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No. 2005/10

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CFS Working Paper No. 2005/10

## On the Optimal Progressivity of the Income Tax Code\*

Juan Carlos Conesa<sup>2</sup> and Dirk Krueger<sup>1</sup>

March 17, 2005

### Abstract:

This paper computes the optimal progressivity of the income tax code in a dynamic general equilibrium model with household heterogeneity in which uninsurable labor productivity risk gives rise to a nontrivial income and wealth distribution. A progressive tax system serves as a partial substitute for missing insurance markets and enhances an equal distribution of economic welfare. These beneficial effects of a progressive tax system have to be traded off against the efficiency loss arising from distorting endogenous labor supply and capital accumulation decisions.

Using a utilitarian steady state social welfare criterion we find that the optimal US income tax is well approximated by a flat tax rate of 17.2% and a fixed deduction of about \$9,400. The steady state welfare gains from a fundamental tax reform towards this tax system are equivalent to 1.7% higher consumption in each state of the world. An explicit computation of the transition path induced by a reform of the current towards the optimal tax system indicates that a majority of the population currently alive (roughly 62%) would experience welfare gains, suggesting that such fundamental income tax reform is not only desirable, but may also be politically feasible.

**JEL Classification:** E62, H21, H24

**Keywords:** Progressive Taxation, Optimal Taxation, Flat Taxes, Social Insurance, Transition

\*This work has benefitted from useful comments by the editor Charles Plosser, an anonymous referee, Marco Bassetto, Martin Floden, Jonathan Heathcote, Tim Kehoe, Victor Rios-Rull and Tom Sargent, as well as seminar participants at CEMFI, CERGE-EI, Complutense, EUI, IDEI, UPF, UPN, UPO, UPV, the Hoover Institution and the 2001 SED Meetings in Stockholm. Conesa acknowledges financial support from Ministerio de Educación y Ciencia (SEC2003-06080), Generalitat de Catalunya, and Fundación Ramón Areces, and Krueger acknowledges financial support under NSF grant SES-0004376. All remaining errors are our own.

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## 1. Introduction

Progressive income taxes play two potentially beneficial roles in affecting consumption, saving and labor supply allocations across households and over time. First, they help to enhance a more equal distribution of income, and therefore, possibly, wealth, consumption and welfare. Second, in the absence of formal or informal private insurance markets against idiosyncratic uncertainty progressive taxes provide a partial substitute for these missing markets and therefore may lead to less volatile household consumption over time.

However, progressive taxation has the undesirable effect that it distorts incentives for labor supply and saving (capital accumulation) decisions of private households and firms. The policy maker thus faces nontrivial trade-offs when designing the income tax code.

On the theoretical side, several papers characterize the optimal tax system when two of these effects are present. The seminal paper by Mirrlees (1971) focuses on the traditional tension between equity and labor supply efficiency, whereas Mirrlees (1974) and Varian (1980) investigate the trade-offs between labor supply efficiency and social insurance stemming from progressive taxation. Aiyagari (1995) shows that, in the presence of uninsurable idiosyncratic uncertainty the zero capital tax result by Judd (1985) and Chamley (1986), derived from the desired efficiency of capital accumulation, is overturned in favor of positive capital taxation. Aiyagari's result is due to the fact that positive capital taxes cure overaccumulation of capital in the light of uninsurable idiosyncratic income shocks, rather than influence the risk

allocation directly. Golosov et. al. (2001) present a model with idiosyncratic income shocks and private information where positive capital taxes, despite distorting the capital accumulation decision, are optimal because they improve the allocation of income risk by alleviating the effects that the informational frictions have on consumption allocations. Albanesi and Sleet (2003) take a similar approach as Golosov et al. (2003) in characterizing (analytically and computationally) efficient allocations in a private information economy and show how to decentralize these allocations with income- and wealth-specific taxes.

Common to these papers is that, in order to insure analytical tractability, they focus on a particular trade-off and derive the qualitative implications for the optimal tax code. In contrast, in this paper we *quantitatively* characterize the optimal progressivity of the income tax code in an economic environment where *all three* effects of progressive taxes (the insurance, equity and efficiency effects) are present simultaneously.

In our overlapping generations economy agents are born with different innate earnings ability and face idiosyncratic, serially correlated income shocks as in Huggett (1993) and Aiyagari (1994). These income shocks are uninsurable by assumption; the only asset that is being traded for self-insurance purposes is a one-period risk-free bond which cannot be shortened. In each period of their finite lives agents make a labor-leisure and a consumption-saving decision, which is affected by the tax code. The government has to finance a fixed exogenous amount of government spending via proportional consumption taxes, taken as

given in the analysis, and income taxes, which are the subject of our study. We restrict the income tax code to lie in a particular class of functional forms. This functional form, which has its theoretical foundation in the equal sacrifice approach (see Berliant and Gouveia (1993)), has two appealing features. First, it provides a close approximation to the actual US income tax code, as demonstrated by Gouveia and Strauss (1994). Second, it provides a flexible functional form, nesting a proportional tax code, a wide variety of progressive tax codes and a variety of regressive tax codes such as a poll tax, with few parameters, which makes numerical optimization over the income tax code feasible.

The social welfare criterion we use in order to evaluate different income tax codes is steady state ex-ante expected utility of a newborn agent, before it is known with which ability level (and thus earnings potential) that agent will be born (i.e. looking upon her future life behind the Rawlsian veil of ignorance). Thus, progressive taxes play a positive role in achieving a more equal distribution of income and welfare (or in other words, they provide insurance against being born as a low-ability type). They also provide a partial substitute for missing insurance markets against idiosyncratic income shocks during a person's life. On the other hand, labor-leisure and consumption-saving decisions are distorted by the potential presence of tax progressivity.

Our first main quantitative result is that the optimal income tax code is well approximated by a proportional income tax with a constant marginal tax rate of 17.2% and a

fixed deduction of roughly \$9,400. Under such a tax code aggregate labor supply is 0.54% higher and aggregate output is 0.64% higher than in the benchmark tax system, calibrated to roughly match the US system. This is true even though average hours worked decline by about 1%, reflecting a shift of labor supply from low-productivity to high-productivity individuals. Households with annual income below around \$18,200 and above \$65,000 would pay lower total income taxes as compared to the benchmark, whereas the middle class, the households with incomes between \$18,200 and \$65,000 face a substantially higher income tax bill. The implied steady state welfare gains from such a tax reform are sizeable, in the order of magnitude equivalent to a uniform 1.7% increase in consumption across *all* agents and *all* states of the world.

The intuition for this result, which supports voices arguing for flat tax reform such as Hall and Rabushka (1995), is that lower marginal tax rates for high-income people increase labor supply and savings incentives, whereas the desired amount of redistribution and insurance is accomplished by the fixed deduction. That the desire for redistribution and insurance, nevertheless, is quantitatively important is reflected in our finding that a pure flat tax, without deduction, leads to a *reduction* in welfare by close to 1%, compared to the US. benchmark, even though aggregate output increases by 9.0% compared to the benchmark.

These results suggest that sizeable welfare benefits are forgone by passing on a fundamental tax reform. Our second main quantitative result, based on an explicit computation of the

transition path induced by such a reform, indicates that such a reform is not only desirable, in a steady state welfare criterion sense, but may also be politically feasible. In particular, we find that a majority of 62% of all agents currently alive would obtain welfare gains from such a reform. As our steady state findings suggest, households located around the median of the labor earnings and wealth distribution tend to suffer most from the reform; our analysis thus suggests that the middle class may be the biggest opponent to the proposed tax reform.

Several other studies attempt to quantify the trade-offs involved with reforming the (income) tax code in models with consumer heterogeneity. Castañeda et al. (1999) and Ventura (1999) use a model similar to ours in order to compare in detail the steady state macroeconomic and distributional implications of the current progressive tax system with those of a proportional (flat) tax system. We add to this literature the normative dimension of discussing *optimal* income taxation (with the implied cost of having to take a stand on a particular social welfare functional), as well as an explicit consideration of transitional dynamics induced by a potential tax reform. Domeij and Heathcote (2004) investigate the allocational and welfare effects of abolishing capital and income taxes, taking full account of the transition, but also do not optimize over the possible set of policies. Saez (2001) investigates the optimal progressivity of capital income taxes; in particular he focuses on the tax treatment of the top tail of the wealth distribution. In order to derive analytical results labor income is exogenous, deterministic and not taxed in his model, so that the labor supply and insurance aspects of

progressive taxation are absent by construction. Finally, Caucutt et al. (2003) and Benabou (2002) study the effects of the progressivity of the tax code on human capital accumulation and economic growth. Their analyses devote more detail to endogenizing economic growth than our study, but allows only limited cross-sectional heterogeneity and intertemporal trade; by stressing distributional and risk allocation aspects we view our analysis as complementary to theirs.

