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Pension Accounts: Evidence from the  
German Pension Reform**

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## **Money-Back Guarantees in Individual Pension Accounts: Evidence from the German Pension Reform\***

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### **Abstract:**

The German Retirement Saving Act instituted a new funded system of supplementary pensions coupled with a general reduction in the level of state pay-as-you-go old-age pensions. In order to qualify for tax relief, the providers of supplementary savings products must offer a guarantee of the nominal value at retirement of contributions paid into these saving accounts. This paper explores how this “money-back” guarantee works and evaluates alternative designs for guarantee structures, including a life cycle model (dynamic asset allocation), a plan with a pre-specified blend of equity and bond investments (static asset allocation), and some type of portfolio insurance. We use a simulation methodology to compare hedging effectiveness and hedging costs associated with the provision of the money-back guarantee. In addition, the guarantee has important implications for regulators who must find an appropriate solvency system for such saving schemes.

**JEL Classification:** G11, G23, G28

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## 1. Introduction

The German Retirement Saving Act (“Altersvermögensgesetz”<sup>1</sup>), which passed the German legislative body in May of 2001, instituted a new funded system of supplementary pensions coupled with a general reduction in the level of state pay-as-you old-age pensions. The goal of this new pension system is to cap and to stabilise the contributions of German employees to the state pension system, which are currently 19.1% of their salary. Therefore, for compulsory members of the state pension systems which are not already in retirement the maximum “first pillar” state pension level is gradually cut from currently 70% to 67% until 2030 of the last net salary before retirement.<sup>2</sup> To compensate for the cut in state pension payouts, individuals will be able to invest voluntarily and on a pre-tax basis a part of their income in individual pension accounts (“Altersvorsorgevertrag”, IPA).<sup>3</sup> Additional incentives to invest into the IPAs are given by the government in the form of a tax relief on pension contributions, direct subsidies for low income earners and extra contributions for children. In order to get the full benefits, households will have to invest about 1% of their income (up to the social security ceiling) into the pension system in 2002, increasing every two years by 1% reaching a maximum of 4% in 2008. The investment income during the accumulation period is not subject to income tax, whereas the payments in the distribution phase arising from the IPA will be fully subject to income tax.

In general, individuals are free to make IPA pension investments in a wide array of products offered by private-sector financial institutions. This allows participants to choose an investment portfolio that is consistent with their individual preferences for risk and return. However, in order to qualify for a tax credit, the IPA products have to satisfy a number of criteria. These conditions are codified in a special law concerning the certification of individual pension products (“Altersvorsorge-Zertifizierungsgesetz”) and supervised by a special authority (“Zertifizierungsstelle”) belonging to the German Federal Financial Supervisory Agency. The intention of the certification requirements is twofold:

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<sup>1</sup> This Act is also known as “Riester Reform”. Walter Riester is the German Labour Minister who is responsible for the reform of the pension system in the year 2001. More formally, the reform as a whole alters several existing laws including (among others) the social security law, the income tax law, the occupational pensions law, the social welfare law, the civil law, the law governing investment management companies, and the law governing insurance companies.

<sup>2</sup> Similar to that, the maximum pension for civil servants is being reduced from currently 75% to 71.75% of the last salary.

<sup>3</sup> In addition, the government promotes also the “second pillar” occupational pension system, e.g. by establishing a new funding vehicle called “Pensionsfonds”.

- First, the government wants to ensure that individuals only use the (tax supported) accumulated savings for a lifelong income stream in the post-retirement phase, and not for consumption during pre-retirement.
- Second, private (and often uninformed) investors paying into the new individual pension plans should be protected against the risk of making “too bad” investment decisions.

In the spirit of the first intention, investments in the personal pension accounts must be preserved until employees reach the age of 60<sup>4</sup>, and no distributions may be made during the accumulation period. When the age of retirement is reached, the accumulated assets be drawn down in the form of a lifelong annuity or a capital withdrawal plan which must (partly) revert into an annuity at the age of 85.<sup>5</sup> To provide transparency, the providers of IPA’s must disclose the nature and level of fees (e.g. to cover distribution and/or administrative costs). If distribution fees are not charged as a percentage of the periodic contribution into the plan, they must be spread equally over a period of at least ten years.<sup>6</sup> During the accumulation phase, the policyholder has the right to suspend the contract as well as to terminate the contract by switching the cash value of the policy to a new provider.

In line with a certain minimum level of investor protection, only regulated financial institution, like banks, life insurance companies and mutual fund companies, are allowed to offer IPAs. In principle, these providers are free in the design of an IPA. In particular, the Certification Act imposes no restrictions concerning the assets in which the providers invest the contribution to back the pension accounts.<sup>7</sup> In addition, the supervisory authority does not check whether the risk and return characteristics of an IPA are economically feasible. However, the provider of an IPA must promise the plan participant that the contract cash

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<sup>4</sup> An exemption is, that a part of the pension plan (min € 10,000 and max. € 50,000) can be withdrawn during the accumulation phase to finance own house. This amount must be paid back (at a zero interest rate) into the IPA before the beginning of the distribution phase.

<sup>5</sup> In the case of a life annuity the provider must promise lifelong constant or increasing (monthly) payments to the annuitant. In the case of a capital withdrawal plan (typically offered by mutual fund and/or bank providers) at least 60% of the accumulated assets (but not less than the contributions paid into the IPA) must be used for constant or rising periodic payments. At latest at the age of 85 the balance must revert into a life annuity, whereas the benefits cannot be less than the last payment received before that age. In addition, not more than 40% of the accumulated assets can be used for a withdrawal plan with variable pension payments (reflecting the return of a specific asset portfolio).

<sup>6</sup> Especially for traditional life insurance policies, it is conventional (until now) that distribution costs are charged as front end loads on the first premiums (via the so-called zillmer-adjustment) resulting in no or low early cash values for the policyholder.

<sup>7</sup> An exemption are financial derivatives (e.g. option, futures, swaps), which can be used within an IPA for hedging purposes only.

value at retirement is at least equal to the contributions (including all extra payments by the government) paid into the IPA.<sup>8</sup> This “money-back” guarantee, which was the core of an intense and controversial debate during the social security reform in Germany, is the focus of this paper. Advocates of the guarantee argue that it protects plan participants against a portion of the downside volatility of capital market returns by providing them with a minimum rate of return with respect to their lifetime contributions. However, the guarantee shapes the design of saving products offered by the providers and raises the question about the economic costs of such a promise.<sup>9</sup> Depending on the assets used to back the pension accounts, providers may be exposed to shortfall risk, due to adverse movements in capital markets. If at retirement, the value of the pension assets is lower than the sum of the contributions paid into the plan, the IPA provider must fill the gap with its own equity capital. The problem faced by money managers is therefore to find a product design, in conjunction with an appropriate investment strategy, that protects the credibility of the guarantee in scenarios of negative investment returns (hedging effectiveness), while still allowing for sufficient upside potential if capital markets are booming (and thus avoiding excessive hedging costs). In addition, the guarantee has important implications for regulators who must find an effective and efficient solvency system for such saving schemes, especially for mutual funds.

The objective of this paper is to explore how this money-back guarantee works for products which are offered by the German mutual fund industry. We evaluate alternative designs for guarantee structures including a life cycle model (dynamic asset allocation), a plan with a pre-specified blend of equity and bond investments (static asset allocation), and some type of portfolio insurance. We use simulation to compare hedging effectiveness and hedging costs associated with the provision of the money-back guarantee.

## **2. Long Term Shortfall-Risk and Return of Saving Plans**

In order to make appropriate investment decisions under uncertainty, individuals must be able to compare the risk and rewards of different asset classes. However, policymakers, regulators, and providers are also interested in the long run performance of financial assets that back the

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<sup>8</sup> Not more than 15% of total contributions can be deducted from the principal guarantee level, if the IPA include insurance coverage against disability. In the case of a switch to a new provider during the accumulation phase, the policyholder gets from the new provider a guarantee on the policy’s cash value at the time of transfer plus new premiums.

<sup>9</sup> See also Lachance/Mitchell (2002)

new pension products. The impact of the investment horizon on the risk of the various financial assets is still a subject of intense and controversial debate within the academic community and among investment professionals.<sup>10</sup> For example, a popular statement is that stocks have a lower downside risk in the long run than in the short run. A practical guideline based on this argument is, that people should invest a higher fraction of their money in stocks the younger they are, independent of preferences.<sup>11</sup> If the time horizon is long enough, this approach would imply that people should invest 100% in stocks. To justify this view, proponents call on the law of large numbers, which (seemingly) forces a phenomenon called “time diversification”. Intuitively, this means that over a sufficiently long investment horizon losses resulting from the high downside fluctuations will be compensated by gains resulting from the high upside fluctuations of short-term stock returns. Some investment advisors substantiate this argument empirically by pointing to historical returns and demonstrating that stocks have outperformed bonds for every 10-, 15 or 20-year investment period on record.

