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A Synergetic Approach to the Dynamics of Financial Markets

Synergetics,[1] a fascinating interdisciplinary science initially proposed by Hermann Haken in the late 1960s, is a framework for understanding the interaction effects of very large complex systems, with an emphasis on explaining how self-organized macroscopic phenomena can emerge as a result of these underlying interactions. An especially exciting aspect is that entirely new and distinct properties of the system can emerge somewhat spontaneously. The approach has seen great success in a host of fields ranging from physics and chemistry to brain science and economics.

Commonalities of synergetic systems are such that they are open systems consisting of very many interacting sub systems. They are far from equilibrium, and there are often dynamics that emerge on different time scales so that slower moving patterns, or 'modes' can enslave or dominate, the faster moving dynamics. These dominant modes show up as the macroscopic organized patterns or behavior of the system and are therefore called order parameters. Self-organized macroscopic structure is not always present in the system but is usually brought on when an external variable called the control parameter (such as for example the temperature, energy flux, or some other characteristic of the environment) surpasses a critical threshold.

The most often used example to elucidate the general concepts of synergetics is that of laser light. Here, the interacting subsystems are the excited electrons of the atoms. All else equal, these will each transition form an outer to inner orbit independently of the others, and in aggregate white light will be emitted. But if energy is pumped into the system above a certain threshold, a coherent field emerges which interacts with the electrons in such way that they follow the coherent wave and all make lock-step transitions so that the light emitted is of the same wavelength, namely laser light. Another example comes from fluid dynamics. If a pan of oil is heated slowly from below, under certain conditions a hexagonal pattern will appear. It is almost as if the oil molecules know exactly where to line up in the pan to create this pattern. But in fact, it is again an example of a slower mode emerging under certain environmental conditions, which enslaves the faster dynamics of the system resulting in a macroscopic self-organized structure.

An additional interesting aspect of synergetics is the interplay between deterministic and random forces. Indeed, the presence of random fluctuations in the system allows for the macroscopic properties of the system to evolve into new (meta)stable states as the control parameter changes. What is actually happening is that the old state becomes unstable, much as a pencil balancing on its tip. The pencil can in principle stay on its tip if carefully set up, but small fluctuations will

1 See Hermann Haken: *Synergetics: An Introduction*, Berlin/New York, NY: Springer 1977.

act upon it and it will ultimately fall in a certain direction that in fact depends on the fluctuations. In a similar fashion, the presence and structure of noise play essential roles in driving the evolution of a synergetic system.

Sometimes, as in the case of the laser, the self-organizing dynamics of the order parameter can be derived from microscopic first principles. In many other cases, however, this is not possible and instead, a high level description of the stochastic dynamics of the order parameter suffices. In yet other cases, an intermediate level of description can be attained, starting not with the very basic, most finely grained dynamics of the sub-systems, but with a slightly higher level of system dynamics, referred to as the mesoscopic level is possible. In this paper, the cooperative dynamics of financial markets will be discussed in such a context.

Indeed, financial markets exhibit all the properties of a synergetic system: Traders can be seen as the sub-systems, who all interact with each other and with external information so as to ultimately give rise to dynamics which determine the price of traded assets. Dynamics of the individual assets exhibit universal features in the structure of the noise (or 'volatility') of the prices. Then, under certain conditions when fear or uncertainty in the environment become large, the financial markets appear to behave in a self-organized, highly correlated fashion, giving rise to market panic.

The questions of synergetics as it pertains to the dynamics of financial markets (which is the focus in the present contribution) is related to – yet quite different from – the tantalizing discussion on the role of synergetics and economics. In recent years, we find quite a few articles discussing this topic. Those models are primarily concerned with explaining how a national economy can be interpreted from the point of view of synergetics. It is perhaps interesting to note that many of these articles stem from German and European authors[2] who are, probably for obvious reasons of locality, more exposed to the general ideas of synergetics. In the US, for example, we see fewer of these discussions; I believe there is also a terminology issue which does not necessarily mean that studies along the lines of Haken's synergetics are not present, the author may be unaware of them or they may just not always be referred to by that name.