The paper is organized as follows: in Section 2 we describe the economic environment and define equilibrium. Section 3 contains a discussion of the functional forms and the parameterization employed in the quantitative analysis. In Section 4 we describe our computational experiments and in Section 5 we summarize our results concerning the optimal tax system and steady state welfare consequences of a tax reform. Section 6 is devoted to a discussion of the allocative and welfare consequences of a transition from the actual tax system to the optimal system derived in Section 5. Conclusions can be found in Section 7.

## **2. The Economic Environment**

### **2.1. Demographics**

Time is discrete and the economy is populated by  $J$  overlapping generations. In each period a continuum of new agents is born, whose mass grows at a constant rate  $n$ . Each agent faces a positive probability of death in every period. Let  $\psi_j = \text{prob}(\text{alive at } j + 1 | \text{alive at } j)$

denote the conditional survival probability from age  $j$  to age  $j + 1$ . At age  $J$  agents die with probability one, i.e.  $\psi_J = 0$ . Therefore, even in the absence of altruistic bequest motives, in our economy a fraction of the population leaves accidental bequests. These are denoted by  $Tr_t$  and confiscated by the government as general revenue. At a certain age  $j_r$  agents retire and receive social security payments  $SS_t$  at an exogenously specified replacement rate  $b_t$  of current average wages. Social security payments are financed by proportional labor income taxes  $\tau_{ss,t}$ .

## 2.2. Endowments and Preferences

Individuals are endowed with one unit of productive time in each period of their life and enter the economy with no assets. They spend their time supplying labor to a competitive labor market or consuming leisure. Individuals are heterogeneous along three dimensions that affect their labor productivity and hence their wage.

First, agents of different ages differ in their average, age-specific labor productivity  $\varepsilon_j$ . For agents older than  $j_r$  (retired agents) we assume  $\varepsilon_j = 0$ . Furthermore, individuals are born with different abilities  $\alpha_i$  which, in addition to age, determine their average deterministic labor productivity. We assume that agents are born as one of  $M$  possible ability types  $i \in \mathbf{I}$ , and that this ability does not change over an agents' lifetime,<sup>1</sup> so that agents, after

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<sup>1</sup>Ability in our model stands in for innate ability as well as for education and other characteristics of an individual that are developed before entry in the labor market, affect a persons' wage and do not change over

the realization of their ability, differ in their current and future earnings potential. The probability of being born with ability  $\alpha_i$  is denoted by  $p_i > 0$ . This feature of the model, together with a social welfare function that values equity, gives a welfare-enhancing role to redistributive fiscal policies.

Finally, workers of same age and ability face idiosyncratic uncertainty with respect to their individual labor productivity. Let by  $\eta_t \in \mathbf{E}$  denote a generic realization of this idiosyncratic labor productivity uncertainty at period  $t$ . The stochastic process for labor productivity status is identical and independent across agents and follows a finite-state Markov chain with stationary transitions over time, i.e.

$$Q_t(\eta, E) = Prob(\eta_{t+1} \in E | \eta_t = \eta) = Q(\eta, E). \quad (1)$$

We assume that  $Q$  consists of only strictly positive entries (as will be true in our calibration). This assumption also assures that there exists a unique, strictly positive, invariant distribution associated with  $Q$  which we denote by  $\Pi$  (see Stokey and Lucas (1989), theorem 11.2). All individuals start their life with average stochastic productivity  $\bar{\eta} = \sum_{\eta} \eta \Pi(\eta)$ , where  $\bar{\eta} \in \mathbf{E}$ . Different realizations of the stochastic process then give rise to cross-sectional productivity, income and wealth distributions that become more dispersed as a cohort ages. In the absence of explicit insurance markets for labor productivity risk a progressive tax system may be an

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a persons' life cycle.

effective, publicly administered tool to share this idiosyncratic risk across agents.

At any given time individuals are characterized by  $(a_t, \eta_t, i, j)$ , where  $a_t$  are asset holdings (of one period, risk-free bonds),  $\eta_t$  is stochastic labor productivity status at date  $t$ ,  $i$  is ability type and  $j$  is age. An agent of type  $(a_t, \eta_t, i, j)$  deciding to work  $\ell_j$  hours commands pre-tax labor income  $\varepsilon_j \alpha_i \eta_t \ell_j w_t$ , where  $w_t$  is the wage per efficiency unit of labor. Let by  $\Phi_t(a_t, \eta_t, i, j)$  denote the measure of agents of type  $(a_t, \eta_t, i, j)$  at date  $t$ .

Preferences over consumption and leisure  $\{c_j, (1 - \ell_j)\}_{j=1}^J$  are assumed to be representable by a standard time-separable utility function of the form

$$E \left\{ \sum_{j=1}^J \beta^{j-1} \frac{(c_j^\gamma (1 - \ell_j)^{1-\gamma})^{1-\sigma}}{1 - \sigma} \right\}, \quad (2)$$

where  $\beta$  is the time discount factor,  $\gamma$  is a share parameter measuring the importance of consumption relative to leisure, and  $\sigma$  controls the degree of risk aversion.<sup>2</sup> Expectations are taken with respect to the stochastic processes governing idiosyncratic labor productivity and the time of death.

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<sup>2</sup>The coefficient of relative risk aversion with our utility specification is given by

$$\frac{-cu_{cc}}{u_c} = \sigma\gamma + 1 - \gamma.$$

### 2.3. Technology

We assume that the aggregate technology can be represented by a standard Cobb-Douglas production function. The aggregate resource constraint is given by

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq K_t^\alpha (A_t N_t)^{1-\alpha} \quad (3)$$

where  $K_t$ ,  $C_t$  and  $N_t$  represent the aggregate capital stock, aggregate consumption and aggregate labor input (measured in efficiency units) in period  $t$ , and  $\alpha$  denotes the capital share. The term  $A_t = (1 + g)^{t-1} A_1$  captures labor augmenting technological progress. The depreciation rate for physical capital is denoted by  $\delta$ . As standard with a constant returns to scale technology and perfect competition without loss of generality we assume the existence of a representative firm operating this technology.

### 2.4. Government Policy

The government engages in three activities in our economy: it spends money, it levies taxes and it runs a balanced budget social security system. The social security system is defined by benefits  $SS_t$  for each retired household, independent of that household's earnings history. The social security tax rate  $\tau_{ss,t}$  is set to assure period-by-period budget balance of the system.<sup>3</sup>

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<sup>3</sup>The current US system ties benefits to past contributions, albeit in a very progressive form. To the extent that our social security system is too progressive, compared to the data, this would bias our tax results towards less progressivity of the system in the model. Note however, that in the actual system payroll

We take the social security system as exogenously given and not as subject of optimization of the policy maker.

Furthermore the government faces a sequence of exogenously given government consumption  $\{G_t\}_{t=1}^{\infty}$  and has three fiscal instruments to finance this expenditure. First it levies a proportional tax  $\tau_{c,t}$  on consumption expenditures, which we take as exogenously given in our analysis. Second, accidental bequests  $Tr_t$  accrue to the government (which one may interpret as estate taxes that do not distort bequest behavior, since our life cycle structure does not model it explicitly). Finally, the government can tax each individual's income<sup>4</sup>,  $y_t = (1 - 0.5\tau_{ss,t})w_t\alpha_i\varepsilon_j\eta\ell_t + r_t a_t$ , where  $w_t$  and  $r_t$  denote the wage per efficiency unit of labor and the risk free interest rate, respectively.<sup>5</sup> We impose the following restrictions on income taxes. First, tax rates cannot be personalized as we are assuming anonymity of the tax code. Second, the government cannot condition tax rates on the source of income, i.e. cannot tax labor and capital income at different rates.<sup>6</sup> Apart from these restrictions, however, income taxes to be paid can be made an arbitrary function of individual income in a given period.

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taxes are capped at some income level, whereas in our model all labor income is subject to the payroll tax, counteracting the potential bias just described. We made this simplifying assumption so that average indexed monthly earnings do not become an additional continuous state variable.

<sup>4</sup>The half of social security taxes paid by employees is part of taxable income in the U.S. under current law. Our definition of taxable income reflects this.

<sup>5</sup>After retirement, taxable income equals  $y_t = SS_t + r_t a_t$ .

<sup>6</sup>For a study that discusses the effects of changing the mix of capital and labor income taxes, see Domeij and Heathcote (2001).

We denote the tax code by  $T(\cdot)$ , where  $T(y)$  is the total income tax liability if pre-tax income equals  $y$ .

When studying the optimal progressivity of the income tax code, the problem of the government then consists of choosing the optimal tax function  $T(\cdot)$ , subject to the constraint that this function can only depend on individual income and balances the budget, keeping fixed the stream of government expenditures and the consumption tax rate.