Nevertheless, it is well known in the academic literature that this is a misleading argument. For example, Samuelson (1963) uses utility theory, Levy/Cohen (1998) use stochastic dominance, and Bodie (1995, 2001) applies option pricing theory to demonstrate the logical flaw in this conjecture. In addition, the use of historical return series implies that the 10-, 15- or 20-year periods used are strongly overlapping and the resulting rollover multi-period returns have a high degree of correlation, both of which result in a serious estimation bias.

## 2.1. *Shortfall Risk Measures*

In this section, we provide additional evidence concerning the impact of the time horizon on the risk of the major financial asset classes, i.e. stocks and bonds in the German context. To do so, we use alternative shortfall risk measures. The concept of shortfall risk is associated with the possibility of “something bad happening”, in other words, falling short compared to a required target (benchmark) return.<sup>12</sup> Returns below the target (losses) are considered to be

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<sup>10</sup> For surveys cf. Albrecht/Maurer/Ruckpaul (2001) and Kritzman/Rich (1998).

<sup>11</sup> Cf. Bodie (2001) and Bodie (2002) for a critique of these simple arguments.

<sup>12</sup> The concept of shortfall risk was introduced in finance by Roy (1952) and Kataoka (1963), expanded and theoretically justified by Bawa (1975), and Fishburn (1977), (1982), (1984). It is widely applied to investment asset allocation by Leibowitz/Bader/Kogelman (1996) and used by

undesirable or risky, while returns above the target (gains) are desirable or non-risky. In this sense, shortfall risk measures are called “relative” or “pure” measures of risk.

A popular measure to examine the downside risk of different investment vehicles is the shortfall probability. Formally, let  $R$  denote the cumulative (multiyear) return of an investment at a specific point in time. Then the shortfall probability is given by

$$SP = \text{Prob}(R < z). \quad (1)$$

where  $z$  is the target (benchmark) which translates the total investment returns into gains or losses. In the special case of a money back guarantee, the target is set equal to zero, i.e. the shortfall occurs when the cash value of the policy is lower than the premiums paid into the saving plan. Despite the popularity of this risk measure in the investment industry, it has a major shortcoming. As Bodie (2001, p. 308) points out it “completely ignores how large the potential shortfall might be”. If the same investment strategy can be repeated many times, the shortfall probability only answers the question “how often” a loss might occur, but not “how bad” such a loss might be.

To provide information about the potential extent of a loss, we calculate the Mean Excess Loss (MEL), also known as conditional shortfall expectation, as an additional measure to evaluate the long term shortfall-risk of financial assets. Formally, this risk index is given by

$$\text{MEL} = E[z - R | R < z] \quad (2)$$

and it indicates the expected loss with respect to the benchmark, under the condition that a shortfall occurs. Therefore, given a loss, the MEL answers the question “how bad on average” the loss will be.<sup>13</sup> In this sense, the MEL can be considered a worst case-risk measure, since the measure only considers the consequences of the mean shortfall-level assuming that a shortfall happens.

A shortfall risk measure which connects the probability and the extent of the conditional shortfall in an intuitive way is the shortfall expectation (SE):

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Albrecht/Maurer/Ruckpaul (2001), Asness (1996), Butler/Domian (1991), Leibowitz/Krasker (1988) and Zimmermann (1991) to judge the long term risk of stocks and bonds.

<sup>13</sup> The MEL is closely connected with the Tail Conditional Expectation, which is given by  $TCE = E(R | R < z) = z - \text{MEL}$ . The TCE has some favourable features, e.g. it is (in contrast to the shortfall probability) a coherent risk measure with respect to the axioms developed by Artzner et al. (1999). In addition the MEL is a suitable version of the mean excess- respectively mean excess loss-function  $E(X - z | X > z)$  considered in extreme value theory. For extreme-value methods in financial risk management cf. Borkovic/Kluppelberg (2000) and Embrechts/Resnick/Samorodnitsky (1999). Very early, Gürtler (1929) introduced the MEL as “Mathematisches Risiko” to evaluate the underwriting risk of insurance companies.

$$SE = E[\max(z - R, 0)] = SP * MEL \quad (3)$$

The shortfall expectation is the sum of losses weighted by their probabilities, and hence it is a measure of the unconditional “average loss”. As equation (3) shows, the mean shortfall level is simply the product of the shortfall probability and the mean level of shortfall given the occurrence of a shortfall. In addition, the SE is, in a certain way, related to the price of an insurance contract which would cover the shortfall. For example, the provider may have the possibility of transferring the shortfall risk to the capital market by using appropriate arbitrage-free put options. Then the shortfall expectation between the cash value of the pension assets and the guarantee payment with respect to the risk adjusted (“martingale”) probabilities discounted back at the risk-free interest rate, results in the (modified) Black/Scholes (1973) option pricing formula.<sup>14</sup> If the provider transfers the shortfall risk to a reinsurance company, the shortfall expectation could be seen as an important element of an appropriate premium.

## 2.2. Calibration

In the remainder of this section, we quantify and compare the shortfall risk (in the sense defined above) with respect to the preservation of principal of two saving plans. The first invests the contribution into stock index fund units, represented by the German stock index (DAX). The other saving plan is based on bond index fund units, which are represented by the German bond index (REXP). The DAX stands for an index portfolio of German blue chips, and the REXP for portfolio of German government bonds. Each of these indices is adjusted for capital gains as well as dividends and coupon payments (on a pre-tax basis). We assume a series of equal contributions paid at the beginning of each month up to the end of the accumulation period, which ranges from 1 to 20 years.

To gain information about the relevant risk measures, we employ an *ex ante* approach by imposing an exogenous structure on the probability distribution governing the uncertainty of future asset returns. With such a model, it is possible to look into the future and compute the risk measures in which we are interested. Due to the complexity of the underlying payment structure of saving plans, there are no analytical closed form expressions for these risk-

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<sup>14</sup> The Black/Scholes formula follows directly only for a single lump sum pension payment and not for a series of contributions; see also Lachance/Mitchell (2002).

measures.<sup>15</sup> Therefore, we use Monte-Carlo simulation to generate a large number of paths for the evolution of the saving plans.

The relevant statistics for the shortfall risk measures are then evaluated on the basis of these scenarios. The stochastic dynamics of the (uncertain) market values of investment fund units are modeled as geometric Brownian motion, which is standard in financial economics and can be traced back to Bachelier (1900). This implies that the log returns of each type of index fund are i.i.d. normally distributed. For the estimation of the process parameters (drift/diffusion), we use the historical monthly log-returns of the DAX and REXP over the period 1:1973-12:2001. The mean log rates of return for stocks (bonds) are 0.7967% p.m. (0.5683% p.m.) and the corresponding standard deviation 5.58% p.m. (1.12% p.m.). To take potential administration costs into account we subtract the equivalent of 0.5% p.a. from the monthly average return on the investments.<sup>16</sup> Compatible with the current German mutual fund fee structure, we take marketing costs into consideration by assuming front end sales charges of 5% for the stock and 3% for the bond fund units.

With respect to these parameters and consistent with the model of a geometric Brownian motion, 3,000,000 random paths for the development of the pension plan with an investment horizon of 20 years (240 months) are generated.<sup>17</sup> For each simulation path  $i$  ( $i = 1, \dots, 3,000,000$ ) we compute for each month  $t$  ( $t = 1, \dots, 240$ ) the (uncertain) compounded (multiyear) return, *i.e.*

$$R_{i,t} = \frac{V_{i,t} - P_t}{P_t}. \quad (4)$$

Here  $V_{i,t}$  stands for the cash value of the IPA in month  $t$  ( $t = 1, \dots, T$ ) in simulation run  $i$  ( $i = 1, \dots, n$ ) and  $P_t$  for the sum of contributions paid until month  $t$ . According to the money-back guarantee, we set the benchmark return equal  $z = 0$ . With respect to this target, the relevant risk parameters are then determined on the basis of the spectrum of possible future developments.

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<sup>15</sup> As shown by Albrecht/Maurer/Ruckpaul (2001) the risk measures can be derived analytically in the case of a lump sum investment.