The Dynamics of Financial Markets

Financial markets represent perhaps some of the most complex systems in existence with the added pitfall that it is nearly impossible to perform experiments on or with them. In addition, the system is inherently highly non-stationary. The price formation process is clearly the result of both local and global interactions on a multitude of different time scales. Individuals all over the world (and, increasingly, their respec-

2 See Viktor Yakimstov: "Theoretical Basis for Synergetics of Economic Processes", in: *European Scientific Journal* 10 (2014) 4, pp. 83–89. Andreas Liening: "Synergetics – Fundamental Attributes of the Theory of Self-Organization and its Meaning for Economics", in: *Modern Economy* 5 (2014), pp. 41–847.

tive algorithms) post orders to buy or sell a particular stock at a particular price. The motivation can be purely need based and utilitarian, but it can also be purely speculative, as with a trader who holds the view that the stock will go up or down over a certain time horizon. Some traders care about time horizons of months or years, while others are solely interested in time horizons on the order of microseconds. Transactions are then cleared at a certain price at a given time, either by passing through the hands of a specialist on the trading floor or automatically on the many electronic markets which have sprouted in recent years. Most of these transactions have been recorded for at least two decades on a tick by tick basis, and this has resulted in an ever-growing database of, among other things, the historical prices and traded volumes of assets. The 'price' of a stock at a given time is considered to be the price at which the most recent transaction occurred. Note that in a fragmented market (such as the foreign exchange market) or in markets where the flow of information occurs at varying speeds, there is not necessarily one single definition of the price available, but this interesting feature is not one we shall not delve into here.

Apart from the fundamental properties of the company whose stock is being traded, factors such as supply and demand clearly must affect the price of stocks, as well as general trends in the particular industry in question. Stock specific events, such as mergers and acquisitions, have a big impact, as do world events, such as wars, terrorist attacks, and natural disasters. Recent examples of this are the dramatic events seen in 2007 and 2008 which were perhaps caused by fundamental flaws in our credit-based economy. The European debt crisis is another example, as is government intervention.

Stocks are, for the most part, traded on a central limit order book, such as the New York Stock Exchange. Modeling the intricate dynamics and micro-structure of this order book is a field of study which has gained traction in recent years.[3] If we borrow terminology from physics, this level of description could be seen as similar to the microscopic level. However, it is often more tractable to use a mesoscopic description which aims at describing the price process as a stochastic Langevin equation where the key feature is how to capture the volatility, or noise, that drives the process.

This is the most important effect since stock price changes (or returns) are essentially unpredictable from moment to moment. Therefore, the deterministic part of the equation is less interesting from a modeling point of view. This mesoscopic view fits well into the framework of description employed within the field of synergetics and constitutes the level of description that we shall explore here.

For many years and in a large body of the financial literature, the random nature of price time series was modeled as a simple Brownian motion by most. The first to

3 See Jean-Philippe Bouchaud/Yuval Gefen/Marc Potters/Matthieu Wyart: "Fluctuations and Response in Financial Markets: The Subtle Nature of Random Price Changes", in: *Quantitative Finance* 4 (2004), pp. 176–190. James Doyne Farmer/Austin Gerig/Fabrizio Lillo/Mike Szabolcs: "Market Efficiency and the Long-Memory of Supply and Demand: Is Price Impact Variable and Permanent or Fixed and Temporary?", in: *Quantitative Finance* 6 (2006), pp. 107–112.