## 2.5. Market Structure

We assume that workers cannot insure against idiosyncratic labor income uncertainty by trading explicit insurance contracts. Also annuity markets insuring idiosyncratic mortality risk are assumed to be missing. However, agents trade one-period risk free bonds to self-insure against the risk of low labor productivity in the future. The possibility of self-insurance is limited, however, by the assumed inability of agents to sell the bond short; that is, we impose a stringent borrowing constraint upon all agents. In the presence of survival uncertainty, this feature of the model prevents agents from dying in debt with positive probability.<sup>7</sup>

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<sup>7</sup>If agents were allowed to borrow up to a limit, it may be optimal for an agent with a low survival probability to borrow up to the limit, since with high probability she would not have to pay back this debt. Clearly, such strategic behavior would be avoided if lenders could provide loans at different interest rates, depending on survival probabilities (i.e. age). In order to keep the asset market structure simple and tractable we therefore decided to prevent agents from borrowing altogether, very much in line with much of the incomplete markets literature in macroeconomics; see Aiyagari (1994) or Krusell and Smith (1998) for

## 2.6. Definition of Competitive Equilibrium

In this section we will define a competitive equilibrium and a balanced growth path. Individual state variables are individual asset holdings  $a$ , individual labor productivity status  $\eta$ , individual ability type  $i$  and age  $j$ . The aggregate state of the economy at time  $t$  is completely described by the joint measure  $\Phi_t$  over asset positions, labor productivity status, ability and age.

Therefore let  $a \in \mathbf{R}_+$ ,  $\eta \in \mathbf{E} = \{\eta_1, \eta_2, \dots, \eta_n\}$ ,  $i \in \mathbf{I} = \{1, \dots, M\}$ ,  $j \in \mathbf{J} = \{1, 2, \dots, J\}$ , and let  $\mathbf{S} = \mathbf{R}_+ \times \mathbf{E} \times \mathbf{J}$ . Let  $\mathbf{B}(\mathbf{R}_+)$  be the Borel  $\sigma$ -algebra of  $\mathbf{R}_+$  and  $\mathbf{P}(\mathbf{E})$ ,  $\mathbf{P}(\mathbf{I})$ ,  $\mathbf{P}(\mathbf{J})$  the power sets of  $\mathbf{E}$ ,  $\mathbf{I}$  and  $\mathbf{J}$ , respectively. Let  $\mathbf{M}$  be the set of all finite measures over the measurable space  $(\mathbf{S}, \mathbf{B}(\mathbf{R}_+) \times \mathbf{P}(\mathbf{E}) \times \mathbf{P}(\mathbf{I}) \times \mathbf{P}(\mathbf{J}))$ .

**Definition 1.** *Given a sequence of social security replacement rates  $\{b_t\}_{t=1}^\infty$ , consumption tax rates  $\{\tau_{c,t}\}_{t=1}^\infty$  and government expenditures  $\{G_t\}_{t=1}^\infty$  and initial conditions  $K_1$  and  $\Phi_1$ , a competitive equilibrium is a sequence of functions for the household,  $\{v_t, c_t, a'_t, \ell_t : \mathbf{S} \rightarrow \mathbf{R}_+\}_{t=1}^\infty$ , of production plans for the firm,  $\{N_t, K_t\}_{t=1}^\infty$ , government income tax functions  $\{T_t : \mathbf{R}_+ \rightarrow \mathbf{R}_+\}_{t=1}^\infty$ , social security taxes  $\{\tau_{ss,t}\}_{t=1}^\infty$  and benefits  $\{SS_t\}_{t=1}^\infty$ , prices  $\{w_t, r_t\}_{t=1}^\infty$ , transfers  $\{Tr_t\}_{t=1}^\infty$ , and measures  $\{\Phi_t\}_{t=1}^\infty$ , with  $\Phi_t \in \mathbf{M}$  such that:*

1. *given prices, policies, transfers and initial conditions, for each  $t$ ,  $v_t$  solves the functional*
- 
- representative examples.

equation (with  $c_t$ ,  $a'_t$  and  $\ell_t$  as associated policy functions):

$$v_t(a, \eta, i, j) = \max_{c, a', \ell} \{u(c, \ell) + \beta \psi_j \int v_{t+1}(a', \eta', i, j+1) Q(\eta, d\eta')\} \quad (4)$$

subject to:

$$\begin{aligned} c + a' &= (1 - \tau_{ss,t}) w_t \varepsilon_j \alpha_i \eta \ell + (1 + r_t) a \\ &\quad - T_t[(1 - 0.5\tau_{ss,t}) w_t \varepsilon_j \alpha_i \eta \ell + r_t a], \text{ for } j < j_r, \end{aligned} \quad (5)$$

$$\begin{aligned} c + a' &= SS_t + (1 + r_t) a \\ &\quad - T_t[SS_t + r_t a], \text{ for } j \geq j_r, \end{aligned} \quad (6)$$

$$a' \geq 0, c \geq 0, 0 \leq \ell \leq 1. \quad (7)$$

2. Prices  $w_t$  and  $r_t$  satisfy:

$$r_t = \theta \alpha \left( \frac{A_t N_t}{K_t} \right)^{1-\alpha} - \delta, \quad (8)$$

$$w_t = \theta (1 - \alpha) A_t \left( \frac{K_t}{A_t N_t} \right)^\alpha. \quad (9)$$

3. The social security policies satisfy

$$SS_t = b_t \frac{w_t N_t}{\int \Phi_t(da \times d\eta \times di \times \{1, \dots, j_r - 1\})} \quad (10)$$

$$\tau_{ss,t} = \frac{SS_t}{w_t N_t} \int \Phi_t(da \times d\eta \times di \times \{j_r, \dots, J\}). \quad (11)$$

4. Transfers are given by:

$$Tr_{t+1} = \int (1 - \psi_j) a'_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) \quad (12)$$

5. Government budget balance:

$$\begin{aligned} G_t = & \int T_t[(1 - 0.5\tau_{ss,t})w_t\varepsilon_j\alpha_i\eta\ell_t(a, \eta, i, j) + r_t(a + Tr_t)]\Phi_t(da \times d\eta \times di \times \{1, \dots, j_r - 1\}) + \\ & \int T_t[SS_t + r_t(a + Tr_t)]\Phi_t(da \times d\eta \times di \times \{j_r, \dots, J\}) + \\ & \tau_{c,t} \int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) + (1 + r_t)Tr_t \end{aligned} \quad (13)$$

6. Market clearing:

$$K_t = \int a \Phi_t(da \times d\eta \times di \times dj) \quad (14)$$

$$N_t = \int \varepsilon_j \alpha_i \eta \ell_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) \quad (15)$$

$$\begin{aligned} \int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) + \int a'_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) + G_t = \\ K_t^\alpha (A_t N_t)^{1-\alpha} + (1 - \delta)K_t \end{aligned} \quad (16)$$

7. Law of Motion:

$$\Phi_{t+1} = H_t(\Phi_t) \quad (17)$$

where the function  $H_t : \mathbf{M} \rightarrow \mathbf{M}$  can be written explicitly as:

1. for all  $\mathcal{J}$  such that  $1 \notin \mathcal{J}$ :

$$\Phi_{t+1}(A \times E \times \mathcal{I} \times \mathcal{J}) = \int P_t((a, \eta, i, j); A \times E \times \mathcal{I} \times \mathcal{J}) \Phi_t(da \times d\eta \times di \times dj) \quad (18)$$

where

$$P_t((a, \eta, i, j); A \times E \times \mathcal{I} \times \mathcal{J}) = \begin{cases} Q(e, E)\psi_j & \text{if } a'_t(a, \eta, i, j) \in A, i \in \mathcal{I}, j+1 \in \mathcal{J} \\ 0 & \text{else} \end{cases} \quad (19)$$

2.

$$\Phi_{t+1}((A \times E \times \mathcal{I} \times \{1\})) = (1+n)^t \begin{cases} \sum_{i \in \mathcal{I}} p_i & \text{if } 0 \in A, \bar{\eta} \in E \\ 0 & \text{else} \end{cases} \quad (20)$$

**Definition 2.** A *Balanced Growth Path* is a competitive equilibrium in which  $b_t = b_1$ ,  $\tau_{c,t} = \tau_{c,1}$ ,  $G_t = ((1+g)(1+n))^{t-1} G_1$ ,  $a'_t(\cdot) = (1+g)^{t-1} a'_1(\cdot)$ ,  $c_t(\cdot) = (1+g)^{t-1} c_1(\cdot)$ ,  $\ell_t(\cdot) = \ell_1(\cdot)$ ,  $N_t = (1+n)^{t-1} N_1$ ,  $K_t = ((1+g)(1+n))^{t-1} K_1$ ,  $T_t = (1+g)^{t-1} T_1$ ,  $\tau_{ss,t} = \tau_{ss,1}$ ,  $SS_t = (1+g)^{t-1} SS_1$ ,  $r_t = r_1$ ,  $w_t = (1+g)^{t-1} w_1$ ,  $Tr_t = (1+g)^{t-1} Tr_1$  for all  $t \geq 1$  and  $\Phi_t((1+g)^{t-1} A, E, \mathcal{I}, \mathcal{J}) = (1+n)^{t-1} \Phi_1(A, E, \mathcal{I}, \mathcal{J})$  for all  $t$  and all  $A \in \mathbf{R}_+$ . That is, per capita variables and functions grow at constant gross growth rate  $1+g$ , aggregate variables grow at constant gross growth rate  $(1+n)(1+g)$  and all other variables (and functions) are time-invariant.<sup>8</sup>

Note that, in order to represent this economy on a computer, one first has to carry out the standard normalizations by dividing the utility function and the budget constraint by  $A_t$  to make the household recursive problem stationary.<sup>9</sup>

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<sup>8</sup>The notation  $T_t = (1+g)^{t-1} T_1$  should be interpreted as follows: an agent with income  $y$  in period 1, faces the same *average and marginal* tax rate as an agent with income  $(1+g)^{t-1} y$  in period  $t$ .