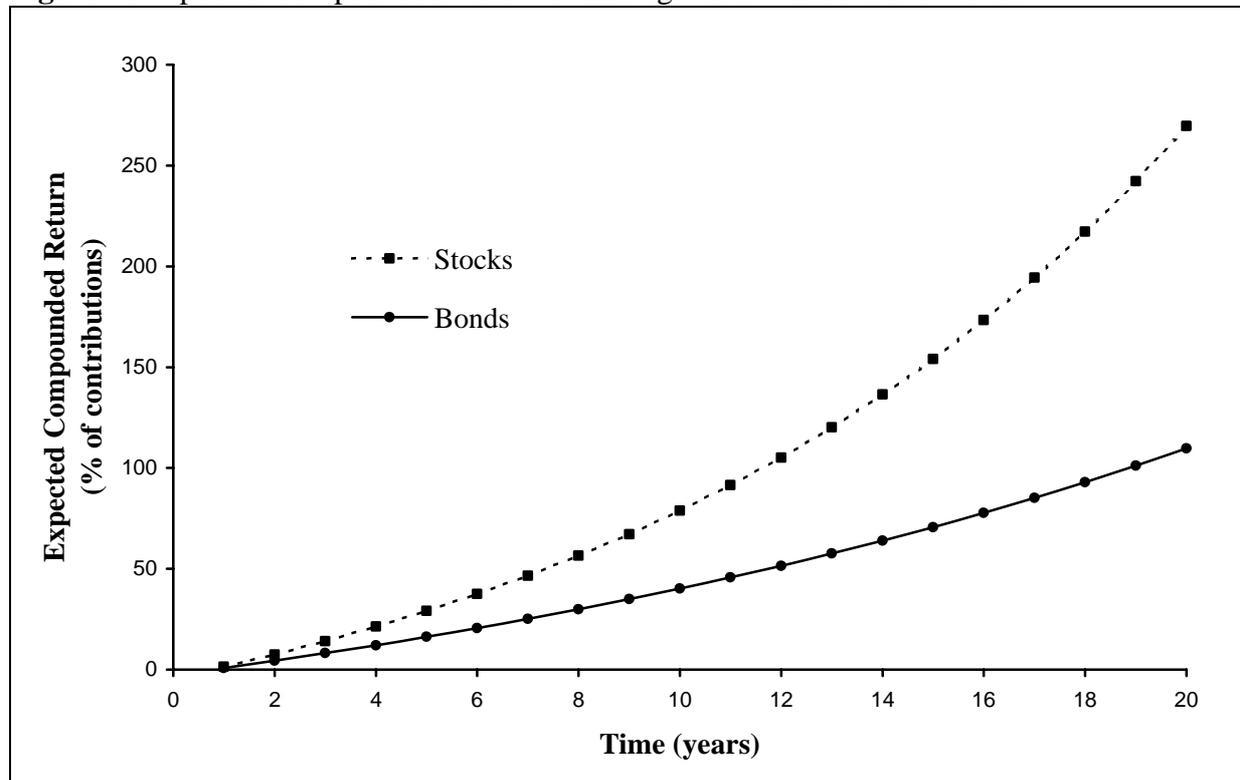
<sup>16</sup> Feldstein/Ranguelova/Samwick (2001, p. 60) use a similar procedure to account for potential administration costs.

<sup>17</sup> The large number of simulation paths is necessary to receive a precise picture of the worst case risk measure MEL, especially when the shortfall probability is low.

### 2.3. Results

We start with the results for the development of the expected (multiyear) return and the shortfall probability of the stock and bond index fund over time. The graphs in Figure 1 suggest the well-known result that a German investor would have the potential to receive a substantially higher expected return by investing in stocks instead of bonds. For example, in the case of stocks at the end of a 20-year accumulation period, the investor can expect a compounded return with respect to his contributions of 270%. For a saving plan based on bond index funds, the expected return is only 109%. However, purchasing such an investment exposes the plan participant to the volatility and therefore the downside risk of financial markets.

**Figure 1:** Expected Compounded Return of Saving Plans in Stocks and Bonds

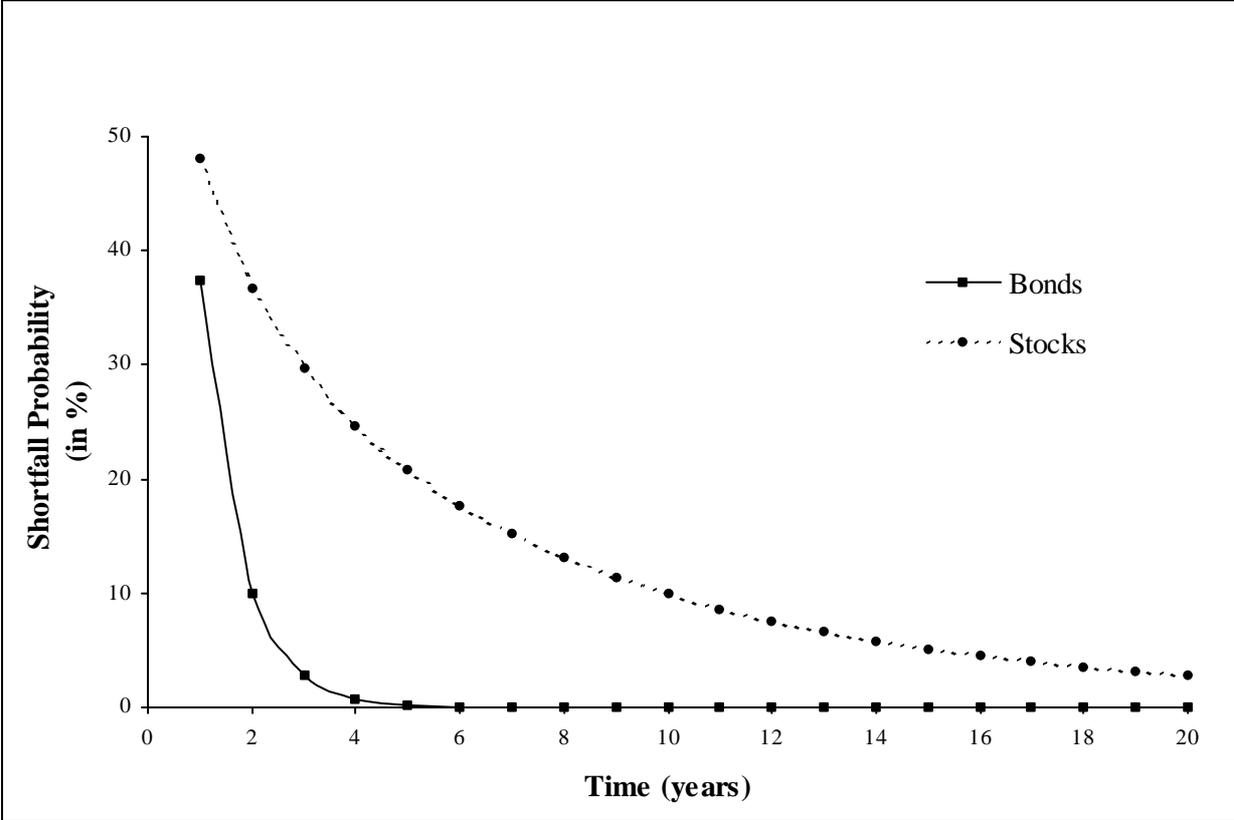


Source: Authors' Computations

Next we illustrate the results for the development of the shortfall probability of a saving plan using stock and bond index funds over time. Figure 2 shows the well-known effect of time diversification, i.e. the fact that the risk of not maintaining nominal capital decreases monotonically with an increasing investment period, for bonds and stocks. However, the rate and the extent of the risk reduction differ notably between the two investment vehicles. For bond index funds, the shortfall probability is 37% for a yearly investment, and close to zero

(i.e. lower than 0.1%) with an investment horizon of seven years onwards. In contrast, the shortfall probability of a stock index fund does not converge as rapidly towards zero. Thus, even for longer time horizons, the shortfall probability remains at a substantial level. For example, the shortfall probability for a 12-month saving plan is 48.09%, and for accumulation period of 20 years it is still 2.72%. In principle, these results confirm a characteristic which Leibowitz/Krasker (1988) call persistence of risk.

