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Private equity-, stock- and mixed asset-portfolios: A bootstrap approach to determine performance characteristics, diversification benefits and optimal portfolio allocations

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Abstract:

In this article, we investigate risk return characteristics and diversification benefits when private equity is used as a portfolio component. We use a unique dataset describing 642 US-American portfolio companies with 3620 private equity investments. Information about precisely dated cash flows at the company level enables for the first time a cash flow equivalent and simultaneous investment simulation in stocks, as well as the construction of stock portfolios for benchmarking purposes. With respect to the methodology involved, we construct private equity, stock-benchmark and mixed-asset portfolios using bootstrap simulations.

For the late 1990s we find a dramatic increase in the extent to which private equity outperforms stock investment. In earlier years private equity was underperforming its stock benchmarks. Within the overall class of private equity, returns on earlier private equity investment categories, like venture capital, show on average higher variations and even higher rates of failure. It is in this category in particular that high average portfolio returns are generated solely by the ability to select a few extremely well performing companies, thus compensating for lost investments.

There is a high marginal diversifiable risk reduction of about 80% when the portfolio size is increased to include 15 investments. When the portfolio size is increased from 15 to 200 there are few marginal risk diversification effects on the one hand, but a large increase in managing expenditure on the other, so that an actual average portfolio size between 20 and 28 investments seems to be well balanced. We provide empirical evidence that the non-diversifiable risk that a constrained investor, who is exclusively investing in private equity, has to hold exceeds that of constrained stock investors and also the market risk.

From the viewpoint of unconstrained investors with complete investment freedom, risk can be optimally reduced by constructing mixed asset portfolios. According to the various private equity subcategories analyzed, there are big differences in optimal allocations to this asset class for minimizing mixed-asset portfolio variance or maximizing performance ratios. We observe optimal portfolio weightings to be between 3% and 65%.

JEL Code: G11

Key Words: Venture Capital, Private Equity, Performance, Return, Risk, Portfolio, Fund, Diversification, Efficient Frontier, Allocation

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1. Introduction

Modern portfolio theory quantifies the benefits of diversification and demonstrates opportunities for improving the performance characteristics of portfolios by combining assets. Over the last years private equity (PE) has become more reputable within the field of alternative asset classes. By 1996 at the latest, worldwide capital commitments to PE funds were experiencing large increases. Nevertheless, the acceptance of PE as a capital investment alternative has lagged behind that of other asset classes like stocks, bonds or even real estate. Reservations about investing in private equity may be caused by several factors. Mainly, it has been low market transparency combined with the complexity of understanding both this market segment and the benefits of portfolio allocations to this asset class that have led to a certain reluctance to treat that enigma, namely private equity as an investment opportunity.

Academic research that analyzes private equity with respect to portfolio management is rare. There are a large number of studies devoted to investigating the characteristics of stocks, bonds or real estate as portfolio components, but the unsatisfactory data situation does not particularly encourage the analysis of private equity assets in this context. Given a relatively inefficient private equity market, the logic of portfolio choice is, therefore, based mostly only on the premise that not all eggs should be placed in one basket. Very few approaches exist which try to make recommendations about optimal portfolio compositions which include private equity. Our innovative approach aims to remedy this situation.

In this paper we analyze several private equity categories, we derive performance characteristics using empirical methods and find evidence on the benefits of adding private equity to a mixed-asset portfolio. This investigation is based on a dataset which is unique with respect to the depth of information it contains. The dataset provides to date exact information about 3620 investments made by 123 funds from 37 investment managers. Our analysis is based on actual cash flows to and from portfolio companies. We begin by simulating investments in individual benchmark stocks with the same timing. Thus, we observe exact benchmark performances in relation to private equity investments, both of which are measured by the internal rate of return (IRR). Due to the lack of suitable worldwide stock universes that can deliver benchmark stocks over the whole period from 1970 to 2002, we have had to reduce the sample used to 643 US-American portfolio companies financed between 1980 and 2002. Each private equity investment is allocated to one benchmark stock from the Russell 2000 stock universe. Comparable PE and benchmark stock returns are used for bootstrap simulations of several pure and mixed asset portfolios in order to observe changes in risk-return characteristics.

Specifically, the analysis is organized in three steps. First, we assess several private equity categories and provide descriptive statistics. Since with respect to the private equity category we observe higher performance variation of venture capital investments combined with a higher average but a lower median return, we recognize the increased need for skilful investment choices when focusing on venture capital. In earlier times benchmark stock investments outperformed PE investments in terms of mean IRRs. It is only since the late 1990s that the overall private equity market has started performing much better. Research has yet to reveal whether this development is a result of a learning process or of improved market mechanisms like the establishment of better exit markets or the emergence of advisors. Will this outperformance continue, or is this merely a temporary bubble? If this is a temporary "exogenous shock", then the individual investment ability of the PE manager will be decisive in ensuring that the higher non-diversifiable pe-market risk is compensated. As a result, in the 'post bubble' market of the early 21st century we actually observe a run on participating established private equity funds. Investors have become cautious; they now search for high quality funds and refuse to invest in newcomers. Is this evidence of the investors' belief that the PE market is once again, on average, underperforming and of their need to find above average funds that will outperform the benchmark?

In a second step we explore differences in the naive risk diversification of portfolios consisting of private equity or stocks. If we increase the portfolio size to number 15 investments we observe strong marginal risk diversification. The complete reduction of diversifiable risk requires an inclusion of at least 200 investments in the portfolio. Whilst there is a small marginal risk reduction when the portfolio size is increased from 15 to 200, there is also a large increase in managing expenditure; so the actual average realworld portfolio size of between 20 and 28 investments seems to be well balanced. Following the new approaches of asset pricing published by Malkiel/Xu (2000) or Jones/Rhodes-Kropf (2002), we assume that a PE manager or a stock portfolio manager is constrained by his statute from holding all security classes and is, therefore, also precluded from holding the market portfolio predicted by the CAPM. We show using empirical methods that the non-diversifiable risk a constrained PE investor has to hold exceeds that of stock investors and also exceeds the market risk. It is referred to as the idiosyncratic risk premium of private equity portfolios.

From the view of non-constrained investors, risk can be optimally reduced by constructing mixed asset portfolios. In a third step we, therefore, derive optimal allocations to private equity and its benchmark stocks in mixed-asset portfolios. Depending on the private equity category involved, we look at changing optimal weightings of private equity in order to construct ‘minimum variance’ or ‘maximum performance ratio’ portfolios. Risk is reduced below the risk level that a constrained stock or PE investor has to hold. Results are robust if we use gross and net performance variables. Finally, a summary and conclusion are given.

2. Related literature

Due to the unsatisfactory data situation research determining optimal PE portfolio constellations is rare. Therefore, we focus our literature review on related research, which has been carried out for two other asset classes: stocks and real estate. Given the similar characteristics of an investment in these asset classes, a comparison of empirical results indicating diversification benefits is worthwhile.

a. Optimal portfolio size

STOCK PORTFOLIOS

Starting with Evans and Archner (1968), the financial literature demonstrates on an empirical basis the naive diversification effects of pure stock portfolios when the number of assets included is increased. The authors first of all show that the connection between increasing portfolio size and portfolio risk takes the form of a rapidly decreasing asymptotic function.[1] They refute the notion that there is any economic justification for a portfolio that includes more than ten securities.[2] Portfolios were built by a random security selection and a mean portfolio return calculation taken from a database of 470 stocks. Evidence derived from similar empirical methods is given by Fisher and Lorie (1970) and Elton and Gruber (1977). They show that there is a reduction in diversifiable risk of between 84% and 88% if the stock portfolio size is increased by only 8 stocks. However, both studies find that there are further diversification effects if the portfolio size is increased by more than 8 stocks.[3] Recent research supports the efficiency of including more than 10 securities. An analysis of share price data between 1955 and 1984 by Poon, Tayler and Ward (1992) shows a further 23.86% risk reduction when the portfolio size is

[1] See figure 3 and 8 for similiar results
[2] See Evans/ Archner (1968), pp. 766
[3] See Fischer / Lorie (1970), pp. 116 and Elton / Gruber (1977), pp. 426.

raised from 10 to 25 stocks.[4] Hellevik and Hermann (1996) investigate naive risk diversification of securities traded on the German stock exchange between 1974 and 1994. They find in nearly all cases a risk diversification of 80% if the portfolio reaches a size between 9 and 19 securities.[5] Other studies by Tole (1982), Newbold and Poon (1993), and De Vassal (2001) contradict the usual assumption that a portfolio of max. 25 stocks in size would be sufficiently diversified.[6] Using stocks from the Russel 1000 index to simulate portfolios De Vassal makes no certain recommendations about the optimal portfolio size, but he determines a portfolio size of up to 100 securities to be useful.

REAL ESTATE PORFOLIOS

There are various characteristics of private equity which correspond to those of real estate. Both assets are not traded in a permanent marketplace with quoted market prices, and both incorporate low liquidity and indivisibility. Furthermore, an investment in these asset classes is characterized by high transaction costs and information that is both limited in its public availability and has a highly asymmetric distribution .[7] In view of these similarities we give a review of real estate literature dealing with risk/return topics.

Both Miles and McCue (1984) and Grissom, Kuhle and Walther (1987) find non-systematic risk diversification effects of between 83% and 90%, respectively, when the portfolio size is increased to include 10 real estate objects.[8] The marginal risk diversification decreases rapidly if the portfolio size is raised beyond 10 properties. More recent studies recommend a larger portfolio size in order to achieve optimal risk diversification. Brown (1997) finds risk diversification to be at the same level as systematic risk for a portfolio size which ranges from 30 up to several hundred properties. He takes into consideration the high dispersion of individual real estate performances.[9] Byrne and Lee (1999), using similar methodology, support these findings. They recommend a portfolio size of at least 200 properties.[10] Byrne and Lee (2000) even find empirical evidence to suggest that 400-500 properties are needed to reduce the risk of a property portfolio down to the market level.[11] Miles and McCue (1984), Hartzell,

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4 See Poon / Taylor / Ward (1992), pp. 93.
5 See Hellevik / Herman (1993), pp. 12.
6 See Tole (1982), pp. 9; Newbould / Poon (1993), pp. 86; De Vasal (2001), pp. 35.
7 See Kallberg, Lui and Greig (1996), pp. 359 ff.
8 See Miles / McCue (1984), pp. 63; Grissom, Kuhle and Walther (1987), pp. 71.
9 See Brown (1997), pp. 136 ff.
10 See Byrne/Lee (1999), pp. 18.
11 See Byrne/Lee (2000), pp. 12; In their recent study Byrne and Lee (2001) determine different risk attitudes connected with the management of large or small portfolios. They show next to decreasing non-systematic risk

Heckman and Miles (1987), and Brown (1997) compare the relative levels of non-systematic risk of real estate and stock portfolios. They indicate a non-systematic risk level between 90% and 94% for real estate and between 62% and 70% for stocks.[12] According to the authors, these differences show a greater need for holding real estate portfolios which are larger than stock portfolios in order to reach individual market risk levels.

PRIVATE EQUITY PORTFOLIOS

Due to the lag in suitable PE data there is only limited empirical research available which reveals the diversification effects of PE portfolios with increasing size. The management complexity of PE portfolios is mentioned by Statman (1987), Kanniainen and Keyschnigg (2000), and Cumming (2001). They suppose that the threshold of optimal portfolio size is reached when a further increase would lead to a rise in marginal costs which is higher than that in marginal benefits. They determine factors influencing the portfolio size, but do not offer any recommendations with respect to optimal PE portfolio sizes.[13] Nor do any other studies provide empirically-based recommendations as are made for other asset classes. Instead they are limited to investigations concerning the actual portfolio size of PE portfolios without answering the question about whether these are the optimal portfolio constellations. The 178 funds of CEPRES' data sample (sample status, May 2003) have included an average of about 25 and a median of about 20.5 portfolio companies.[14]

b. Optimal asset allocations to mixed portfolios

STOCKS, BONDS AND REAL ESTATE

A number of studies have presented evidence which argues that real estate is offering investors diversification benefits. Kuhle (1987) investigates the risk/return characteristics of mixed stock and real estate portfolios. He uses data from 26 Real Estate Investment Trusts (REIT) and 42 common stocks to build mixed portfolios with changing asset allocations. He calculates performance ratios as the ratio between return and risk to examine the return/risk characteristics of mixed portfolios. His results show that the

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an increasing systematic risk with increasing portfolio size. They attribute this effect to a larger number of riskier investments held in large portfolios.

[12] See Miles / McCue (1984), pp. 66; Hartzell / Heckman / Miles (1987), pp. 248; Brown (1997), pp. 138

[13] See Statman (1987), pp. 354; Kanniainen / Keuschnigg (2000), pp. 5; Cumming (2001), pp. 3.

[14] This is similar to the number of one specific fund's portfolio companies observed by other authors. Recent studies made by Ljungqvist / Richardson (2003) observed an average number of 22 portfolio companies in one fund, Reid / Terry / Smith (1997) indicate an average number of 28 (according to a survey of 20 funds).

overall performance of mixed asset portfolios is not significantly different from that of portfolios consisting only of common stocks.[15] The exceptions are those mixed asset portfolios that contain at least a 2/3 share of REITs. However, these results are distorted because Kuhle constructs portfolios from data that combines single stocks and already diversified real estate portfolios (REITs).[16] Other studies published at this time, like Webb and Rubens (1987) or Webb, Curico and Rubens (1988), present similar results. They consider a 43% or respectively 66% investment in real estate to be the optimal allocation.[17] Brown and Schuck (1996) estimate via bootstrap simulations ex ante standard deviations, returns and correlations between stocks and real estate. They find an optimal allocation to real estate to be around 40% (portfolio size: 1000 assets). The impact of a changing overall portfolio size is also examined. The mean weighting of real estate which is required to achieve a minimum variance portfolio decreases to 14.2% with decreasing portfolio size (1 asset).[18] Liang, Meyer and Webb (1996) also used bootstrap simulations to build mixed portfolios. They cannot provide any reliable recommendation concerning the optimum composition of mixed-asset portfolios.[19] Making adjustments to their methodology, Ziobrowski, Cheng and Ziobrowski (1997) produce different conclusions. They show that investors with a low risk preference should not invest more than 10% of their portfolio's capital in real estate. This corresponds to the average investment size in real estate by institutional investors.[20] Data of superior quality are used by Kallberg, Lui and Greig (1996). They use exact cash flow data of real estate investments to calculate the modified internal rate of return (MIRR). In conclusion, they find a real estate allocation up to 9 per cent to be optimal, when they model the efficient frontier indicating the best risk/return characteristics of mixed stock, bond and real estate portfolios.[21] Ziobrowski and Ziobrowski (1997) also generate portfolios of financial and real estate assets and determine the efficient frontier. They recommend a higher allocation of about 20-30% to real estate. These findings correspond to those of Brinson, Diermeier and Schlarbaum (1986), whose results recommend a 20% investment share in real estate.[22] Using expected returns derived from an equilibrium model, Ennis and Burik (1991) find that the most

[15] See Kuhle (1987), pp. 6.
[16] See Georgiev (2002), pp. 3 and 5. They show that the market prices used here for the REITs do not necessarily represent the underlying market value of the underlying assets and have a higher correlation to stock than to real estate performances.
[17] See Webb / Rubens (1997),pp 13 and Webb / Curico / Rubens (1988), pp. 446.
[18] See Brown/ Schuck (1996),pp. 68.
[19] See Liang/ Meyer/ Webb (1996), pp. 205.
[20] See Ziobrowski/ Cheng/ Ziobrowski (1997), pp. 703.
[21] See Kallberg/ Lui/ Greig (1996), pp. 367.
[22] See Brinson, Diermeier and Schlarbaum (1986), pp. 22

efficiently diversified portfolios include real estate investments in the range of 10% to 15% of total assets.[23]

PRIVATE EQUITY

The low transparency of private equity markets and, the resulting unsatisfactory data situation make an exact comparison of the returns on PE and other asset classes difficult. In general, we have no annual returns for PE investments that are comparable to those of other assets with continually quoted market prices. There is only a series of cash flows with no intermediate values, which given a full distribution allows the annualized internal rate of return (IRR) to be calculated over the entire life of the investment. There are no studies based on real cash flows that calculate PE IRRs and at the same time the cash inflow equivalent IRRs of investments in other asset classes. Most studies use proxies to simulate characteristics of PE investments. The following studies have attempted to quantify the return/risk characteristics of PE, and some authors try to give a recommendation about the optimal allocation to PE.