<sup>9</sup>See e.g. Aiyagari and McGrattan (1998) for a detailed discussion of this normalization, for a model that

### 3. Functional Forms and Calibration of the Benchmark Economy

In this section we discuss the functional form assumptions and the parameterization of the model that we employ in our quantitative analysis.

#### 3.1. Demographics

The demographic parameters have been set so that the model economy has a stationary demographic structure matching that of the US economy. Agents enter the economy at age 20 (model age 1), retire at age 65 (model age 46) and die with certainty at age 100 (model age 81). The survival probabilities are taken from Faber (1982). Finally, the population growth rate is set to an annual rate of 1.1%, the long-run average for the US. Our demographic parameters are summarized in Table I.

[Table 1 about here]

The maximum age  $J$ , the population growth rate and the survival probabilities together determine the population structure in the model. We chose  $J$  so that the model delivers a ratio of people older than 65 over population of working age as observed in the data.

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is very similar to ours.

### 3.2. Preferences

We assume that preferences over consumption and leisure can be represented by a period utility function of the form:

$$U(c, \ell) = \frac{(c^\gamma(1 - \ell)^{1-\gamma})^{1-\sigma}}{1 - \sigma}. \quad (21)$$

In order to calibrate the preference parameters we proceed as follows. First, we fix the coefficient of relative risk aversion to  $\sigma = 4$ . Then the discount factor  $\beta$  is chosen so that the equilibrium of our benchmark economy implies a capital-output ratio of 2.7 as observed in the data.<sup>10</sup> The share of consumption in the utility function is chosen so that individuals in active age work on average  $\frac{1}{3}$  of their discretionary time. The choice of  $\sigma$  and  $\gamma$  implies a coefficient of relative risk aversion of approximately 2.

Preference parameters are summarized in Table II.

[Table 2 about here]

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<sup>10</sup>For model parameters that are calibrated using data and equilibrium observations of the model it is understood that all parameters jointly determine equilibrium quantities of the model. Our discussion relates a parameter to that equilibrium target which is affected most, in a quantitative sense, by the particular parameter choice.

Our measure of capital includes nonresidential fixed assets (equipment, software and nonresidential structures) as well as private residential structures and consumer durable goods. The data comes from the 2000 BEA Fixed Assets and Durable Goods tables.

### 3.3. Endowments

In each period agents are endowed with one unit of time. Their labor productivity is composed of a type specific component depending on ability  $\alpha_i$ , an age-specific average component  $\varepsilon_j$  and a idiosyncratic stochastic component  $\eta_t$  in a multiplicative fashion. The deterministic component of efficiency units of labor is taken from Hansen (1993). It features a hump over the life cycle, with peak at age 50.

Our principle for calibrating the ability component  $\alpha_i$  and the stochastic process governing idiosyncratic productivity  $\eta_t$  is to reproduce a cross-sectional age-dependent earnings variance within our benchmark model that matches the statistics reported by Storesletten et al. (2004), derived from the PSID. For the ability component of labor productivity we choose two types,  $M = 2$  with equal mass,  $p_i = 0.5$  for  $i = 1, 2$ . The types' ability levels  $(\alpha_1, \alpha_2)$  are chosen to match Storesletten et al.'s (2004) finding that the variance of log-earnings of a cohort that just entered the labor market is equal to 0.27, which yields as  $\alpha_1 = e^{-\sqrt{0.24}}$  and  $\alpha_2 = e^{\sqrt{0.24}}$ . Thus wages for high ability agents are, on average, 46% higher than median wages and wages of low ability agents, correspondingly, 46% lower than median wages.<sup>11</sup> We summarize the calibration of ability in Table III.

[Table 3 about here]

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<sup>11</sup>Note that we cannot simply take Storesletten et al.'s (2004) estimates for their stochastic income process, since they estimate a process for labor income, whereas in our model labor supply is endogenous.

For the stochastic idiosyncratic productivity component we make the following assumptions. Households start their working life at the average productivity level  $\bar{\eta}$ , as described in section 2.2; from then on their productivity levels are governed by a seven state Markov chain, whose states summarized in Table IV, and whose transition matrix is characterized by high persistence; its second largest eigenvalue equals 0.865. With this stochastic process for labor productivity the cross-sectional variance of log-earnings implied by our model increases roughly linearly with age of the cohort, with an increase of about 0.0165 per year the cohort ages, as in Storesletten et al.'s (2004) data.<sup>12</sup>

[Tabel 4 about here]

### 3.4. Technology

We assume that the aggregate production function is of Cobb-Douglas form:

$$F(K_t, N_t) = K_t^\alpha (A_t N_t)^{1-\alpha} \quad (22)$$

with capital share  $\alpha$ , where we choose  $\alpha = 0.36$ , in accordance with the long-run capital share for the US economy. The depreciation rate is set to  $\delta = 6.58\%$  so that in the balanced

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<sup>12</sup>We obtained this stochastic process by first specifying a simple continuous-state  $AR(1)$  process for log-productivity  $\log(\eta)$  that was then discretized into a seven state Markov chain using Tauchen and Hussey's (1991) method. The persistence and variance parameters of the  $AR(1)$  were chosen to achieve the linear increase in the variance of log-earnings (with age of the cohort), with slope of 0.0165, as reported in the main text.

growth path the benchmark economy implies an investment to output ratio equal roughly to 25.5% as in the data.<sup>13</sup> Finally, since in a balanced growth path per capita GDP is growing at rate  $g$ , we choose  $g = 1.75\%$  to match the long-run growth rate of per capita GDP for US data. Technology parameters are summarized in Table V.

[Table 5 about here]

### 3.5. Government Policies and the Income Tax Function

In order to parameterize the actual tax code we proceed as follows. First, we fix the proportional consumption tax rate to  $\tau_c = 5.2\%$ , which is the consumption tax rate found by Mendoza et al. (1994) for the US. The level of government consumption,  $G$ , is chosen so that in a balanced growth path the government consumption share of GDP is 17%, as in the data.

The social security system is chosen so that the replacement rate (ratio of retirement pension to the average wage) is 50%. The implied payroll tax required to finance benefits, under the assumption of a balanced budget for the social security system, is uniquely pinned down by our assumptions about demographics, and is equal to  $\tau_{ss} = 12.4\%$ , as currently for the US, excluding Medicare.

The principal focus of this paper is the income tax code. We want to use an income tax code that provides a good approximation to the actual current tax code for the US and then,

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<sup>13</sup>Notice that investment into consumer durables is included in aggregate gross investment when we compute this ratio in the data.

in our policy experiment, vary this tax code in order to find the hypothetical optimal tax code, given a particular social welfare function.

We use a functional form for the income tax code that is theoretically motivated by the equal sacrifice principle (see Gouveia and Strauss (1994)) and is fairly flexible in that it encompasses a wide range of progressive, proportional and regressive tax schedules. Letting  $T(y)$  denote total taxes paid by an individual with pre-tax income  $y$ , the tax code is restricted to the functional form

$$T(y) = a_0 \left( y - (y^{-a_1} + a_2)^{-\frac{1}{a_1}} \right) \quad (23)$$

where  $(a_0, a_1, a_2)$  are parameters.

Note that  $\lim_{y \rightarrow \infty} \frac{T(y)}{y} = \lim_{y \rightarrow \infty} T'(y) = a_0$  so that the limiting marginal and average tax rate equals  $a_0$ . For  $a_1 = -1$ , we obtain a constant tax independent of income  $T(y) = -a_0 a_2$ , for  $a_1 \rightarrow 0$  we have a purely proportional system  $T(y) = a_0 y$ . Finally, for  $a_1 > 0$  we have a progressive system since:

$$t(y) = \frac{T(y)}{y} = a_0 \left( 1 - (1 + a_2 y^{a_1})^{-\frac{1}{a_1}} \right) \quad (24)$$

$$T'(y) = a_0 \left( 1 - (1 + a_2 y^{a_1})^{-\frac{1}{a_1} - 1} \right) \quad (25)$$

and thus average (as well as marginal) taxes are a strictly increasing function of income  $y$ .