propose such a model was Louis Bachelier in his thesis in 1900, which lay largely undiscovered until much later when Fischer Black and Myron Scholes wrote their famous paper in 1973 based on a very similar model.[4] They made important contributions in particular to the pricing of options, for which they received the Nobel Prize. Options are traded instruments that give the holder the right, not the obligation, to buy a stock at a later date at a certain price called the strike price. In Black's and Scholes's work, the log price is assumed to follow a Gaussian distribution. Even today, many trading assumptions and risk control notions are based off of that prior. However, in more recent years, there has been a large body of work documenting in quite some detail and with statistical accuracy (due largely to the vast amount of observations available) that the time series of financial market data show some intriguing statistical properties that deviate substantially from the Gaussian assumption. These features are referred to as stylized facts.[5] It is interesting to note that many of the stylized facts appear to be universal, in the sense that they are exhibited by a variety of financial instruments relating to commodities as diverse as wheat, currencies (such as the Euro-Dollar rate), and individual stocks. Some are also exhibited over various periods in history (and can therefore be seen as stationary); others are exhibited on multiple time scales (and so can be seen as self-similar).

Other interesting properties pertain to the dynamics and statistics of the cross-section of financial instruments. Ultimately, the goal is to comprehend and model the joint stochastic process of the price formation dynamics of the collection of stocks across time. Therefore, investigating various statistical properties both across time and across stocks is essential. The challenge then lies in coming up with a model that captures the dynamics inherent in the data. In addition, it is desirable that such a model be somewhat intuitive, parsimonious, and somewhat easy to use from a mathematical point of view.

Next follows a brief review of a class of models that have been proposed in recent years, aimed at modeling the stock price dynamics in such a way as to capture as many of the statistical properties of real financial data as possible.[6] It will be shown how these models could be used for important applications such as the pricing of options and other derivative instruments. In the first part of this article, the focus is on financial time series, and in the second part cross-sectional dynamics across a universe of stocks are discussed. In all cases, it becomes apparent that notions of nonlinear cooperative feedback appear to be essential ingredients in this very complex real-world system.

4 See Fischer Black/Myron Scholes: "The Pricing of Options and Corporate Liabilities", in: *The Journal of Political Economy* 81 (1973), pp. 637–654.

5 See Jean-Philippe Bouchaud/Marc Potters: *Theory of Financial Risks and Derivative Pricing*, Cambridge: Cambridge University Press 2004. Parameswaran Gopikrishnan/Vasiliki Plerou/Luís A. Nunes Amaral/Martin Meyer/H. Eugene Stanley: "Scaling of the Distribution of Fluctuations of Financial Market Indices", in: *Physical Review E* 60 (1999), pp. 5305–5316.

6 See Lisa Borland: "A Theory of Non-Gaussian Option Pricing", in: *Quantitative Finance* 2 (2002), pp. 415–431. Lisa Borland/Jean-Philippe Bouchaud: "A Non-Gaussian Option Pricing Model with Skew", in: *Quantitative Finance* 4 (2004), pp. 499–514.

Stylized Facts across Time

The time series of stock returns, defined as relative price changes or log price changes, typically exhibits intermittent clusters of higher versus lower magnitude returns, as shown in the bottom part of figure 1. This phenomenon is known as volatility clustering. As a consequence, the probability distribution of these financial returns is typically fat-tailed. In fact, it has been shown that the distribution of both intra-day and daily returns can be very well fit by a power-law tail of about -3 (referred to by some as the cubic law of finance).[7] This tail index is consistent with that of a Student's-t distribution with 3–5 degrees of freedom (equivalent to a so-called Tsallis distribution with index q = 1.4–1.5). This very same distribution fits to returns from a wide variety of financial instruments, such as stocks, currencies, and commodities, with much the same tail index (see figure 2). Because of this rather universal behavior, the non-Gaussian distribution is an important stylized fact. Therefore, any model of financial data should clearly try to capture at least this important feature. A related stylized fact is constituted by the observation that, as the time lag over which returns are calculated is increased, this power-law behavior of the distribution of returns persists for quite a while. In fact, the distribution only becomes Gaussian after the time lag approaches the order of a few months.[6] This feature is a signature of long-range memory being involved in the process, implying that while the random innovations from event to event are uncorrelated, they are not necessarily independent.