Probably the most famous investigation is that by Cochrane (2003), who analyzes a dataset from 1987 to 2000. He matches under certain assumptions information from two separate databases to calculate venture capital backed company returns. Cochrane only observes returns of portfolio companies which go public or out of business, but not of those that remain private.[24] He corrects for this sample bias by using maximum likelihood estimates to identify and measure the increasing probability of going public or being acquired. Without a selection bias correction he finds arithmetic average returns of 69.8%, and with bias correction average returns that decrease to 59%. Ljungquist and Richardson (2003) try to quantify PE fund performance using a superior data basis, but offer no evidence with respect to the optimal allocation to PE in mixed-asset portfolios. They use data collected from the portfolio of only one single investor. This dataset is in danger of including a selection bias. They explore IRRs measured on the basis of actual funds' cash flow data. They do not have data at the company level and only make an approximation about whether the analyzed funds are completely realized.[25] Results indicate a simple weighted mean IRR for mature funds of around 20%. Using a repeated valuation model to correct for selection bias in the reporting of values, Quigley and Woodward (2002) try to

[23] See Ennis/Burik (1991), pp. 27.

[24] See Cochrane (2003), table 1, next to the missing data of companies which remain private (45.5%), Corranes' sample just includes 9% lost investments. This is less than the number of lost investments in our sample and does not correspond to the usual default rates in venture capital.

[25] Ljungquist and Richardson (2003), pp. 6 ff.

build a VC index for the period between 1987 and the first quarter of 2001. They focus on individual portfolio companies and find for mixed-asset portfolios (PE, stocks and bonds) an optimal allocation to PE to be between 10% and 15%.[26] McFall Lamm and Ghaleb-Harter show that an investor should invest between 19% and 51% in PE.[27] Bader (1996) recommends under varying assumptions a PE allocation between 10% and 39%.[28] Pradhuman, Kan and Chbani (2001) and Merrill Lynch (1995) all use small caps to proxy PE investments. These studies indicate benefits by investing 15% or 10% of total capital, respectively, in PE.[29] Superior data are used by Chen, Baierl and Kaplan (2002) to construct an efficient frontier. They use real IRR data from 148 PE funds that have been liquidated as of June 30, 1999.The earliest date of investment is January 1, 1960. No exact cash flow data were available to calculate comparable performances for other asset classes taking into consideration the exact investment timing. Their results produce an efficient frontier consisting of VC and the S&P 500 index that justifies allocations between 2% and 9% to VC for constructing the minimum variance portfolio or maximum Sharpe ratio portfolio.[30]

3. Data and methodology

Benchmarking PE investment performances against those of other asset classes is difficult. In contrast to quoted assets with daily market prices there are only two occasions, the date of investment and the date of divestment, when a market-determined value is known for PE investments. If the historical series of cash flows over the entire life of investment after liquidation is known, then the annualized internal rate of return (IRR) can be calculated. Using an interim IRR based on net asset values to get annual rate of returns is just an estimation of reality. Thus, up until today with recent datasets, an exact comparison of PE (measured by the IRR as the annualized internal rate of return over the entire life of investment) and stock investment performance (measured by a volatile annual rate of return) was not possible.

The dataset we use contains exact and complete information about 3619 PE investments made between 1970 and the end of 2002. We have access to precisely dated cash flows down to the company level, the exact assignment of every company to its fund and its investment manager and, furthermore, a large amount of investment manager,

[26] Quigley/ Woodward (2002), pp. 22.
[27] McFall, Lamm and Ghaleb-Harter (2001), pp. 75.
[28] Bader (1996), pp. 208-210.
[29] Pradhuman, Kan and Chbani (2001), pp. 35.
[30] Chen, Baierl and Kaplan (2002), pp. 88.

fund, and company-specific information. We are not confronted by selection bias like Cochrane owing to missing company data. Our sample includes all investment information made by the funds investigated, including those which remain private, are written off or are lost. This data is derived from the records of CEPRES' Private Equity Analyzer which collects detailed PE data on a completely anonymous basis. Therefore, we do not know anything about the identity of the company, fund or investment manager. Nevertheless, the sample is well balanced. Table 1 presents the origin of the samples' investment manager and portfolio companies. There is possibly a certain survivorship bias because data are derived mainly from those PE managers who have reported over the last years. From these general partners we also obtain information which describes their former mature funds. Unfortunately, we have no information about fund managers who were not in business until the mid 1990s.

To avoid "estimation biases" due to the subjective valuation treatment we concentrate our study on completely liquidated investments with real cash flow history. This follows the approach used by Cochrane (2003).[31] Therefore, the overall dataset is reduced to 1539 completely realized, lost or written off investments.

Information about the amount and date of all cash flows to and from the PE investments enables a cash flow equivalent and simultaneous investment simulation in stocks. For every single PE investment we choose another benchmark stock to simulate comparable performance (both measured in IRR).[32] In order to choose the right benchmark it is essential to find stocks from one homogeneous universe. There is no uniform small cap stock universe in Europe, South America and Asia that covers at least the last twenty years. All major stock indices covering small caps emerged in the last ten years. We, therefore, confine our research to US-American PE portfolio companies between 1980 and 2002 and draw benchmarks to US-American small cap stocks quoted within this period. The dataset is reduced to 642 US-American PE investments. The comparison between the complete and the reduced sample shows no significant differences in performance. The hypothesis of no difference in mean IRRs is not rejected. Table 2 presents a very low t-value. The sample includes PE investments from all financing stages: early-, expansion-, later stage, mbo/lbo, turnaround and mezzanine. Table 3 presents the sample's exact composition. The sample's portfolio companies are operating in a well-balanced industry range. The reduced sample's composition does not

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[31] See Chorane (2003), pp.3

[32] As it is recommended in Ehrhardt/ Koerstein (2001), pp. 455 or Barber/ Lyon (1997).

deviate far from that of the complete sample. Thus, when we use the reduced sample we expect to find similar results to those obtained from analyzing the whole sample.

Following the methodology of De Vassal (2001), we match a sample of firms which had been original constituents of the small-cap universe, Russell 2000. The Russell 2000 measures the performance of the 2000 smallest companies of the Russell 3000 index and represents approximately 8% of its market capitalization. The Russell 3000 represents approximately 98% of the U.S. equity market available for investment. We divided the PE sample into two investment periods of 10 and 12 years between 1980 and 1990 and 1990 and 2002, respectively. To benchmark PE investments which were made in the 80s we use original constituents of the Russell 2000 in its composition of the year 1980 and their total return performance through to the end of 2002. PE investments made between 1990 and 2002 are allocated to original constituents of the Russell 2000 in its composition of the year 1990. For stocks of companies that did not survive the entire holding period of the allocated PE investment, we recognize the total return until the last reported stock price and simulate, in addition, a reinvestment in another size and industry-matched stock from the same index composition.

Each PE portfolio company is acting in the same industry as its benchmark stock. To avoid size effects we ranked all PE investments according to their investment size and all stocks according to their market capitalization. By means of this ranking we allocated each PE investment to a comparable benchmark stock. Owing to large deviations in the stocks' market capitalization and the PE investment costs, an exact size match was not possible.[33]

In accordance with the described methodology, every PE investment is allocated to one stock in the same industry. A simulated investment of each PE cash flow in the allocated benchmark stock at same cash flow date and with a simultaneous divestment delivers exact benchmark IRRs. In particular, we created an investment in the benchmark stock with the same timing and amount, whenever a draw down or distribution on private equity company level occurred. The amount was translated in to a number of shares of the benchmark company by dividing the investment cash flow by the stock's current market price, i.e. buying shares for the equivalent amount at the current quote.

[33] The average market capitalisation of one stock was around 29 million US-Dollar, the average financing costs of one private equity investment were around 9 million US-dollar.

$$Shares_{yt}^{Index} = \frac{negative\ cash\ flow_{yt}}{Stock\_market\_price_t} \qquad t = 1,2,....,T$$

Here t denotes the current month and y refers to a particular company.

Whenever a cash outflow, i.e. a disbursement to the fund's investors occurred, we divided the positive company cash flow by the sum of all positive company cash flows to weigh the particular cash flow. We multiplied this fraction with the sum of all shares of the index and the current index level. As a result, a positive cash flow was created for the IRR calculation of the benchmark stock.

$$positive\ cash\ flow_t^{benchmark\_stock} = \frac{PE\_company\ cash\ flow_{yt}}{\sum_1^T PE\_positive\ company\ cash\ flow_{yt}} * \sum_1^T company\_shares * market\_price$$

Here t denotes the current month and y refers to a particular company.

This approach creates a cash flow pattern for benchmark stocks that mimics the pattern of the underlying private equity company investment. Using this cash flow pattern, we can easily compute the IRR for the respective benchmark stock and thus obtain the performance the investor would have achieved if he had invested in the benchmarked company. Each PE IRR (PE_n) has its counterpart benchmark IRR (S_n). There is no dilution of benchmark performances due to different investment periods or different performance measures. We overcome the usual problems of non-performance comparability between these two asset classes.

We simulate portfolios built from a changing number of PE or stock investments by using the bootstrap methodology. We construct portfolios from equally weighted real PE investments and measure their cross-section mean return, volatility and other descriptive statistics. This approach does not take into account the different single starting points and the different capital weightings of investments carried out by real-life private equity funds. In this paper, however, we aim to simulate portfolios in order to examine the overall properties and dynamics of private equity investment performance patterns and not in order to evaluate the investment managers' ability with respect to timing and capital weighting.[34] The bootstrap methodology used is described in the appendix.

4. Assessing private equity and its stock benchmark risk/return characteristics

[34] Further discussion about the suitability of different methodologies, see Burgel (1998), pp. 33.

SAMPLE PERIOD – Investments between 1980 and 2002

We begin by examining the complete sample comprising US PE portfolio companies financed between 1980 and the end of 2002. The first panel of table 4 shows the descriptive statistics of PE investments and of benchmark investments in stocks. Each set represents one portfolio consisting of 642 investments that are completely comparable in terms of timing and cash inflow. On the one hand these are investments in PE portfolio companies and on the other hand in stocks. The mean return of PE investments shows an IRR of 36,49%. This in line with expectations, is more than three times higher than that of equal investments in stocks (11,59%). Besides higher mean returns, PE investments are characterized by higher cross section volatility. However, PE's standard deviation of 242% is only about twice as high as that of stocks [103%]. The high returns of PE portfolios are generated on the whole by only a few high performing companies. In the case of PE, the sample therefore shows higher maximum returns and a wider range of investment performances (PE: 3026%/ stocks: 1943%). Figure 1 shows the typical IRR distribution of PE investments. In contrast to the benchmark, PE performance distribution is characterized by two peaks. This is the result of a high number of lost investments, a relatively low number of modestly performing investments, but a large number of well and some extremely well performing investments. [35]

SAMPLE PERIOD – Investments between 1980 and 1990

Reducing the sample to investments made between 1980 and 1990 we see deterioration in the performance of both PE and stocks. The mean return of PE is nearly zero and underperforms benchmark stocks. Analytically, this is the consequence of fewer extremely high performing PE investments. The maximum performance of PE investments made in the 1980s does not exceed an IRR of 642%. A median return of 8.77%, however, exceeds the median benchmark return. This reflects, despite a large number of totally lost investments, the overall high figure for modestly performing PE investments. However, extremely well performing investments, which are able to compensate in terms of mean IRR for the huge amount of failures, on average does not remain. Within this period PE markets had not really been established. It was not until the 1990s that an efficient and professional PE market was developed by consultants with knowledge of M&A, by the establishment of new, well functioning and more liquid exit-

[35] This corresponds partly to the findings of Cochrane (2001),pp. 10. Analyzing only the returns to ipo or trade sale, he determines a few outstanding returns of thousands of percent and many relatively more modest returns.

channels, and last but not least by the academic investigation of these topics, e.g. entrepreneurship. In the period between 1980 and 1990 the correlation between PE and the benchmark stocks' performance is lower than that measured over the full sample period [-0,044].

SAMPLE PERIOD – Investments between 1990 and 2002

Investments which were made in the last decade (vintage year 1990 to 2002) exhibit different characteristics. Our sample data reflects the well performing capital market movements of the 1990s. High performing PE investments, especially those realized during the bubble between 1996 and 2001, made it possible to reach a mean IRR of 56.8%. The variation in PE investment performances (standard deviation 295%) increases as well. Even if the benchmark investments also show a relatively high performance of 16.39%, PE outperforms the stock investments. In this period, the correlation between PE and stock investments also increases from -0.044 to -0.016. This matches the result of Longin and Solnik (1995), which demonstrate empirically the rise of a correlation between national stock markets in periods of high volatility.[36] Especially in the last years of the 20th century the return volatility of all asset markets was increasing.

SAMPLE PERIOD – Realization date before and after January 1997

We observe extraordinary returns in the last decade. One reason may be the booming years between the end of 1996 and 2001. In all probability there will be no similar recovery in the near future that will raise performances to reach former absolute heights. Nevertheless, we have to ask whether PE in the future will still outperform stocks as a result of more developed PE markets, or whether the former outperformance of PE relative stocks was only the result of a larger bubble of PE markets compared to the stock markets. PE performance is largely determined by the condition of the exit markets. To test robustness we exclude from the overall sample all investments realized in the boom years after 1996. Table 4.1 panel 4 presents a drop in average PE return to 7.9%, but also shows a relatively high median return of 17.6%. The median return is higher than that of the benchmark investments. However, there is a greater degree of skewness to the left and maximum performances do not reach the same heights as the benchmark investments. Before 1997, PE seemed to be characterized by a constantly high number of modestly

[36] See Longin / Solnik (1995), pp. 16.

performing investments, together with fewer extremely well performing investments with 4 digit IRRs. Even if there was a constant frequency of lost investments over time, in the early years the number of extremely well performing investments was too low - and the relative number of lost investments too high - to outperform the stock benchmark in terms of mean IRR. The high number of modestly performing investments could not compensate for the high number of lost investments. Investments which had been realized in the boom years after 1997 outperformed the benchmark in terms of mean and median IRR. The return outperformance was priced by a strong increase in return volatility. Further analysis will show to what extent this risk was diversifiable.

To avoid selection bias we take note of all realized investments including those which were written off or lost. Nevertheless, there could be a potential for bias. Badly performing investments from previous years with no chance to exit may still be held by the fund and will dilute the overall fund's' performance. Such company data are only partly considered by using complete investment information of all mature funds. To test for robustness we reduce the sample to those investments which are taken exclusively from realized, and in a second step from almost realized (at least 70% of funds' investments are completely realized), funds (see table 4.1 panel 6, 7). Thus we integrate all investments made by the funds into the analysis and avoid potential selection bias. Results confirm the findings of the analyses which are made for investments realized before 1997. Most funds which started to invest in the late 1990s have not yet been completely realized and, therefore, their investments are not taken into account in this sample subset. The extremely high returns of the last few years are not recognized if we analyze this subset. Table 4.1 panel 7 describes investments taken from completely realized funds. Most funds which were raised in the late 1990s contain at least one non-completely realized investment. Consequently, all investments done by those funds are excluded from the analysis. Consequently, the sample "Investments taken from fully realized funds" includes investments conducted mainly in the 1980s and very early 1990s. Funds which had been raised in this period are usually realized by today. As a result, the performance data are similar to those of the sample that describes investments from the 1980s. We cannot, however, evaluate the exact performance of funds which have been raised in the late 90tis and are still not completely realized but have profited from the exit environment of the last years.