Gouveia and Strauss (1994) use this parametric class of tax function to approximate the current US system and obtain values of  $a_0 = 0.258$  and  $a_1 = 0.768$ . The parameter  $a_2$  is

chosen so that the government balances its budget in the balanced growth path. Note that  $a_2$  is not invariant to units of measurement: if one scales all variables by a fixed factor, one has to adjust the parameter  $a_2$  in order to preserve the same tax function.<sup>14</sup> The policy parameters employed as benchmark are summarized in Table VI.

[Table 6 about here]

#### 4. The Computational Experiment

We will define the optimal tax code as that tax code (within the parametric class chosen) with the highest ex-ante steady state expected utility of a newborn. With a given tax code  $T$ , parameterized by  $(a_0, a_1, a_2)$  is associated a balanced growth path with invariant measure  $\Phi_T(a, \eta, i, j)$  and value function  $v_T(a, \eta, i, j)$ . Our social welfare function is then given by:

$$\begin{aligned} SWF(T) &= \int_{\{(a, \eta, i, j): a=0, j=1\}} v_T(a, \eta, i, j) d\Phi_T \\ &= \sum_{i \in \mathbf{I}} p_i v_T(a = 0, \eta = \bar{\eta}, i, j = 1) \end{aligned} \quad (26)$$

and we aim at determining:

$$T^* = \arg \max_{(a_0, a_1)} SWF(T) \quad (27)$$

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<sup>14</sup>The parameter  $a_2$  depends on units in the following sense. Suppose we scale income by a factor  $\lambda > 0$  (i.e. change the units of measurement). In order to let the tax system be unaffected by this change one has to adjust  $a_2$  correspondingly:  $a_2 y^{a_1} = \tilde{a}_2 (\lambda y)^{a_1}$  and therefore  $\tilde{a}_2 = a_2 \lambda^{-a_1}$ .

Numerically, this is done by constructing a grid in the space of policy parameters  $(a_0, a_1)$ , computing the equilibrium and the associated expected utility of a newborn for every grid point and finding the welfare-maximizing  $(a_0, a_1)$ -combination. In conjunction with this analysis we will compare macroeconomic aggregates in the balanced growth path associated with the optimal tax code with those arising in the balanced growth path of the benchmark tax code. This analysis can be found in the next section.

The first stage of our quantitative analysis is confined to a positive and normative comparison of balanced growth path allocations. In the second stage of our analysis, we explicitly compute the transitional dynamics induced by a reform from the benchmark economy towards the optimal tax code. In particular, starting from the initial balanced growth path we induce an unexpected change of  $(a_0, a_1)$  to their optimal levels (optimal in the sense of the first part of our analysis), and adjust  $a_2$  along the transition path in order to guarantee government budget balance in every period. In Section 6 we first discuss the time paths of aggregate variables along the transition towards the new steady state. We then identify the winners and losers of the reform by computing the welfare consequences for agents of different ages and economic status that are alive at the time of the implementation of the tax reform. A brief discussion of the implied political economic consequences implied by the welfare calculations concludes our quantitative analysis.

## 5. The Optimal Tax Code

We find that the optimal tax code, as defined above, is described by  $a_0 = 0.172$ ,  $a_1 = 19$ . Such a tax code is roughly equivalent to a proportional tax of 17.2% with a fixed deduction of about \$9,400. Note that this optimal tax code, with flat marginal tax rate and sizeable deduction comes close to the tax reform proposal advanced by Hall and Rabushka (1995), who suggested a constant marginal tax rate of 19% with a deduction of \$22,500 (for a family of four) for the U.S. economy.

Figures 1 and 2 display the average and marginal tax rates implied by the optimal income tax code and, as comparison, of the benchmark income tax code.

[Figure 1 about here]

We see that marginal tax rates (and consequently average tax rates) in the optimal tax system are considerably lower for households in the upper tail of the income distribution, as compared to the benchmark system. Also, due to the fixed deduction marginal tax rates are (roughly) 0 for the first \$9,400 of income under the optimal system.

[Figure 2 about here]

In order to assess how tax burdens differ in both system, in Figure 3 we plot the total dollar amount a household with particular income would pay less (or more) in income taxes under the new, as compared to the old system. We see that households with small and large incomes see their income tax burdens reduced, those with high incomes significantly, whereas households in the middle of the income distribution face a higher income tax bill. For example, a household with yearly income of \$40,000 would pay more than \$400 more in income taxes per year under the new, compared to the old tax system.

[Figure 3 about here]

In order to obtain a better understanding for the economic forces underlying the results concerning the optimal tax code, in Table VII we compare the main macroeconomic aggregates associated with the optimal tax code with those obtained under the benchmark tax system. In order to isolate the efficiency from the insurance and redistribution effect it is also instructive to present the corresponding numbers for a pure proportional tax system, without exemption level. Finally, in column 4 we report results from subjecting households to the tax reform, but keeping prices at their benchmark economy level, in order to isolate the effects of a higher steady state capital stock and thus wages.

To compare welfare across different tax systems, we compute (as consumption equivalent variation *CEV*) the uniform percentage decrease in consumption, at each date and in each

event (and fixed labor-leisure allocation), needed to make a household indifferent between being born into the balanced growth path associated with a particular tax system and being born into the benchmark balanced growth path. Positive  $CEV$  thus reflect a welfare increase due to a tax reform, compared to the benchmark system.<sup>15</sup>

[Table 7 about here]

Notice that the optimal tax code implies higher labor supply and, higher capital accumulation than the benchmark economy, and as a result GDP per capita is 0.64% higher. This reflects the reduced disincentive effects to work and save for the households at the high end of the income distribution, due to vastly reduced marginal income tax rates for that group. For the labor supply decision, note that average hours worked actually decline by roughly one percent after the reform, but total labor supply increases by half a percent, which is explained by the fact that it is high-ability, high-productivity agents who expand their labor supply in response to lower marginal tax rates for their income brackets. Finally, due to the increase in economic activity triggered by the tax reform the fraction of GDP devoted to government consumption and the average tax rate required to fund government outlays shrinks, leaving a higher *fraction* of already higher output for private consumption and investment. In total,

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<sup>15</sup>All welfare numbers in the remainder of this paper refer to this welfare measure. For total labor supply  $N$ , the capital stock  $K$ , output  $Y$ , wages  $w$  and consumption  $C$  we report percentage deviations from the values for the benchmark economy.

aggregate consumption increases by a substantial 0.7%.

Providing better incentives to work and save comes at the price of creating a more dispersed income, wealth and consumption distribution. Table VII documents this with the Gini coefficients for income, consumption and wealth. First, under the optimal tax system pre-tax income is more unequally distributed since high-ability, high productivity agents work and save disproportionately more under the new tax code (see also the wealth Gini). But the optimal tax system prescribes a substantial deduction of \$9,400, so that the after-tax income Gini increases by roughly the same points as the pre-tax income Gini. More unequally distributed income leads to an increase in consumption inequality under the optimal, compared to the benchmark system. Despite this slight increase in consumption inequality, the balanced growth path welfare gains of a tax reform amount to roughly 1.7%, in consumption equivalent variation. It is important to point out here that our benchmark economy is able to account for most, but not all of the observed wealth inequality. Castañeda et al. (2003) report a Gini coefficient of wealth of 0.78, whereas in our benchmark economy it amounts to 0.644. The divergence between the model and the data stems from the fact that the model is incapable of generating sufficiently high wealth concentration at the very top of the distribution, partially because it rules out saving for bequests by construction. As Castañeda et al. (2003) suggest, in a model like ours the presence of a pay-as-you-go social security system is crucial for our relative success of creating substantial wealth inequality, since it significantly

reduces the incentives of young and middle-aged agents to accumulate assets and thus leads to a large fraction of agents in these age cohorts with no financial assets.<sup>16</sup>

The comparison of the actual with a pure proportional tax system without deduction shows even more dramatically that in this economy there exists a strong social desire for redistribution and insurance, which the optimal tax system reflects with the fixed deduction.<sup>17</sup> Even though GDP per capita and aggregate consumption are almost 9% higher in a purely proportional system as compared the benchmark BGP, social welfare is lower under purely proportional taxes. This is due to the fact that a purely proportional system does not provide insurance against being born as a low type (ex-post one may call this insurance redistribution). Second, it does not provide insurance against idiosyncratic income fluctuations: even agents born into the new BGP with high ability (and average productivity) experience welfare gains

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<sup>16</sup>If we increase the dispersion of the type-specific productivity shocks  $\alpha_i$  further, in order to obtain a wealth dispersion in the benchmark economy that is closer to US data (as reported by Castaneda et al., 2003), and then repeat our steady state welfare comparison, we find even larger steady state welfare gains from the tax reform. This is mainly due to the fact that the ex-ante redistribution motive is strengthened in the new calibration, since the low productivity type is so income poor now that it benefits even more from the deduction. The dynamics of the transition path is similar to the one reported for our calibration in section 6 below.