**Figure 2:** Shortfall Probability Against a (nominal) 0% Target Rate of Return in Stock and Bond Saving Plans

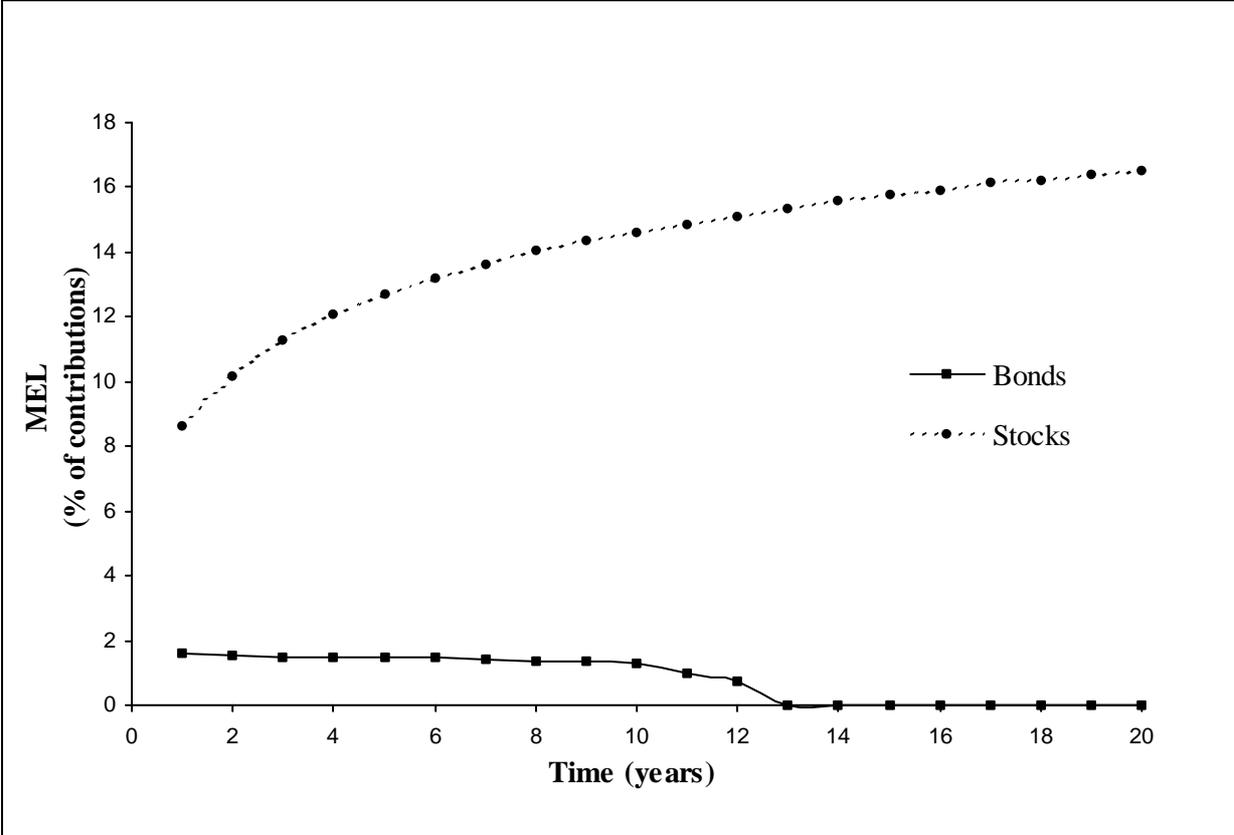


Source: Authors’ Computations

Corresponding results for the mean excess loss (MEL) are presented in Figure 3. Saving plans in stocks have an MEL that increases monotonically with the length of the accumulation period, in contrast to bonds. For example, for an accumulation period of one year (i.e. 12 months) the conditional expected loss is 8.62% of the sum of the contribution paid into the stock pension plan, while for a holding period of 20 years (i.e. 240 months) this risk index increases to 16.53%. For a pension plan using bond index funds, the MEL is 1.63% after one year, while for accumulation periods of 13 years onwards, none of the 3,000,000 simulation paths produces a shortfall. Hence, with respect to the magnitude of a potential shortfall, the popular argument that stocks become less risky in the long run is not true.

This result is in line with Samuelson’s (1963) finding concerning the fallacy of the law of large numbers. In addition, these results make clear that the use of the shortfall probability alone is a misleading risk measure of stock investments in the long run. The worst-case aspect of a long term investment in stocks is partly hidden by only taking the shortfall probability into consideration. Recently Bodie (2002) provided the following very intuitive explanation for this result “(..) the probability of a bad thing happening is only part of the risk equation.

**Figure 3:** Conditional Mean Expected Loss (MEL) Against a (nominal) 0% Target Rate of Return in Stock and Bond Saving Plans



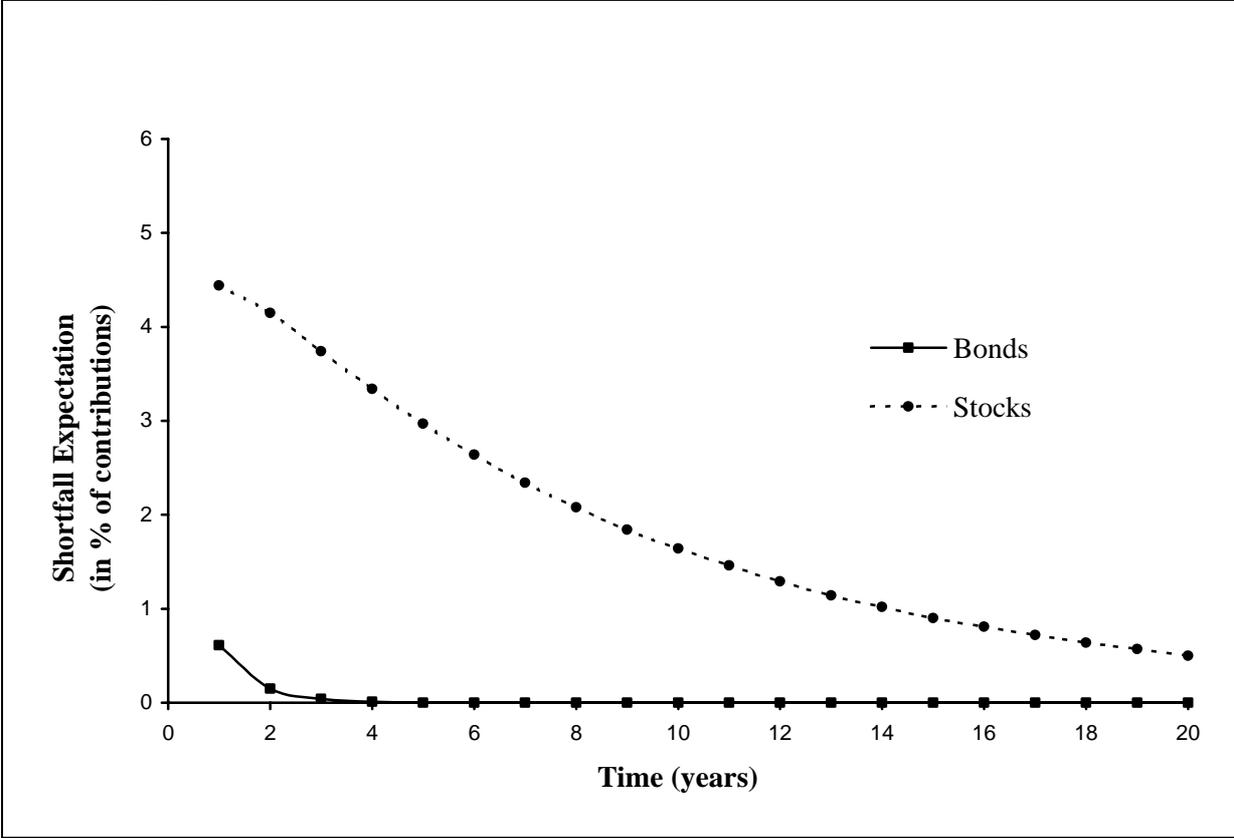
Source: Authors’ Computations

The other part is the severity of that bad thing, and the further out you go, the more severe it could be.” Thus, the elucidation of the worst-case risk embodied in a long-term investment in stocks represents an additional piece of information that might be essential for investors.

Figure 4 shows how the unconditional shortfall expectation develops over time. For a saving plan in bond index funds, both the probability of loss and the mean excess loss decrease with the length of the time horizon. Because the shortfall expectation measures the net effect of both risk components, it is also decreasing in time. For a stock-based saving plan, this risk measure is also decreasing, i.e. the decreasing shortfall probability over-compensates the increasing MEL, to a certain extent. However, in contrast to bonds, we can observe a risk

persistence-characteristic in the stock fund: even for very long time horizons, the shortfall expectation remains at a substantial level.

**Figure 4:** Expected Shortfall Against a (Nominal) 0% Target Rate of Return in Stock and Bond Saving Plans



Source: Authors' Computations

In summary, even for long investment horizons, a pure stock investment is not free of the downside risk of losing money. Hence it is not possible to perfectly smooth the negative short-run fluctuations of stock returns over long horizons and simultaneously to keep expected excess returns with certainty. Consequently, assets with low volatility and low expected returns, like bonds, are not superfluous in the design of long-term saving products. Insurance contracts covering the shortfall of a principal guarantees are not costless for a pure stock investment, even for long investment horizons (see Lachance/Mitchell 2002).