SAMPLE PERIOD - S*ummary*

To summarize: it is only in the late 1990s that we observe private equity investments outperforming their public market investment equivalent. We do not know yet if this development is a result of a learning process or of improved market mechanisms, such as the establishment of better exit markets or the emergence of advisors, or whether it is merely due toly a temporary bubble. This will be an important factor which helps to decide the future development of the whole PE industry. If the overall industry does not function well, the individual skills of PE investment managers will be increasingly decisive as a factor for success. A market clearance of low quality PE managers will help to save the industry's reputation! Due to the large capital amounts in the market, which have to be allocated to private equity, it is however uncertain, whether a market clearance will be possible. The real world shows, that the institutional investment pressure forces non privileged market participants without invitation to A-funds to invest in B-funds. As a result, the overall performance will be still modest and could underperform the traditional markets. Today, private equity funds are again beginning to raise an expected overall amount of 60 billion US-dollars (expectations 2004) in venture capital that was sidelined after the Nasdaq plunged in 2000. Market experts argue, that again "we are seeing some effects of the overhang at play".[37] Due to the high competition for the best deals, companies that definitely will not contribute to an extraordinary pe market outperformance are again financed as the way out of the capital overhang.

INVESTMENT STAGES – Venture Capital vs. Buy Outs

Table 4.2 shows descriptive statistics of sample subsets which represent investments in different financing stages. We observe strongly increased average returns for pure venture capital investments compared to all other private equity investments without VC, such as MBO/LBO and turnaround investments (all PE without venture capital = PE w/t VC). The PE w/t VC mean IRR does not exceed its benchmark by much. The higher mean return of venture capital is connected to a higher variation (313% versus 51%) of returns within a wider range of possible outcomes (max. performance of VC: 2962% versus PE w/t VC: 148%). Our results do not completely correspond to those of Ljungquist and Richardson (2003). They analyze performance at the fund level and derive an outperformance for funds with investment focus on buyouts (against venture capital).[38] Although average returns in our sample are lower, the median return of PE w/t VC is

[37] See Francisco Bamby, *Navigating a bounce in venture capital*, CBS.MarketWatch.com, March 18 2004, pp.1
[38] See Ljungquist and Richardson (2003), pp.21; they observe an average return of 21.83 % for buyout funds versus 14.08 for venture funds.

much higher than that of pure venture capital investments and of its benchmark. Venture capital receives high average returns owing only to some extremely well performing outliers (Figure 2). Due to the high number of lost investments (25.48%), the right company selection and the capital weightings are decisive for the overall outperformance of VC funds. Given equal weightings, we observe for venture capital investments an average return of 47.81%. The subsample PE w/t VC has a lot of modestly performing investments and an average IRR of 7.62%. The right portfolio composition is not as dependent on the manager's ability to choose highflyers. Furthermore, there is a big difference between the correlations of both PE categories compared to their benchmark stocks. In contrast to PE w/t VC, which has a high correlation of 0.25 to stock investments, VC is practically uncorrelated to the stock benchmark.

Compared to the overall venture capital sample, purely early stage investments are characterized by a decreasing average outperformance (21.39% versus 37.12%) against the benchmark. The median return is even negative. This is the result of a very high frequency of lost investments (38%). Only a few extremely well-performing investments contribute to a high average return. Nevertheless, the overall sample's best performing investments are early stage investments. This leads to a wide range and high variation of investment returns. The portfolio manager's ability to select well-performing investments determines the overall early stage-fund performance. The chance of selecting badly performing portfolio companies is high during early stage investing.

INVESTMENT STAGES – Mezzanine vs. Non-Mezzanine

Our final analysis is presented in panels 4 and 5 of table 4.2. We explore performance differences dividing the full sample into mezzanine-financed and non-mezzanine-financed investments. We observe similar results as before. Mezzanine investments are characterized by a lower mean return outperformance (14.37% versus 27.54%) against its benchmark, but a higher outperformance of median returns (22.43% versus 5.88%). The reason is similar to that of preceding explanations. Mezzanine investments are exposed to lower risk than the sample's other investments. Thus, we observe lower variation in returns within a smaller range of outcomes and a lower frequency of total lost investments (only 4%).

5. Effects of naive diversification when portfolio size is increased

In this section we explore the diversification benefits of PE portfolios with increasing size and compare them to those of benchmark portfolios that are composed of public market return equivalents. Owing to our available data with information down to the company level, for the first time it has become possible to construct own portfolios with varying size. We perform a bootstrap simulation of portfolios each including a certain number of investments. After deriving 5,000 bootstrap samples for each portfolio size, we then calculate cross-section variation over the samples' outcomes.

THE RISK – Frequency distribution of portfolio returns

Figure 3 shows changes in standard deviation over the bootstrap portfolios when the portfolio size is increased. Figures 4 to 6 represent the frequency distributions of returns on funds consisting of different numbers of investments. As described in the preceding section, all PE investments exhibit higher performance dispersion and therefore higher absolute risk than their stock benchmark investments. Although there is a stronger diversification of absolute risk for PE portfolios compared to the stock portfolios, in this section, we observe similar relative naive diversification effects. The distribution of portfolio returns approaches normal distribution with increasing size. Since there is a relativly high rate of total lost investments within the private equity asset class, building portfolios decreases the risk of failure. It is already the case with a portfolio consisting of only 5 PE investments that there is a almost zero per cent probability of total loss. The probability of negative absolute returns, however, is not decreasing – but increasing - before more than 5 investments are included in the portfolio (see table 5). This is not caused by the relatively high number of lost or negatively performing, but rather by the small number of extremely well performing investments. Portfolios have to reach a minimum size to increase the probability that at least one high performing investment is included. This is in accordance with the common assumptions made in the context of PE investment. Though we find similar decreases in the standard deviation of PE and stock portfolios with increasing size, table 5 shows some differences when the quartile distribution is analyzed for returns on both asset classes. Whilst we find a similar probability of negative returns (35.22% versus 30.35%), PE is characterized by a high number of investments that perform worse than minus fifty per cent. This is the result of many failing investments with an IRR of minus one hundred per cent. If we increase portfolio size, the number of PE portfolios performing worse than minus 50 per cent does not decrease as fast as the number of stock portfolios exposed to that worse performance.

However, in all cases there is a higher number of negative performing stocks (<0%) than of PE portfolios.

THE (NON-)DIVERSIFIABLE RISK – Standard deviation of portfolio returns

In recent times new approaches to asset pricing, which deviate slightly from the traditional assumptions made by the CAPM, have been published. Malkiel/Xu (2000) or Jones/Rhodes-Kropf (2002) argue that in practice the assumption that investors can hold any combination of the market portfolio and risk free assets is often violated. They indicate that these so called *constrained investors* are unable to hold the market portfolio for reasons such as transaction costs, liquidity constraints or other exogenous factors.[39] With respect to portfolio construction within the PE sector, a significant length of time is required to assess the deal flow and make investment decisions. The PE manager identifies only a small number of investments, which will be included in his certain portfolio. Furthermore, the investments are highly illiquid and transaction costs are excessively high. Jones/Rhodes-Kropf (2002) show that, dependent on the number of portfolio constituents, PE managers are exposed to changing, but real, levels of idiosyncratic risk. Because of their PE investment statute PE managers are *constrained* and unable to hold the market portfolio.[40] Frequently they are subject to investment restrictions which even relate to private investment. Therefore, even if the PE manager increases the portfolio size to an infinite number of private equity portfolio constituents, he still faces limited market non-diversifiable risk, the so-called “PE market” risk. With respect to the combination of asset classes, the “PE market” portfolio that is available to a *constrained* PE manager is less diversified than the market portfolio. In keeping with this assumption, the constrained stock manager holds the “stock-market” portfolio in case of full diversification (with respect to the number of stocks and their industries) occurring within his asset class.[41]

Increasing the overall number of portfolio constituents, we give empirical evidence of naive risk diversification down to the level of stock- or PE market risk.[42] Table 6 shows that the “PE market” risk exceeds the “stock-market” risk. This is in line with the hypothesis of Jones /Rhodes-Kropf (2002). We show empirically that the non-

[39] Malkiel, G.M./Xu,Y. (2000)
[40] For a discussion of investment statutes and restrictions see Feinendegen, S. / Schmidt, D.M. / Wahrenburg, M. (2002) and Schmidt, D.M. / Wahrenburg, M. (2003).
[41] See Malkiel/ Xu (2000), pp.2 ff. for general discussion
[42] See Jones /Rhodes-Kropf (2002), pp. 4

diversifiable risk that a constrained PE investor has to hold exceeds that of stock investors as well as the market risk [this we show in the next section].

Those PE portfolios where investments are realized after 1996 have an especially high non-diversifiable market risk of about 23.78%. In particular, we observe, for example, constrained-market risk levels for PE portfolios realized between 1980 and the end of 96 of about 6.1%, and for portfolios constructed after 1990, with realizations occurring mainly after 1996, of about 20.8%. Moreover, mezzanine portfolios have the lowest non-diversifiable risk (5%).

It is consistent with these results that the diversifiable risk of PE portfolios exceeds that of stock portfolios (in absolute measures). The relative diversifiable risk, however, calculated as the ratio between diversifiable and full risk, shows small variations between the PE and benchmark portfolios. We obtain relative diversifiable risk levels that range between 92.2% and 94.8% of full risk. Nevertheless, it is apparent that portfolios consisting exclusively of mezzanine-financed companies have a lower relatively diversifiable risk than their benchmark portfolios. In contrast, all other private equity portfolios have slightly higher relatively diversifiable risk levels than their benchmark portfolios. For these private equity categories we observe that greater benefits are to be gained from increasing the portfolio size than is the case for stock or mezzanine portfolios.

The empirical research described above exclusively investigates naive diversification effects of stock or real restate portfolios. A large number of papers have tried to answer the question how many investments are needed to obtain a well-diversified portfolio. Exploring the optimal portfolio size of private equity portfolios on an empirical basis is more complicated. Besides the fact that it is necessary to have access to data on single investments and not just on overall fund performances, there are other factors which are important with respect to determining the limits of portfolio size. In contrast to stocks or real estate portfolios, the management of each additional private equity investment is connected with an extraordinarily high degree of effort. Therefore, according to Statman (1987) optimal portfolio size is not only restricted by decreasing marginal risk diversification, but also by the increasing management effort or limited managing capacity involved. In addition to the restrictions by investment statute to investing in only one asset class, transaction costs are another reason why the investment-manager is not able to hold optimally diversified portfolios.[43] Jones and Rhodes-Kropf

[43] See Malkiel/ Xu (2000), pp.2 ff.

(2002) show empirically that this higher level of idiosyncratic risk, which the constrained PE investment manager has to face, is priced.[44] We show that there is also an idiosyncratic risk premium which a constrained PE investment manager has to hold even if he is fully diversified within his asset class. It exceeds the risk that has to be held by a constrained stock manager. The descriptive statistics of section 4, which describe the outperformance of private equity investments relative to benchmark stocks, do not always confirm a proper pricing of the higher idiosyncratic risk levels that PE managers have to hold owing to certain investment restrictions. Especially in the 1980s and the early 1990s the constrained PE investor did not receive any compensation for holding more risk. PE was underperforming the market. In recent years the higher level of idiosyncratic risk has been priced by the market. PE has outperformed stock investments.

THE DIVERSIFIABLE RISK

We find for all PE and stock portfolios at least a 90% diversifiable risk reduction for a portfolio comprising 50 investments. Tables 7 (1. and 2.) gives an overview of risk diversification effects with increasing portfolio size for sample subsets. On the basis of separate analysis with respect to the investment stage, we explore faster diversifiable risk reduction for portfolios consisting of investments from an earlier investment stage. Table 7.2 displays the benefits from diversification with respect to investment stage. It indicates, for example, that for a portfolio size of 50 investments there is a 94.31% diversifiable risk reduction for early stage portfolios or a 92.3% reduction for VC portfolios versus a 91.7% reduction for PE w/t VC portfolios. The benchmark stock portfolios do not exhibit these clear gradations but the process of change is similar. It is likely that the effects vary relative to the exactness of benchmark stock choice with respect to investment size. Portfolios containing “small size” investments seem to diversify risk faster. There is a stronger and faster diversification of company-specific risk when the portfolio size is increased.

Almost complete diversifiable risk reduction is achieved by a portfolio size of 200 investments. For a portfolio size of 200 we observe for both PE and benchmark stock portfolios a diversifiable risk reduction of at least 99.7%. However, if the initial portfolio size is 15 investments further marginal risk reduction is small when the portfolio size is raised. An inclusion of 15 investments reduces diversifiable risk of both PE and

[44] See Jones /Rhodes-Kropf (2002), pp. 2

benchmark portfolios already by about almost 80% (see table 7).[45] Again, for a portfolio size of 15 we find a diversifiable risk reduction of at least somewhere in the region of 78.4%. In the real world, as noted in section 2., we observe an average portfolio size that is between 20 and 28 investments. In this respect portfolio managers would appear to be able to find the balance between risk diversification and portfolio management expenditure.

We observed different diversifiable risk reduction effects by analyzing certain private equity categories. Comparing the level of diversifiable risk reduction with increasing size of PE and benchmark stock portfolios, we also detect differences within each subsample category. To test for differences in diversifiable risk reduction, we orientate our analysis on that of Kuhle (1987) or Kuhle and Moorehead (1989) with respect to methodology. We use the Z-test to determine the statistical difference between the mean value of PE's diversifiable risk reduction level and that of its benchmark. The null hypothesis of "no difference in diversifiable risk reduction level between PE and the benchmark stock portfolios with increasing portfolio size" is rejected for all subsamples at a 1% level of significance. Table 7 presents the respective z-values with $Z = \frac{\overline{X} - 0}{\delta / \sqrt{8}}$ and $\overline{X}$ = mean $\sum_{i=1}^{8} x^i - y^i$ (x = diversifiable risk reduction level of PE portfolios at size-step i; y = diversifiable risk reduction level of benchmark portfolio at size-step i; i=1,2…8 for portfolio size =2,5….200). The results indicate slower diversifiable risk reduction for later stage private equity portfolios (subsamples PE w/t VC and mezzanine with negative z-values) than is observed for their benchmark portfolios. On the contrary, there is a faster reduction of diversifiable risk when portfolios that exclusively contain venture capital investments (subsamples 'VC', 'early stage VC' and 'Full sample w/t Mezzanine' [here overweight of vc investments in sample] with positive z-values) are increased compared to the benchmark portfolios. This is in line with previous results. Portfolios composed of small sized investments are characterized by a fast reduction of diversifiable risk.

In addition to the differences presented here in diversification effects between PE portfolios with different compositions, this analysis clearly shows the difficulties involved in using small cap stocks as a private equity proxy. It is too simple and too inexact to proxy for private equity by using small caps. In the next section we investigate optimal asset allocations for mixed PE and stock portfolios and we pick up a topic that starting

[45] This corresponds to findings of Hellevik and Hermann (1996) with respect to stocks and Miles and McCue

with Merrill Lynch (1995) has always been simplified by using small caps to proxy for PE. Despite the existence of some methodological hurdles, we use original private equity IRR data with correct benchmark IRRs in order to revise former empirical findings.

6. Optimal asset allocation

According to Jones /Rhodes-Kropf constrained investors have to hold an idiosyncratic risk premium because of investment restrictions and limited management capacity. In this section, we try to find optimal portfolio compositions from the view of non-constrained investors and determine unconstrained market risks.[46] Following the methodology described in the appendix, we simulate mixed asset portfolios to determine the optimal portfolio allocation to stocks and private equity. Using data of different samples' subsets we show results on the basis of historical data and increased transparency and we try to give recommendations for future allocations.

