange multiplier. The first-order conditions imply that the optimal capital labor ratio is the same for all $j \in [0,1]$, i.e.,

$$\frac{K_{jt}}{L_{jt}} = \frac{W_t}{P_t r_t} (\phi - 1).$$

Plugging the optimal capital labor ratio into the technology of firm j and solving for the factor inputs yields the factor demand functions

$$L_{jt} = \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}-1} I_{jt} \tag{38}$$

$$K_{jt} = \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}} I_{jt}. \tag{39}$$

Firm j demands these amounts of labor and capital, respectively, to combine them to I_{jt}, which yields Y_{jt} units of output. Accordingly, the firm's cost function to produce I_{jt} is

$$MC_t I_{jt} = W_t \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}-1} I_{jt} + P_t r_t \left(\frac{W_t}{P_t r_t} (\phi - 1) \right)^{\frac{1}{\phi}} I_{jt}, \tag{40}$$

where MC_t denotes nominal marginal (or average) costs. This equation can be rearranged to obtain equation (8) in the main text.

A.2   Price-Setting Problem of Firms

The price-setting problem of the firm j, see equation (9), implies that the optimal product price is given by

$$P_{jt}^{\star} = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{E_t \sum_{i=0}^{\infty} (\alpha(1 - \delta))^i \Omega_{t,t+i} Y_{t+i} \left(\Xi_{t,t+i} / P_{t+i} \right)^{-\theta} \frac{MC_{t+i}/P_{t+i}}{A_{t+i} Q_{t-s_{jt}} G_{jt+i}}}{E_t \sum_{i=0}^{\infty} (\alpha(1 - \delta))^i \Omega_{t,t+i} Y_{t+i} \left(\Xi_{t,t+i} / P_{t+i} \right)^{1-\theta}}. \tag{41}$$

Rewriting this equation yields

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)$$

$$= \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{1-\theta}}. \quad (42)$$

The multi-period growth rate of the cohort effect relative to the experience effect corresponds to

$$\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}} = \frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}},$$

for $i > 0$, and equals unity for $i = 0$. Hence, this growth rate is independent of the index j, because when going forward in time, firms are subject to the same experience effect. Thus, we can rewrite the equation (42) according to

$$\frac{P_{jt}^{\star}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t},$$

where the numerator N_t is given by

$$N_t = E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\left(\frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}}\right).$$

The numerator evolves recursively as shown by equation (11). The denominator D_t also evolves recursively, and jointly this yields the recursive pricing equations (10)-(12).

A.3 First-Order Conditions to the Household Problem

The first-order conditions that belong to the household problem comprise the household's budget constraint, a no-Ponzi scheme condition, the transversality condition, and the following equations:

$$\frac{W_t}{P_t} = -\frac{U_{Lt}}{U_{Ct}}$$

$$\Omega_{t,t+1} = \beta\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}}{U_{Ct}}$$

$$1 = E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$$

$$1 = E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right].$$

Here, we denote by $U(.)$ the period utility function. Our assumption that $U(C_t, L_t) = ([C_t V(L_t)]^{1-\sigma} - 1)/(1-\sigma)$ implies

$$U_{Ct} = C_t^{-\sigma}V(L_t)^{1-\sigma}$$

$$U_{Lt} = C_t^{1-\sigma}V(L_t)^{-\sigma}V_{Lt},$$

where $U_{Ct} = \partial U(C_t, L_t)/\partial C_t$ and $V_{Lt} = \partial V(L_t)/\partial L_t$.

A.4 Recursive Evolution of the Price Level

Plugging the weighted average price of a cohort, equation (13), into the price level, equation (14), yields that

$$P_t^{1-\theta} = \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta} + \sum_{s=1}^{\infty}(1-\delta)^s\delta\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P_{t-s,t-k}^\star)^{1-\theta} + \alpha^s(\Xi_{t-s,t}P_{t-s,t-s}^\star)^{1-\theta}\right].$$

Telescoping the sums yields:

$$P_t^{1-\theta} = \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta}$$
$$+ \delta(1-\delta)^1\left[(1-\alpha)(\Xi_{t,t}P_{t-1,t}^\star)^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1,t-1}^\star)^{1-\theta}\right]$$
$$+ \delta(1-\delta)^2\left[(1-\alpha)(\Xi_{t,t}P_{t-2,t}^\star)^{1-\theta} + (1-\alpha)\alpha(\Xi_{t-1,t}P_{t-2,t-1}^\star)^{1-\theta} + \alpha^2(\Xi_{t-2,t}P_{t-2,t-2}^\star)^{1-\theta}\right]$$
$$+ \dots .$$

Collecting optimal prices that were set at the same date in square brackets yields:

$$P_t^{1-\theta} =$$
$$\delta\Xi_{t,t}^{1-\theta}\left[(P_{t,t}^\star)^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-1,t}^\star)^{1-\theta} + (1-\delta)(P_{t-2,t}^\star)^{1-\theta} + (1-\delta)^2(P_{t-3,t}^\star)^{1-\theta} + \dots\right\}\right]$$
$$+ [\alpha(1-\delta)]\delta\Xi_{t-1,t}^{1-\theta}\left[(P_{t-1,t-1}^\star)^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-2,t-1}^\star)^{1-\theta} + (1-\delta)(P_{t-3,t-1}^\star)^{1-\theta} + \dots\right\}\right]$$
$$+ \dots .$$

Using equation (15) and the definition of p_t^e in equation (17), we can replace the terms in curly brackets in the previous equation by p_t^e. This yields

$$P_t^{1-\theta} = \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_t^e)^{\theta-1}}{\delta} - 1\right\}\right]$$
$$+ [\alpha(1-\delta)]^1\delta(\Xi_{t-1,t}P_{t-1,t-1}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-1}^e)^{\theta-1}}{\delta} - 1\right\}\right]$$
$$+ [\alpha(1-\delta)]^2\delta(\Xi_{t-2,t}P_{t-2,t-2}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-2}^e)^{\theta-1}}{\delta} - 1\right\}\right] + \dots .$$

Rearranging the previous equation yields

$$P_t^{1-\theta} = (\Xi_{t,t}P_{t,t}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right]$$
$$+ \alpha(1-\delta)(\Xi_{t-1,t})^{1-\theta}\left\{(\Xi_{t-1,t-1}P_{t-1,t-1}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right]\right.$$
$$\left. + \alpha(1-\delta)(\Xi_{t-2,t-1}P_{t-2,t-2}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots\right\}.$$

The term in curly brackets in the previous equation corresponds to $P_{t-1}^{1-\theta}$, which yields the price level equation (16) in the main text.

A.5 Equilibrium Definition

We are now in a position to define the market equilibrium:

Definition 1 *An equilibrium is a state-contingent path for* $\{(P_{jt}, L_{jt}, K_{jt})$ *for* $j \in [0, 1]$, $W_t, r_t, i_t, C_t, K_{t+1}, L_t, B_t, T_t\}_{t=0}^{\infty}$ *such that*

1. *the firms' choices* $\{P_{jt}, L_{jt}, K_{jt}\}_{t=0}^{\infty}$ *maximize profits for all* $j \in [0, 1]$, *given the price adjustment frictions,*

2. *the household's choices* $\{C_t, K_{t+1}, L_t, B_t\}_{t=0}^{\infty}$ *maximize expected household utility,*

3. *the government flow budget constraint holds each period, and*

4. *the markets for capital, labor, final and intermediate goods and government bonds clear,*

   given the initial values $B_{-1}(1 + i_{-1}), K_0, P_{j,-1}$, *and* $A_{-1}Q_{-1-s_{j,-1}}G_{j,-1}$, *with* $j \in [0, 1]$.

A.6 Aggregate Technology and Aggregate Productivity

To derive the aggregate technology, we combine firms' technology to produce the differentiated product in equation (2) with product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to obtain

$$\frac{Y_t}{A_t Q_t} \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}} \right) \left(\frac{P_{jt}}{P_t} \right)^{-\theta} = \left(\frac{K_{jt}}{L_{jt}} \right)^{1-\frac{1}{\phi}} L_{jt} - F_t.$$

Integrating over all firms with $j \in [0, 1]$, using labor market clearing, $L_t = \int_0^1 L_{jt} \, \mathrm{dj}$, and the fact that optimizing firms maintain the same (and hence the aggregate) capital labor ratio yields

$$\frac{Y_t}{A_t Q_t} \int_0^1 \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}} \right) \left(\frac{P_{jt}}{P_t} \right)^{-\theta} \mathrm{dj} = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t.$$

Rearranging this equation and defining the (inverse) endogenous component of aggregate productivity as in equation (19) in the main text yields the aggregate technology (18).

To derive the recursive representation of Δ_t shown in equation (20), we rewrite equation (19) according to

$$\frac{\Delta_t}{P_t^{\theta}} = \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}} \right) (P_{jt})^{-\theta} \mathrm{dj},$$

using the processes describing the evolution of Q_t and G_{jt}. As for the price level, we proceed with the aggregation in two steps. First, we aggregate the optimal prices of all firms operating within a particular cohort. Second, we aggregate all cohorts in the economy. To this end, we rewrite Δ_t/P_t^{θ} in the previous equation according to

$$\frac{\Delta_t}{P_t^{\theta}} = \sum_{s=0}^{\infty} (1-\delta)^s \delta \widehat{\Lambda}_t(s), \tag{43}$$

using

$$
\widehat{\Lambda}_t(s) = \begin{cases} \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right) \left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P_{t-s,t-k}^{\star})^{-\theta} + \alpha^s(\Xi_{t-s,t}P_{t-s,t-s}^{\star})^{-\theta}\right] & \text{if} \quad s \geq 1, \\ (\Xi_{t,t}P_{t,t}^{\star})^{-\theta} & \text{if} \quad s = 0. \end{cases}
$$

Substituting out for $\widehat{\Lambda}_t(s)$ in equation (43) yields

$$
\frac{\Delta_t}{P_t^\theta} = \delta(\Xi_{t,t}P_{t,t}^{\star})^{-\theta}
$$
$$
+ \delta\sum_{s=1}^{\infty}(1-\delta)^s\left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}}\right)\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P_{t-s,t-k}^{\star})^{-\theta} + \alpha^s(\Xi_{t-s,t}P_{t-s,t-s}^{\star})^{-\theta}\right].
$$

We rearrange the previous equation following corresponding steps to those in appendix A.4. This yields

$$
\frac{\Delta_t}{P_t^\theta} = (\Xi_{t,t}P_{t,t}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right]
$$
$$
+ \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t}P_{t-1,t-1}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right]
$$
$$
+ [\alpha(1-\delta)]^2\left(\frac{q_t q_{t-1}}{g_t g_{t-1}}\right)(\Xi_{t-2,t}P_{t-2,t-2}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots .
$$

We rearrange the previous equation further to obtain that

$$
\frac{\Delta_t}{P_t^\theta} = (\Xi_{t,t}P_{t,t}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right]
$$
$$
+ \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t})^{-\theta}\Bigg\{(P_{t-1,t-1}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right]
$$
$$
+ \alpha(1-\delta)\left(\frac{q_{t-1}}{g_{t-1}}\right)(\Xi_{t-2,t-1}P_{t-2,t-2}^{\star})^{-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \dots \Bigg\}.
$$

The term in curly brackets in the previous equation is equal to $\Delta_{t-1}/P_{t-1}^\theta$, which yields

$$
\frac{\Delta_t}{P_t^\theta} = \left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right](\Xi_{t,t}P_{t,t}^{\star})^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t})^{-\theta}\frac{\Delta_{t-1}}{P_{t-1}^\theta}.
$$

Multiplying the previous equation by P_t^θ yields equation (20) in the main text.