For low volatility assets like a portfolio of government bonds, the probability and the severity of losing money decreases over time. Over long investment horizons, the price to insure the downside risk of a principal guarantee for a pure bonds investment is very low (close to zero). Hence, bond pension plans are very effective vehicles for producing principal guarantees. However, this does not mean that with a pure bond pension plan, the economic costs of downside protection is zero. Providers of bond based IPA's must give up a substantial part of

the upside returns that are possible with stocks. From an *ex ante* point of view, a measure of these economic (hedging) costs - in the sense of a smaller upside potential – can be seen as the difference between the expected return of both investment vehicles.<sup>18</sup>

### **3. Regulatory Framework of Money-Back Guarantees for IPA: The Case of Mutual Funds**

The money-back guarantee as described in the German Certification Act can be represented as a fixed liability of the provider, when it issues the IPA. If the cash value of the financial assets backing the liabilities at the beginning of the retirement phase is lower than the sum of the contributions paid into the policy, the provider must fill the gap with equity capital. From this point of view, it is clear that the money-back guarantee should be subject to solvency regulation.

Saving products offered by commercial banks (e.g. saving accounts) or insurance companies (e.g. life insurance products) in Germany are usually designed (at least in part) with fixed interest rates. However, this is not the case for mutual funds. The fundamental idea of a collective investment scheme such as a mutual fund is to collect money from many private investors via the offering of fund units, and to invest this money in a well-diversified portfolio of stocks, bonds, and/or real estate. The units of the mutual fund are liquid in the sense that they are traded on an active secondary market (e.g. for so-called exchange-traded-funds) or investors can ask for redemption of their holdings to net-asset value prices at any point in time. The investment management company usually assumes no obligation other than that of investing the funds in a reasonable and prudent manner, solely in the interest of the investors. It provides no guarantees with respect to a rate of investment return. Hence, the investor bears all capital market risk and receives the full reward of the financial asset that backs the mutual fund units. Because the balance sheets of mutual fund providers are not exposed to financial market fluctuations, they are excluded from risk-based solvency capital regulation requirements in Germany, in contrast to insurance companies and commercial banks.<sup>19</sup>

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<sup>18</sup> Despite the fact that the expected return is the most common measure of the “reward”, “return” or “value” of financial investments it is – especially in a downside risk context - possible to measure the upside potential more directly, c.f. Holthausen (1981) or Albrecht/Maurer/Möller (1998).

<sup>19</sup> According to German Investment Company Law (KAGG), the minimum equity capital for investment fund management companies (i.e. the provider of the pension products) is € 2.5 Millions.

By contrast, if the provider of an IPA is an investment management company which uses its own mutual funds, the German Federal Banking Supervisory Authority (BAKred)<sup>20</sup> requires (conditional) solvency capital because of the statutory “money back” guarantee. This solvency requirement, published in December 2001, can be modelled in the following way. Let  $V_t$  denote the cash value of an IPA at time  $t$ , and let  $P_t$  be the sum of the contributions (including all extra payments by the government) paid into the policy until time  $t$ . Furthermore, let  $r_f(t, T) =: r_{f,t}$  be the yield at time  $t$  on a zero coupon bond maturing at time  $T$  (i.e. the planned age of retirement), taken from the current term structure of German interest rates.

For each IPA, the investment management company must build solvency capital equal to 8% of the total contributions paid into the plan<sup>21</sup> only, in each period  $t$  in which the risk-adjusted cash value of the policy is lower than the present value of the contribution:

$$\frac{V_t}{\exp(2.33\sigma)} \leq \frac{P_t}{(1 + r_{f,t})^{T-t-1}}. \quad (5)$$

In this formula<sup>22</sup>  $\sigma$  stands for the monthly volatility of the mutual fund units backing the pension account. The volatility must be estimated from historical time series returns of the fund unit prices using a window between two and five years. If the policy consists of more than one type of mutual fund (e.g. equity and bond funds),  $\sigma$  is computed as the weighted sum of the individual fund volatilities according to the current asset allocation of the policy.

The economic rationale behind this formula is as follows. At every point in time, the IPA issuer has the safe investment alternative of investing some part of the contributions in zero bonds, so that at the end of the investment period at time  $T$  the proceeds would equal the participant’s contributions during the accumulation phase. The necessary amount to meet the total contribution guarantee of  $P_t$  at time  $t$  is  $P_t / (1 + r_{f,t})^{T-t}$ , which is the right hand side of formula (5). If the provider does not use zero bonds, but instead employs only stocks to back the IPA, nothing happens as long as the cash value of the policy is “substantially” higher than the present value of the contributions. “Substantially” higher means, under the German

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<sup>20</sup> Since Mai 2002 the BAKred is a Department of the new German Federal Financial Supervisory Agency.

<sup>21</sup> In addition to the solvency equity capital, investment management companies must build supplementary reserves, if the difference between the present value of the contributions and the risk adjusted cash value of the policy exceeds 8% of the total contributions.

<sup>22</sup> The formula is explained in more detail in the appendix

solvency rule, that given a current cash value of  $V_t$  there is a probability of only 1% (note 2.33 is the 99% quantile of the standard normal distribution) that the uncertain cash value of the policy one month later  $V_{t+1}$  is lower than the present value of the contributions. This explains the risk adjustment on the left hand side of the solvency formula.

Hence, without capital requirements, an underfunding of the principal liability during the accumulation period is possible. The amount to which such an underfunding is allowed depends on the volatility of the pension assets and the time remaining to the end of the accumulation period. For example (see Table 1), if the monthly returns of the pension assets have a volatility of 7.22% per month which annualized is about 25% per year (a typical value for German stock funds), the risk-free interest rate is 4% per annum, and the remaining accumulation period is 30 years (360 months), then the critical level is only 35.8%. This means that as long as the cash value of the policy exceeds 35.8% of the contribution paid into the plan, no risk-based-solvency capital is necessary. If the time to retirement is only five years (60 months), the critical level increases to 97.2%. However, the provider has the possibility of reducing the volatility of the IPA and the possible amount of underfunding by investing more of the pension assets in low volatility assets such as bonds.

**Table 1: Critical Level of Under Funding (as % of Contributions) with Respect to the Solvency Formula (5)**

End of plan (Years)	Volatility (per month)								
	<b>0.29%</b>	<b>0.58%</b>	<b>0.87%</b>	<b>1.15%</b>	<b>1.44%</b>	<b>2.89%</b>	<b>5.77%</b>	<b>7.22%</b>	
<b>30</b>	30.5%	30.7%	30.9%	31.1%	31.3%	32.4%	34.6%	35.8%	
<b>25</b>	37.2%	37.5%	37.7%	38.0%	38.2%	39.5%	42.3%	43.7%	
<b>20</b>	45.4%	45.8%	46.1%	46.4%	46.7%	48.3%	51.6%	53.4%	
<b>15</b>	55.5%	55.9%	56.2%	56.6%	57.0%	59.0%	63.1%	65.2%	
<b>10</b>	67.8%	68.2%	68.7%	69.1%	69.6%	72.0%	77.0%	79.6%	
<b>5</b>	82.7%	83.3%	83.8%	84.4%	85.0%	87.9%	94.0%	97.2%	
<b>3</b>	89.6%	90.2%	90.8%	91.4%	92.0%	95.2%	101.8%	105.3%	
<b>2</b>	93.3%	93.9%	94.5%	95.2%	95.8%	99.1%	106.0%	109.6%	
<b>1</b>	97.1%	97.7%	98.4%	99.0%	99.7%	103.1%	110.3%	114.1%	

Source: Authors' Computations

In summary, with an appropriate asset allocation and depending on the age of the participant, it is possible for the provider of mutual fund-based IPA to avoid capital requirements without jeopardizing the credibility of the principal guarantee. However, the burden of such a

conditional solvency system is the implementation of an efficient risk monitoring system for each IPA.

#### **4. Hedging Costs and Hedging Effectiveness of Mutual Fund Products**

In view of our results concerning the long-run risks of pure stock investments, and given the regulatory environment placing a significant capital charge on a fund with too much shortfall risk, it is clear that a sensible strategy for a mutual fund must contain some element of risk management or hedging. As mentioned above, the problem is to provide sufficient credibility for the promised payments (hedging effectiveness) while at the same time reducing the upside potential of the investment as little as possible, to keep hedging costs low. Note that the term “hedging costs” refers neither to the regulatory capital the mutual fund company has to provide nor to potential expenditures for the purchase of derivative contracts. The only source of hedging costs for the products considered below is a reduction in average expected wealth or, equivalently, in the total return on the contributions paid into the IPA.