By way of illustration, figure 7 presents risk return characteristics of portfolios with a portfolio size of 20 investments and changing private equity allocations. Each line represents the efficient frontier of portfolios built from private equity and stock investments from different sample subsets. With respect to the individual risk/return preferences, investments in different asset categories with different allocations make sense. The unconstrained investor is able to reduce risk below the level of sole "stock-" or "PE market" risk by combining both assets. The asset classes' return and risk differences and their correlations affects the optimal portfolio weighting of private equity. If we include private equity investments taken from the full sample, from the pure VC sample or from the full sample w/t mezzanine we find similar efficient frontier lines. Mixed portfolios built between 1990 and 2002 show the highest returns, portfolios with an investment period between 1980 and 1990 exhibit the worst performance characteristics. This is in keeping with the results of section 4. An allocation to mezzanine investments increases portfolio returns with moderate risk bearing.

THE MINIMUM VARIANCE PORTFOLIO

Determining minimum variance portfolios, table 8 shows according to the PE sample used that the optimal weighting of private equity lies between 3% and 46% of

(1984) with respect to real estate.

[46] Under the Assumption that stocks and private equity would be the complete investment universe.

portfolio investments.[47] We reveal for portfolios that are built with randomly chosen private equity investments out of the overall sample an optimal allocation to private equity of about 15%. Increasing the overall portfolio size from 1 to 200 investments leads, we find, to slight changes in optimal allocation to private equity and an enormous decrease in portfolio risk. Figure 9 shows efficient frontiers and diversification effects when portfolio size is increased. By way of an example, we find that an addition of 5 private equity investments to an existing portfolio that already contains 5 investments would reduce cross-section standard deviation by about 34%. An addition of 5 stocks, however, would reduce the risk level by about 68%, and an optimal allocation to PE and stocks even would reduce risk by about 78%.

The optimal portfolio weighting of private equity in order to achieve minimum variance compositions is mainly determined by the relation of private equity and stock portfolio standard deviations and their correlation. Although there is a relatively high correlation of 0.25 between PE w/t VC and the benchmark stocks, we find that the highest allocation of about 46% to that category gives rise to the minimum variance portfolio. This is caused by the relatively small return volatility of later staged private equity. If we investigate private equity samples consisting of other investments - those made between the years 1980 until 1990, realized up until 1996, with mezzanine-financed investments, or samples of funds that are realized up to 70 per cent - we find that it is optimal to allocate between 28% and 43% to private equity in order to minimize volatility. Due to the extremely high return volatility of pure VC or early stage investments compared to their benchmarks, no more than a small allocation (down to 3%) to those investments should be made.

In all cases it is advantageous to include private equity in mixed portfolios. As shown in figure 8, the mixed asset risk is smaller than the constrained market risks of stocks or PE. The differences between the mixed asset market risk (as shown in table 8 last column) and the constrained market risks where the investor is restricted to investing in stocks or PE (as shown in table 6 each second line), is that part of idiosyncratic risk that the investor has to hold because of his investment restriction. The constrained investor has to hold a higher share of idiosyncratic risk.

THE MAXIMUM RETURN/RISK-RATIO-PORTFOLIO

[47] Due to equally weighted amounts of invested capital in every single company it is inconsequential whether portfolio weights are measured as a relative number of overall portfolio investments or, using weightings, by the relative proportion of overall invested capital.

To determine optimal portfolio compositions, we measure further risk return characteristics by calculating a performance ratio. With respect to the methodology we follow Kuhle (1987) and interpret the portfolio quality according to the ratio of average return and cross section standard deviation over the portfolio's investments. This performance ratio is similar to the well-known Sharpe ratio. Utilizing this measure we determine different optimal portfolio weights ranging between 5% and 65% (table 9). Under the premise of maximizing performance ratios, the relation of overreturn to overvolatility (compared to its benchmark stock portfolios) is that factor which determines the optimal PE/stock proportions in the portfolio. Apart from two subsets, in our sample all private equity portfolios compared to their respective benchmark show a level of excess return which is relatively higher than that of excess volatility. As a result, portfolio compositions which maximize performance ratios call for high PE allocations. Table 9 presents optimal portfolio weightings of private equity for maximizing performance ratios. All PE portfolios, except those formed with investments from the subset “Vintage year 1980-90”, are composed optimally with weightings between 27% and 65%. Portfolio weightings are higher than those which would have been optimal for minimizing investment return volatility. By way of an example, if we analyze *the complete sample,* figure 10 shows the course of changing performance ratios when PE weightings are increased. The asset compositions of minimum variance portfolios do not correspond to those with maximum performance ratios. The peak with a maximum performance ratio is reached by allocating about 35% to private equity. In particular, starting without any allocation to PE the performance ratio is increased by 68% when we allocate 35% to private equity. Figure 11 shows these changes of performance ratios on a log scale. Furthermore, the figure illustrates that it is preferable to invest exclusively in private equity – which is diversified within its asset class- than exclusively in benchmark stocks.

If portfolios are evaluated on the basis of performance ratios, a portfolio allocation to mezzanine investments produces the best results (performance ratio between 35% and 486% according to portfolio size, table 9.2). On the grounds of the excellent return-risk ratios expressed by the empirical data, this analysis recommends that mezzanine portfolio weightings be increased up to 65%.

PE investments made between 1980 and 1990, or realized between 1980 and 1996, exhibit worse return risk characteristics. They are characterized by smaller returns and higher risks than their benchmark stocks. With respect to maximum performance ratios, our portfolio simulation suggests smaller optimal PE weightings than were recommended in order to obtain minimum variance portfolios.

THE PORTFOLIO SIZE AND THE OPTIMAL PORTFOLIO ALLOCATION

Mainly with respect to 'minimum variance portfolio' but also to 'maximum performance ratio portfolio' measures, we find that the optimal PE portfolio weightings change with increasing portfolio size (table 8 and 9 lines "weights"). Owing to slightly different relative levels of maximum diversifiable risk between PE and stocks (table 6, each 3. line), there are different levels of PE and stock volatility with progressive portfolio size increases (table 7). When constructing mixed asset portfolios, the changing extent of risk diversification of PE compared to stock portfolios with increasing size influences the optimal allocation to private equity. With increasing portfolio size the optimal weight of private equity rises if PE portfolios expel a higher relative share of diversifiable risk. This is illustrated in table 6, panel "Only VC", which presents a relation between the full risk of VC and stocks of about 349.66 %. When the portfolio size of both portfolios is increased to 200 investments we find a proportion between the PE and stock portfolios' non-diversifiable risk of about 287% (table 6, column 6). We find a higher relative ratio of diversifiable to full risk for pure VC portfolios than for the benchmark stock portfolios. Therefore, both diversifiable risk and absolute risk diversification effects are higher with increasing portfolio size. Owing to these larger risk diversification benefits of certain PE portfolios with increasing size, it is preferable to increase the allocation to PE with increasing portfolio size. In tables 8.2 and 9.2 the panel "only VC" shows that the optimal portfolio weighting of VC increases from 6% to 11% and from 27% to 34%, respectively, with increasing mixed portfolio size - between 1 and 200 investments. On the other hand, we find a tendency for decreasing PE optimal weightings in mixed portfolios, if there is a smaller diversifiable risk level for the PE subset compared to its benchmark (e.g. subset "mezzanine", table 8.2). These findings correspond exactly to those of Brown (1997) and Brown and Schuck (1996). On the one hand, they find relatively higher non-systematic risk levels of real estate compared to stock portfolios. On the other hand, when the size of mixed stock and real estate portfolios is increased they establish higher allocations to real estate with respect to optimizing risk/return characteristics.

7. Extensions

a. Net performances

As mentioned above, when determining portfolio characteristics we do not take into account any management fee. The difference between fixed annual fees concerning the management of private equity or stock portfolios is not so essential that it would cause the results to be biased to any significant extent. The carried interest payments, however, will change the performance levels of private equity portfolios. To test for variation in returns, we calculate net IRRs for the full sample. This calculation suffers from two major problems: First, the level of carried interest payments is not completely standardized.[48] Furthermore the calculation method is highly varying between the fund managers (e.g. deal by deal or carry payments after full funds capital repayment). Second, there are difficulties in simulating carried interest payments, which in reality are charged on the overall fund's pooled cash flow series. In this analysis, we calculate net IRRs utilization derived on the basis of deal-by-deal carry calculation. We determine net IRR distributions at the company level by subtracting a 20% carried interest on the capital gains of each single company after 100% distribution of capital which was invested in this company. Usually, the carried interest is calculated on the fund's cumulated companies' cash flows. Deal by deal carry payments are only seldomly negotiated, if the fund manager's bargaining power is extremely high. Hence, our results are absolutely worst case scenarios in the sense of investors' return. In reality, negative performing investments with long lasting negative cash flows delay the overall funds' distributions and therefore the need to pay carried interest. On the basis of a portfolio simulation including individual i.i.d. distributed company returns we cannot take this fact into account. We calculate carried interest payments considering the individual cash flow time series of each portfolio company separately. Thus, we calculate portfolio descriptive statistics using each single investment's net IRR. Utilizing this deal-by-deal carry calculation, we neglect the fact that negative cash flows of investments with negative IRRs are reducing the fund's overall capital gains and therefore the amount of carried interest payments on fund basis.

Because of these difficulties while measuring net real IRRs, we only present these as an extension in table 10. We preferred to make former calculations on the basis of gross cash flows instead of taking into account distortions as a result of wrongly calculated carried interest charges. In any event, table 10 presents net IRRs of the sample "vintage year 1980-2002". Average IRRs decrease by around 30% , median IRRs only by around 16%. The greater decrease of mean IRRs is caused by a larger absolute cut of extremely high capital gains and the problems discussed before. Performance characteristics will not change hardly since the standard deviation also decreases by around 18%.

Since it is less the intention of this paper to give a precise account of previous returns than it is to study the effects of a combination of asset classes, we base our analysis on measures of real gross returns.To test for robustness we repeat calculations using the PE net returns. Table 11 shows optimal weightings of private equity with respect to minimizing the portfolio variance. Because of the smaller deviations of private equity portfolio returns, when performing calculations on the basis of net performances, it is recommended that a slightly increased share be allocated to private equity (from 16% to 20%). Portfolio volatility and the returns of minimal variance portfolios correspond exactly to those derived on the basis of gross performance parameters. Moreover, the market risk of fully diversified portfolios - diversified that is in terms of the number of portfolio constituents, not the number of asset classes - corresponds to that which was calculated previously. In revealing maximum performance ratios we find economically similar values and the optimal private equity portfolio weightings with respect to maximizing performance ratios also do not differ from former values.

The fact that similar results are obtained when using net (simulated) or gross performance parameters furthermore justifies the use of gross values in order to determine the characteristics and dynamics of private equity and mixed asset portfolios.

b. Alternative performance measurements

Although the internal rate of return represents the standard performance measure for private equity investments and is even recommended by the EVCA, it is associated with some shortcomings with respect to evaluating the investment return. On the whole, when calculating the IRR the reinvestment rate of distributed capital and the investment rate of unbounded capital are assumed to correspond to the internal rate of return. Return parameters tend to adopt extreme values. On the one hand, extremely well performing investments expel very high IRRs owing to a reinvestment assumption with high project rates. On the other hand, an implicated low reinvestment rate decreases overall IRR of badly performing investments. To test robustness we investigate private equity and benchmark stock returns following Kallberg, Lui and Greig (1996) by calculating the

[48] See Gompers/Lerner (1996) for the US and Schmidt/Wahrenburg (2003) for Europe

modified IRR (MIRR) with an assumed unbounded capital market-reinvestment rate of 5%. Table 12 presents MIRR descriptive statistics. We obtain more moderate average return measures of about 14.37% and 4.26% for private equity and stocks, respectively. Due to an on average lower reinvestment rate, maximum returns decrease to 1409% and 21.7% for private equity and benchmark stocks, respectively. Besides lower return parameters, we indicate similar relative outperformances for private equity against its benchmark. Relative outperformance of about 237% based on MIRR calculation is even slightly higher than that based on common IRR calculation (214%). Due to these similar ratios we do not expect any strong variations in the results of the analysis carried out in section 6 and 7 where calculations were performed with the IRR as the return measure. However, whilst indicating these differences in performance parameters describing the same investments, we are also aware of the importance of comparing benchmark returns on the basis of the same return measurement.

8. Summary

In this paper we investigate risk and return aspects of private equity investments. Our analysis is based on a representative sample of 642 US-American private equity portfolio companies with exact cash flow information. Information about the amount and date of all cash flows to and from the PE investments enables a cash flow equivalent and simultaneous investment in benchmark stocks. Thus, for every PE investment we observe an exact benchmark performance. By applying bootstrap simulations we observe risk-return characteristics of portfolios with changing constituents.

In analyzing the different subsets of private equity, we find higher performance variation within earlier stage categories. The pure venture capital sample is characterized not only by extremely well performing investments, but also by a high rate of lost investments. In addition to a higher average performance of venture capital portfolios, managing these portfolios requires skilful investment selection. High portfolio performance depends on the ability to pick high performing outliers. Later stage private equity, like MBO/LBOs or mezzanine investments, are distinguished by smaller performance variations and lost rates. With respect to these portfolios, we observe smaller average but higher median returns. It is easier to select investments with moderate performance.

Time series analysis shows that it was only in the late 1990s that the overall private equity market was performing extremely well in terms of mean IRR. In the late 20th century we find a dramatic increase in the extent to which private equity outperforms stock investment. In earlier years private equity was underperforming its stock benchmarks. Future studies have to determine whether this development is the result of a learning process or of improved market mechanisms such as the establishment of better exit markets or the emergence of advisors, or simply due to a temporary bubble. Findings on these topics will be an important factor in determining the future development of the whole PE industry. If the overall industry does not function well, the individual skills of the PE investment manager will play an increasingly decisive role. A market clearance of low quality PE managers is with respect to the large capital in the market, which has to be allocated to pe, not a realistic assumption, but would help to save the industry's reputation!

In a special scenario we assume that a PE or a stock portfolio manager is *constrained* by his statute from holding all security classes. Constrained portfolios consisting of at least 200 investments have a bearing on “PE“ or “stock-market” risk. We show empirically that the non-diversifiable risk which a constrained PE investor has to hold exceeds that of stock investors by between 6% and 517%. Following the new approaches of Jones/Rhodes-Kropf (2003), we interpret this as some measure of the idiosyncratic risk premium of private equity portfolios.

For PE and stock portfolios almost 80% of diversifiable risk is reduced when the portfolio size is increased to 15 investments. In fact, we and other authors observe the real world average PE portfolio size to be somewhere between 20 and 28 investments. With respect to portfolio size, portfolio managers seem to be able to find the balance between risk diversification and portfolio management expenditure.

In a second scenario we reveal optimal portfolio compositions from the view of *unconstrained* investors by simulating mixed-asset portfolios. With respect to both the ‘minimum variance’ and ‘maximum performance ratio’ measures a mixed asset portfolio allocation to private equity proves to be advantageous. The *unconstrained* investor is able to reduce risk below the level of sole “stock-“or “PE market” risk by combining both asset classes. With reference to the private equity sample used, we establish via bootstrap simulation optimal mixed-asset portfolio weightings of private equity to be between 3% and 65%. With a recommended portfolio weighting of 65%, mezzanine investments are best suited to optimizing mixed asset portfolios with respect to the received performance ratio.

In two scenarios the results reveal the necessity of choosing suitable investment categories, selecting well-performing investments, and finding the right proportions in mixed asset portfolios.

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Tables

Table 1 – Origin

This table shows the origin of complete samples' investment manager and portfolio companies [in number of investment manager or companies]. The investment managers are located exclusively in Europe and the US.

	Investment Manager	Company
North America	18	1694
South America		16
Europe	19	1339
Asia		101
No statement		470

Table 2 – Performance comparison of complete and reduced samples

The mean performance comparison between the complete and the reduced sample does not show any significant deviations. The reduced sample includes all US-American fully-realized investments [in percent of the whole sample].