A.7 Consolidated Budget Constraint

Consolidating the household's and the government's budget constraints shown in the main text yields

$$
C_t + K_{t+1} = (1-d)K_t + r_t K_t + \frac{W_t}{P_t}L_t + \frac{\int_0^1\Theta_{jt}\,\mathrm{dj}}{P_t} - \tau\left(\frac{\int_0^1 P_{jt}Y_{jt}\mathrm{dj}}{P_t}\right). \tag{44}
$$

To compute aggregate firm profits denoted by $\int_0^1\Theta_{jt}\,\mathrm{dj}$, we use marginal costs in equation (40) and combine them with the factor demands for L_{jt} and K_{jt}, equations (38) and (39),

44

which yields that $MC_t I_{jt} = W_t L_{jt} + P_t r_t K_{jt}$. We use this equation and product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to rewrite aggregate firm profits according to

$$
\begin{aligned}
\int_0^1 \Theta_{jt} \ \mathrm{dj} &= (1+\tau) \int_0^1 P_{jt} Y_{jt} \ \mathrm{dj} - \int_0^1 MC_t I_{jt} \ \mathrm{dj} \\
&= (1+\tau) \int_0^1 P_{jt} Y_{jt} \ \mathrm{dj} - \int_0^1 (W_t L_{jt} + P_t r_t K_{jt}) \ \mathrm{dj} \\
&= (1+\tau) P_t Y_t - W_t L_t - P_t r_t K_t,
\end{aligned}
$$

with $L_t = \int_0^1 L_{jt} \ \mathrm{dj}$ and $K_t = \int_0^1 K_{jt} \ \mathrm{dj}$. Thus, the consolidated budget constraint (44) reduces to

$$
K_{t+1} = (1-d)K_t + Y_t - C_t.
$$

Dividing the previous equation by trend growth Γ_t^e yields

$$
\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t,
$$

where $\gamma_t^e = \Gamma_t^e/\Gamma_{t-1}^e$ denotes the gross trend growth rate.

A.8   Transformed Sticky-Price Economy

We define $p_t^\star = P_{t,t}^\star/P_t$ and $mc_t = MC_t/(P_t(\Gamma_t^e)^{1/\phi})$ and $w_t = W_t/(P_t \Gamma_t^e)$ and $c_t = C_t/\Gamma_t^e$. We also use that $p_t^e = 1/\Delta_t^e$, which follows from the equations (17) and (26). This yields

the following equations that describe the transformed sticky-price economy.

$$1 = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{1-\theta} + \alpha(1-\delta)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta-1} \tag{45}$$

$$\Delta_t = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1} \tag{46}$$

$$p_t^\star = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t} \tag{47}$$

$$N_t = \frac{mc_t}{\Delta_t^e} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{48}$$

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right] \tag{49}$$

$$mc_t = \left(\frac{w_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}} \tag{50}$$

$$r_t k_t = (\phi-1)w_t L_t \tag{51}$$

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - f\right) \tag{52}$$

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t \tag{53}$$

$$\gamma_t^e = (a_t q_t \Delta_{t-1}^e/\Delta_t^e)^{\phi} \tag{54}$$

$$(\Delta_t^e)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta} \tag{55}$$

$$w_t = -c_t\left(\frac{V_{Lt}}{V(L_t)}\right) \tag{56}$$

$$1 = E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right] \tag{57}$$

$$1 = E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right] \tag{58}$$

$$\Omega_{t,t+1} = \beta\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma} \tag{59}$$

After adding a description of monetary policy and a price indexation rule, these seventeen equations determine the paths of the seventeen variables $i_t, \Pi_t, y_t, c_t, k_t, L_t, r_t, w_t, mc_t, \gamma_t^e, \Delta_t, \Delta_t^e, p_t^\star, \Xi_{t-1,t}, N_t, D_t, \Omega_{t-1,t}$ given the four exogenous shocks q_t, g_t, a_t, ξ_t.

A.9 Steady State in the Transformed Sticky-Price Economy

We consider a steady state in the transformed sticky-price economy, in which g and q are constant and the government maintains a constant inflation rate Π, which also implies a constant rate of price indexation Ξ.

To solve for the model variables in this steady state, we first solve for the ratio Δ/Δ^e as a function of model parameters and the inflation rate Π only. To this end, we derive an expression for $p^\star$ as a function of Δ using the equations (45) and (46). Both equations

can be rearranged to obtain, respectively,

$$(1 - \alpha(1-\delta)(\Pi/\Xi)^{\theta-1}) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{1-\theta} \tag{60}$$

$$\Delta\left(1 - \alpha(1-\delta)(\Pi/\Xi)^\theta(g/q)^{-1}\right) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{-\theta}. \tag{61}$$

Dividing the equation (60) by the equation (61) yields

$$p^\star = \Delta^{-1}\left(\frac{1 - \alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)(\Pi/\Xi)^\theta(g/q)^{-1}}\right). \tag{62}$$

We substitute this expression for $p^\star$ into the equation (61), which yields

$$\left(\frac{\Delta}{\Delta^e}\right)^{1-\theta} = \frac{\alpha\delta(\Delta^e)^{\theta-1} + 1 - \alpha}{1 - \alpha(1-\delta)(\Pi/\Xi)^\theta(g/q)^{-1}}\left(\frac{1 - \alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)(\Pi/\Xi)^\theta(g/q)^{-1}}\right)^{-\theta}.$$

We use equation (55) to substitute for $(\Delta^e)^{\theta-1}$ on the right hand side of the previous equation and rearrange the result to obtain

$$\frac{\Delta(\Pi)}{\Delta^e} = \left(\frac{1 - \alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}\left(\frac{1 - \alpha(1-\delta)(g/q)^{\theta-1}}{1 - \alpha(1-\delta)(\Pi/\Xi)^\theta(g/q)^{-1}}\right), \tag{63}$$

where we have indicated that $\Delta(\Pi)$ depends on the steady-state inflation rate Π. For later use, we define the relative price distortion as

$$\rho(\Pi) = \frac{\Delta^e}{\Delta(\Pi)}. \tag{64}$$

Combining the pricing equations (47) to (49) yields

$$\frac{1}{mc} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1}{p^\star\Delta^e}\right)\left(\frac{1 - \alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}](\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}](\Pi/\Xi)^\theta(g/q)^{-1}}\right).$$

Using the expression for $p^\star$ in equation (62) to substitute for $p^\star$ in the previous equation and the solution for $\Delta(\Pi)/\Delta^e$ in equation (63), we thus obtain a solution for $1/mc$. Again for later use, we denote the average markup by $\mu = 1/mc$ and thus obtain the solution

$$\mu(\Pi) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1 - \alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{\theta-1}}\left(\frac{1 - \alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}](\Pi/\Xi)^{\theta-1}}{1 - \alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}](\Pi/\Xi)^\theta(g/q)^{-1}}\right). \tag{65}$$

Again, we indicate here that $\mu(\Pi)$ depends on the steady-state inflation rate.

Now, we rewrite marginal costs in equation (50) as

$$mc = \left(\frac{w}{r}(\phi-1)\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right),$$

and use equation (51) to obtain $mc = \left(\frac{k}{L}\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right)$ or

$$r = \mu(\Pi)^{-1}\left(1 - \frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{-\frac{1}{\phi}}, \tag{66}$$

after also using $\mu = 1/mc$. Analogous steps for the wage rate also imply

$$w = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}. \tag{67}$$

Furthermore, the aggregate technology (52), the aggregate resource constraint (53) and the household's optimality conditions (56) to (59) imply the following four equations:

$$y = \rho(\Pi) \left(\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} L - f\right)$$

$$w = c\left(-\frac{V_L}{V(L)}\right)$$

$$r = \frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d$$

$$y = c + (\gamma^e - 1 + d)k,$$

where we have used $\rho(\Pi) = \Delta^e/\Delta(\Pi)$. To simplify these four equations further, we use the equations (66) and (67) to substitute out for w and r. Then, we express all the remaining variables relative to hours worked, which yields the following four equations:

$$\frac{y}{L} = \rho(\Pi) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} \tag{68}$$

$$\frac{c}{L} = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{LV_L}\right) \tag{69}$$

$$\frac{k}{L} = \mu(\Pi)^{-1} \left(1 - \frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-1} \tag{70}$$

$$\frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d)\frac{k}{L}. \tag{71}$$

We now show that these four equations determine the four variables y, c, L, k, given a steady-state inflation rate Π and assuming that the ratio of fixed costs over output, f/y, is a calibrated parameter.

First, we solve for hours worked as a function of Π by substituting the equations (68) to (70) into equation (71). This yields

$$\mu(\Pi)\rho(\Pi) \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} = \left(\frac{1}{\phi}\right) \left(-\frac{V(L)}{LV_L}\right) + \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right) \left(1 - \frac{1}{\phi}\right),$$

or

$$\left(-\frac{V(L)}{LV_L}\right) = \phi\mu(\Pi)\rho(\Pi) \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} - (\phi - 1) \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right)$$

$$= \mathcal{L}(\Pi),$$

where $\mathcal{L}(\Pi)$ abbreviates the right-hand-side term, which is a function of the steady-state inflation rate. The previous equation provides an implicit solution for L. We obtain an explicit solution for L, if we assume a functional form for $V(L)$. Using that $V(L) = 1 - \psi L^\nu$, with $\nu > 1$ and $\psi > 0$ yields

$$-\frac{V(L)}{LV_L} = \frac{1 - \psi L^\nu}{\psi \nu L^\nu}$$

and hence

$$L(\Pi) = \left(\frac{1}{\psi + \psi \nu \mathcal{L}(\Pi)}\right)^{1/\nu}, \tag{72}$$

where we have indicated that in general, steady-state hours worked L depend on the steady-state inflation rate Π through $\mathcal{L}(\Pi)$. Recall that in order to compute $\mathcal{L}(\Pi)$, the equations (63), (64) and (65) are required. The solutions for k, c, and y can be recursively computed from the equations (68) to (70). These solutions are

$$k(\Pi) = \mu(\Pi)^{-\phi} \left(1 - \frac{1}{\phi}\right)^\phi \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-\phi} L \tag{73}$$

$$c(\Pi) = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{V_L}\right) \tag{74}$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \tag{75}$$

Again, we indicate that these solutions depend on the steady-state inflation rate.

B   Planner Problem and Its Solution

The planner allocates resources across firms and time by maximizing expected discounted household utility subject to firms' technologies and feasibility constraints. The planner problem can be solved in two steps. The first step determines the allocation of given amounts of capital and labor between heterogenous firms at date t. The second step determines the allocation of aggregate capital, consumption and labor over time. Endogenous variables in the planner solution are indicated by superscript e.