<sup>17</sup>The fact that a proportional income tax of only 9.9% is needed to finance government spending of 15.6% of GDP is due to the fact that savings incentives, and thus revenues from assets of deceased households are substantial (of course, in addition to sizeable consumption taxes) with purely proportional taxes.

of only 4.5%, compared to the 9% increase in average consumption.<sup>18</sup> But since agents born with low ability suffer welfare losses of 2.8% in terms of consumption equivalent variation and the utilitarian social welfare function weighs utility losses of low-utility agents more heavily than utility gains of high-utility households, overall steady state welfare under a purely proportional system is lower than under the benchmark.<sup>19</sup>

Our social welfare criterion is steady state expected welfare of a newborn agent. One may object that this welfare criterion ignores the transition cost required to build up a higher capital stock under the new tax system. Why do agents whose welfare enters the social welfare function benefit from a higher aggregate capital stock, the transition cost of building it up we ignore? Because a higher capital stock means higher wages, thus higher consumption and higher welfare. One way to assess the potential bias from ignoring the transition cost of building up the capital stock is to evaluate how big the steady state welfare gains would have been hadn't wages increased between the old steady state and the steady state with the new, optimal tax system. That is, we compute a new steady state with the new tax system, but fixing wages (and returns to capital at their old steady state level). Thus the only economic variables that have changed between the old and new steady state for all agents (and thus the newborn agents who matter for social welfare) is the tax system. This exercise distributes the welfare gains we report in the paper into one part that is due to higher capital and thus higher

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<sup>18</sup>Also note that these agents work more with proportional taxes than under the benchmark system.

<sup>19</sup>In other words, our social welfare function captures egalitarian concerns.

wages, and a part that is due to efficiency gains from lower marginal tax rates on labor income, possibly better insurance against income shocks and more equitable distribution between the different types of the population. The latter parts will still be present in our comparison of the old and the new steady state with fixed prices, whereas the part that is due to higher capital and thus higher wages will not be.

Column 4 of table 7 presents the results from this exercise. We see that while the welfare gains are not quite as large when abstracting from the increase in steady state capital and thus wages, the bulk of the welfare gains from the tax reform (1.6% of the original 1.7%) remain intact, because they are due to efficiency gains from lower marginal tax rates on labor income and a more equitable distribution of welfare across the two types.<sup>20</sup> In fact, this result is not entirely surprising as the welfare results from a purely proportional tax system shows. That system generates a substantially higher steady state capital stock and wages, yet the welfare effects, even ignoring the transitional costs to build up that capital stock, are detrimental. Thus our tax system is not primarily optimal because it merely provides low marginal taxes and thus higher steady state capital; otherwise a purely proportional tax system should do even better, which it doesn't.

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<sup>20</sup>Note that the Gini coefficient for consumption is higher under the new tax system (both with equilibrium and fixed prices) than the old tax system, which indicates less insurance in the new, compared to the old system. This adverse effect is more than offset by the redistribution effect across types and lower distortions of the labor-leisure choice.

In fact, decomposing the welfare consequences of the reform (1.7% on the aggregate) by type we observe that the reform benefits the poorer type by 2.47%, in terms of consumption, and leads to small welfare losses of the richer type, by  $-0.3\%$ . This suggests that redistribution across type (ex-ante insurance) is an important factor in our welfare gains from tax reform. The poorer type has income for most of her life that puts her into the income range where taxes are reduced (with high probability), whereas the richer type is likely to be a middle class household that faces a higher tax bill (with small probability of making it to higher income ranges where the new tax system features tax cuts relative to the benchmark system). The fact that the poor type gains more than the rich type loses is due to the efficiency and capital deepening effects of the tax reform.<sup>21</sup>

## 6. Transition to the Optimal Tax Code and Welfare Implications

In this section we shift attention to the quantitative implications of reforming the tax code towards the optimal found in the previous section, taking full account of the transition path induced by the tax reform. To do so we have to take a stand on how the tax code is adjusted as the economy moves from the old to the new balanced growth path. We assume that

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<sup>21</sup>Holding wages constant isolates the capital deepening effect, which turns out to be fairly uniform across the two types: the welfare gains are lowered to 2.36% for the low type and to  $-0.4\%$  for the high type). Thus not only in the aggregate, but also type by type the capital deepening effect accounts for 0.1 percentage points of the welfare increase due to our tax reform.

in period 1 the economy is in the BGP of the benchmark economy. Then, in period 2 an unanticipated tax reform is carried out, imposing  $a_0 = 0.172$  and  $a_1 = 19$  from then on. The parameter  $a_2$  is adjusted in every period to satisfy budget balance at each date of the transition. Effectively, this amounts to a reform that imposes a constant marginal tax rate of 17.2% and the fixed deduction is adjusted in every period so that the government collects enough revenues in order to finance a constant level of public consumption. Over time the economy converges to its new BGP, and the tax system to the one defined and computed as optimal in the previous section.<sup>22</sup>

### 6.1. Dynamics of Aggregate Variables

Figure 4 documents the evolution of macroeconomic aggregates along the transition path.

[Figure 4 about here]

On impact of the reform the aggregate capital stock is predetermined by the savings decisions in the previous period. Therefore, since the tax reform was unanticipated, the aggregate capital stock does not react immediately to the reform. However, aggregate labor supply increases on impact, due to the new lower marginal income tax rates for high-income

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<sup>22</sup>A detailed description of the computational method employed to compute transition paths for our economy is available upon requests by the authors. The method is identical to the one used by and documented in Conesa and Krueger (1999) for a similar economy.

earners. In the first reform period the increase in labor supply amounts to more than 0.6%, and converges to its new BGP level, which is roughly 0.5% higher than in the initial BGP. As a consequence of the increase in labor supply and an initially fixed capital stock the capital-labor ratio falls and interest rates rise on impact. Then, as capital accumulation picks up and the capital stock increases by about 0.81% in the new, as compared to the old BGP, the interest rate falls below the initial BGP level (and wages increase above the initial BGP level, see Figure 5).

[Figure 5 about here]

Notice that total GDP and thus per capita GDP is monotonically increasing along the transition path, due to both the expansion of labor supply and stronger capital accumulation. With per capita income the income tax base grows along the transition path (over and above the long-run growth rate of the economy,  $g$ ). This transitional acceleration of economic growth explains why the tax deduction can be monotonically expanded over time as well, without increasing marginal taxes, reducing government outlays or violating the government budget constraint.

Figure 6 displays the time pattern of the deduction along the transition path. It mirrors the evolution of income per capita, starts at a level around \$9,300 and monotonically increases until its final steady state value above \$9,400.

[Figure 6 about here]

## 6.2. Welfare Consequences of the Reform

Explicitly considering the transitional dynamics induced by a fundamental tax reform allows us to evaluate the welfare implications of such a reform *for all individuals alive* at the moment the reform is implemented (rather than for agents born *directly* into the new balanced growth path, as in the previous section). Such an analysis may also shed some light on the political feasibility of a hypothetical reform.

Who gains from a potential reform? As Figure 7 suggests, the income-richest and agents at the bottom of the income distribution. Individuals with bad productivity realizations and very few assets gain from the reform because of the deduction whereas individuals with average productivity realizations and few assets lose with the reform. Finally, individuals with high productivity and/or large asset holdings and thus high labor and/or capital income invariably benefit from the lower marginal tax rates induced by the reform. In the light of Figure 3 (which shows the change of the tax burden across income) and Figure 6 (which demonstrates that the tax system along the transition path mimics that of the final BGP fairly closely) these results are not unexpected. Note that, as agents age, their average labor productivity increases; thus, conditional on the same asset position, 45 year old households are income richer than newborn households.

[Figure 7 about here]

However, even though a reform towards a flat tax with deduction promises significant bal-

anced growth path welfare gains, our numerical analysis of the transition path indicates that a significant fraction of the population alive at the date at which the reform is undertaken will experience welfare losses from the reform, roughly 38%. Figure 8 displays the fraction of the population in each age cohort that experiences welfare gains. Despite that, a clear majority, composed of income-poor and income-rich households would benefit from the reform, even taking transitional costs into account. This suggests that a fundamental tax reform of the type discussed in this paper may not only be desirable from a steady state utilitarian perspective, but may also be politically feasible.

[Figure 8 about here]

Notice that the fraction of individuals with welfare gains is fairly constant among working cohorts, but then increases with age of retired cohorts. Among the newborn generation, low type households benefit and high types lose from the reform. This is due to the fact that young individuals are born with the average productivity shock and have no assets. This places the low types well within the range of income for which the large deduction of the new tax code dominates the previous status quo. The high types, in contrast, start their life as part of the middle class (and with very persistent income shocks, in expectation stay there for a while) which suffers from the tax reform. For older generations, after retirement assets are decumulated and thus incomes start to decline further and further with age, placing the elderly into the income-poor category that benefits from the generous deduction under the

new tax code.