	Complete Sample [only realized investments]	*Reduced Sample*
mean IRR	36.71%	36.49%
Variance	7.26	5.83
sample size	1539	642
degrees of freedom	1330	
t-value	0.018607	

Table 3 – Sample description

This table presents the sample description of both the complete sample consisting of 3619 world-wide private equity investments and the reduced sample consisting of 642 US-American investments

Sample Description							
Industry	Technology	Health Care	Consumer Discretionary & Services	Materials & Processing	Other Energy	Financial Services	Others
Reduced sample	40.44%	14.73%	9.87%	12.23%	2.19%	2.04%	18.50%
Complete sample	39.55%	15.9%	8.67%	11.08%	1.93%	2.09%	20.78%
	Private Equity						
	venture Capital						
Investment stage	Early	Expansion	Later	Recapitalization	MBO/MBI/LBO	Acquisition Financing	
Reduced sample	39.01%	20.25%	17.28%	6.17%	11.85%	5.43%	
Complete sample	38.77%	16.10%	13.15%	3.71%	24.20%	4.07%	
Number of financing rounds [reduced sample]					2114		
Proportion of mezzanine-financed investments [reduced. sample]					19.50%		
Volume of invested capital [reduced sample]					$4,101,649,064		

Table 4.1 – Descriptive Statistics [time periods]

This table shows the descriptive IRR statistics of the samples' private equity (PE) and the stock benchmark investment returns according to their investment date. Each panel describes the return characteristics for the individual sub-sample. Each private equity investment return is allocated to one completely comparable stock investment return.

Sample	Vintage Years 1980-02 (Full sample)		Vintage Years 1980-90		Vintage Years 1990-02		Realized before the end of 96		Realized after 96		Investments taken from funds, which are up to 70% realized		Investments taken from fully-realized funds	
	PE	*Stocks*	*PE*	*Stocks*	*PE*	*Stocks*	*PE*	*Stocks*	*PE*	*Stock*	*PE*	*Stocks*	*Pe*	*Stocks*
Mean IRR	36,49%	11,59%	0,03%	3,39%	57,82%	16,39%	7,89%	9,80%	59,05%	12,90%	6,11%	5,47%	2,82%	6,38%
Median IRR	14,55%	2,74%	8,77%	1,90%	22,18%	3,41%	17,60%	3,84%	12,94%	1,71%	13,14%	2,81%	13,40%	0,39%
Modus	-1	-1	-1	-1	-1	0	-1	-1	-1	0	-1	-1	-1	-1
St. Deviation	2,42	1,03	0,85	0,59	2,95	1,21	0,87	0,71	3,11	1,22	0,78	0,55	0,84	0,72
Variance	5,83	1,05	0,72	0,35	8,71	1,46	0,75	0,51	9,68	1,48	0,61	0,31	0,71	0,52
Kurtosis	74,20	176,85	15,83	98,79	50,28	141,19	11,48	48,82	45,36	155,15	13,60	80,80	22,88	65,92
Skewness	7,55	11,42	2,61	8,27	6,34	10,46	2,06	5,83	6,06	11,31	2,18	6,89	3,36	7,10
Range	3062%	1943%	748%	831%	3062%	1943%	748%	831%	3061,53%	1942,59%	748%	831%	748,13%	830,85%
Minimum IRR	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%
Maximum IRR	2961,53%	1842,59%	648,13%	730,85%	2961,53%	1842,59%	648,13%	730,85%	2961,53%	1842,59%	648,13%	730,85%	648,13%	730,85%
Frequency of total lost	19,16%	0,78%	21,10%	0,84%	18,02%	0,007%	20,28%	0,01%	18,28%	0,55%	15,90%	0,007%	17,07%	0,61%
Sample size	642	642	237	237	405	405	280	280	361	361	396	396	164	164
Correlation	-0,011		-0,0446008		-0,01609108		-0,04739192		-0,00840265		-0,00855593		-0,11878351	

Table 4.2 – Descriptive Statistics [investment stage]

This table shows the descriptive IRR statistics of the samples' private equity (PE) and the stock benchmark investment returns according to their investment stage. Each panel describes the return characteristics for the individual sub-sample. Each private equity investment return is allocated to one completely comparable stock investment return.

Sample	Private Equity w/t VC		Venture Capital only		Early Stage investments		Mezzanine		Full sample w/t Mezzanine	
	PE	*Stocks*	*PE*	*Stocks*	*PE*	*Stocks*	*PE*	*Stocks*	*PE*	*Stocks*
Mean IRR	7,62%	6,90%	47,81%	10,69%	22,57%	1,18%	22,47%	8,10%	40,03%	12,49%
Median IRR	22,56%	2,73%	8,70%	1,96%	-27,83%	0,63%	25,16%	2,73%	8,77%	2,89%
Modus	-0,99	#NV	-1	0	-1	-1	-1	#NV	-1	0
St. Deviation	0,51	0,42	3,13	0,92	3,14	0,49	0,72	0,54	2,67	1,11
Variance	0,26	0,17	9,79	0,84	9,86	0,24	0,51	0,29	7,13	1,24
Kurtosis	0,85	4,49	51,12	59,98	53,58	11,93	7,54	15,85	60,97	158,06
skewness	-0,23	1,28	6,52	6,80	6,67	2,19	1,69	2,62	6,90	11,03
Range	249%	276%	3062%	1091%	3062%	390%	458%	468%	3062%	1943%
Minimum IRR	-100,00%	-89,96%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%	-100,00%
Maximum IRR	148,78%	185,55%	2961,53%	991,27%	2961,53%	290,09%	357,75%	367,70%	2961,53%	1842,59%
Frequency of total lost	0,042%	0%	25,48%	0,012%	38,61	1,9%	4%	0,8%	22,24%	0,58%
Sample size	71	71	310	310	158	158	125	125	517	517
Correlation	0,25541717		0,00630298		-0,00651703		0,048		-0,0128108	
percentage realized until 1996	28,16%		60,60%		53,06%					
percentage invested since 1996	28,16%		24,74%		25,85%					
percentage realized since 1996	71,83%		73,53%		70,88%					

Table 5 – Deciles performance frequency distribution

Frequency distribution of the bootstrap portfolios' returns (bootstrap sample = 5000). Bootstrap-Portfolios represented here have different sizes N=1,5,20,200. Stock portfolios correspond to the private equity portfolios with respect to both the characteristics of included stocks like industry and capitalization and to the timing of benchmark investment simulation. The panel IRR Class represents the deciles of the simulated portfolio return.

	N=1				N=5				N=20				N=200			
	Private Equity		Stocks		Private Equity		Stocks		Private Equity		Stocks		Private Equity		Stocks	
IRR class	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequency*	*Cumulated%*	*Frequ.*	*Cumulated%*
-100%	922	18,44%	32	,64%	2	,04%	0	,00%	0	,00%	0	,00%	0	,00%	0	,00%
-90%	129	21,02%	39	1,42%	2	,08%	0	,00%	0	,00%	0	,00%	0	,00%	0	,00%
-80%	41	21,84%	50	2,42%	8	,24%	0	,00%	0	,00%	0	,00%	0	,00%	0	,00%
-70%	77	23,38%	99	4,40%	38	1,00%	0	,00%	0	,00%	0	,00%	0	,00%	0	,00%
-60%	72	24,82%	122	6,84%	42	1,84%	0	,00%	0	,00%	0	,00%	0	,00%	0	,00%
-50%	33	25,48%	92	8,68%	128	4,40%	7	,14%	1	,02%	0	,00%	0	,00%	0	,00%
-40%	86	27,20%	105	10,78%	189	8,18%	22	,58%	5	,12%	0	,00%	0	,00%	0	,00%
-30%	71	28,62%	217	15,13%	280	13,79%	94	2,46%	39	,90%	1	,02%	0	,00%	0	,00%
-20%	119	31,00%	313	21,39%	422	22,23%	267	7,80%	208	5,06%	11	,24%	0	,00%	0	,00%
-10%	211	35,22%	448	30,35%	461	31,45%	645	20,71%	370	12,46%	233	4,90%	0	,00%	0	,00%
0%	236	39,94%	693	44,22%	498	41,42%	1079	42,30%	583	24,13%	1206	29,02%	19	,38%	106	2,12%
10%	296	45,86%	878	61,78%	507	51,56%	1175	65,81%	719	38,52%	1644	61,90%	203	4,44%	2269	47,52%
20%	397	53,80%	618	74,15%	464	60,84%	749	80,79%	567	49,86%	863	79,16%	639	17,23%	2001	87,56%
30%	633	66,46%	374	81,63%	336	67,57%	379	88,38%	493	59,72%	360	86,36%	1097	39,18%	550	98,56%
40%	414	74,74%	300	87,64%	307	73,71%	161	91,60%	354	66,81%	253	91,42%	1159	62,36%	65	99,86%
50%	294	80,62%	210	91,84%	214	77,99%	94	93,48%	249	71,79%	129	94,00%	894	80,25%	7	100,00%
60%	188	84,38%	79	93,42%	163	81,25%	64	94,76%	206	75,91%	95	95,90%	579	91,84%	0	100,00%
70%	77	85,92%	78	94,98%	118	83,61%	42	95,60%	194	79,79%	32	96,54%	247	96,78%	0	100,00%
80%	77	87,46%	37	95,72%	71	85,03%	28	96,16%	171	83,21%	26	97,06%	103	98,84%	0	100,00%
90%	58	88,62%	13	95,98%	66	86,35%	8	96,32%	164	86,49%	32	97,70%	49	99,82%	0	100,00%
100%	67	89,96%	40	96,78%	49	87,33%	8	96,48%	113	88,76%	53	98,76%	6	99,94%	0	100,00%
110%	26	90,48%	18	97,14%	41	88,16%	16	96,80%	86	90,48%	33	99,42%	3	100,00%	0	100,00%
120%	12	90,72%	9	97,32%	29	88,74%	18	97,16%	54	91,56%	9	99,60%	0	100,00%	0	100,00%
130%	66	92,04%	19	97,70%	28	89,30%	9	97,34%	48	92,52%	7	99,74%	0	100,00%	0	100,00%
140%	29	92,62%	17	98,04%	30	89,90%	17	97,68%	58	93,68%	5	99,84%	0	100,00%	0	100,00%
150%	41	93,44%	11	98,26%	30	90,50%	10	97,88%	55	94,78%	4	99,92%	0	100,00%	0	100,00%
160%	27	93,98%	0	98,26%	28	91,06%	12	98,12%	69	96,16%	1	99,94%	0	100,00%	0	100,00%
170%	0	93,98%	0	98,26%	37	91,80%	6	98,24%	46	97,08%	1	99,96%	0	100,00%	0	100,00%
180%	24	94,46%	0	98,26%	26	92,32%	1	98,26%	38	97,84%	1	99,98%	0	100,00%	0	100,00%
190%	8	94,62%	7	98,40%	16	92,64%	3	98,32%	23	98,30%	0	99,98%	0	100,00%	0	100,00%
200%	10	94,82%	0	98,40%	15	92,94%	13	98,58%	18	98,66%	0	99,98%	0	100,00%	0	100,00%
over 200%	259	100,00%	80	100,00%	353	100,00%	71	100,00%	67	100,00%	1	100,00%	0	100,00%	0	100,00%

Table 6 – Comparison of "total-", "private equity-" and "stock market" risks

The total risk is measured as the standard deviation of the private equity or the stock investment returns if no portfolios are built. The constrained market risk represents the standard deviation of in terms of the number of included investments (at least 200) fully-diversified portfolios, where no further risk diversification is possible by including more investments to the portfolio. In this scenario we compare pure private equity and pure stock portfolios. The analysis is done for all sub-samples. The relative diversifiable risk represents diversifiable risk as a proportion of total risk.

	Private Equity	**Stock**	**Ratio**	**Private Equity**	**Stock**	**Ratio**
Sample	*Vintage Years 1980-02*			*Private Equity without VC*		
Total risk	260%	98%	264%	51%	42%	120%
Constrained market risks	17,1%	7,1%	232%	29,7%	28%	106%
rel. diversifiable risk [as portion of overall risk]	93,40%	92,50%		94,20%	93,38%	
Sample	*Vintage Years 1980-90*			*Venture Capital only*		
Total risk	87%	54%	160%	330%	94%	349,66%
Constrained market risks	5,8%	4,1%	139%	6,7%	19,3%	287%
rel. diversifiable [as portion of overall risk]	93,33%	92,30%		94,15%	92,89%	
Sample	*Vintage Years 1990-02*			*Early Stage investments*		
Total risk	298%	121%	245%	328%	50%	651,68%
Constrained market risks	20,8%	8,4%	246%	20,8%	3,3%	617%
rel. diversifiable risk [as portion of overall risk]	93,01%	93,05%		93,65%	93,29%	
Sample	*Realized before the end of 96*			*Mezzanine*		
Total risk	88%	75%	116,6%	71%	56%	125,9%
Constrained market risks	6,1%	4,9%	124%	5%	3,7%	134%
rel. diversifiable risk [as portion of overall risk]	93,04%	93,45%		93,29%	93,37%	
Sample	*Realized after 96*			*Private Equity without Mezzanine*		
Total risk	321,7%	111,8%	287,/%	291%	117%	248%
Constrained market risks	23,78%	8,75%	271,7%	18,6%	7,8%	237%
rel. diversifiable [as portion of overall risk]	92,6%	92,17%		93,56%	93,36%	
Sample	*Investments taken from funds, which are up to 70% realized*			*Investments taken from fully-realized Funds*		
Total risk	76%	56%	136%	82,4%	70,7%	116%
Constrained market risks	5,2%	3,9%	133,97%	6%	5,2%	115%
rel. diversifiable risk [as portion of overall risk]	93,15%	92,95%		92,71%	92,64%	

Table 7.1 – Relative diversifiable risk reduction

This table presents the results for the bootstrap simulation of private equity and benchmark portfolios with increasing size. The table presents the reduction of diversifiable risk. When we include 200 investments to the pure private equity (line 1) or the stock portfolio (line 2), all diversifiable risk is excluded. Z-values are given with $Z = \frac{\overline{X} - 0}{\delta / \sqrt{8}}$ and $\overline{X}$ =mean $\sum_{i=1}^{8} x^i - y^i$ (x= non-systematic risk reduction level of PE portfolios at size-step i; y = non-systematic risk reduction level of benchmark portfolio at size-step i; i=1,2…8 for portfolio size =2,5….200) for testing the null hypothesis "no difference in non-systematic risk reduction level between PE and the benchmark stock portfolios with increasing portfolio size". The differences in the ratio of systematic risk and the ratio of full risk shows different diversifiable risk reduction benefits of private equity and their benchmark portfolios. With respect to changing investment horizons, we observed similar results for all subsets.

Portfolio Size [number of investments]	2	5	10	15	20	50	100	200
Sample	*Vintage Years 1980-02*							
diversifiable risk reduction w.r.t. portfolio size	40,42%	62,58%	76,69%	81,51%	85,37%	92,85%	96,98%	100,0%
diversifiable risk reduction w.r.t. portfolio size [benchmark]	40,19%	57,08%	72,08%	80,51%	83,37%	92,42%	96,81%	100,0%
Z-value (difference for diversifiable risk reduction with increasing portfolio size)								6,48

Table 7.2 – Relative diversifiable risk reduction [investment stage]

This table presents the results for the bootstrap simulation of private equity and benchmark portfolios with increasing size. The table presents the reduction of diversifiable risk. When we include 200 investments to the pure private equity (line 1) or the stock portfolio (line 2) all diversifiable risk is excluded. Z-values are given with $Z = \frac{\overline{X} - 0}{\delta / \sqrt{8}}$ and $\overline{X}$ =mean $\sum_{i=1}^{8} x^i - y^i$ (x= non-systematic risk reduction level of PE portfolios at size-step i; y = non-systematic risk reduction level of benchmark portfolio at size-step i; i=1,2…8 for portfolio size =2,5….200) for testing the null hypothesis "no difference in non-systematic risk reduction level between PE and the benchmark stock portfolios with increasing portfolio size". The differences in the ratio of systematic risk and the ratio of full risk shows different diversifiable risk reduction benefits of private equity and their benchmark portfolios. With respect to changing investment horizons, we observed similar results for all subsets.