B.1   Intratemporal Planner Problem

The intratemporal problem corresponds to

$$\max_{L^e_{jt}, K^e_{jt}} \left(\int_0^1 (Y^e_{jt})^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad Y^e_{jt} = A_t Q_{t-s_{jt}} G_{jt} \left((K^e_{jt})^{1-\frac{1}{\phi}} (L^e_{jt})^{\frac{1}{\phi}} - F_t\right),$$

and given L^e_t and K^e_t, with $L^e_t = \int_0^1 L^e_{jt}\,\mathrm{dj}$ and $K^e_t = \int_0^1 K^e_{jt}\,\mathrm{dj}$. Optimality conditions yield $K^e_{jt}/L^e_{jt} = K^e_t/L^e_t$ and hence that all firms maintain the same capital labor ratio. Thus, the

problem can be recast in terms of the optimal mix of input factors, $I_{jt}^e = (K_{jt}^e)^{1-1/\phi}(L_{jt}^e)^{1/\phi}$:

$$\max_{I_{jt}^e} \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(I_{jt}^e - F_t \right) \right]^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad I_t^e = \int_0^1 I_{jt}^e \, dj,$$

with $I_t^e = (K_t^e)^{1-1/\phi}(L_t^e)^{1/\phi}$ being given. Equating the first-order conditions to this problem for two different firms j and k to each other yields the condition

$$Z_{jt} \left[Z_{jt} \left(I_{jt}^e - F_t \right) \right]^{-\frac{1}{\theta}} = Z_{kt} \left[Z_{kt} \left(I_{kt}^e - F_t \right) \right]^{-\frac{1}{\theta}},$$

where $Z_{jt} = Q_{t-s_{jt}} G_{jt}$ denotes productivity of the firm j at date t. Rearranging this condition yields $I_{jt}^e - F_t = (Z_{jt}/Z_{kt})^{\theta-1} (I_{kt}^e - F_t)$, and aggregating this equation over all j's yields

$$I_{kt}^e - F_t = \frac{(G_{kt} Q_{t-s_{kt}}/Q_t)^{\theta-1}}{\int_0^1 (G_{jt} Q_{t-s_{jt}}/Q_t)^{\theta-1} \, dj} (I_t^e - F_t). \tag{76}$$

Thus, the optimal input mix of the firm k net of fixed costs is proportional to the optimal aggregate input mix net of fixed costs, and the factor of proportionality corresponds to the (weighed) productivity of the firm k relative to the (weighed) aggregate productivity in the economy. Thus, equation (76) shows that the productivity distribution determines the efficient allocation of the optimal input mix across firms.

To obtain the aggregate technology in the planner economy, we combine equation (76) with equation (2) and the Dixit-Stiglitz aggregator (1). This yields

$$Y_t^e = \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(\frac{(Q_{t-s_{jt}} G_{jt})^{\theta-1}}{\int_0^1 (Q_{t-s_{jt}} G_{jt})^{\theta-1} \, dj} (I_t^e - F_t) \right) \right]^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}.$$

Simplifying this equation yields the aggregate technology in the planner economy,

$$Y_t^e = \frac{A_t Q_t}{\Delta_t^e} \left((K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}} - F_t \right), \tag{77}$$

where the efficient productivity adjustment factor is defined as

$$1/\Delta_t^e = \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}}/Q_t \right)^{\theta-1} dj \right)^{\frac{1}{\theta-1}} \tag{78}$$

and evolves recursively. To see this, rewrite equation (78) as

$$(1/\Delta_t^e)^{\theta-1} = \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}} \right)^{1-\theta} dj$$

$$= \delta \left\{ 1 + \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}} \right)^{1-\theta} \right\}$$

$$= \delta \left\{ 1 + (1-\delta) \left(\frac{q_t}{g_t} \right)^{1-\theta} + (1-\delta)^2 \left(\frac{q_t q_{t-1}}{g_t g_{t-1}} \right)^{1-\theta} + \dots \right\}$$

$$= (p_t^e)^{\theta-1}.$$

The last step follows from backward-iterating equation (17) and implies that the efficient productivity adjustment factor equals the relative price of firms hit by a δ-shock in period t in the economy with flexible prices,

$$1/\Delta_t^e = p_t^e. \tag{79}$$

It follows also from equation (17) that Δ_t^e evolves recursively as shown in equation (26). The intratemporal planner allocation then consists of equation (76), which determines the efficient allocation of the optimal input mix across firms, and equations (77) and (26), which describe the aggregate consequences of the efficient allocation at the firm level.

B.2 Intertemporal Planner Problem

The intertemporal allocation maximizes expected discounted household utility subject to the intertemporal feasibility condition,

$$\max_{\{C_t^e, L_t^e, K_{t+1}^e\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t \xi_t U(C_t^e, L_t^e) \quad s.t. \tag{80}$$

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + \frac{A_t Q_t}{\Delta_t^e}\left((K_t^e)^{1-\frac{1}{\phi}}(L_t^e)^{\frac{1}{\phi}} - F_t\right), \tag{81}$$

with $U(.)$ denoting the period utility function and Δ_t^e given by equation (26). The first order conditions to this problem comprise the feasibility constraint and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{82}$$

$$1 = \beta E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}^e}{U_{Ct}^e}\left(Y_{Kt+1}^e + 1 - d\right)\right], \tag{83}$$

denoting by Y_{Kt}^e the marginal product of capital and by Y_{Lt}^e the marginal product of labor. Thus, the planner allocation for aggregate variables is characterized by the aggregate technology, equation (77), the efficient adjustment factor, equation (26), the feasibility condition, equation (81), and the two first-order conditions (82) and (83).

C Proof of Proposition 1

To show that condition (27) holds under flexible prices, we divide equation (16) by $P_t^{1-\theta}$ and impose $\alpha = 0$ to find out that the optimal relative price $p_t^\star$ of firms experiencing a δ-shock in period t is equal to p_t^e. This and the equations (47) to (49) determining the optimal relative price of firms experiencing a δ-shock in t imply with $\alpha = 0$ that

$$p_t^e = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with the equation (79) yields

$$1 = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) mc_t, \tag{84}$$

which shows that real detrended marginal costs are constant in the economy with flexible prices. From equation (10) it follows that the optimal relative price of the firm j (independently of the number of periods s_{jt} elapsed since the last δ-shock) in the flexible-price model is

$$\frac{P_{jt}^{\star}}{P_t} (G_{jt} Q_{t-s_{jt}}/Q_t) = \left(\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} \right) \frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with equation (84), we obtain condition (27) in the main text. Plugging this condition into equation (19) shows that the flexible-price equilibrium implements $\Delta_t = \Delta_t^e$. Thus, the aggregate production function in equation (18) in the flexible-price equilibrium is given by

$$Y_t = \frac{A_t Q_t}{\Delta_t^e} \left((K_t)^{1 - \frac{1}{\phi}} (L_t)^{\frac{1}{\phi}} - F_t \right), \tag{85}$$

with $F_t = f \cdot (\Gamma_t^e)^{1 - 1/\phi}$ and $\Gamma_t^e = (A_t Q_t / \Delta_t^e)^\phi$, and the resource constraint (derived in Appendix A.7) is given by

$$K_{t+1} = (1 - d)K_t + Y_t - C_t. \tag{86}$$

The two equations (85) and (86) are the same constraints faced by the planner under efficient allocation. Combined with the fact that the household decisions in the flexible price economy are undistorted in the presence of the corrective sales subsidy, it follows that the allocation of aggregate consumption, capital, labor, and output in the flexible-price equilibrium is identical to efficient allocation.

D Proof of Proposition 2

Establishing (1): First, we show that firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as in the flexible-price economy. Let superscript e denote allocations and prices in the flexible-price economy, which we have shown reproduces the efficient allocation. Under flexible prices ($\alpha = 0$) and given condition 1, the optimal relative price implied by equation (10) for firms with a δ-shock in period t is given by

$$p_t^e = \frac{(P_{t,t}^{\star})^e}{P_t^e} = \frac{MC_t^e}{P_t^e A_t Q_t}.$$

Under sticky prices ($\alpha > 0$) and the efficient allocation, combining this equation with equation (11) implies

$$\frac{N_t}{p_t^e} = 1 + \alpha(1 - \delta)E_t \left[\Omega_{t,t+1}^e \frac{Y_{t+1}^e}{Y_t^e} \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right)^\theta \left(\frac{q_{t+1}}{g_{t+1}} \right) \left(\frac{p_{t+1}^e}{p_t^e} \right) \left(\frac{N_{t+1}}{p_{t+1}^e} \right) \right]. \tag{87}$$

Furthermore, equation (12) implies

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega^e_{t,t+1}\frac{Y^e_{t+1}}{Y^e_t}\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right]. \tag{88}$$

Firms hit by a δ-shock in period t in the sticky-price economy choose the same optimal relative price as firms receiving a δ-shock in period t in the flexible-price economy, i.e., $P^\star_{t,t}/P_t = N_t/D_t = p^e_t$ or equivalently $N_t/p^e_t = D_t$, if it holds that

$$\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)\left(\frac{q_{t+1}}{g_{t+1}}\right)\left(\frac{p^e_{t+1}}{p^e_t}\right) = 1, \tag{89}$$

which follows from comparing the equations (87) and (88). To show that equation (89) holds under the optimal inflation rate stated in proposition 2, we lag this equation by one period and rearrange it to obtain

$$\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)p^e_t = p^e_{t-1}\frac{g_t}{q_t}.$$

Combining this equation with equation (17) implies that the optimal inflation rate as defined in equation (28) satisfies equation (89).

Establishing (2): To show that, under the optimal inflation rate, firms that are subject to a Calvo shock in period t and hence can adjust their price do not find it optimal to change their price, we need to establish that

$$P^\star_{t-k,t} = \Xi^\star_{t-k,t}P^\star_{t-k,t-k}, \tag{90}$$

for all $k > 0$. Dividing this equation by the (optimal) aggregate price level $P^\star_{t-k}$ and using the result from step (1), i.e., $P^\star_{t,t}/P^\star_t = p^e_t$, we obtain

$$\frac{P^\star_{t-k,t}}{P^\star_{t-k}} = \Xi^\star_{t-k,t}\left(\frac{P^\star_{t-k,t-k}}{P^\star_{t-k}}\right) = \Xi^\star_{t-k,t}p^e_{t-k}.$$

Using equation (15), we can rewrite the previous equation as

$$\frac{P^\star_{t,t}}{P^\star_t}\left(\frac{q_t\times\cdots\times q_{t-k+1}}{g_t\times\cdots\times g_{t-k+1}}\right)\frac{P^\star_t}{P^\star_{t-k}} = \Xi^\star_{t-k,t}p^e_{t-k}.$$

Again using $P^\star_{t,t}/P^\star_t = p^e_t$ and that $\Xi_{t-k,t} = \prod_{j=1}^k \Xi_{t-k+j-1,t-k+j}$ further delivers

$$\left(\frac{p^e_t}{p^e_{t-k}}\right)\left(\frac{q_t\times\cdots\times q_{t-k+1}}{g_t\times\cdots\times g_{t-k+1}}\right)\left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}}\times\cdots\times\frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}}\right) = 1.$$

Rewriting the previous equation as

$$\left(\frac{\Pi^\star_t}{\Xi^\star_{t-1,t}}\frac{q_t}{g_t}\frac{p^e_t}{p^e_{t-1}}\right)\times\left(\frac{\Pi^\star_{t-1}}{\Xi^\star_{t-2,t-1}}\frac{q_{t-1}}{g_{t-1}}\frac{p^e_{t-1}}{p^e_{t-2}}\right)\times\cdots\times\left(\frac{\Pi^\star_{t+1-k}}{\Xi^\star_{t-k,t+1-k}}\frac{q_{t+1-k}}{g_{t+1-k}}\frac{p^e_{t+1-k}}{p^e_{t-k}}\right) = 1$$

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shows that each term in parenthesis is equal to unity under the optimal inflation rate, which follows from equation (89). This establishes that firms that can adjust their price maintain the indexed price as given by equation (90).