Volatility is typically defined as the square root of the squared variation of returns. Obviously, this definition is not unique because one can choose to calculate the squared variation over an arbitrary period of time. But as it turns out, the statistical properties of volatility are not that sensitive to the exact choice of a time frame. If financial data were Gaussian distributed, then the volatility would be the standard deviation of that distribution. In fact, this assumption is still made both by practitioners and in theoretical works in mathematical finance, although there is increasing awareness about the shortfalls of such an assumption. In practice, for example, the devastating effects of the Gaussian assumption were seen in August of 2007 and October of 2008, when investors panicked claiming that '25-sigma events' were wiping them out. In fact, these types of statements are only true if the underlying distribution is assumed to be Gaussian. If a heavy-tailed distribution is assumed instead, then the behavior of the markets as we have seen them in recent years is to be expected with a probability of about one or two extreme events per decade. Clearly, it is extremely important for hedging and risk control purposes to have a richer understanding of volatility.

The time series of the volatility of the Dow Jones index for the past century exhibits clear periods of lower and higher volatility typically clustered together. Note

7 See Xavier Gabaix/Parameswaran Gopikrishnan/Vasiliki Plerou/H. Eugene Stanley: "A Theory of Power-Law Distributions in Financial Market Fluctuations", in: *Nature* 423 (2003), pp. 267–270.

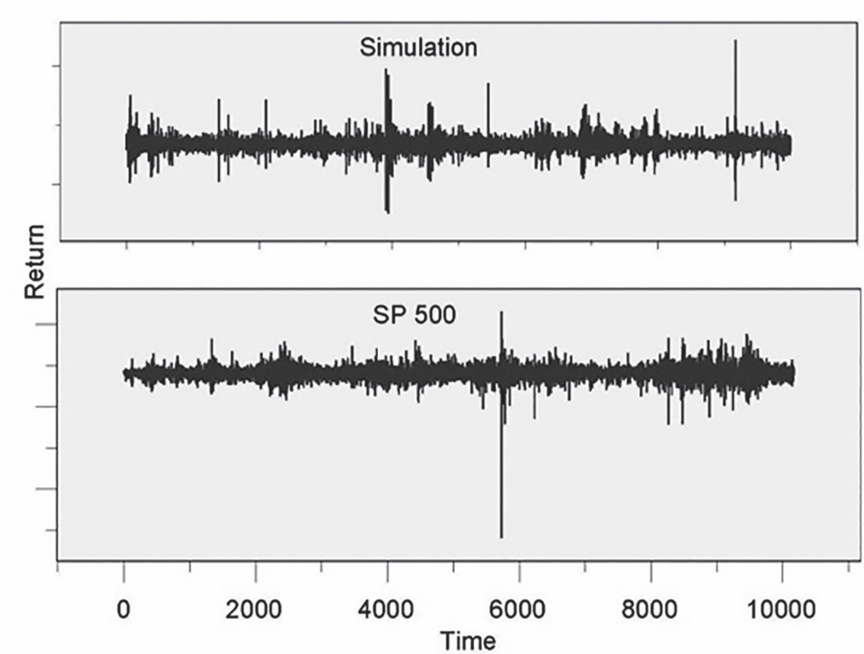

Figure 1: The time series of returns of the Dow Jones index since 1928 (bottom). The top shows the time series of returns resulting from a cooperative multi-timescale feedback model, which captures most of the interesting statistical features of real returns, such as the clustering of volatility.

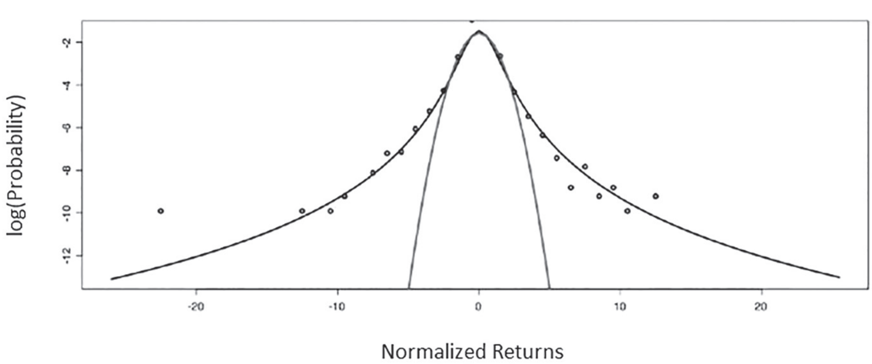

Figure 2: The empirical distribution of real market returns (dots) is shown together with a Gaussian distribution of the same standard deviation (red) and a fit with a fat-tailed Student's-t distribution with 5 degrees of freedom (Tsallis distribution of index q = 1.5) as shown by the black line.