Because of the substantial positive correlation of the financial assets backing the pension plans, an IPA provider cannot manage the risk resulting from guarantees by using traditional insurance pooling techniques.<sup>23</sup> Hence it is necessary to manage underlying risk of the principal guarantee for each IPA individually.

##### *4.1. Methodology*

We focus on products currently offered by German mutual fund companies, and we compare them to the simple strategies of investing in stocks or bonds exclusively. In total, we analyze five strategies with respect to their long-run risk-return profile:

a. **Pure stock strategy**

This strategy was discussed above with respect to its long-run risks. Given the parameter values used in our simulation study, this strategy will likely produce the highest expected wealth at the end of the investment period. On the other hand, this strategy can be quite costly for the mutual fund company if it must put up substantial solvency capital to render credible its payment promises.

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<sup>23</sup> See also Lachance/Mitchell (2002)

b. Pure bond strategy

A pure bond strategy follows the opposite approach. To reduce the risk of falling short of the promised wealth at the end of the accumulation period, this strategy invests only in bonds or broadly diversified government bond portfolios. One might expect that this reduces or even completely eliminates the shortfall risk, but this benefit also comes at the cost of lower expected returns.

c. Static portfolio strategy

This strategy is a mixture of the pure bond and the pure stock strategy. The portfolio remains unchanged over the whole period, and it contains both stocks and bonds from the start. With reference to the typical asset allocation of German retirement funds (AS-Funds)<sup>24</sup> our simulations for the 15-year horizon use an equally weighted stock and bond portfolio, whereas for the 30-year investment period we use 75% stocks and 25% bonds.

d. Life-cycle strategy

Popular advice often given to investors is to alter the portfolio composition with age. People are usually advised to hold a larger share of the portfolio in stocks when young, and then to shift into bonds later on. The idea behind this strategy is that it would be hard to compensate unfavorable movements on the stock market occurring late in the accumulation period, since little time is left, so that this type of risk could be avoided by investing in bonds. The life-cycle strategy is an unconditional ‘hedge’ in the sense that more volatile return opportunities are generally considered too dangerous late in the investment period, irrespective of the performance of stocks before the rebalancing date. We implement this strategy by defining fixed points in time at which the portfolio composition is changed, with more and more weight on bonds instead of stocks. The exact dates and compositions are as follows: For an investment horizon of 15 years the plan is assumed to start with 40 percent of the allocations going into equity and 60 percent into bonds. After five years this allocation changes, and for the remaining time 10 percent go into stocks and 90 percent into bonds. In the case of a 30 year plan there is an initial period of ten years with pure stock investment, followed by five years with an allocation of 70 percent equity and 30 percent bonds. After another five years the allocation of the contributions is again changed to 40 percent equity and 60 percent bonds. Over the remaining ten years 90 percent of the contributions go into bond funds and the remaining 10 percent into stocks.

e. Conditional hedging strategy

This strategy aims at combining the performance advantage of a pure stock strategy with the risk-reducing effect of a pure bond strategy. As opposed to the life cycle strategy, however, the decision to shift from one investment into the other is not driven by an exogenous variable like age, but rather by the performance of the respective investments. For this reason, such a strategy represents a “conditional hedging” approach. Usually one starts out with a pure stock investment and shifts to bonds as soon as a certain critical level of wealth is reached. In this case, subsequent contributions go in bonds until the safety level is again exceeded, when the strategy switches back to a 100 percent stock investment. An important parameter for this type of strategy is the critical level of wealth at which the investment rule (for subsequent contributions) changes. To link this critical

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<sup>24</sup> AS-Fonds (Altersvorsorge-Sondervermögen) are special mutual fund products regulated in the Investment Management Company Act, which the German government introduced in 1998 for retirement saving. In contrast to usual balanced funds, AS-Fonds can invest into real estate, require a saving plan of at least 18 years, and are subject to some quantitative investment restrictions. For more details see Laux/Siebel (1999).

value to the intervention line set by the regulatory authorities in Germany (see formula 5), we set the critical level of wealth (as an example) to 75% above the intervention value defined according to the solvency formula (5).

A possibility not discussed up to now is the use of derivative assets to protect the value of an investment plan against shortfall risk. The appropriate instrument here would be a put option on the value of the plan with a strike price equal to the sum of the nominal payments. However, the application of put options in this context is not without problems. First of all, due to the very long maturity of the savings plans, any option would be very expensive, and the cost would have to be paid up front, i.e. at the beginning of the accumulation period, which raises the financing question. Second, it seems unlikely that a put with such a long time to maturity would be offered at all, so that a roll-over strategy would become necessary, with all the risks involved in terms of prices and liquidity. Third, for the put option to be of real value to the institution holding it, it would have to be made sure that the seller can actually cover its liabilities at the end of the accumulation period. This does not seem without problems in practice, so that there would always be doubts concerning the actual risk-reduction potential of such an option. Finally, there is a significant operational problem in using put options, since all the accounts have to be protected individually. This means that for every IPA, the provider would have to hold a put option with the appropriate strike price and time to maturity. This seems by far too costly and complicated for the typical institution, so that hedging strategies using ‘physical’ financial derivatives will not be considered further in the following analysis.

We analyze the five strategies described above in terms of wealth levels (or total returns) and required regulatory capital. Since there are no closed-form expressions for the statistics of interest, we use Monte Carlo simulation to generate a large number of paths for the evolution of the savings plans. The relevant statistics for total returns (relative to a benchmark) and regulatory capital are then evaluated on the basis of these scenarios. The key ingredient in such a simulation is a suitable model to describe the dynamics of the relevant funds and the short rate of interest. For the funds we use the standard capital market model, representing asset price movements by means of correlated Wiener processes. The dynamics of the short rate are given by the one-factor model suggested by Cox/Ingersoll/Ross (CIR, 1985). While we assume constant correlations between the risk factors, the covariances will vary due to the fact that the conditional standard deviation for the short rate will in general not be equal to the unconditional value.

The time series used to estimate the process parameters (mean returns, volatilities, correlations) are the monthly log returns of the German stock index DAX representing the stock index fund, the log returns of the bond performance index REXP as the bond index fund as well as the 1-year interest rate as a proxy for the short rate. Parameters were estimated via a maximum-likelihood approach, the estimates are presented in table 2 and 3. As discussed above we subtracted the equivalent of 0.5% p.a. from the monthly average return on the investments to take potential administration costs into account.

**Table 2: Descriptive Statistics for Risk Factors 1:1973-12:2001**

Asset	Mean (% p.m.)	Volatility (% p.m.)	Correlations	
			Stocks	Bonds
Stocks	0.7967	5.5800	1	0.2051
Bonds	0.5683	1.1200		1

Source: Authors' Computations

**Table 3: Descriptive Statistics for the Short Rate Process 1:1973-12:2001**

Kappa	Theta	Sigma	Correlations Innovations with	
			Stock Returns	Bond Returns
0.1494	0.0539	0.0511	0.1417	-0.7009

The process estimated is the CIR process  $dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$ , where Kappa ( $\kappa$ ) is the speed of mean-reversion, Theta ( $\theta$ ) is the long-run mean of the short rate, and Sigma ( $\sigma$ ) is the volatility of changes in the short rate.  $dW_t$  is the increment of a standard Wiener process.

Source: Authors' Computations.

The CIR process is very popular in interest rate modeling. This is mainly due to the fact that it is able to generate both mean-reversion in interest rates as well as non-negative rates with probability one. Since the process exhibits mean-reversion, the sign of the drift component (i.e., the expected change in the short rate over the next time interval) depends on whether the process is currently above or below its long-run mean. How quickly the process reverts back to this long-run mean is determined by the speed of mean-reversion. In table 3  $\kappa$  (kappa) represents this speed of mean reversion,  $\theta$  (theta) stands for the long-run mean of the interest rate, while  $\sigma$  (sigma) denotes the volatility of changes in the short rate. The market price of interest rate risk was set equal to zero for reasons of simplicity.

To ensure stability of the simulation results, we base our analysis on 3,000,000 simulations for each of the respective strategies. We then compute

- statistics related to hedging costs: the mean of the total return generated by the respective strategies for the different points in time (months)
- statistics related to hedging effectiveness: the shortfall risk and the required regulatory capital for the respective strategies.