Establishing (3): We can establish the fact that the condition 2 causes initial prices to reflect initial relative productivities as follows. The pricing equations (10)-(12) imply under flexible prices and no markup distortion that

$$\frac{P_{jt}^\star}{P_t} \left(\frac{Q_{t-s_{jt}} G_{jt}}{Q_t} \right) = \frac{MC_t}{P_t A_t Q_t}.$$

For a firm hit by a δ-shock in period t, this equation yields

$$p_t^e = \frac{MC_t}{P_t A_t Q_t}.$$

Combining both previous equations yields

$$\frac{P_{jt}^\star}{P_t} = \left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}} \right) p_t^e.$$

Plugging this equation into the aggregate price level, $P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta} \, \mathrm{d}j$, yields

$$1 = \int_0^1 \left(\frac{Q_t}{Q_{t-s_{jt}} G_{jt}} \right)^{1-\theta} (p_t^e)^{1-\theta} \mathrm{d}j.$$

Rewriting this equation and using $p_t^e = 1/\Delta_t^e$ yields equation (25) for $t = -1$.

E   Discontinuity of the Optimal Inflation Rate

This appendix compares the optimal inflation rate in an economy with δ-shocks ($\delta > 0$) to the economy in the absence of such shocks ($\delta = 0$). We refer to the first economy as the δ-economy and to the latter as the 0-economy. Comparing these two economies is not as straightforward as it might initially appear: even if both economies are subject to the same fundamental shocks (a_t, q_t, g_t, ξ_t), the efficient allocation displays a discontinuity when considering the limit $\delta \to 0$. The discontinuity arises because aggregate productivity growth in the δ-economy is driven by $a_t q_t$, while it is driven by $a_t g_t$ in the 0-economy.

To properly deal with this issue, we construct a δ-economy whose efficient aggregate allocation (consumption, hours, capital) is identical to the efficient aggregate allocation in the 0-economy.[37] We then compare the optimal inflation rates in these two economies and show that the optimal inflation rate for the δ-economy differs from the optimal inflation rate for the 0-economy, even for the limit $\delta \to 0$.

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[37] The two economies do of course differ in their underlying firm-level dynamics.

Let a_t^δ, q_t^δ, g_t^δ denote the productivity disturbances in the δ-economy and let $A_{-1}^\delta G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta$ for $j \in [0,1]$ denote the initial distribution of firm productivities. This, together with the process $\{\delta_{jt}\}_{t=0}^\infty$ for all $j \in [0,1]$, determines the entire state-contingent values for A_t^δ, Q_t^δ, G_{jt}^δ, and $Q_{t-s_{jt}}^\delta$ for all $j \in [0,1]$ and all $t \geq 0$.

Next, consider the 0-economy and suppose it starts with the same initial capital stock as the δ-economy. For the 0-economy, we normalize $Q_{t-s_{jt}}^0 \equiv 1$ for all $j \in [0,1]$ and all t and then set the initial firm productivity distribution in the 0-economy equal to that in the δ-economy by choosing the initial conditions

$$A_{-1}^0 = A_{-1}^\delta,$$
$$G_{j,-1}^0 = G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta.$$

Finally, let the process for common TFP in the 0-economy be given by

$$A_t^0 = A_t^\delta \left(\int_0^1 \left(Q_{t-s_{jt}}^\delta G_{jt}^\delta \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \left(\int_0^1 \left(G_{jt}^0 \right)^{\theta-1} \mathrm{dj} \right)^{\frac{-1}{\theta-1}},$$

where G_{jt}^0 is generated by an arbitrary process g_t^0, e.g., $g_t^0 = g_t^\delta$. In this setting, it is easily verified that aggregate productivity associated with the efficient allocation, defined as

$$A_t Q_t / \Delta_t^e = A_t Q_t \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}} / Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}},$$

is the same in the δ-economy and the 0-economy.[38] We then have the following result:

Proposition 6 *Under the assumptions stated in this section, the efficient allocations in the two economies, the δ-economy and the 0-economy, satisfy*

$$C_t^\delta = C_t^0, L_t^\delta = L_t^0, K_t^\delta = K_t^0$$

for all $t \geq 0$ and all possible realizations of the disturbances.

Proof. Since $A_t^\delta Q_t^\delta / \Delta_t^{e,\delta} = A_t^0 Q_t^0 / \Delta_t^{e,0}$ for all t, it follows from the planner's problem (23)-(24) and the fact that the initial capital stock is identical that both economies share the same efficient allocation. ∎

The following proposition shows that (generically) the optimal inflation rate discontinuously jumps when moving from the 0-economy to the δ-economy, even if both economies are identical in terms of their efficient aggregate dynamics:[39]

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[38] The fact that $A_t Q_t / \Delta_t^e$ is equal to aggregate productivity in the efficient allocation follows from equations (24) and (25).

[39] Recall that the optimal inflation rates implement the efficient aggregate allocations in these economies.

Lemma 3 *Under the assumptions stated in this section and provided conditions 1 and 2 hold, the optimal inflation rate in the 0-economy is $\Pi_t^{\star,0} = 1$ for all t. The optimal inflation rate in the δ-economy is given by equation (28); in particular, for $g_t^\delta = g$ and $q_t^\delta = q$, and in the absence of price indexation, the optimal rate of inflation in the δ-economy satisfies $\lim_{t\to\infty} \Pi_t^{\star,\delta} = g/q$.*

Proof. The results directly follow from proposition 2 and lemma 1. ∎

The previous result illustrates the fragility of the optimality of strict price stability in standard sticky-price models, once non-trivial firm-level productivity trends are taken into account. Moreover, in combination with proposition 6, the result shows that two economies that can be identical in terms of their aggregate efficient allocations may require different inflation rates for implementing these allocations.

F   Proof of Proposition 3

Under the assumptions stated in the proposition, it is straightforward to show that the relative price distortion $\rho(\Pi)$ and the markup distortion $\mu(\Pi)$, which are defined in equations (63), (64) and (65), are inversely proportional to each other,

$$\mu(\Pi) = 1/\rho(\Pi).$$

As a result, the solution of L determined in equation (72) in appendix A.9 simplifies to

$$L = \left(\frac{1}{\psi(1+\nu)} \right)^{1/\nu},$$

because $\mathcal{L}(\Pi) = 1$ and, therefore, L no longer depends on the steady-state inflation rate Π. This result implies that $L(1) = L(\Pi^\star)$, as stated in proposition 3.

In this case, the solutions for capital and consumption, equations (73) and (74), imply

$$k(\Pi) = \rho(\Pi)^\phi \left(1 - \frac{1}{\phi}\right)^\phi (\gamma^e - 1 + d)^{-\phi} L,$$

$$c(\Pi) = \rho(\Pi)^\phi \left(\frac{1}{\phi}\right) \left(1 - \frac{1}{\phi}\right)^{\phi-1} (\gamma^e - 1 + d)^{1-\phi} \left(-\frac{V(L)}{V_L}\right),$$

where we explicitly indicate that steady-state capital and consumption depend on Π.

Comparing steady-state consumption for the policy implementing the optimal inflation rate $\Pi^\star$ and the alternative policy implementing strict price stability in economies without price indexation yields

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{\rho(1)}{\rho(\Pi^\star)} \right)^\phi.$$

Equations (63) and (64) imply that the relative price distortion $\rho(\Pi^\star) = 1$. This yields

$$\frac{c(1)}{c(\Pi^\star)} = \rho(1)^\phi,$$

$$= \left(\frac{\Delta^e}{\Delta(1)}\right)^\phi$$

$$= \left(\frac{1 - \alpha(1 - \delta)(g/q)^{\theta-1}}{1 - \alpha(1 - \delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1 - \alpha(1 - \delta)(g/q)^{-1}}{1 - \alpha(1 - \delta)(g/q)^{\theta-1}}\right)^\phi,$$

which is the expression in proposition 3.

To show that $c(1)/c(\Pi^\star) \leq 1$, note that $c(1)/c(\Pi^\star) = 1$, if $g = q$ and hence $\Pi^\star = 1$. To show that the inequality holds strictly, $c(1)/c(\Pi^\star) < 1$, for $g \neq q$, we take the derivative of $c(1)/c(\Pi^\star)$ with respect to g/q. This yields

$$\frac{\partial}{\partial(g/q)}\left(\frac{c(1)}{c(\Pi^\star)}\right) = \left[\frac{c(1)}{c(\Pi^\star)}\right]\left[\frac{\alpha(1 - \delta)\phi}{(g/q)^2}\right]\frac{1 - (g/q)^\theta}{\left[1 - \alpha(1 - \delta)(g/q)^{-1}\right]\left[1 - \alpha(1 - \delta)(g/q)^{\theta-1}\right]}.$$

Terms in square brackets are positive, because we have assumed that $(1 - \delta)(g/q)^{\theta-1} < 1$ (see equation (7)), $\alpha < 1$, and $g/q > \alpha(1-\delta)$. Therefore, the derivative is strictly positive if $1 - (g/q)^\theta > 0$ and thus $g/q < 1$. The derivative is strictly negative if $1 - (g/q)^\theta < 0$ and thus $g/q > 1$. The derivative is zero if $g/q = 1$.

G   Data Appendix

G.1   LBD Database

We use data from 1986 to 2013 dropping observations of establishments that were present already in the sample in 1976 for which age information is not available. We only consider establishments with at least one paid employee and truncate employment observations above the 99% percentile in a given industry and year. This leaves us with 147 million establishment-employment observations in our estimation sample.

To improve the consistency of the mapping from SIC codes to NAICS codes, we follow the same establishments over the SIC-NAICS changeover to reverse engineer proper SIC codes for the considered industry z. Using this procedure, only about 0.2 million observations cannot be allocated to a NAICS code, because they have a coarse industry code under SIC. Since their employment share is negligible (0.02%), this should not affect our estimates.

G.2 Sectoral Disaggregation and Sector Weights ψ_z and γ_z^e/γ^e

We use the value added series of the BEA GDP-by-Industry data for 71 industries and focus on the 65 private industries.[40]

To compute the sectoral trend growth rate γ_z^e entering lemma 2, we use the chain-type quantity indexes for value added by industry for the years 1976 to 2013, which is the time span for which the LBD data is available. For a few industries (retail trade, hospitals, nursing and residential care facilities), data is only available from 1997 onwards. For these industries, we use data for the period 1997 to 2013.

To compute the aggregate trend growth rate γ^e entering lemma 2, we use the chain-type quantity index for private industries for the years 1976 to 2013.

To compute expenditure shares ψ_z for $z = 1, \ldots Z$, we use the expenditure shares as implied by the GDP statistics for the year 2013.

To compute time-varying expenditure shares ψ_{zt} for $z = 1, \ldots Z$, we use the expenditure shares as implied by the GDP statistics for the respective year. For a few sectors (retail trade, hospitals, nursing and residential care facilities), expenditure shares are not available for the period 1976 to 1996. We impute these shares using the distribution that we observe in 1997.

Table 2 below reports how we map the LBD NAICS codes into the 65 BEA private industries ($z = 1, 2, \ldots 65$).

Table 2: BEA-NAICS Mapping

z	BEA Code	BEA Title	Related 2007 NAICS Codes
1	111CA	Farms	111, 112
2	113FF	Forestry, fishing, and related activities	113, 114, 115
3	211	Oil and gas extraction	211
4	212	Mining, except oil and gas	212
5	213	Support activities for mining	213
6	22	Utilities	221
7	23	Construction	230, 233
8	321	Wood products	321
9	327	Nonmetallic mineral products	327
10	331	Primary metals	331
11	332	Fabricated metal products	332
12	333	Machinery	333
13	334	Computer and electronic products	334
14	335	Electrical equipment, appliances, and components	335
15	3361MV	Motor vehicles, bodies and trailers, and parts	3361, 3362, 3363
16	3364OT	Other transportation equipment	3364, 3365, 3366, 3369
17	337	Furniture and related products	337
18	339	Miscellaneous manufacturing	339
19	311FT	Food and beverage and tobacco products	311, 312
20	313TT	Textile mills and textile product mills	313, 314

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[40] The data is available at http://www.bea.gov/industry/gdpbyind_data.htm and was retrieved on August 24, 2016.