that while the distribution of returns is fat-tailed, volatility itself follows a distribution that is close to log-normal. Interestingly, the same type of statistics is valid for volatility calculated either daily or every 5 minute within the day, implying a self-similar structure. The fact that volatility is self-similar on different time scales is a universal feature. The clustering feature that is observed is a signature that memory is inherent in volatility; if volatility is high, it will persist for a while. This memory can be quantified by looking at the how the autocorrelation of volatility persists over increasing lags in time. Causality is also a feature inherent in the structure of volatility. In other words, if you take a time series of volatility, the statistical properties of future volatility conditioned on the past are not the same as the properties of past volatility conditioned on the future. This reinforces the notion that volatility has a memory. There are some other interesting statistical features that relate to conditional volatility. Specifically, if you look at the probability of observing the volatility of a certain magnitude given that a volatility shock larger than a set threshold was just observed, what you will see translates into an instance of the Omori law for volatility[8]. Just as earthquakes are followed by aftershocks, volatility shocks are followed by other larger than normal shocks. Another stylized fact of volatility that exhibits some temporal asymmetry is the so-called leverage effect, which describes the positive correlation between negative returns and volatility: large negative price drops will give rise to subsequently higher volatility.

Stock Price Models

Several different models have been proposed in an attempt to capture fat tails and volatility clustering which do not exist in the Gaussian, Bachelier or Black-Scholes models Popular approaches include Lévy processes[8], which induce jumps and thus fat tails on short time scales, but convolve too quickly to the Gaussian distribution as the time scale increases. Stochastic volatility models, such as the Heston model[9], where the volatility is assumed to follow its own mean-reverting stochastic process, reproduce fat tails, but do not account for the long-term memory observed in the data. The same holds true for the simplest of Robert Fry Engle's Nobel Prize winning GARCH models[10] in which the volatility is essentially an autoregressive function of past returns. Multi-fractal stochastic volatility models (similar to cascade models of turbulent flow) reproduce many of the stylized facts and are thus promising candidates for more accurate representations of these phenomena with the

8 See Fabrizio Lillo/Rosario N. Mantegna: "Power law relaxation in a complex system: Omori law after a financial market crash", in: *Physical Review E* 68 (2003), DOI: 016119.

9 See Steven L. Heston: "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options", in: *The Review of Financial Studies* 6 (1993), pp. 327–343.

10 See Tim Bollerslev/Robert Fry Engle/Daniel B. Nelson: "ARCH Models", in: Robert Fry Engle/Daniel L. McFadden (eds.): *Handbook of Econometrics*, Vol. 4, Amsterdam: Elsevier Science 1994, pp. 2961–3038.