The model assumes equal contributions into the plan occurring at the beginning of each month, and a front-end load of 5% proportional to the unit price for the stock fund and 3% for the bond fund. These loads are comparable to the current German mutual fund fee structure.  $V_{i,t}$  denotes the uncertain total wealth of the IPA in month  $t$  ( $t = 1, \dots, T$ ) in simulation run  $i$  ( $i = 1, \dots, n$ ), and  $z_{i,t}$  represents the critical level of wealth in month  $t$  according to formula (5) determined by the federal banking supervisory authority (BAKred).<sup>25</sup> If  $P_t$  represents the sum of payments into the plan until time  $t$ , the average compounded (multiyear) return ( $EW_t$ ) of the IPA at the end of month  $t$  is given by:

$$EW_t = \frac{1}{n} \sum_{i=1}^n \frac{V_{i,t} - P_t}{P_t}. \quad (6)$$

The probability of a solvency capital charge ( $CP_t$ ) in month  $t$  is estimated by:

$$CP_t = \frac{1}{n} \sum_{i=1}^n \max[z_{i,t} - V_{i,t}]^0 I_{(-\infty, z_{i,t})}(V_{i,t}), \quad (7)$$

where the indicator variable  $I_{(a, b)}(X)$  is equal to one if  $X \in (a, b)$  and zero otherwise. The mean solvency capital charge ( $MC_t$ ) at time  $t$  month after the beginning of the plan normalized by the sum of the contributions  $P_t$  paid into the IPA, is given by

$$MC_t = \frac{1}{n} \sum_{i=1}^n \frac{C_{i,t}}{P_t}. \quad (8)$$

According to the regulatory authorities, the solvency capital charge  $C_{i,t}$  depends on how far the mutual funds based IPA wealth falls short of the critical level. The rule says that the capital charge is at least 8% when wealth falls below the critical level. If the amount of the shortfall exceeds 8%, the capital charge is increased accordingly to cover the gap. Hence  $C_{i,t}$  must to be calculated according to the following formula:

$$C_{i,t} = \begin{cases} 0.08 \cdot P_t & 0 < 1 - V_{i,t} / z_{i,t} \leq 0.08 \\ (1 - V_{i,t} / z_{i,t}) \cdot P_t & 1 - V_{i,t} / z_{i,t} > 0.08 \\ 0 & 1 - V_{i,t} / z_{i,t} < 0 \end{cases}. \quad (9)$$

<sup>25</sup> C.f. BAKred (2001).

The mean conditional capital charge ( $MCC_t$ ) at month  $t$  given that a capital charge has occurred is computed according to:

$$MCC_t = MC_t/CP_t. \quad (10)$$

## 4.2. Results

Our results for the expected total return of savings plans based on the different investment strategies are given in Table 4. It is no surprise that the pure stock strategy does best in terms of this measure, since stocks have the highest expected monthly return. This also causes the differences between the respective strategies to increase with time. Nevertheless, it is interesting to note how close the conditional hedge strategy comes in terms of expected total return. Even after 30 years, the difference to the pure stock strategy is only about 3.5 percent of the contributions paid. This can be taken as a first indication that this type of strategy might be an interesting compromise between the return potential of a pure stock strategy and the risk-avoiding property of a pure bond approach.

**Table 4: Expected Total Return (in % of contributions)**

The table gives the expected total compounded return for the different IPA plans at different points in time for a 30 year accumulation period (numbers in parentheses are for an accumulation period of 15 years). For example, an entry of 16.26 for the 100% bond strategy in year 5 means that the value of an IPA with an investment horizon of 30 years is after five years on average 16.26% higher than the sum of the contributions over the first five years.

Year	1	5	10	15	30
100% bond	0.80 (0.80)	16.26 (16.26)	40.17 (40.17)	70.67 (70.67)	225.38 (—)
100% stock	1.38 (1.38)	29.09 (29.09)	78.78 (78.78)	154.06 (154.06)	731.60 (—)
Static	1.72 (1.08)	26.31 (22.44)	68.79 (58.03)	130.38 (107.36)	554.59 (—)
Life-cycle	1.38 (1.03)	29.09 (21.39)	78.78 (46.73)	140.13 (81.36)	384.93 (—)
Cond. Hedge	1.38 (1.33)	29.09 (26.07)	78.77 (67.40)	153.93 (126.54)	728.06 (—)

Source: Authors' Computations

Nevertheless, expected wealth is just one measure to be considered; any sensible comparison of the given products must also focus on risk measures. The risk of the different strategies is measured by the regulatory capital charge that the mutual fund company adopting these strategies would face. Table 5 indicates that, for an investment horizon of 30 years, no regulatory capital is needed over the first five years for any of the strategies. The pure bond

strategy can even be regarded as entirely risk-free with respect to regulatory capital charges. The life-cycle approach also exhibits very low capital charges on average. This strategy seems to be an interesting alternative to a pure bond investment, given its advantage in terms of expected return.

**Table 5: Mean Regulatory Capital Charge (as % of Contributions)**

The table gives the average regulatory capital that has to be put up for the different IPA plans at different points in time for a 30 year accumulation period (numbers in parentheses are for an accumulation period of 15 years). For example, an entry of 0.04 for the static strategy in year 30 means that in this year on average 0.04 percent of the contributions made over 30 years have to be provided as regulatory capital.

Strategy	Year				
	1	5	10	15	30
100% bond	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
100% stock	0 (0)	0 (0.07)	<0.01 (0.68)	0.01 (1.67)	0.28 (—)
Static	0 (0)	0 (0)	0 (0.01)	0.01 (0.08)	0.04 (—)
Life-cycle	0 (0)	0 (0)	<0.01 (0)	<0.01 (<0.01)	0 (—)
Cond. Hedge	0 (0)	0 (<0.01)	<0.01 (0.01)	<0.01 (0.06)	0.08 (—)

Source: Authors' Computations

As expected, the pure stock strategy balances its high return potential with an 'expensive' regulatory capital level. It requires more than three times the regulatory capital than the conditional hedge strategy, which is in second place with respect to this criterion. Furthermore, Table 5 provides insight into the impact of the investment horizon. Long-term strategies generally exhibit lower risk than the 15-year plans with the only exception being the life-cycle strategy. The fundamental reason for longer-term strategies requiring less regulatory capital than shorter-term strategies is the discounting embedded in the critical solvency ratio set by the regulatory authorities. This means that the required minimum wealth level of a plan is the lower the longer the remaining time to maturity of the plan. For the life-cycle strategy, however, there are two effects to be considered. For the shorter horizon plan, the period of pure stock investment is rather short, so the risk in general decreases. To take the most pronounced example for the usual impact of the investment horizon, consider the pure stock strategy. The average capital increases dramatically compared to the 30 year-plan, so that this approach looks very costly when it is implemented over this short investment horizon.

The probability that the mutual fund company will be required to put capital aside is given in Table 6, and the results are qualitatively similar to Table 5. The pure bond strategy never forces the provider to put up capital, whereas the pure stock does with a significant likelihood. Again, time is an important factor here. For the 30 year horizon, the probability of a capital charge for any strategy never exceeds 1.4 percent, but for a 15 year pure stock plan, this probability is almost 9 percent at the end of the investment period. For the other strategies, the ratios of 15-year to 30-year probabilities are also quite high, so that if a plan actually exhibits the risk of a capital charge, this risk tends to increase for shorter horizons.

**Table 6: Probability of a Regulatory Capital Charge (in %)**

The table gives the average frequency (or probability) of the event that regulatory capital has to be put up for the different IPA plans at different points in time for a 30 year accumulation period (numbers in parentheses are for an accumulation period of 15 years). For example, an entry of 0.07 for the 100% stock strategy in year 15 means that in this year in 0.07 percent of the cases regulatory capital had to be provided. Numbers in parentheses represent results for an investment horizon of 15 years.

Strategy	Year				
	1	5	10	15	30
100% bond	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
100% stock	0 (0)	0 (0.67)	<0.01 (4.78)	0.07 (8.98)	1.40 (—)
Static	0 (0)	0 (0)	0 (0.06)	<0.01 (0.74)	0.25 (—)
Life-cycle	0 (0)	0 (0)	<0.01 (0)	<0.01 (<0.01)	0 (—)
Cond. Hedge	0 (0)	0 (<0.01)	<0.01 (0.08)	0.01 (0.62)	0.64 (—)

Source: Authors' Computations

We also note that there is an important difference between shortfall probability and the probability of a regulatory capital charge. As shown above, the shortfall probability of a pure stock investment actually falls with a longer investment horizon, whereas the probability of a capital charge goes up. Again this is due to the fact that the critical level of wealth set by the German regulatory authorities contains a discounting component. Thus this critical level will go up with decreasing time to maturity, thereby causing a higher likelihood for a capital charge.