Table 2—continued from previous page

z	BEA Code	BEA Title	Related 2007 NAICS Codes
21	315AL	Apparel and leather and allied products	315, 316
22	322	Paper products	322
23	323	Printing and related support activities	323
24	324	Petroleum and coal products	324
25	325	Chemical products	325
26	326	Plastics and rubber products	326
27	42	Wholesale trade	42
28	441	Motor vehicle and parts dealers	441
29	445	Food and beverage stores	445
30	452	General merchandise stores	452
31	4A0	Other retail	442, 443, 444, 446, 447, 448, 451, 453, 454
32	481	Air transportation	481
33	482	Rail transportation	482
34	483	Water transportation	483
35	484	Truck transportation	484
36	485	Transit and ground passenger transportation	485
37	486	Pipeline transportation	486
38	487OS	Other transportation and support activities	487, 488, 492
39	493	Warehousing and storage	493
40	511	Publishing industries, except internet (includes software)	511
41	512	Motion picture and sound recording industries	512
42	513	Broadcasting and telecommunications	513, 515, 517
43	514	Data processing, internet publishing, and other information services	514, 518, 519
44	521CI	Federal Reserve banks, credit intermediation, and related activities	521, 522
45	523	Securities, commodity contracts, and investments	523
46	524	Insurance carriers and related activities	524
47	525	Funds, trusts, and other financial vehicles	525
48	531	Real estate	531
49	532RL	Rental and leasing services and lessors of intangible assets	532, 533
50	5411	Legal services	5411
51	5415	Computer systems design and related services	5415
52	5412OP	Miscellaneous professional, scientific, and technical services	5412, 5413, 5414, 5416, 5417, 5418, 5419
53	55	Management of companies and enterprises	55
54	561	Administrative and support services	561
55	562	Waste management and remediation services	562
56	61	Educational services	611
57	621	Ambulatory health care services	621
58	622	Hospitals	622
59	623	Nursing and residential care facilities	623
60	624	Social assistance	624
61	711AS	Performing arts, spectator sports, museums, and related activities	711, 712
62	713	Amusements, gambling, and recreation industries	713
63	721	Accommodation	721
64	722	Food services and drinking places	722
65	81	Other services, except government	811, 812, 813, 814

G.3 Sectoral Results for the Year 2007

The following table reports for the year 2007 a set of descriptive statistics and the regression outcomes.

Table 3: Descriptive Statistics and Regression Results, LBD, 2007

z	J	$\sum_j L_j$	Descriptive statistics				Linear specification			Linear-quadratic specification				
			$E(\ln L_j)$	$SD(\ln L_j)$	$E(s_j)$	$SD(s_j)$	$100\cdot\hat{\eta}$	$100\cdot SE(\hat{\eta})$	R^2 (in %)	$100\cdot\hat{\hat{\eta}}$	$100\cdot SE(\hat{\hat{\eta}})$	$100\cdot\hat{\mu}$	$100\cdot SE(\hat{\mu})$	R^2 (in %)
1	114200	642400	1.06	1.03	11.33	6.89	1.80	0.04	1.45	1.27	0.12	0.02	0.01	1.47
2	25900	203400	1.27	1.13	11.36	9.13	2.27	0.08	3.35	2.57	0.23	−0.01	0.01	3.36
3	6900	89700	1.48	1.33	12.88	10.73	0.48	0.15	0.15	−0.69	0.57	0.04	0.02	0.22
4	6600	172500	2.31	1.36	14.84	11.88	2.22	0.14	3.75	3.21	0.55	−0.03	0.02	3.80
5	10200	213200	1.89	1.47	9.23	9.24	1.73	0.16	1.18	0.13	0.53	0.06	0.02	1.27
6	21100	561900	2.12	1.48	18.27	12.10	2.67	0.08	4.79	−0.39	0.40	0.09	0.01	5.06
7	687700	5490800	1.34	1.14	10.10	9.22	3.27	0.01	7.04	4.36	0.05	−0.04	0.00	7.11
8	15700	448600	2.42	1.42	14.69	12.01	3.52	0.09	8.92	4.22	0.30	−0.02	0.01	8.95
9	16700	402800	2.34	1.33	16.26	13.53	2.72	0.07	7.63	2.09	0.26	0.02	0.01	7.66
10	4800	366200	3.19	1.67	20.08	14.82	4.48	0.15	15.82	6.77	0.58	−0.05	0.01	16.11
11	57100	1297500	2.24	1.35	17.21	12.92	3.27	0.04	9.84	4.24	0.15	−0.02	0.00	9.91
12	24700	874100	2.51	1.47	18.59	13.11	3.29	0.07	8.63	3.30	0.24	0.00	0.01	8.63
13	13400	719500	2.67	1.64	15.47	11.94	4.11	0.11	8.91	4.51	0.37	−0.01	0.01	8.91
14	5800	334200	2.82	1.65	17.82	13.18	4.68	0.15	14.03	4.84	0.54	0.00	0.01	14.04
15	7900	725700	3.05	1.82	16.02	12.72	3.52	0.16	6.03	1.71	0.54	0.04	0.01	6.18
16	4300	352600	2.75	1.83	14.54	12.75	5.40	0.20	14.12	5.39	0.68	0.00	0.02	14.12
17	20000	373600	1.89	1.36	13.41	10.93	4.00	0.08	10.40	3.63	0.26	0.01	0.01	10.41
18	28200	467900	1.76	1.34	14.38	11.53	3.37	0.07	8.42	3.03	0.21	0.01	0.01	8.43
19	27700	1235800	2.50	1.58	15.59	13.09	4.85	0.07	16.14	3.73	0.23	0.03	0.01	16.22
20	9100	248800	2.04	1.51	15.08	12.17	4.25	0.12	11.66	2.40	0.41	0.05	0.01	11.87
21	9300	175100	1.96	1.35	10.47	10.96	2.77	0.13	5.02	1.20	0.38	0.04	0.01	5.22
22	4900	372900	3.49	1.48	22.14	14.62	4.20	0.13	17.11	4.30	0.53	0.00	0.01	17.11
23	31500	499000	1.84	1.28	16.25	12.01	2.94	0.06	7.58	2.41	0.19	0.01	0.00	7.60
24	2200	80100	2.22	1.52	17.54	14.29	3.70	0.21	12.18	0.38	0.77	0.08	0.02	12.96
25	13100	626800	2.72	1.56	17.60	13.74	3.60	0.09	10.04	4.77	0.34	−0.03	0.01	10.12
26	13800	735600	3.05	1.49	17.63	12.48	3.27	0.10	7.49	4.01	0.34	−0.02	0.01	7.52
27	400800	4596200	1.63	1.22	12.60	10.38	3.29	0.02	7.80	3.19	0.06	0.00	0.00	7.80
28	118400	1695500	1.93	1.20	12.90	10.37	2.56	0.03	4.88	4.05	0.13	−0.05	0.00	5.00
29	130900	2531700	1.88	1.41	11.20	10.09	4.15	0.04	8.78	7.31	0.14	−0.10	0.00	9.17
30	44900	2565300	2.76	1.62	11.13	9.62	5.66	0.07	11.34	11.26	0.27	−0.18	0.01	12.26
31	741800	6797900	1.69	1.03	11.51	9.86	1.58	0.01	2.31	3.81	0.04	−0.07	0.00	2.71
32	5400	217700	2.08	1.67	10.93	9.97	5.46	0.22	10.63	7.57	0.77	−0.07	0.02	10.76
33	1200	49000	2.27	1.68	9.92	8.39	4.83	0.56	5.86	13.49	1.84	−0.28	0.06	7.78

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Table 3—continued from previous page

z	J	Descriptive statistics					Linear specification			Linear-quadratic specification				
		$\sum_j L_j$	$E(\ln L_j)$	$SD(\ln L_j)$	$E(s_j)$	$SD(s_j)$	$100 \cdot \hat{\eta}$	$100 \cdot SE(\hat{\eta})$	R^2 (in %)	$100 \cdot \hat{\hat{\eta}}$	$100 \cdot SE(\hat{\hat{\eta}})$	$100 \cdot \hat{\mu}$	$100 \cdot SE(\hat{\mu})$	R^2 (in %)
34	1600	49800	2.07	1.60	12.08	10.42	4.58	0.37	8.89	6.87	1.38	−0.07	0.04	9.06
35	103300	1060600	1.37	1.28	9.30	8.96	4.68	0.04	10.70	6.19	0.14	−0.05	0.00	10.82
36	16300	398700	2.02	1.55	10.41	9.47	4.26	0.12	6.78	4.86	0.42	−0.02	0.01	6.80
37	2600	30900	1.78	1.18	10.56	10.66	2.01	0.21	3.32	3.95	0.81	−0.06	0.03	3.55
38	52000	810600	1.70	1.35	9.29	8.64	3.35	0.07	4.60	4.83	0.21	−0.05	0.01	4.70
39	14300	574500	2.46	1.54	10.36	9.68	1.68	0.13	1.12	4.60	0.44	−0.09	0.01	1.46
40	29800	711400	2.00	1.46	12.13	11.47	3.66	0.07	8.21	2.27	0.22	0.04	0.01	8.35
41	19000	210400	1.43	1.34	9.61	8.51	2.42	0.11	2.35	5.52	0.35	−0.11	0.01	2.80
42	58000	1074800	1.78	1.43	8.39	9.03	3.60	0.06	5.16	8.97	0.22	−0.18	0.01	6.22
43	23800	427000	1.70	1.45	9.96	10.38	3.74	0.09	7.20	7.93	0.34	−0.13	0.01	7.81
44	219400	2164200	1.70	1.05	10.10	10.32	2.81	0.02	7.64	5.18	0.08	−0.08	0.00	8.07
45	79000	525300	1.05	1.09	7.62	7.73	2.47	0.05	3.06	3.38	0.14	−0.03	0.01	3.12
46	165700	1268900	1.20	1.10	11.45	10.04	2.24	0.03	4.19	1.79	0.10	0.01	0.00	4.20
47	7500	58500	0.94	1.18	9.51	10.65	3.55	0.12	10.32	3.68	0.47	0.00	0.01	10.33
48	266100	1157200	0.93	0.93	9.72	9.40	1.61	0.02	2.65	2.62	0.06	−0.03	0.00	2.75
49	62800	508400	1.61	0.99	9.57	8.37	2.37	0.05	4.00	5.98	0.14	−0.13	0.00	5.08
50	172800	889600	1.05	0.98	12.41	9.75	1.53	0.02	2.33	2.27	0.09	−0.02	0.00	2.37
51	461600	3495500	1.24	1.14	9.56	8.63	3.00	0.02	5.13	4.00	0.06	−0.04	0.00	5.19
52	95100	811900	1.14	1.24	6.12	5.77	3.79	0.07	3.12	6.24	0.18	−0.11	0.01	3.34
53	50000	2132900	2.31	1.69	10.77	10.03	3.84	0.07	5.18	5.30	0.24	−0.05	0.01	5.26
54	309800	5779000	1.68	1.43	8.97	8.29	2.48	0.03	2.06	4.00	0.10	−0.05	0.00	2.15
55	20400	309200	1.90	1.27	10.10	9.01	3.04	0.10	4.68	4.49	0.32	−0.05	0.01	4.78
56	90100	6748600	2.44	1.83	14.11	11.91	8.88	0.04	33.24	−1.12	0.19	0.29	0.01	35.34
57	505500	4373400	1.56	1.06	11.98	9.90	0.93	0.02	0.75	3.64	0.06	−0.09	0.00	1.24
58	6900	5018900	5.75	1.59	19.82	11.53	6.22	0.15	20.38	20.14	0.63	−0.39	0.02	25.95
59	72800	2780600	2.71	1.44	11.08	9.47	5.11	0.05	11.34	3.17	0.18	0.06	0.01	11.48
60	142500	2075000	1.94	1.23	11.56	9.89	2.83	0.03	5.13	4.37	0.12	−0.05	0.00	5.25
61	39500	376700	1.16	1.25	11.23	9.77	3.66	0.06	8.21	1.18	0.22	0.08	0.01	8.53
62	61500	1125900	2.01	1.35	11.87	10.96	2.39	0.05	3.79	4.99	0.20	−0.08	0.01	4.08
63	55700	1318400	2.18	1.38	10.63	9.35	0.23	0.06	0.02	3.10	0.22	−0.09	0.01	0.36
64	505500	8893400	2.29	1.16	9.13	8.66	2.40	0.02	3.22	5.36	0.06	−0.11	0.00	3.72
65	691400	4396100	1.30	1.00	14.16	11.06	2.26	0.01	6.24	1.97	0.04	0.01	0.00	6.24