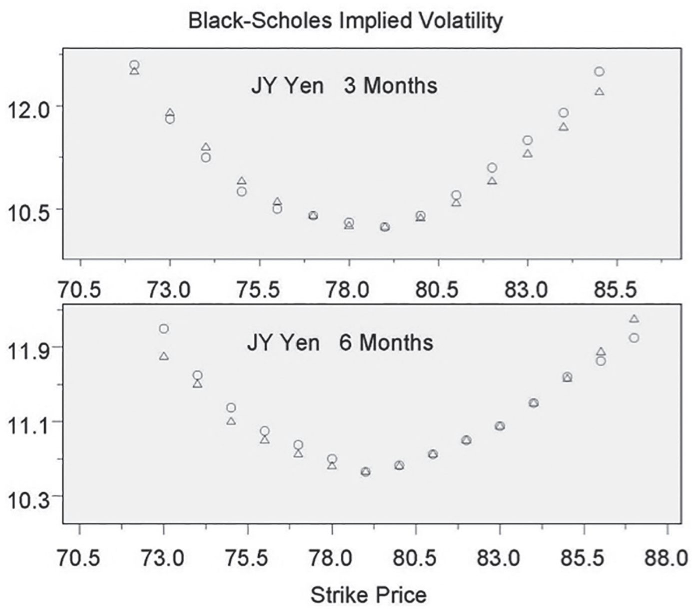

Figure 3: The Black-Scholes model for option pricing is based on a Gaussian distribution. The volatilities that need to be used with that model to capture real prices vary with the option strike price (the price at which the holder has the right to purchase the underlying stock) in such a way to compensate for tail effects. The non-Gaussian statistical feedback model takes into account nonlinear cooperative effects and yields option prices that closely match the market with just one volatility parameter. For comparative purposes, the implied Black-Scholes volatilities of that model (circles) are shown here with implied volatilities of real option prices (triangles).

limitation that they are strictly symmetric in terms of time reversal, a stark contrast to the empirical evidence.[11] In addition, most of the above-mentioned models are difficult if not impossible to deal with analytically. Analytic tractability is desirable for reasons such as efficiently calculating the fair price of options or other financial derivatives traded globally in high volumes.

A few years ago, a rather realistic model of stock returns based on a statistical feedback model was developed. It was inspired by ideas from non-extensive thermodynamics[12] that aimed at including cooperative effects in the spirit of synerget-

11 See Jean-François Muzy/Jean Delour/Emmanuel Bacry: “Modelling Fluctuations of Financial Time Series: From Cascade Process to Stochastic Volatility Model”, in: *The European Physical Journal B* 17 (2000), pp. 537–548.

12 See Constantino Tsallis: *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World*, New York, NY: Springer 2009.

ics. In order to incorporate the afore-mentioned power-law statistics on the distributions of real returns, which are very stable over time scales ranging from minutes to weeks, and which only slowly converge to Gaussian statistics in very long time scales, it was proposed that the fluctuations driving stock returns could be modeled by a statistical feedback process. This was formulated in such a manner that the volatility at a given instant was inversely proportional to the probability of observing the most recent price, given an initial price level at time 0. Hence, unusually big (rare) returns lead to larger subsequent volatility, and vice versa, more normal returns lead to more subdued volatility.

Therefore, the statistical feedback term can be seen as capturing the market sentiment or collective behavior of market participants. Intuitively, this means that, if the market players observe unusually high deviations of the log price change, the effective volatility will be high due to general market panic. Conversely, traders will react more moderately if the price change is close to its more typical or less extreme values. As a result, the model exhibits intermittent behavior consistent with that observed in the effective volatility of markets. To incorporate the subtle effect of skew and the stylized fact known as the leverage effect, we further extended the stock price process to include asymmetries.[13] This model was used to derive generalized formulae for pricing options in the presence of fat tails. The model exhibits remarkably good agreement with real option prices (figure 3).

The Multi-Timescale Model

Although very successful for pricing options, the statistical feedback model is still not entirely realistic. The main reason is that there is one single characteristic time in that model. In particular, the effective volatility at each time is related to the conditional probability of observing an outcome of the process at time t given what was observed at time t = 0. For option pricing, this is perfectly reasonable because one is interested in the probability of the price reaching a certain value at some point in the future based entirely on one's current knowledge. However, this is a shortcoming when it comes to modelling real stock returns. Particularly in real markets, traders drive the price of the stock based on their own trading horizon. There are traders who react to each tick the stock makes and others who react to what they believe to be relevant on the horizon of a year or more, and, of course, there is the entire spectrum in between. Therefore, an optimal model of real price movements should attempt to capture this existence of multiple time scales and long-range memories.