Portfolio size [number of investments]	2	5	10	15	20	50	100	200
Sample	*Private Equity without VC*							
diversifiable risk reduction w.r.t. portfolio size	32,46%	58,64%	72,44%	78,39%	82,02%	91,71%	96,37%	99,92%
diversifiable risk reduction w.r.t. portfolio size [benchmark]	34,06%	62,06%	74,51%	80,30%	84,51%	92,77%	96,74%	99,95%
Z-value (difference for non-systematic risk reduction with increasing portfolio size)								-10,987
Sample	*Venture Capital Only*							
diversifiable risk reduction w.r.t portfolio size	31,16%	61,07%	74,43%	80,02%	83,47%	92,31%	96,67%	99,94%
diversifiable risk reduction w.r.t. portfolio size [benchmark]	29,21%	60,15%	73,58%	79,43%	83,56%	92,17%	96,69%	99,95%
Z-value (difference non-systematic risk reduction with increasing portfolio size)								6,225
Sample	*Early Stage investments*							
diversifiable risk reduction w.r.t. portfolio size	30,86%	59,27%	73,19%	80,13%	83,17%	94,31%	96,74%	99,96%
diversifiable risk reduction w.r.t. portfolio size [benchmark]	27,66%	58,96%	72,32%	78,91%	82,87%	94,59%	96,63%	99,93%
Z-value (difference non-systematic risk reduction with increasing portfolio size)								5,174
Sample	*Mezzanine*							
diversifiable risk reduction w.r.t. portfolio size	31,72%	59,69%	73,92%	80,03%	83,60%	92,58%	96,86%	100,00%
diversifiable risk reduction w.r.t. portfolio size [benchmark]	35,07%	62,87%	74,26%	80,55%	84,40%	92,54%	97,02%	99,86%
Z-value (difference non-systematic risk reduction with increasing portfolio size)								-5,69
Sample	*Private Equity without Mezzanine*							
diversifiable risk reduction w.r.t portfolio size	38,39%	62,18%	76,86%	81,52%	85,53%	92,76%	97,07%	100,0%
diversifiable risk reduction w.r.t portfolio size [benchmark]	26,24%	63,18%	74,94%	81,09%	85,27%	92,81%	97,04%	100,0%
Z-value (difference non-systematic risk reduction with increasing portfolio size)								3,17

Table 8.1 – Mixed asset portfolios: minimum variance portfolios [sub-samples according to time period]

This table presents the results for the bootstrap simulation of mixed asset portfolios combining private equity and stocks. Bootstrap samples = 5000. N=mixed asset portfolio size. Portfolios are built in order to minimize portfolio return cross section variance. According to the private equity sample used, this table represents the optimal portfolio allocation to private equity in order to obtain minimum variance portfolios. Min. standard deviation is decreasing with increasing portfolio size. Market risk is reached when 200 investments in optimal portfolio weightings are included. The level of the mixed asset market risk is below the level of "PE-" or "stock-market risk". Correlation is measured between the return of each single private equity bootstrap portfolio (out of 5000) and its benchmark portfolios. With respect to investment timing and company characteristics, each single investment used to build benchmark portfolios corresponds exactly to one private equity investment.

N	1	5	10	20	50	100	200 ***(approximated market risk***)
Sample	Overall sample [Vintage Years 1980-02]						
Private Equity Weighting	12,00%	16,00%	17,00%	16,00%	15,00%	15,00%	16,00%
Min. St. Dev.	92,31%	42,51%	29,81%	20,79%	13,13%	9,48%	***6,66%***
Return	14,29%	15,46%	15,74%	15,53%	14,97%	15,29%	15,37%
Correlation	-0,011						
Sample	*Vintage Years 1980-90*						
Private Equity Weighting	28,00%	32,00%	33,00%	34,00%	34,00%	34,00%	34,00%
Min. St. Dev.	45,61%	21,31%	15,12%	10,68%	6,70%	4,64%	***3,37%***
Return	2,11%	2,66%	2,19%	2,23%	2,14%	2,20%	2,24%
Correlation	-0,045						
Sample	*Vintage Years 1990-02*						
Private Equity Weighting	15,00%	15,00%	15,00%	15,00%	14,00%	15,00%	14,00%
Min. St. Dev.	112,03%	48,37%	34,65%	24,95%	15,76%	11,28%	***7,86%***
Return	23,59%	21,75%	22,77%	22,57%	22,17%	22,67%	22,10%
Correlation	-0,016						
Sample	*Realized before the end of 96*						
Private Equity Weighting	43,00%	40,00%	41,00%	40,00%	40,00%	40,00%	39,00%
Min. St. Dev.	55,42%	24,31%	16,39%	11,74%	7,56%	5,42%	***3,76%***
Return	9,18%	8,95%	8,91%	8,94%	9,01%	8,94%	8,90%
Correlation	-0,047						
Sample	*Realized after 1996*						
Private Equity Weighting	11,00%	10,00%	11,00%	11,00%	11,00%	11,00%	11,00%
Min. St. Dev.	105,50%	45,78%	32,68%	24,01%	15,55%	11,48%	***8,34%***
Return	18,51%	17,13%	17,60%	17,90%	17,79%	17,78%	17,76%
Correlation	-0,0045						
Sample	*Investments taken from funds which are up to 70% realized*						
Private Equity Weighting	35,00%	33,00%	33,00%	33,00%	36,00%	34,00%	34,00%
Min. St. Dev.	44,41%	19,54%	13,81%	10,16%	6,37%	4,54%	***3,19%***
Return	5,98%	5,27%	5,33%	5,70%	5,68%	5,71%	5,65%
Correlation	-0,009						
Sample	*Investments taken from fully-realized funds*						
Private Equity Weighting	41,00%	44,00%	44,00%	44,00%	44,00%	44,00%	44,00%
Min. St. Dev.	51,05%	22,75%	16,36%	11,42%	7,28%	5,22%	***3,65***%
Return	4,95%	4,89%	4,70%	4,90%	4,73%	4,77%	4,81%
Correlation	-0,096						

Table 8.2 - Mixed asset portfolios: minimum variance portfolios [sub-samples according to investment stage]

This table presents the results for the bootstrap simulation of mixed asset portfolios combining private equity and stocks. Bootstrap samples = 5000. N=mixed asset portfolio size. Portfolios are built in order to minimize portfolio return cross section variance. According to the private equity sample used, this table represents the optimal portfolio allocation to private equity in order to obtain minimum variance portfolios. Min. standard deviation is decreasing with increasing portfolio size. Market risk is reached when 200 investments in optimal portfolio weights are included. The level of the mixed asset market risk is below the level of "PE" or "stock-market risk". Correlation is measured between the return of each single private equity bootstrap portfolio (out of 5000) and its benchmark portfolios. With respect to investment timing and company characteristics, each single investment used to build benchmark portfolios corresponds exactly to one private equity investment.

N	1	5	10	20	50	100	***200*** ***(approximated market risk)***
Sample	*Private Equity without VC*						
Private Equity Weighting	37,00%	37,00%	35,00%	35,00%	38,00%	39,00%	46,00%
Min. St. Dev.	36,95%	15,66%	11,28%	7,77%	4,78%	3,38%	***2,32%***
Return	7,51%	6,82%	6,96%	6,87%	6,95%	6,97%	7,06%
Correlation	0,255						
Sample	*Venture Capital Only*						
Private Equity Weighting	6,00%	8,00%	9,00%	9,00%	9,00%	11,00%	11,00%
Min. St. Dev.	90,86%	39,99%	28,55%	20,12%	12,89%	8,98%	***6,34%***
Return	14,17%	14,11%	14,47%	14,77%	14,67%	15,39%	15,39%
Correlation	0,0063						
Sample	*Early Stage investments*						
Private Equity Weighting	3,00%	3,00%	3,00%	3,00%	2,00%	3,00%	3,00%
Min. St. Dev.	49,77%	22,36%	16,20%	11,24%	5,86%	4,85%	***3,34%***
Return	1,94%	2,88%	3,09%	3,17%	2,81%	3,09%	3,09%
Correlation	-0,0065						
Sample	*Mezzanine*						
Private Equity Weighting	37,00%	35,00%	37,00%	35,00%	37,00%	37,00%	36,00%
Min. St. Dev.	45,23%	18,97%	13,96%	9,90%	6,23%	4,40%	***3,13%***
Return	14,27%	12,84%	13,82%	12,88%	13,53%	13,48%	13,29%
Correlation	0,048						
Sample	*Private Equity without Mezzanine*						
Private Equity Weighting	15,00%	14,00%	16,00%	15,00%	14,00%	15,00%	14,00%
Min. St. Dev.	108,13%	44,78%	32,52%	22,10%	14,57%	10,08%	***6,96%***
Return	17,94%	15,71%	16,76%	16,30%	16,28%	16,64%	16,20%
Correlation	-0,013						

Table 9.1 - Mixed asset portfolios: maximum performance ratio [subamples according to period]

This table presents the results for the bootstrap simulation of mixed asset portfolios including private equity and stocks. Bootstrap sample = 5000. N=mixed asset portfolio size. Portfolios are built in order to maximize performance ratio (return/risk). According to the private equity sample used, this table represents the optimal portfolio allocation to private equity in order to obtain maximum performance ratio portfolios. With respect to investment timing and company characteristics, each single investment used to build benchmark portfolios corresponds exactly to one private equity investment.

N	1	5	10	20	50	100	200
Sample	Overall sample [Vintage Years 1980-02]						
Private Equity Weighting	34,00%	37,00%	38,00%	36,00%	35,00%	36,00%	37,00%
Max Performance Ratio	18,54%	42,32%	60,68%	86,00%	131,85%	187,40%	266,24%
Sample	Vintage Years 1980-90						
Private Equity Weighting	5,00%	8,00%	3,00%	2,00%	0,00%	0,00%	2,00%
Max Performance Ratio	5,24%	14,19%	18,35%	25,79%	40,12%	59,95%	80,98%
Sample	Vintage Years 1990-02						
Private Equity Weighting	38,00%	38,00%	36,00%	37,00%	36,00%	38,00%	38,00%
Max Performance Ratio	25,21%	53,68%	77,18%	106,82%	166,71%	238,45%	338,09%
Sample	Realized before the end of 96						
Private Equity Weighting	35,00%	35,00%	36,00%	34,00%	35,00%	34,00%	34,00%
Max Performance Ratio	16,74%	37,03%	54,67%	76,73%	119,89%	166,38%	238,20%
Sample	Realized after 1997						
Private Equity Weighting	38,00%	38,00%	39,00%	40,00%	41,00%	43,00%	45,00%
Max Performance Ratio	23,54%	51,29%	72,59%	102,50%	159,77%	220,52%	306,25%
Sample	*Investments taken from funds, which are up to 70% realized*						
Private Equity Weighting	36,00%	37,00%	35,00%	36,00%	37,00%	36,00%	36,00%
Max Performance Ratio	13,46%	27,03%	38,66%	56,26%	89,23%	125,83%	177,61%
Sample	Investments taken from fully-realized funds						
Private Equity Weighting	26,00%	30,00%	28,00%	29,00%	28,00%	28,00%	29,00%
Max Performance Ratio	10,21%	22,41%	30,58%	44,83%	68,80%	97,37%	138,97%

Table 9.2 - Mixed asset portfolios: maximum performance ratio [sub-samples according to period]

This table presents the results for the bootstrap simulation of mixed asset portfolios including private equity and stocks. Bootstrap sample = 5000. N=mixed asset portfolio size. Portfolios are built in order to maximize the performance ratio (return/risk). According to the private equity sample used, this table represents the optimal portfolio allocation to private equity in order to obtain maximum performance ratio portfolios. With respect to investment timing and company characteristics, each single investment used to build benchmark portfolios corresponds exactly to one private equity investment.

N	1	5	10	20	50	100	200
Sample	Private Equity without VC						
Private Equity Weighting	41,00%	40,00%	37,00%	40,00%	44,00%	48,00%	53,00%
Max Performance Ratio	20,37%	43,49%	61,79%	88,72%	146,24%	208,06%	306,26%
Sample	Venture Capital Only						
Private Equity Weighting	27,00%	28,00%	29,00%	28,00%	30,00%	31,00%	34,00%
Max Performance Ratio	20,28%	44,09%	62,24%	89,31%	142,28%	207,96%	305,16%

Sample	Early Stage investments						
Private Equity Weighting	19,00%	19,00%	18,00%	18,00%	18,00%	19,00%	19,00%
Max Performance Ratio	7,74%	18,41%	26,58%	39,54%	69,91%	92,27%	134,90%
Sample	Mezzanine						
Private Equity Weighting	62,00%	62,00%	63,00%	65,00%	64,00%	63,00%	65,00%
Max Performance Ratio	34,54%	77,33%	111,46%	150,40%	245,39%	343,19%	485,89%
Sample	Private Equity without Mezzanine						
Private Equity Weighting	36,00%	37,00%	37,00%	35,00%	35,00%	35,00%	34,00%
Max Performance Ratio	19,53%	41,46%	59,59%	84,88%	131,02%	191,27%	272,53%

Table 10 – Net IRR descriptive statistics

We calculated net IRR distributions at the company level by subtracting a 20% carried interest on capital gains after the 100% distribution of invested capital. The management fee is not recognized since similar management fees are incurred while managing stock portfolios. Owing mainly to a charge of carried interest on the basis of cumulated fund cash flows, the results are worst case scenarios. In reality negative performing investments delay the overall positive funds cash flow and therefore the need to pay carried interest. OwingDue to a portfolio simulation we cannot consider this fact. We calculate carried interest payments for each portfolio company.

Net IRR *Sample: vintage year 1980-02*	
Mean	25,62%
Median	12,22%
Modus	-1
St. Deviation	1,98039577
Variance	3,92196741
Kurtosis	74,5760158
Skewness	7,42834613
Range	25,5962366
Minimum	-100,00%
Maximum	2459,62%
Frequency of total lost	19,16%
Sample size	642
	0,15348056

Table 11 – optimal weights (calculated on net IRR basis)

N	1	5	10	20	50	100	200
Sample	Full Sample (net)						
			Min. variance portfolios				
Private Equity Weighting	21,00%	20,00%	25,00%	15,00%	21,00%	21,00%	19,00%
Min. St. Dev.	90,08%	40,04%	29,97%	19,64%	12,44%	9,00%	6,16%
Return	14,44%	14,19%	15,61%	14,42%	14,29%	14,49%	14,25%
			Max. performance ratio portfolios				
Private Equity Weighting	36,00%	36,00%	40,00%	37,00%	36,00%	36,00%	35,00%
Max Performance Ratio	17,08%	38,26%	55,26%	78,07%	122,35%	172,50%	249,65%

Table 12 – MIRR descriptive statistics

Modified IRR calculation is based on an assumed reinvestment rate of 5%.

Modified IRR	*sample "vintage year 1980-02"*	
	PE	*Stocks*
Mean	14,37%	4,26%
Median	11,14%	4,73%
Modus	-1	#NV
St. Deviation	1,33012028	0,05845256
Variance	1,76921997	0,003416702
Kurtosis	43,3290348	22,02522813
Skewness	5,55898307	-2,818507972
Range	15,0992932	0,779563791
Minimum	-100%	-56,26%
Maximum	1409,92%	21,69%
Frequency of total lost	19,16%	0,78%
Sample size	642	642
Confidence Level (95,0%)	0,10308425	0,00453007

Figures

Figure 1 – IRR distribution private equity vs. stocks

Frequency distribution of the complete samples' private equity and their benchmark stocks investment returns [both measured in IRR].