Notes: z denotes the industry number; for table 2; J denotes the number of establishments in industry z; $\sum_j L_j$ denotes total employment; $E(\ln L_j)$ denotes

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the mean of log employment; $SD\left(\ln L_j\right)$ denotes the standard deviation of log employment; $E\left(s_j\right)$ denotes the mean of establishment age; $SD\left(s_j\right)$ denotes the standard deviation of establishment age; $\widehat{\eta}$ denotes the estimated coefficient obtained from the regression (35); $SE\left(\widehat{\eta}\right)$ denotes the standard error of this coefficient; R^2 denotes the regression's R-squared statistic; $\widehat{\widehat{\eta}}$ denotes the estimated coefficient on age when augmenting the regression (35) with a regressor age squared; and $\widehat{\widehat{\mu}}$ denotes the estimated coefficient on age squared.

Online Appendix to "Optimal Trend Inflation"

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This appendix spells out details of the multi-sector model in sections 10 and 11 of the main text and presents proofs for the propositions and lemmas stated in these sections.

1 Decentralized Economy

1.1 Household and Government Budget Constraints

In the multi-sector model, the representative household has the same preferences as in the one-sector model, but faces the modified flow budget constraint

$$C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \sum_{z=1}^{Z}\left(\int_0^1 \frac{\Theta_{jzt}}{P_t}\ \mathrm{dj}\right) + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t,$$

where Θ_{jzt} denotes nominal profits from ownership of firm j in sector $z = 1, \ldots Z$. The government faces the budget constraint

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + \sum_{z=1}^{Z} \tau\left(\int_0^1 \left(\frac{P_{jzt}}{P_t}\right) Y_{jzt}\ \mathrm{dj}\right) - T_t.$$

1.2 Sectoral Technology, Marginal Costs and Price Setting

Sectoral technology. Output Y_{zt} of sector z combines intermediate products $j \in [0, 1]$ according to

$$Y_{zt} = \left(\int_0^1 Y_{jzt}^{\frac{\theta-1}{\theta}}\ dj\right)^{\frac{\theta}{\theta-1}} \quad , \quad \theta > 1\ .$$

Cost minimization yields $P_{zt} = \left(\int_0^1 P_{jzt}^{1-\theta}\ dj\right)^{\frac{1}{1-\theta}}$ and the usual demand functions. Firm j in sector z uses technology $Y_{jzt} = A_{zt}Q_{zt-s_{jzt}}G_{jzt}\left(K_{jzt}^{1-1/\phi}L_{jzt}^{1/\phi} - F_{zt}\right)$. The idiosyncratic δ-shock in sector z occurs at the rate $\delta_z \geq 0$.

Marginal costs. Firm j hires labor and capital at economy-wide and perfectly competitive factor markets. The cost minimization problem of firm j in sector z yields the first order conditions that imply that firms $j \in [0, 1]$ maintain the same optimal capital labor ratio in all sectors and implies that marginal costs correspond to

$$MC_t = \left(\frac{W_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{P_t r_t}{1 - 1/\phi}\right)^{1 - \frac{1}{\phi}}. \tag{1}$$

Price setting. Let P_{jzt} denote the price charged by firm j in sector z in period t. The firms in this sector that receive a δ-shock can freely choose the product price but otherwise can adjust prices only with probability $\alpha_z \in (0,1)$ in each period. Thus, firm j sets its optimal price by solving:

$$\max_{P_{jzt}} E_t \sum_{i=0}^{\infty} (\alpha_z(1-\delta_z))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1+\tau_z)P_{jzt}Y_{jzt+i} - MC_{t+i}I_{jzt+i}\right] \tag{2}$$

$$s.t. \quad Y_{jzt+i} = A_{zt+i}Q_{zt+i}\mathcal{Q}_{jzt+i}\left(I_{jzt+i} - F_{zt+i}\right),$$

$$Y_{jzt+i} = \psi_z \left(\frac{P_{jzt}}{P_{zt+i}}\right)^{-\theta} \left(\frac{P_{zt+i}}{P_{t+i}}\right)^{-1} Y_{t+i}.$$

$I_{jzt} = F_{zt} + Y_{jzt}/(A_{zt}Q_{zt}\mathcal{Q}_{jzt})$ denotes the units of factor inputs $(K_{jzt}^{1-\frac{1}{\phi}}L_{jzt}^{\frac{1}{\phi}})$ required to produce Y_{jzt} units of output, $\mathcal{Q}_{jzt} = Q_{zt-s_{jzt}}G_{jzt}/Q_{zt}$, and $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$. The optimal price of firm j in sector z evolves according to

$$\frac{P_{jzt}^{\star}}{P_t}\mathcal{Q}_{jzt} = \left(\frac{1}{1+\tau}\frac{\theta}{\theta-1}\right)\frac{N_{zt}}{D_{zt}}, \tag{3}$$

$$D_{zt} = 1 + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\left(\frac{P_{zt+1}}{P_{zt}}\right)^{\theta-1}\left(\frac{Y_{t+1}}{Y_t}\right)D_{zt+1}\right], \tag{4}$$

$$N_{zt} = \frac{MC_t}{P_tA_{zt}Q_{zt}} + \alpha_z(1-\delta_z)E_t\left[\Omega_{t,t+1}\left(\frac{P_{zt+1}}{P_{zt}}\right)^{\theta-1}\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{q_{zt+1}}{g_{zt+1}}\right)N_{zt+1}\right]. \tag{5}$$

1.3 Aggregation and Market Clearing

Sectoral and aggregate price levels. We express the sectoral price level recursively following analogous steps to the steps that we use in the one sector model to derive the aggregate price level. This yields

$$P_{zt}^{1-\theta} = \left[\alpha_z\delta_z + (1-\alpha_z)(\Delta_{zt}^e)^{1-\theta}\right](P_{z,t,t}^{\star})^{1-\theta} + \alpha_z(1-\delta_z)(P_{zt-1})^{1-\theta}. \tag{6}$$

Here, $P_{z,t-s,t}^{\star}$ $(P_{z,t,t}^{\star})$ denotes the optimal price of the firm that received a δ shock s (zero) periods ago and belongs to sector z. Δ_{zt}^e denotes the productivity adjustment factor in sector z in the efficient economy, which is derived below. This factor can be shown to evolve recursively as

$$(\Delta_{zt}^e)^{1-\theta} = \delta_z + (1-\delta_z)\left(\Delta_{zt-1}^e q_{zt}/g_{zt}\right)^{1-\theta}. \tag{7}$$

Equation (6) implies that

$$1 = \left[\alpha_z\delta_z + (1-\alpha_z)(\Delta_{zt}^e)^{1-\theta}\right](p_{zt}^{\star})^{1-\theta} + \alpha_z(1-\delta_z)\Pi_{zt}^{\theta-1},$$

using the definitions $p_{zt}^{\star} = P_{z,t,t}^{\star}/P_{zt}$ and $\Pi_{zt} = P_{zt}/P_{zt-1}$. Cobb-Douglas aggregation of sectoral output also implies that the aggregate price level corresponds to

$$P_t = \prod_{z=1}^{Z} \left(\frac{P_{zt}}{\psi_z}\right)^{\psi_z}.$$

Sectoral and aggregate technologies. We define the sectoral productivity factor as

$$\Delta_{zt} = \int_0^1 \left(\frac{1}{\mathcal{Q}_{jzt}}\right) \left(\frac{P_{jzt}}{P_{zt}}\right)^{-\theta} dj.$$

Following corresponding steps as in the one sector economy, this equation can be expressed recursively according to

$$\Delta_{zt} = \left[\alpha_z \delta_z + (1 - \alpha_z)(\Delta_{zt}^e)^{1-\theta}\right] (p_{zt}^{\star})^{-\theta} + \alpha_z(1 - \delta_z)\left(\frac{q_{zt}}{g_{zt}}\right) \Pi_{zt}^{\theta}\Delta_{zt-1}. \tag{8}$$

It can also be shown that the sectoral technology corresponds to

$$Y_{zt} = \frac{A_{zt}Q_{zt}}{\Delta_{zt}}\left(K_{zt}^{1-\frac{1}{\phi}}L_{zt}^{\frac{1}{\phi}} - F_{zt}\right),$$

using the definitions $L_{zt} = \int_0^1 L_{jzt} \, dj$ and $K_{zt} = \int_0^1 K_{jzt} \, dj$. Augmenting economy-wide labor market clearing $L_t = \sum_z L_{zt}$ according to

$$\left(\frac{K_t}{L_t}\right)^{1-\frac{1}{\phi}} L_t = \sum_{z=1}^{Z} \left(\frac{K_{zt}}{L_{zt}}\right)^{1-\frac{1}{\phi}} L_{zt}$$

and rewriting it using sectoral technology and $K_t = \sum_z K_{zt}$ yields

$$K_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - F_t = \sum_z \left(Y_{zt}\frac{\Delta_{zt}}{A_{zt}Q_{zt}}\right),$$

with $F_t = \sum_z F_{zt}$. We then replace sectoral output by aggregate output using demand functions $Y_{zt} = \psi_z \left(P_{zt}/P_t\right)^{-1} Y_t$. This yields the aggregate technology

$$Y_t = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t}\left(K_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - F_t\right), \tag{9}$$

denoting the aggregate productivity factor by

$$\frac{\Delta_t}{(\Gamma_t^e)^{1/\phi}} = \sum_z \psi_z \left(\frac{P_{zt}}{P_t}\right)^{-1}\left(\frac{\Delta_{zt}}{A_{zt}Q_{zt}}\right).$$

Γ_t^e denotes the aggregate growth trend that is derived below.

2 Planner Problem

To isolate the distortions in the allocation of the decentralized economy, we also derive the the first-best allocation from the planner problem. The solution of the planner problem involves the allocation of factor inputs across firms with different levels of productivity within the sector z; the allocation of factor inputs between sectors with different average productivities; and the optimal intertemporal paths of aggregate variables.

2.1 Sectoral and Aggregate Technologies

The within-sector allocation corresponds to the intratemporal allocation in the one sector model, when this allocation is applied to sector z. Thus, sectoral technology in the planned economy corresponds to

$$Y_{zt}^e = \frac{A_{zt} Q_{zt}}{\Delta_{zt}^e} \left((K_{zt}^e)^{1-\frac{1}{\phi}} (L_{zt}^e)^{\frac{1}{\phi}} - F_{zt} \right), \tag{10}$$

where the efficient level of the endogenous component of sectoral productivity is

$$1/\Delta_{zt}^e = \left(\int_0^1 \left(G_{jzt} Q_{t-s_{jzt}} / Q_{zt} \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}},$$

and evolves according to equation (7).