The average conditional regulatory capital charge depicted in Table 7 shows how much capital will be needed given that the cash value of the IPA falls below the critical BaKred value. Note that when the empirical probability of a capital charge is zero, this measure is not

defined. The Table shows results qualitatively similar to the general long-run risk-return profile of the various asset classes. The risk of a pure stock strategy becomes obvious, since if regulatory capital is needed, it will probably be a significant amount. For example, at the end of 30 years the mutual fund company would need on average almost 20 percent of the contributions as regulatory capital in those scenarios where wealth falls below the critical BaKred value. The benefits of flexibility become obvious when the static strategy is compared with the conditional hedge. The conditional hedge produces, on average, higher wealth over the whole investment period and the average conditional regulatory capital is also lower. So if one were to compare the different products on the basis of these two measures only, the static strategy would be dominated. Note, however, that the probability of a capital charge is lower for the static strategy.

**Table 7: Mean Conditional Regulatory Capital Charge (as % of Contributions)**

The table gives the average conditional regulatory capital that has to be put up for the different IPA plans at different points in time for a 30 year accumulation period (numbers in parentheses are for an accumulation period of 15 years), i.e. the average amount of regulatory capital that is necessary, given that regulatory capital has to be put up at all. For example, an entry of 9.00 for the conditional hedge strategy in year 15 means that in this year on average 9.00 percent of the sum of contributions over the first 15 years had to be provided as regulatory capital in those cases where capital had to be put up at all. Note that this number is not defined ('n.def. '), if the empirical probability of having to provide regulatory capital is equal to zero.

Strategy	Year				
	1	5	10	15	30
Pure bond	n.def. (n.def.)	n.def. (n.def.)	n.def. (n.def.)	n.def. (n.def.)	n.def. (n.def.)
Pure stock	n.def. (n.def.)	n.def. (9.92)	9.46 (14.31)	11.59 (18.63)	19.90 (—)
Static	n.def. (n.def.)	n.def. (n.def.)	n.def. (8.75)	9.31 (10.17)	14.71 (—)
Life-cycle	n.def. (n.def.)	n.def. (n.def.)	9.46 (n.def.)	8.04 (8.00)	n.def. (—)
Cond. Hedge	n.def. (n.def.)	n.def. (8.06)	8.00 (8.90)	9.00 (9.95)	13.29 (—)

Source: Authors' Computations

The analyses thus far focus on the statistical output, but it is also important to assess the administrative costs generated by each plan. Cost will not be major for the strategy products such as the pure bond, the pure stock plans, the static, and the life-cycle strategies. However, for the conditional hedge, the need to shift incoming distributions across asset classes depending on how much wealth has been accumulated in the plan, might imply considerable administrative effort. It is therefore of interest to examine the relative frequency of shifts from

one asset class to the other, when a conditional hedge strategy is run. Here again, time is an important factor, since for the 15-year plan the mutual fund company must change the asset class in 98 percent of the paths generated by the simulation, whereas this need arises in only 26 percent of the cases for the longer horizon. In any case, costs must be taken into account when strategies are compared with respect to their practical application.

In summary it is not possible to identify the overall dominating investment product or strategy. With a few exceptions, higher potential in terms of average wealth usually comes at the cost of higher regulatory capital. It is important in this context to look at the average amount of regulatory capital conditional on the event that capital actually has to be put aside. Here it becomes obvious that strategies with a fixed stock investment can produce significant risks. This risk is mitigated when the conditional hedge strategy is employed. Nevertheless, additional administrative cost must be considered.

## **5. Conclusions**

Due to the severe financing problems of standard pay-as-you-go pension systems in many countries alternative vehicles for retirement financing have to be developed. In Germany such a new system was recently installed when the German Retirement Saving Act was passed by the legislative body. The government offers significant tax relief for investment products meeting certain requirements, the most important of these being a guarantee offered by the company selling the product that the cash value of the IPA at the end of the accumulation period is at least as high as the nominal sum of the contributions. To lend sufficient credibility to the payment promises made by institutions providing investment products for these savings plans, the regulatory authorities in Germany have imposed a capital charge in case the value of the savings plan falls below a certain critical level.

At first sight it seems that in order to implement such a principal guarantee complicated and expensive financial products, like derivatives, are needed. However, as we have shown in this paper, there are other ways of achieving a sometimes practically risk-free position without using options or similar instruments. We analyze in detail various strategies, which aim at combining the potentially return-increasing properties of equity investment with the risk-

reducing characteristics of bond investments. These strategies are (in principal) actually implemented in products offered in Germany, so that their analysis is a real-world application of the tools and methods of capital market theory. Of course, the trade-off between return and risk is always at the core of the analysis. However, in the context of this paper it is important to recognize that variance is not the most important measure of risk. As opposed to a more traditional approach we consider shortfall and the need of regulatory capital as the two most important types of risk.

The strategies analyzed in this paper range from simple pure bond or equity investments over mixed equity-bond funds and products offering a change in portfolio composition at pre-defined points in time to highly sophisticated products with conditionally changing investment styles. One of the key results of our study is that these dynamic strategies, switching from stocks into bonds whenever the value of the savings plan falls below some critical solvency ratio set by regulatory authorities, perform for long investment horizons rather well in terms of expected total returns. They come close to pure stock investments with respect to the average value they generate for the investor. However, it is also very important for the financial institution to keep an eye on the expected amount of regulatory capital required by a certain investment strategy. Due to the conditional change in allocation when the critical regulatory value is reached, the expected capital charge is significantly smaller than in the case of a pure equity investment.

Besides the basic type of strategy, the length of the investment horizon is an important factor for the risks and rewards of alternative strategies. In general, the longer the maturity of the plan, the lower the expected capital charge, since the critical level set by the authorities in Germany contains a discount factor, and the higher the expected total return. Nevertheless, it is important to consider other risk variables as well in this

We are far from claiming that one of the strategies discussed in this paper should be seen as uniformly superior to any other. It was our intention to point out the benefits and risks offered by the different types of products to provide a basis for a thorough discussion of the issues involved in product design and regulation.

## Appendix: Derivation of the Solvency Formula

Consider an investment plan where payments into an IPA are made at equally spaced points in time  $t = 0, 1, \dots, T$  (e.g. months). Let  $P_t$  denote the sum of payments up to time  $t$ ,  $T$  the planned terminal date of the plan (equal to the beginning of the payout phase), and  $q(r_{f,t}, T-t) = (1+r_{f,t})^{t-T}$  the discount factor with risk-free rate  $r_{f,t}$  and remaining time to maturity  $T-t$ . Without loss of generality we assume that the investor holds exactly one share of the fund at time  $t$ . We are interested in the solvency ratio  $V_t / P_t$  at time  $t$ , which makes sure that the uncertain market value of the shares  $V_{t+1}$  at time  $t+1$  is less than the sum of payments  $P_t$  into the plan discounted up to time  $T$ , i.e. less than  $P_t q(r_{f,t}, T-t-1)$ , with a probability of at most  $\varepsilon$ .

To be able to quantify this shortfall risk we have to specify a model for the random evolution of the value of the investment shares. Here we make the standard assumption that the dynamics of this value can be described by a geometric Brownian motion. This implies that the relative change in value (i.e. the log-return)  $\ln(V_{t+1}) - \ln(V_t)$  is normally distributed with mean  $m$  and variance  $s^2$ . Formally we obtain the desired solvency ratio as the solution of the following inequality:

$$\text{Prob}[V_{t+1} < P_t q(r_{f,t}, T-t-1)] = \text{Prob}[\ln(V_{t+1}) < \ln(P_t q(r_{f,t}, T-t-1))] \leq \varepsilon \quad (\text{A1})$$

Using the above distributional assumption inequality (A1) is equivalent to

$$\ln(V_t) + m \leq \ln(P_t) + \ln[q(r_{f,t}, T-t-1)] + N_{1-\varepsilon} s, \quad (\text{A2})$$

where  $N_{1-\varepsilon}$  is the  $(1-\varepsilon)$ -quantile of the cumulative standard normal distribution. Under the additional (conservative) assumption<sup>26</sup> that the one-period expected return is equal to zero (i.e.  $m=0$ ) inequality (A2) can be written as

$$V_t / \exp[N_{1-\varepsilon} s] \leq P_t q(r_{f,t}, T-t-1). \quad (\text{A3})$$

Setting  $N_{1-\varepsilon} = 2.33$ , which implies a tolerated shortfall probability of not more than 1%, this represents the equation for the solvency ratio (5) presented in the main text.

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<sup>26</sup> This assumption is indeed conservative, since it increases the shortfall probability compared to the common case  $m > 0$ . Furthermore it is no longer necessary to estimate expected returns (e.g., from historical time series), which are subject to much larger estimation risks than volatilities. See for example Merton (1980).

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