The results of our model indicate that a fundamental tax reform towards a flat tax with deduction may be desirable for generations born in the far future, is likely to gain support from a majority of the population currently alive, but may face stiff opposition from the middle class.

## 7. Conclusion

In this paper we have demonstrated, using a computable stochastic dynamic general equilibrium model, that under a utilitarian social welfare function the optimal income tax code is well approximated by a flat tax rate schedule with a fixed deduction of roughly \$9,400. Such a system implies lower marginal rates for high-productivity individuals as compared to the current US tax code, and thus reduces the distortionary effects on labor supply and capital accumulation originating from high marginal income tax rates for households in the upper tail of the income distribution. At the same time, the deduction pays tribute to the social desire for equity and provides insurance against idiosyncratic labor income uncertainty. Compared to the actual system, under the optimal tax system tax burdens would be reduced for households in the lower and upper tail of the income distribution, whereas the middle class would face a higher income tax bill. The long run welfare gains of a fundamental tax reform towards a flat tax are sizeable, in the order of 1.7% uniformly higher consumption.

Are there large sums of money left on the table by not embarking on such a tax reform? The second part of our analysis, based on an explicit calculation of the transition path induced by such a reform suggests an affirmative answer. More than 60% of the individuals initially alive would experience welfare gains. However, the group of the US population usually referred to as the middle class would almost unambiguously lose out, casting some doubts about whether fundamental tax reform towards a flat tax is politically feasible, and in fact desirable from a normative point of view.

There are several directions in which our analysis could be extended. In deriving our normative results we necessarily have to take a stand on the social welfare function which aggregates well-being of the heterogeneous population in our model. We conjecture that employing an alternative social welfare function which places more weight on individuals at the lower end of the distribution than a utilitarian functional (e.g. a Rawlsian welfare function) would favor a more progressive income tax system than the one we have identified as optimal.

In addition, our findings that the welfare benefits of a flat tax reform are by no means uniform across the population when the transition path is considered explicitly, make it desirable to derive the optimal *evolution* of the income tax system over time. This, however, would require, even in the parametric class of tax functions considered in this paper, to optimize over the tax function parameters in each period until a new, endogenously determined

balanced growth path is reached, which seems computationally infeasible at this point, unless further, somewhat arbitrary, restrictions on the evolution of the tax parameters are imposed.

Finally, while the current US personal income tax code (as well as our current paper) does not distinguish between the source of taxable income when computing tax liabilities, the theoretical literature on optimal capital taxation usually maintains that labor income and capital income can be taxed at different rates. There are at least two reasons why in our current model the optimal capital tax is nonzero, unlike in Judd (1985) and Chamley (1986). Their result does not generalize to a life cycle economy, as demonstrated by Erosa and Gervais (2002) and Garriga (2003). Furthermore, Aiyagari (1995) shows that the presence of idiosyncratic uninsurable uncertainty invalidates the result as well, opening up the question about the optimal capital income tax in our current environment, if labor income taxes are allowed to be progressive and thus can act as insurance mechanism against exactly the uncertainty that drives Aiyagari's result. We investigate this further in ongoing research (see Conesa et al., 2005).

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# Tables

**Table I: Demographics Parameters**

Parameter	Value	Target
Retir. Age: $j_r$	46(65)	Compul. Ret. (assumed)
Max. Age: $J$	81(100)	Certain Death (assumed)
Surv. Prob.	Faber (1982)	Data
Pop. Growth: $n$	1.1%	Data

**Table II: Preferences Parameters**

Parameter	Value	Target
$\beta$	0.9986	$K/Y = 2.7$
$\sigma$	4.0	Fixed
$\gamma$	0.3856	Avg Hours = $\frac{1}{3}$

**Table III: Ability**

Parameter	Value	$p_i$
$\alpha_1$	0.6115	0.5
$\alpha_2$	1.6354	0.5

**Table IV: Stochastic Productivity**

Parameter	Value	$\Pi$
$\eta_1$	0.447	0.034
$\eta_2$	0.589	0.135
$\eta_3$	0.749	0.214
$\eta_4 = \bar{\eta}$	0.942	0.236
$\eta_5$	1.185	0.214
$\eta_6$	1.508	0.135
$\eta_7$	1.986	0.034

**Table V: Technology Parameters**

Parameter	Value	Target
$\alpha$	0.36	Data
$\delta$	6.58%	$I/Y = 25.5\%$
$g$	1.75%	Data

**Table VI: Policy Parameters**

Parameter	Value
$\tau_c$	5.2%
$a_0$	0.258
$a_1$	0.768
$b$	0.5
$\tau_{ss}$	12.4%

**Table VII: Comparison across Tax Codes**

Variable	BENCH.	OPTIMAL	PROP.	Fixed ( $w, r$ )
Parameter $a_0$	0.258	0.172	0.099	0.172
Parameter $a_1$	0.768	19.0	0.0	19.0
Interest Rate $r$	6.77%	6.73%	6.15%	6.77%
Wages $w$	--	0.10%	2.80%	0%
Average Hours Worked	0.333	-0.96%	5.94%	-1.05%
Total Labor Supply $N$	--	0.54%	5.86%	0.45%
Capital Stock $K$	--	0.81%	13.88%	0.45%
GDP $Y$	--	0.64%	8.86%	0.45%
Aggregate Consumption $C$	--	0.70%	8.93%	0.70%
Gov. share in GDP $\frac{G}{Y}$	17.0%	16.89%	15.64%	16.85%
Total Income Tax as % of $Y$	10.66%	10.55%	9.13%	10.54%
Gini Coef. for Pre-tax Income	0.392	0.408	0.400	0.408
Gini Coef. for After-tax Income	0.371	0.386	0.400	0.386
Gini Coef. for Wealth	0.644	0.666	0.667	0.666
Gini Coef. for Consumption	0.280	0.290	0.302	0.290
$ECV$	--	1.7%	-0.95%	1.6%