To obtain the aggregate technology in the planner solution, the planner solves

$$\max_{Y_{zt}^e, L_{zt}^e, K_{zt}^e, \forall z} Y_t^e = \prod_z (Y_{zt}^e)^{\psi_z} \quad s.t. \quad Y_{zt}^e = \frac{A_{zt} Q_{zt}}{\Delta_{zt}^e} \left((K_{zt}^e)^{1-\frac{1}{\phi}} (L_{zt}^e)^{\frac{1}{\phi}} - F_{zt} \right),$$

with $L_t^e = \sum_z L_{zt}^e$ and $K_t^e = \sum_z K_{zt}^e$ and L_t^e, K_t^e given. The solution to this problem yields the aggregate technology

$$Y_t^e = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e} \left((K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}} - F_t \right), \tag{11}$$

with $F_t = \sum_z F_{zt}$ and defining

$$\frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e} = \prod_z \psi_z^{\psi_z} \left(\frac{A_{zt} Q_{zt}}{\Delta_{zt}^e} \right)^{\psi_z}. \tag{12}$$

2.2 Intertemporal First-Best Allocation

The derivation of the intertemporal allocation in the planner problem proceeds along analogous steps as the derivation of the one-sector model. Therefore, the first-best allocation

of aggregate variables implied by the planner solution corresponds to

$$\left(\Delta_{zt}^e\right)^{1-\theta} = \delta_z + (1-\delta_z)\left(\Delta_{zt-1}^e q_{zt}/g_{zt}\right)^{1-\theta}$$

$$\frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e} = \prod_z \psi_z^{\psi_z}\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\psi_z}$$

$$Y_t^e = \frac{(\Gamma_t^e)^{1/\phi}}{\Delta_t^e}\left((K_t^e)^{1-\frac{1}{\phi}}(L_t^e)^{\frac{1}{\phi}} - F_t\right),$$

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{13}$$

$$1 = \beta E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}^e}{U_{Ct}^e}\left(Y_{Kt+1}^e + 1 - d\right)\right],$$

$$K_{t+1}^e = (1-d)K_t^e + Y_t^e - C_t^e,$$

denoting by Y_{Kt}^e the marginal product of capital and by Y_{Lt}^e the marginal product of labor.

2.3   Balanced Growth Path

Let aggregate output grow with the trend Γ_t^e and sectoral output grow with the trend Γ_{zt}^e. The intertemporal feasibility condition implies that K_t^e grows at the same rate as aggregate output, and $K_t^e = \sum_z K_{zt}^e$ implies that K_{zt}^e grows at the same rate as K_t^e, i.e., at rate Γ_t^e.

We express sectoral and aggregate growth trends in terms of productivity parameters by using the sectoral and the aggregate technology in the planner solution. This yields

$$\Gamma_t^e = \prod_{z=1}^Z \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\psi_z\phi},$$

$$\frac{\Gamma_{zt}^e}{\Gamma_t^e} = \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right) / \prod_{z=1}^Z \left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)^{\psi_z}.$$

These two equation further imply that

$$\gamma_t^e = \prod_{z=1}^Z (\gamma_{zt}^e)^{\psi_z},$$

$$\gamma_{zt}^e = (\gamma_t^e)^{1-\frac{1}{\phi}}\left(\frac{a_{zt}q_{zt}\Delta_{zt-1}^e}{\Delta_{zt}^e}\right),$$

using $\gamma_{zt}^e = \Gamma_{zt}^e/\Gamma_{zt-1}^e$ and $\gamma_t^e = \Gamma_t^e/\Gamma_{t-1}^e$. We use the growth trends Γ_t^e and Γ_{zt}^e and the assumption that $F_{zt} = f_z(\Gamma_t^e)^{1-1/\phi}$ to convert the non-stationary variables in the system of equations (13) into stationary variables. This implies that the aggregate productivity factor in the planner solution is constant and corresponds to $1/\Delta^e = \prod_z \psi_z^{\psi_z}$.

3 Steady State in the Decentralized Economy

We use the sectoral and aggregate growth trend Γ_t^e and Γ_{zt}^e from the planner solution to also detrend the non-stationary variables in the decentralized economy. For this transformation, we define the stationary variables $y_t = Y_t/\Gamma_t^e$, $k_t = K_t/\Gamma_t^e$, $c_t = C_t/\Gamma_t^e$, and $w_t = W_t/(P_t\Gamma_t^e)$.

3.1 Steady State with Two Distortions

Then, we rewrite the detrended decentralized economy in a way that shows that only two distortions, the relative price distortion $\rho(\Pi)$ and the markup distortion $\mu(\Pi)$, which both depend on the aggregate inflation rate $\Pi_t = P_t/P_{t-1}$, prevent the decentralized economy from perfectly replicating the planner solution. These steps again are analogous to the steps in the one-sector economy (see Appendix A.8 in the paper). In the steady state, the decentralized multi-sector economy is then represented by the following equations.

$$y = \left(\frac{\rho(\Pi)}{\Delta^e}\right)\left(k^{1-\frac{1}{\phi}}L^{\frac{1}{\phi}} - f\right) \tag{14}$$

$$c\left(-\frac{V_L}{V(L)}\right) = \frac{\mu(\Pi)^{-1}}{\Delta^e}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \tag{15}$$

$$1/[\beta\,(\gamma^e)^{-\sigma}] - 1 + d = \frac{\mu(\Pi)^{-1}}{\Delta^e}\left(1 - \frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{-\frac{1}{\phi}} \tag{16}$$

$$y = c + (\gamma^e - 1 + d)k. \tag{17}$$

Here, V_L denotes the derivative of $V(L)$. Given the aggregate distortions $\rho(\Pi)$ and $\mu(\Pi)$ and the aggregate growth rate γ^e, these equations determine y, k, L and c. The aggregate distortions are determined by the equations (suppressing the argument Π)

$$(\rho\mu)^{-1} = \sum_{z=1}^{Z}\psi_z(\mu_z\rho_z)^{-1}, \tag{18}$$

$$\mu = \prod_{z=1}^{Z}\mu_z^{\psi_z}, \tag{19}$$

and hence are defined in terms of sectoral distortions.

The sectoral relative price distortion is defined as $\rho_z = \Delta_z^e/\Delta_z$. Using the equations (7) and (8), which determine Δ_z^e and Δ_z, respectively, we can express the (inverse) sectoral relative price distortion in the steady state as a function of the aggregate inflation rate,

$$\rho_z(\Pi)^{-1} = \left(\frac{1 - \alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}}{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1}}\right)\left(\frac{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}, \tag{20}$$

which holds for $z = 1, \ldots Z$ and where we have used the fact that the sectoral inflation rate is related to the aggregate inflation rate according to $\Pi_z = (\gamma^e/\gamma_z^e) \Pi$, which follows from product demand $(P_{zt}/P_t) = \psi_z (Y_{zt}/Y_t)^{-1}$.

The sectoral markup distortion is defined as $\mu_z = p_z/mc$. Using the equations (3) to (5) and combining them with the definitions that $p_{zt} = (P_{zt}/P_t) (\Gamma_{zt}^e/\Gamma_t^e)$ and $mc_t = MC_t/(P_t(\Gamma_t^e)^{1/\phi})$, we can also express the sectoral markup distortion in the steady state as a function of the aggregate inflation rate,

$$\mu_z(\Pi) = \left(\frac{1}{1+\tau} \frac{\theta}{\theta-1} \right) \tag{21}$$

$$\left(\frac{1 - \alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)\beta(\gamma^e)^{1-\sigma}[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1}} \right) \left(\frac{1 - \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}}{1 - \alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}} \right)^{\frac{1}{\theta-1}},$$

which holds for $z = 1, \ldots Z$ and where we have used that $\Pi_z = (\gamma^e/\gamma_z^e) \Pi$. To summarize, equations (14) to (21) represent the steady state equations that determine the variables $c, L, y, k, \mu, \rho, \mu_z, \rho_z$ for the sectors $z = 1, \ldots Z$ and given Π.

3.2 Conditions for the Existence of the Steady State

We provide existence conditions for the limiting case in which $\beta(\gamma^e)^{1-\sigma} \to 1$, which is the case for which we derive our main results in the multi-sector economy. First, we impose

$$1 > (1 - \delta_z) (g_z/q_z)^{\theta-1}, \tag{22}$$

for all $z = 1, \ldots Z$, to ensure that Δ_z^e in equation (7) has a well-defined steady state value. Second, we impose conditions that ensure sectoral distortions in equations (20) and (21) that are well defined in the steady state with $\beta(\gamma^e)^{1-\sigma} \to 1$. These conditions are

$$1 > \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta}(g_z/q_z)^{-1},$$
$$1 > \alpha_z(1-\delta_z)[(\gamma^e/\gamma_z^e)\Pi]^{\theta-1}.$$

The additional condition $1 > \alpha_z(1-\delta_z)(g_z/q_z)^{\theta-1}$ is always fulfilled as a result of condition (22) and $\alpha_z < 1$.

4 Proof of Proposition 5

First, we show that under the conditions stated in the proposition, the sectoral relative price distortion and the sectoral markup distortion are inversely equal to each other and that therefore the two aggregate distortions are also inversely equal to each other. Second, we show that as a result, the aggregate steady state inflation rate that maximizes steady state utility can be derived by minimizing the aggregate markup distortion. Third, we

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show that this minimization yields the optimal aggregate steady state inflation rate in the proposition.

4.1 Proportionality of Distortions

In the steady state with $\beta(\gamma^e)^{1-\sigma} \to 1$ and $\frac{1}{1+\tau}\frac{\theta}{\theta-1} = 1$, the sectoral markup distortion in equation (21) can be rearranged according to

$$
\begin{aligned}
\mu_z(\Pi) &= \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1}} \right) \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}} \right)^{\frac{1}{\theta-1}}, \\
&= \left(\frac{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}}{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta}(g_z/q_z)^{-1}} \right) \left(\frac{1 - \alpha_z(1 - \delta_z)[(\gamma^e/\gamma_z^e)\,\Pi]^{\theta-1}}{1 - \alpha_z(1 - \delta_z)(g_z/q_z)^{\theta-1}} \right)^{\frac{\theta}{\theta-1}}, \\
&= \rho_z(\Pi)^{-1},
\end{aligned}
$$

where the last step follows from equation (20). Equation (18) then implies that

$$
\mu(\Pi) = \rho(\Pi)^{-1}.
$$

4.2 Steady State with One Distortion

The inverse relationship between the two aggregate distortions and $\beta(\gamma^e)^{1-\sigma} \to 1$ imply that equations (14) to (17) can be rearranged as

$$
y = \left[\mu(\Pi)^{-1}/\Delta^e\right] \left(k^{1-\frac{1}{\phi}} L^{\frac{1}{\phi}} - f \right) \tag{23}
$$

$$
c \left(-\frac{V_L}{V(L)} \right) = \left[\mu(\Pi)^{-1}/\Delta^e\right] \left(\frac{1}{\phi} \right) \left(\frac{k}{L} \right)^{1-\frac{1}{\phi}} \tag{24}
$$

$$
\gamma^e - 1 + d = \left[\mu(\Pi)^{-1}/\Delta^e\right] \left(1 - \frac{1}{\phi} \right) \left(\frac{k}{L} \right)^{-\frac{1}{\phi}} \tag{25}
$$

$$
y = c + (\gamma^e - 1 + d)k. \tag{26}
$$

Accordingly, the sticky price economy consists of the definition of the aggregate markup in equation (19), the relationship $\mu_z(\Pi) = \rho_z(\Pi)^{-1}$ and equation (20) determining $\rho_z(\Pi)^{-1}$, and the equations (23) to (26).