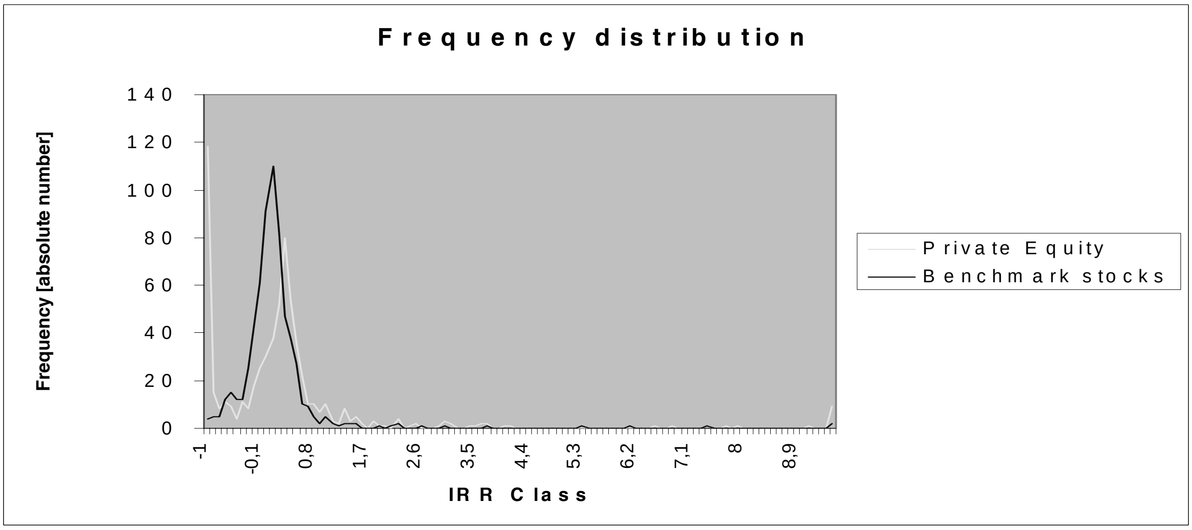


Figure 2 – IRR Distribution private equity

Frequency distribution of the complete samples' 'venture capital' and 'private equity without venture capital' [MBO/LBO/MBI/Turnaround] investment returns [both measured in IRR].

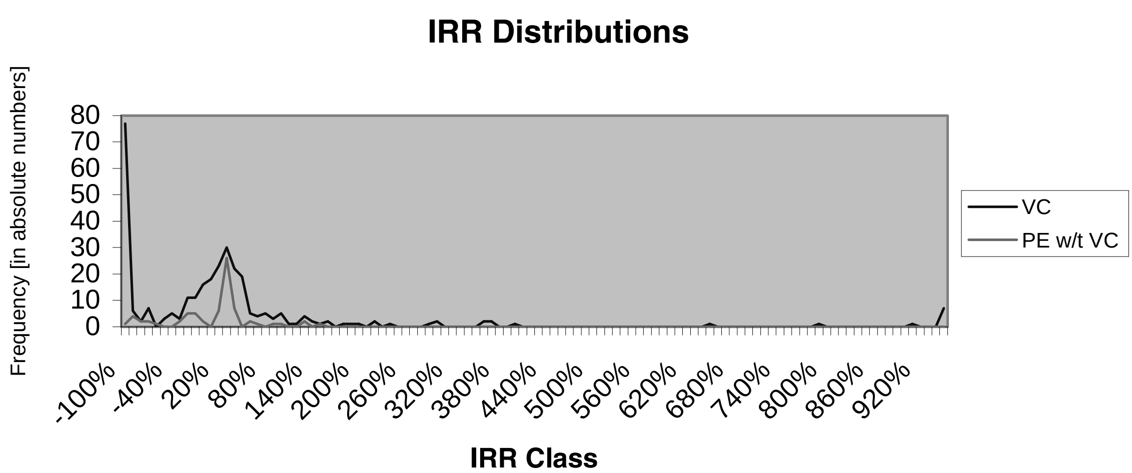


Figure 3 – Naive diversification effects with increasing portfolio size

Portfolio cross section standard deviation diversification effects when increasing the overall portfolio size. Sample: complete sample including 642 US-American private equity investments

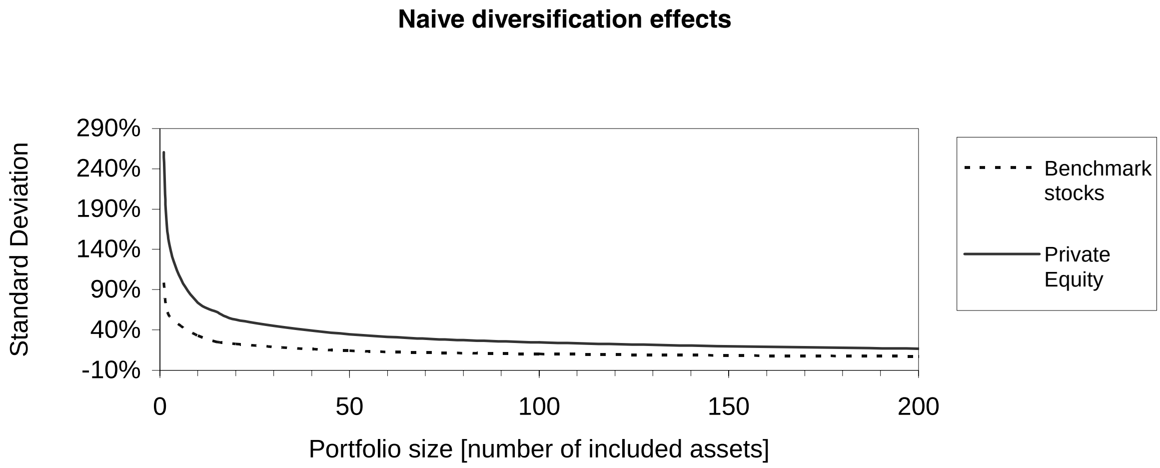


Figure 4 - Portfolio frequency distribution: portfolio size = 5

Frequency distribution of a 5 asset portfolios consisting exclusively of private equity or benchmark stocks. This frequency distribution is taken from the sample “vintage years 1980-2002”.

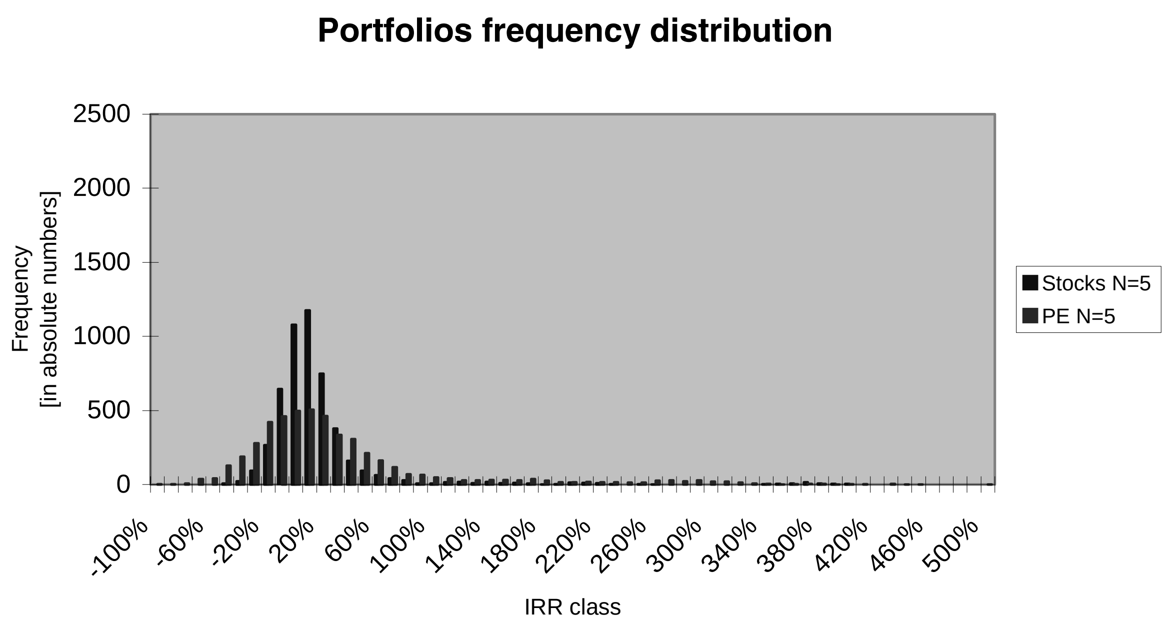


Figure 5 - Portfolio frequency distribution: portfolio size = 20

Frequency distribution of a 20 asset portfolios consisting exclusively of private equity or benchmark stocks. This frequency distribution is taken from the sample "vintage years 1980-2002".

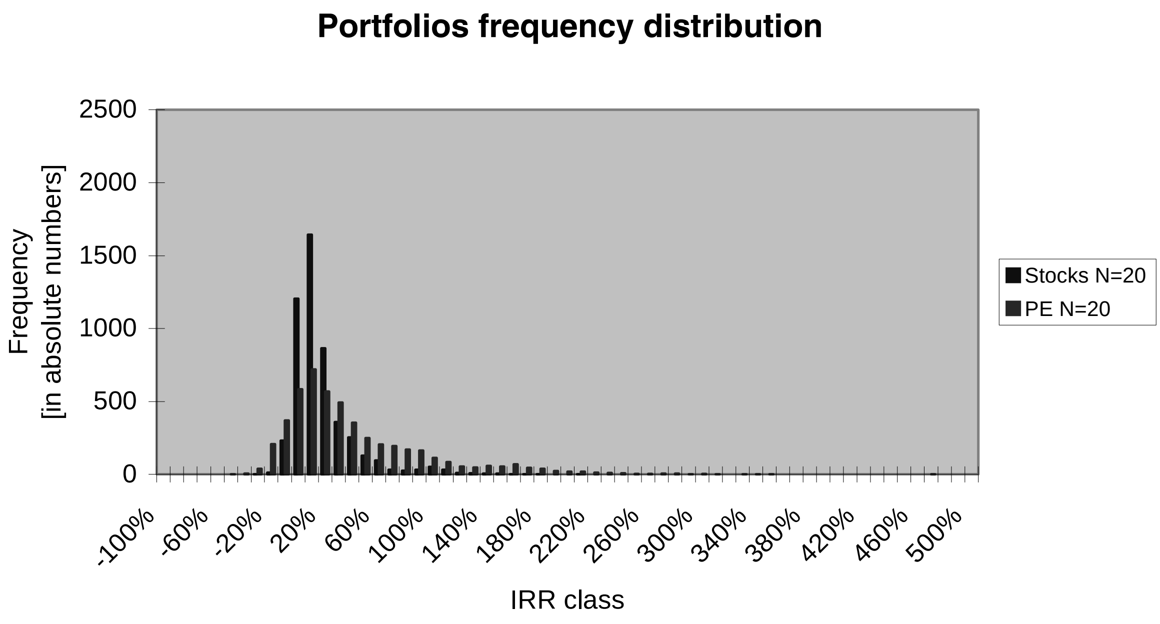


Figure 6 – Portfolio frequency distribution: portfolio size = 200

Frequency distribution of a 200 asset portfolio consisting exclusively of private equity or benchmark stocks. This frequency distribution is taken from the sample "vintage years 1980-2002".

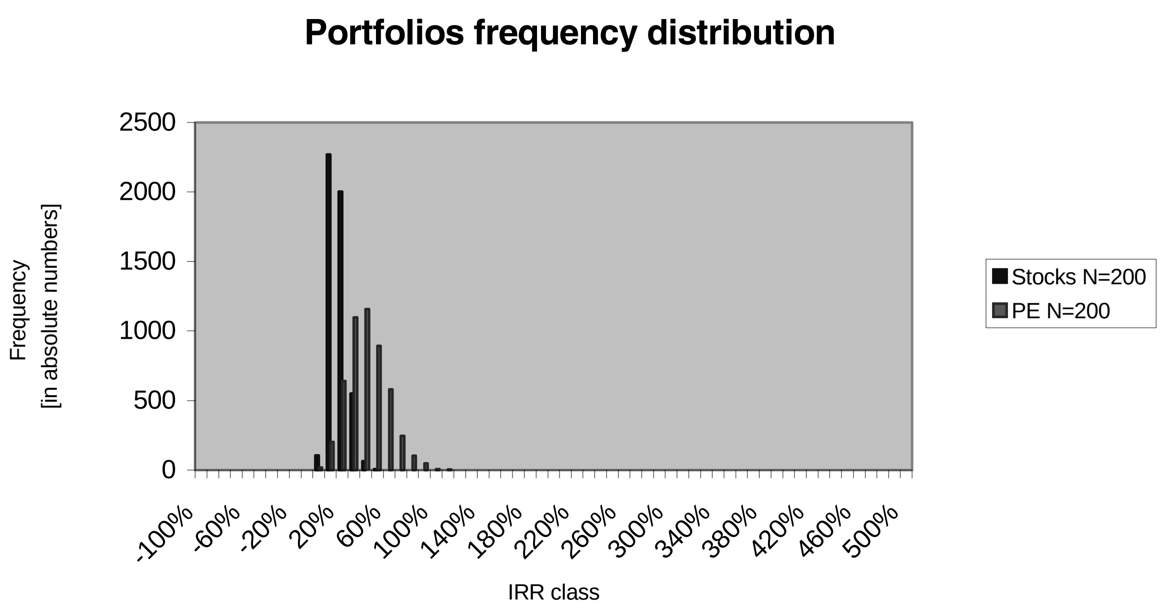


Figure 7 – Efficient frontiers

Efficient frontiers generated in accordance with changing allocations to private equity and benchmark stocks (small caps). Investments of different private equity categories are mixed with stocks to build efficient portfolios. Overall portfolio size: 20 assets.