# Figures

Figure 1: Average Tax Rates under 2 Tax Systems

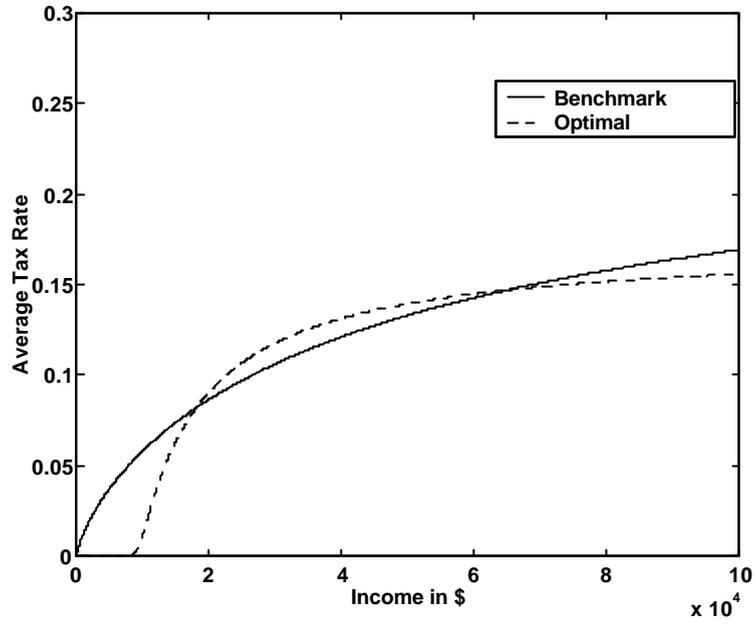


Figure 2: Marginal Tax Rates under 2 Tax Systems

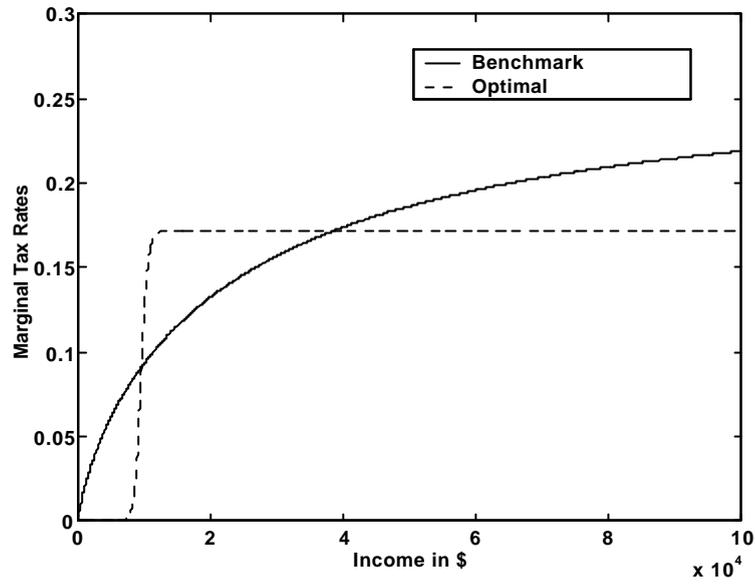


Figure 3: Difference in Total Taxes Paid

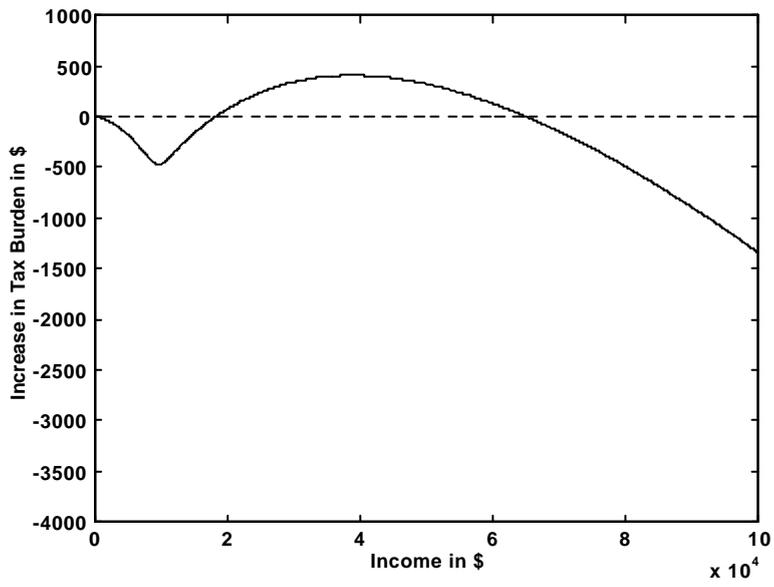


Figure 4: Evolution of Aggregate Quantities along the Transition

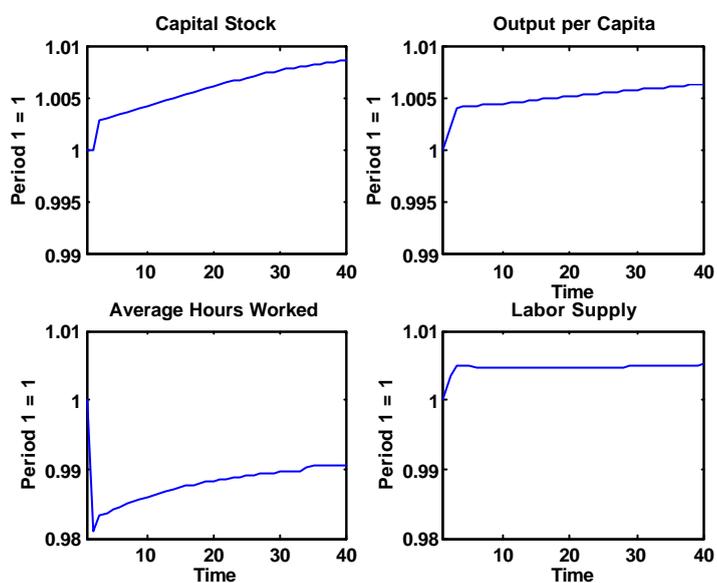


Figure 5: Evolution of Prices along the Transition

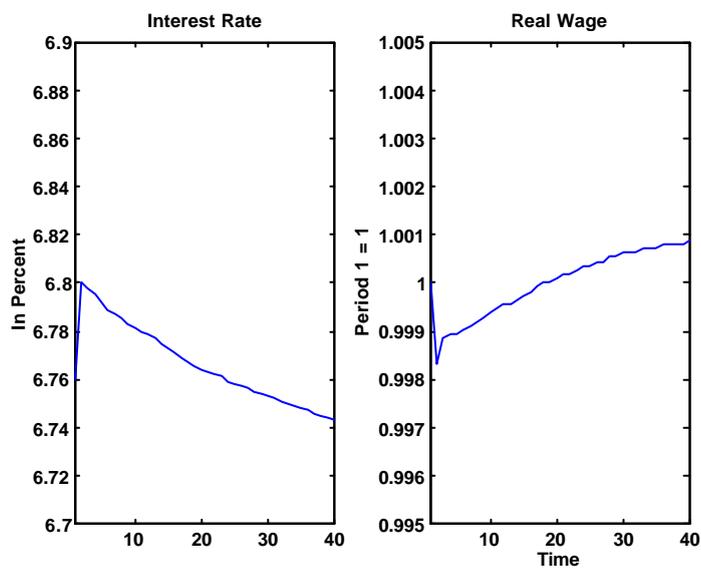


Figure 6: Evolution of the Tax Deduction along the Transition

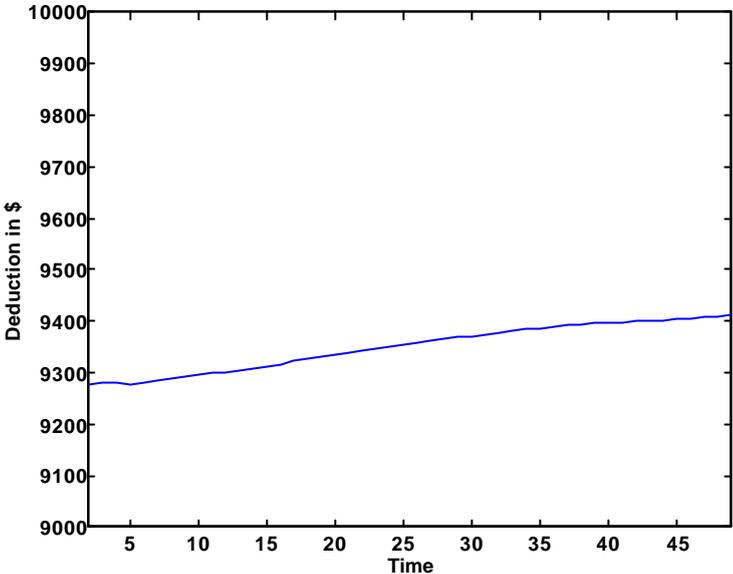


Figure 7: Welfare Consequences

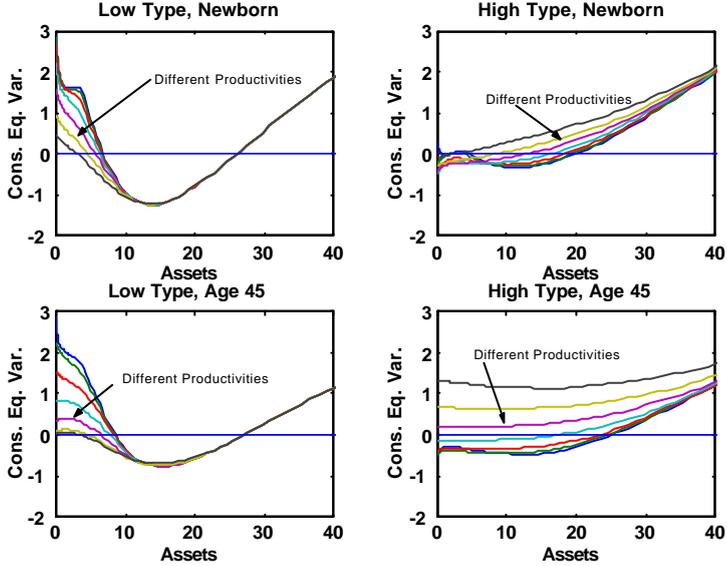
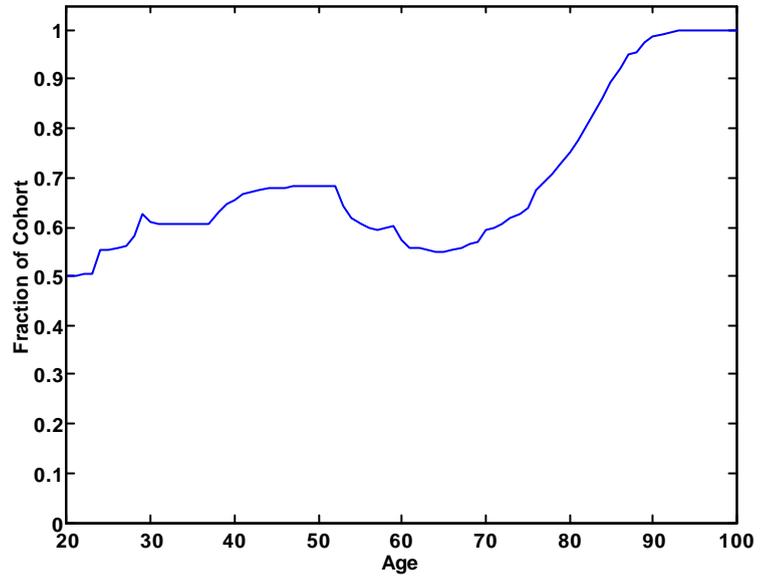


Figure 8: Fraction of Cohort with Welfare Gains



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