In the proposition, we derive the aggregate steady state inflation rate Π that maximizes steady state utility subject to this sticky price economy. Given the structure of this economy, however, it turns out that instead of $\max_\Pi U(c(\Pi), L(\Pi))$, we can directly $\min_\Pi \mu(\Pi)$.

The reason for this is that Π enters equations (23) to (26) only through aggregate productivity $\mu(\Pi)^{-1}/\Delta^e$, which enters the aggregate technology in equation (23), the MPL in equation (24), and the MPC in equation (25). This implies that minimizing $\mu(\Pi)$

shifts the production possibility frontier of the social planner that seeks the optimal $\Pi^\star$ outwards and therefore also maximizes steady state utility.

4.3 Minimizing the Markup Distortion

Equation (19) implies that minimizing the aggregate markup distortion requires that

$$\frac{\partial \mu(\Pi)}{\partial \Pi} = \sum_{z=1}^{Z} \psi_z \mu_z(\Pi)^{\psi_z - 1} [\partial \mu_z(\Pi)/\partial \Pi] \left(\prod_{\neg z} \mu_z(\Pi)^{\psi_z} \right) = 0,$$

using $\neg z$ to denote the set of all sectors except sector z. Simplifying yields

$$\sum_{z=1}^{Z} \psi_z \frac{\partial \mu_z(\Pi)/\partial \Pi}{\mu_z(\Pi)} = 0. \tag{27}$$

We use equation (20), $\mu_z(\Pi) = \rho_z(\Pi)^{-1}$ and shorthand $s_z = \alpha_z(1 - \delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$ to obtain

$$\frac{\partial \mu_z(\Pi)/\partial \Pi}{\mu_z(\Pi)} = \frac{\theta s_z \Pi^{\theta-2} \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right)}{\left(1 - s_z \Pi^\theta \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right) \right) \left(1 - s_z \Pi^{\theta-1} \right)} \left[\Pi - \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right)^{-1} \right].$$

Plugging this expression into equation (27) and multiplying by Π^2 yields

$$\sum_{z=1}^{Z} \left(\frac{\psi_z \theta s_z \Pi^\theta \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right)}{\left(1 - s_z \Pi^\theta \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right) \right) \left(1 - s_z \Pi^{\theta-1} \right)} \right) \left[\Pi - \left(\frac{q_z \gamma^e}{g_z \gamma_z^e} \right)^{-1} \right] = 0. \tag{28}$$

We denote the weight in parenthesis by $\tilde{\omega}_z$, and normalize it so that it sums to unity. This yields the new weight $\omega_z = \tilde{\omega}_z / \sum_{z=1}^{Z} \tilde{\omega}_z$, with $\sum_{z=1}^{Z} \omega_z = 1$. Thus, we obtain

$$\sum_{z=1}^{Z} \omega_z \left[\Pi^\star - \left(\frac{g_z \gamma_z^e}{q_z \gamma^e} \right) \right] = 0, \tag{29}$$

where ω_z is given by the expression in the proposition. Solving equation (29) for $\Pi^\star$ also yields the optimal aggregate steady state inflation rate in the proposition.

5 Proof of Lemma 3

To derive the lemma, we denote $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$ and repeat equation (28) with the new notation:

$$\sum_{z=1}^{Z} \tilde{\omega}_z(\Pi, m_z) \left[\Pi - m_z \right] = 0, \tag{30}$$

with $\tilde{\omega}_z(\Pi, m_z) = \frac{\psi_z \theta s_z \Pi^\theta / m_z}{\left(1 - s_z \Pi^\theta / m_z \right) \left(1 - s_z \Pi^{\theta-1} \right)}$ and $s_z = \alpha_z(1 - \delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$. Expanding equation (30) accurate to the first order at the points $\bar{\Pi}$ and $\bar{m}_z$, with $\bar{\Pi} = \bar{m}_z$, yields

$$\sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z) \left[\Pi - m_z \right] = 0 + O(2).$$

Rewriting this equation yields

$$\Pi^\star = \left(\sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z) \right)^{-1} \sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z) \, m_z + O(2). \tag{31}$$

$\Pi^\star$ is a weighted average of the m_z's for all sectors z and with weights evaluated at the expansion point and normalized to unity. The normalized weight of sector z evaluated at $\bar{\Pi} = \bar{m}_z$ corresponds to

$$\frac{\tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)}{\sum_{z=1}^{Z} \tilde{\omega}_z(\bar{\Pi}, \bar{m}_z)} = \psi_z \left[\frac{\theta s_z \bar{\Pi}^{\theta-1}}{\left(1 - s_z \bar{\Pi}^{\theta-1}\right)^2} \right] \left(\sum_{z=1}^{Z} \psi_z \left[\frac{\theta s_z \bar{\Pi}^{\theta-1}}{\left(1 - s_z \bar{\Pi}^{\theta-1}\right)^2} \right] \right)^{-1},$$

$$= \psi_z,$$

where the second equality follows from the requirement in the lemma that $s_z = \alpha_z(1 - \delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$ is the same for all sectors $z = 1, \ldots Z$ and the fact that $\sum_{z=1}^{Z} \psi_z = 1$. Hence,

$$\Pi^\star = \sum_{z=1}^{Z} \psi_z m_z + O(2),$$

which corresponds to the equation in the lemma after using $m_z = \frac{g_z \gamma_z^e}{q_z \gamma^e}$.

6 Proof of Proposition 6

To derive the proposition, we solve technology $Y_{jzt} = A_{zt} Q_{z,t-s_{jzt}} G_{jzt} \left(K_{jzt}^{1-1/\phi} L_{jzt}^{1/\phi} - F_{zt} \right)$ of the firm j in sector z for labor L_{jzt}. The fact that the optimal capital labor ratio of firm j corresponds to the aggregate capital labor ratio yields

$$L_{jzt} = \left(\frac{K_t}{L_t} \right)^{\frac{1}{\phi}-1} \left(F_{zt} + \frac{Y_{jzt}}{A_{zt} Q_{z,t-s_{jzt}} G_{jzt}} \right). \tag{32}$$

We now replace Y_{jzt} by the firm's product demand $Y_{jzt} = (P_{jzt}/P_{zt})^{-\theta} Y_{zt}$. To express the relative price in product demand by the relative productivity, we use the pricing equations (3) to (5), impose $\alpha_z = 0$, and combine them with the pricing equations for a firm in sector z that receives a δ shock in period t. This yields

$$\frac{P_{jzt}}{P_{zt}} \left(\frac{Q_{zt-s_{jzt}} G_{jzt}}{Q_{zt}} \right) = \frac{P_{z,t,t}^\star}{P_{zt}}. \tag{33}$$

Using the notation $p_{zt}^\star = P_{z,t,t}^\star/P_{zt}$, we obtain from the equation (6) with $\alpha_z = 0$ that $p_{zt}^\star = 1/\Delta_{zt}^e$. Accordingly, we rearrange equation (33) to obtain

$$\frac{P_{jzt}}{P_{zt}} = \left(\frac{A_{zt} Q_{zt}}{\Delta_{zt}^e} \right) \frac{1}{A_{zt} Q_{z,t-s_{jzt}} G_{jzt}}.$$

Plugging this equation for the relative price into product demand yields

$$Y_{jzt} = (\Delta_{zt}^e)^\theta \left(\frac{Q_{z,t-s_{jzt}}G_{jzt}}{Q_{zt}}\right)^\theta Y_{zt},$$

or

$$\frac{Y_{jzt}}{A_{zt}Q_{z,t-s_{jzt}}G_{jzt}} = \left[\left(\frac{A_{zt}Q_{zt}}{\Delta_{zt}^e}\right)\left(\frac{Q_{z,t-s_{jzt}}G_{jzt}}{Q_{zt}}\right)\right]^{-1}(\Delta_{zt}^e)^{\theta-1}\left(\frac{Q_{z,t-s_{jzt}}G_{jzt}}{Q_{zt}}\right)^\theta Y_{zt},$$

$$= \left[\Gamma_{zt}^e(\Gamma_t^e)^{\frac{1}{\phi}-1}\right]^{-1}(\Delta_{zt}^e)^{\theta-1}\left(\frac{Q_{z,t-s_{jzt}}G_{jzt}}{Q_{zt}}\right)^{\theta-1}Y_{zt},$$

or

$$\frac{Y_{jzt}}{A_{zt}Q_{z,t-s_{jzt}}G_{jzt}} = \left(Q_{z,t-s_{jzt}}G_{jzt}/Q_{zt}\right)^{\theta-1}y_{zt}(\Delta_{zt}^e)^{\theta-1}(\Gamma_t^e)^{1-\frac{1}{\phi}}, \tag{34}$$

with $y_{zt} = Y_{zt}/\Gamma_{zt}^e$ and since trend growth in the multi-sector economy implies that $\Gamma_{zt}^e(\Gamma_t^e)^{\frac{1}{\phi}-1} = A_{zt}Q_{zt}/\Delta_{zt}^e$. Substituting equation (34) into equation (32) and using $k_t = K_t/\Gamma_t^e$ and $F_{zt} = f_z(\Gamma_t^e)^{1-\frac{1}{\phi}}$ yields

$$L_{jzt} = \left(f_z + \left(Q_{z,t-s_{jzt}}G_{jzt}/Q_{zt}\right)^{\theta-1}y_{zt}(\Delta_{zt}^e)^{\theta-1}\right)\left(\frac{k_t}{L_t}\right)^{\frac{1}{\phi}-1}, \tag{35}$$

which shows that L_{jzt} grows with the relative productivity $Q_{z,t-s_{jzt}}G_{jzt}/Q_{zt}$. Imposing zero fixed costs $f_z = 0$ and taking the natural logarithm yields

$$\ln(L_{jzt}) = (\theta-1)\ln\left(Q_{z,t-s_{jzt}}G_{jzt}/Q_{zt}\right) + d_{zt}.$$

The composite variable $d_{zt} = \ln\left((k_t/L_t)^{\frac{1}{\phi}-1}y_{zt}(\Delta_{zt}^e)^{\theta-1}\right)$ and varies with time t and sector z. Dropping the sector subscript z to economize on notation thus yields

$$\ln(L_{jt}) = (\theta-1)\ln\left(Q_{t-s_{jt}}G_{jt}/Q_t\right) + d_t. \tag{36}$$

Given our assumptions on the productivity processes, we have

$$\ln\left(Q_{t-s_{jt}}G_{jt}/Q_t\right) = \ln\left(\frac{Q_0 q^{t-s_{jt}}g^{s_{jt}}}{Q_0 q^t}\right) + \ln\left(\prod_{i=1}^{t-s_{jt}}\varepsilon_i^q\right)\left(\prod_{i=t-s_{jt}+1}^t\varepsilon_i^g\right)\bigg/\left(\prod_{i=1}^t\varepsilon_i^q\right),$$

$$= s_{jt}\ln(g/q) + \frac{1}{\theta-1}\epsilon_{jt}, \tag{37}$$

where $\epsilon_{jt} = (\theta-1)\ln\left(\prod_{i=t-s_{jt}+1}^t\varepsilon_i^g\right)\big/\left(\prod_{i=t-s_{jt}+1}^t\varepsilon_i^q\right)$ is a stationary residual under the stated assumptions. Plugging equation (37) into equation (36) delivers the equation in the proposition.

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