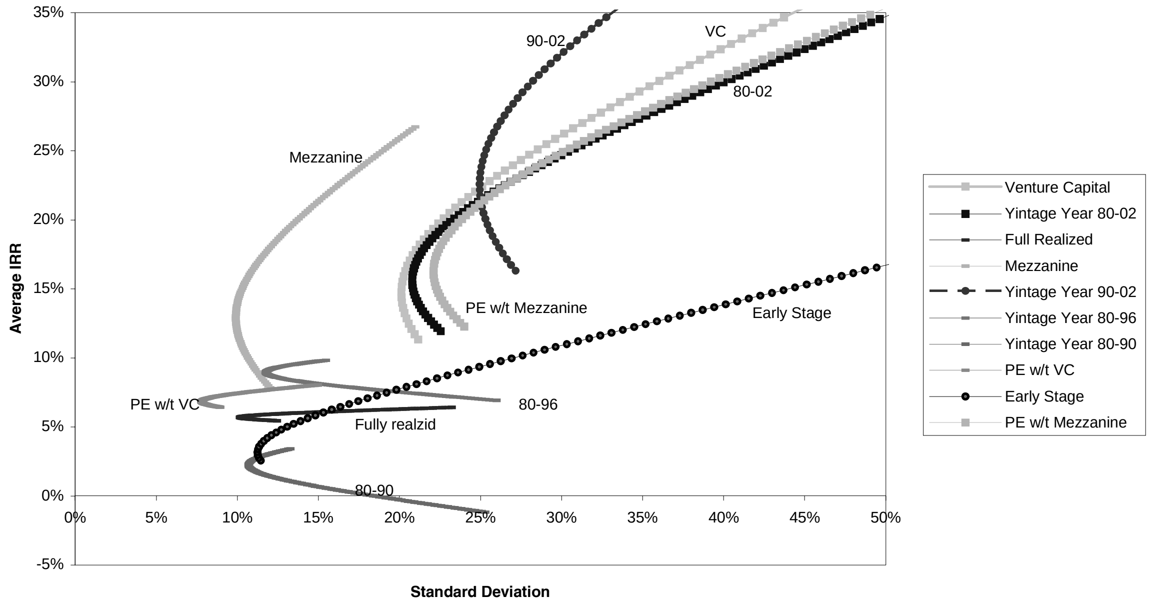


Figure 8 – naive diversification effects

Portfolio cross section standard deviation diversification effects when increasing the overall portfolio size. Sample: complete sample including 642 US-American private equity investments

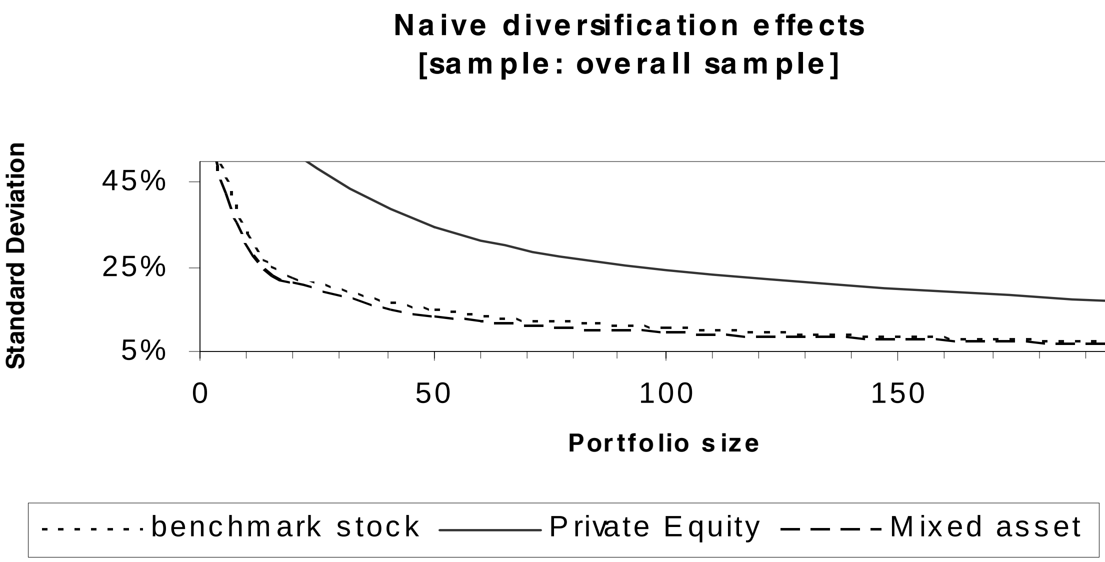


Figure 9 – Diversification effects of mixed portfolios with increasing size

Diversification effects when increasing the portfolio size of mixed asset portfolios. N = overall portfolio size. Return/risk of the following portfolio combinations: **100% stocks**: return=11.34% st. deviation=46.52% [N=5] 32.84% [N=10]%; **100% PE**: return: 37.01% st. deviation=108% [N=5] 74% [N=10]; **83% stocks / 17% PE**: return=15.46% st. deviation=42.51% [N=5] 29.84% [N=10]; **50% stocks / 50% PE**: return =24,18% st. deviation=58.56% [N=5] 40.14% [N=10].

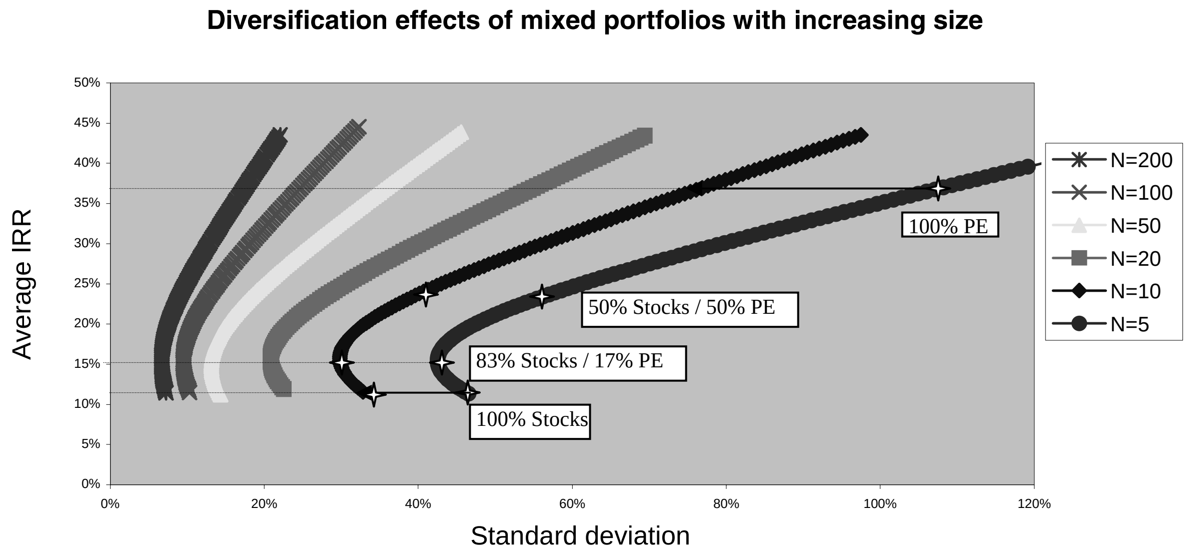


Figure 10 – Performance ratio with increasing portfolio allocation to private equity

Performance ratio (average portfolio return/cross section standard deviation) changes when the allocation to private equity is increased (complete sample out of 642 investments). N= overall portfolio size

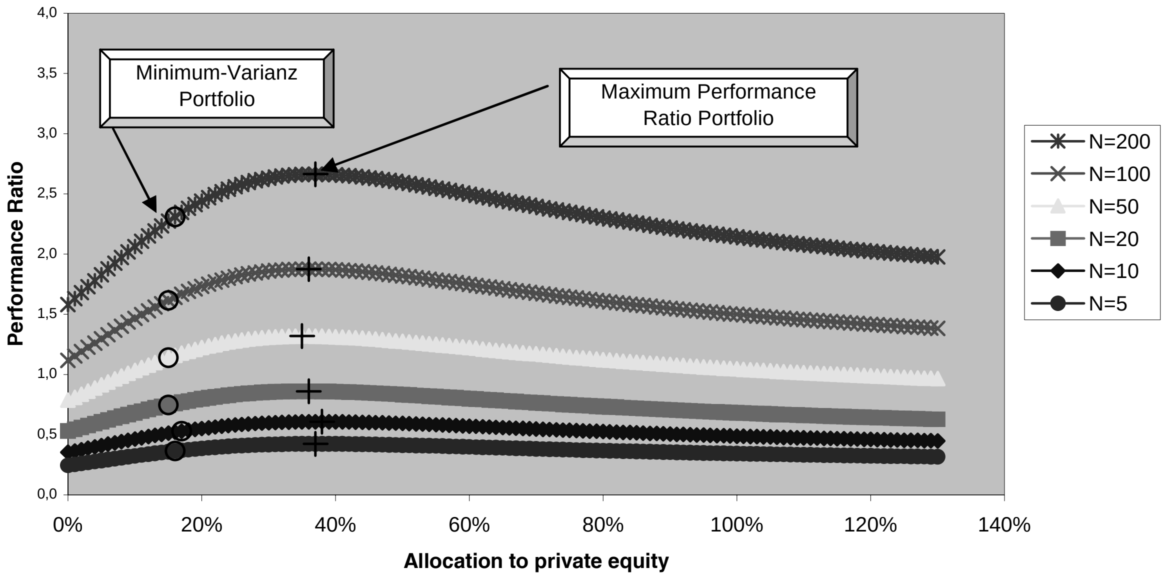
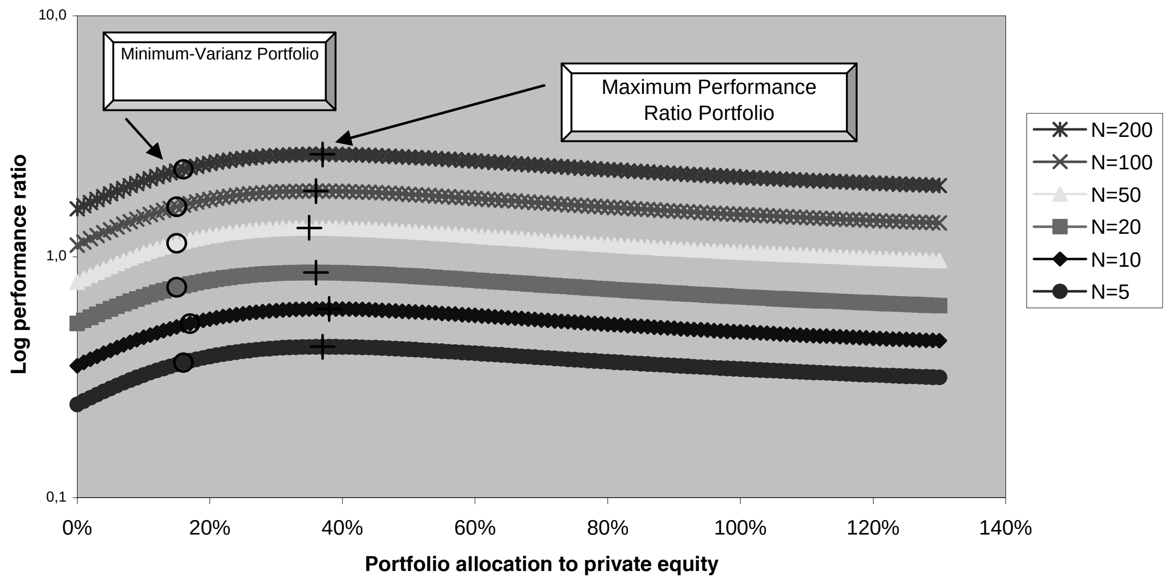


Figure 11 – log. performance ratio with increasing portfolio allocation to private equity

Log performance ratio (average portfolio return/cross section standard deviation) changes when the allocation to private equity is increased (complete sample out of 642 investments). N= overall portfolio size.



Appendix

The bootstrap approach

We use the following algorithm:

Sample: $(PE_{1,}....PE_n)$; $(S_{1,}...S_n)$; for n=642, each from empirical distribution $\hat{F}$, describing individual private equity or stock investments

PE	=	IRR of Private Equity investments
S	=	IRR of simultaneous stock investments
n	=	number of PE and benchmark investment

Simulation of pure private equity portfolios:

First, we select B= 5000 independent bootstrap samples x_m^{*1}, x_m^{*2}, …., x_m^{*B} each consisting of m= 1, 2, 5, 10, 15, 20, 50, 100, 200, 300, 500 (simulated portfolio size) data values, each describing one investment return. x_m^{*} is a vector of m PE investment returns. Each vector $\{x_1^{*b},...x_m^{*b}\}$ is a random sample from $\{PE_1,...PE_n\}$ The sample is drawn with replacement from the empirical distribution $\hat{F}$ (nonparametric estimate of the real population F).[49] We calculate for each vector the average portfolio return, $\bar{x}_m^{*b}$=mean$\sum_{i=1}^{m} x_i^{*b}$, for b = 1…B. [50] As a result we get 5000 portfolio returns each consisting of m investments. We are able to determine the quartile distribution of portfolio returns.

Second, our parameter of interest is θ=t(F). It is the empirical estimate $\hat{\theta}^*$=s($\bar{x}_m^{*b}$)=$\frac{\sum_{i=1}^{B}\bar{x}_m^{*b}}{B}=\mu_{\hat{F}}^m$, b=1,2,..B. We get varying $\mu_{\hat{F}}^m$ for each portfolio size m= 1, 2, 5, 10, …, 500.[51]

Third, we estimate se$_F(\theta)$ by sample standard deviation of the B replications for each portfolio size m=1, 2, 5, 10,…, 500

$$s\hat{e}^m = \sqrt{\frac{\sum_{b=1}^{B}[\hat{\theta}^*(b)-\mu_{\hat{F}}^m]^2}{(B-1)}}\,.^{52}$$

Simulation of stock portfolios:

According to the methodology of PE portfolio simulation we generate the bootstrap sample $y_m^{*1}, y_m^{*2}, \ldots y_m^{*B}$ as a random sample from $\{S_{1,}...S_n\}$. With respect to the chosen

[49] See Efron (1993), pp. 45-47
[50] Shao, J. / Dongsheng, T (1995), pp. 207
[51] Following Efron (1993), pp. 45-47
[52] See Efron (1993), pp. 47

benchmark investments, this bootstrap sample corresponds exactly to the PE bootstrap sample.

Simulation of mixed portfolios with different asset allocations (Private Equity VC and stocks [S])

First, we select B= 5000 independent pairs of bootstrap samples $x_m^{*1}, y_m^{*1}; x_m^{*2}, y_m^{*2}$; ….; x_m^{*B}, y_m^{*B} each consisting of m= 1, 2, 5, 10, 20, …500 data values drawn with replacement from the empirical distribution $\hat{F}$. Each pair x_m^{*b} y_m^{*b} represents the IRRs of both one PE portfolio with size m and its time and cash flow equivalent portfolio investment in benchmark stocks. As before, $\{x_1^{*b},...x_m^{*b}\}$ is a random sample from $\{PE_n,...PE_n\}$ and $\{y_1^{*b},...y_m^{*b}\}$ is the corresponding random sample from $\{S^1,...S^n\}$). We calculate for each bootstrap sample the average return as follows: $\bar{x}_m^{*b}$ =mean $\sum_{i=1}^{m} x_i^{*b}$, $\bar{y}_m^{*b}$ =mean $\sum_{i=1}^{m} y_i^{*b}$; m=1, 2, 5, 10, 20, …500

Second, we built mixed asset portfolios with different portfolio sizes m and PE weightingss α. Each mixed asset portfolio return is calculated as $f^{\alpha,m,b}(\bar{x}_m^{*b}, \bar{y}_m^{*b}) = \alpha\ \bar{x}_m^{*b} + (1-\alpha)\ \bar{y}_m^{*b}$,[53] for m=1, 2, 5,…500 and α=0.01; 0.02; …0.99.

Third, we determine the empirical estimate of average mixed asset portfolio return $\hat{\theta}^*$=s($f^{\alpha,m,b}$ ($\bar{x}_m^{*b}, \bar{y}_m^{*b}$))=$\mu_{\hat{F}}^{m,\alpha}$ over each bootstrap sample: b=5000 with size m=1, 2, 5,…500 and $\alpha = 0.01;....0.99$. Additionally, we estimate $se_F(\theta)$ by sample standard deviation of the B replications for each portfolio size m=1, 2, 5, …500

$$\hat{se}_B^{m,\alpha} = \sqrt{\frac{\sum_{b=1}^{B}[\hat{\theta}^*(b) - \mu_{\hat{F}}^{m,\alpha}]^2}{(B-1)}}$$ for m=1, 2, 5, …500 and $\alpha = 0.01;....;0.5$

Knowing the mean Bootstrap returns and standard deviations for mixed portfolios with changing asset weights, we can outline the efficient frontier. Remember that each mixed portfolio is the weighted composition of one pure PE portfolio and one pure stock portfolio with the same portfolio size. Again, both portfolios consist of the same number of PE or stock investments with exact investment and distribution timing. With respect to timing and all other benchmark criteria, both portfolios are exactly comparable. Therefore, the results are not diluted by diversification effects due to investment date differences. Furthermore, we determine correlations between both asset classes by confronting completely comparable benchmark returns. To test the results for robustness we repeat the steps from above with

[53] See Cochrane, p. 12, for methodology of portfolio formation composed of fraction w in a VC investment and fraction (1-w) in an risk free return

reduced sample ($PE_{1,}....PE_n$) and the corresponding stock performance sample ($S_{1,}...S_n$) (n<642).

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