A natural extension of this model would mean generalizing its scope in order to include feedback over many time scales instead of just one.[14]

13 See Borland/Bouchaud: "A Non-Gaussian Option Pricing Model with Skew" (note 6).
14 See Lisa Borland/Jean-Philippe Bouchaud: "On a Multi-Timescale Statistical Feedback Model for Volatility Fluctuations", in: *The Journal of Investment Strategies* 1 (2011/2012), pp. 65–104.

If one assumes that the mean drift of the stock can be set to zero, then the random price return *dy* at time *i* is constructed as the product of a time-dependent volatility σ_i and a random variable ω_i of zero mean and unit variance:

$$dy_i = \sigma_i d\omega_i \qquad \text{Equation 1}$$

The volatility for a given stock *k* is written to include feedback over multiple time scales i-j:

$$\sigma_i^k = \sigma_0 \left(1 + \sum_{j=1}^{\infty} g_{i-j} (y_j^k - y_i^k)^2\right)^{\frac{1}{2}} \qquad \text{Equation 2}$$

where i and j correspond to time. Here, σ_0 is the baseline volatility and the parameter g is a coupling constant that controls the strength of the feedback. This coupling g scales as an inverse power of the time scale $g = g_0 (i - j)^{-\gamma}$, where *y* determines the decay rate of memory in the system which represents the relative importance between short-term and long-term traders. The model can be calibrated to real stock data and the main stylized facts of stock returns are reproduced. In figure 1, a simulation of that model is shown. The process describes real data very well, with obvious periods of lower and higher volatility clustering together.

Another powerful class of processes which has recently been used to describe the arrival of clustered volatility events is the self-exciting Hawkes process.[15] This is similar in spirit to the multi-timescale feedback model in that the intensity of arrival times of events is modeled by a function that contains a kernel which attributes the weights to be given to past events. Thereby, memory is captured in this process, a key feature of all realistic models of stock price innovations.

Statistical Signatures across Stocks: The Self-Organization of Correlation

Up until now, this text has focused on understanding and modeling the dynamics of stock returns across time. However, in order to grasp the properties of the full joint stochastic process driving markets, we turn our attention to the cross-sectional dynamics across a universe of stocks at any given point in time. Along these

15 See Alan G. Hawkes: "Point Spectra of Some Mutually Exciting Point Processes", in: *Journal of the Royal Statistical Society. Series B (Methodological)* 33 (1971), pp. 438–443. Vassil Chatalbashev/Yifan Liang/Ari Officer/Nikolaos Trichakis: *Exciting Times for Trade Arrivals. MS&E Investment Practice Project Report*, Stanford University 2007, http://users.iems.northwestern.edu/~armbruster/2007msande444/report1a.pdf (accessed March 2015). Stephen J. Hardiman/Nicolas Bercot/Jean-Philippe Bouchaud: "Critical Reflexivity in Financial Markets: A Hawkes Process Analysis", in: *The European Physical Journal B* 86 (2013), pp. 1–9.

lines, there have been several studies in recent years that focus on exploring the structure and dynamics of correlations across the different stocks that comprise the market.[16] In particular, these models explore correlations during market stress or times of bubbles, panic, and crashes. This is important because in such extreme scenarios investors are at the highest risk; their usual models and world views might break down, and the results could be devastating as seen in August 2007, October 2008, or during the so called 'Flash Crash' of May 2010.

Some of the author's contributions in this area have investigated whether there are any particular cross-sectional statistical signatures in these periods of market panic.[17] To get a grasp on the cross-sectional distribution of stock returns at a given time point, one can look at moments such as the mean, standard deviation, skew, and kurtosis, as a function of time. The standard deviation of returns is widely referred to as dispersion. Calculated across 1500 United States stocks and plotted for the time period 1993–2009, the dispersion grows relatively big during the time periods of panic, in accordance with the discussion above. However, the more striking discovery comes when the cross-sectional kurtosis is plotted alongside the dispersion or together with market returns (figure 4). Even by eye, it is quite clear that there is a strong negative correlation between the two quantities, which is in fact about -25%. In times of panic, dispersion is high yet excess kurtosis practically vanishes. In more normal times, the dispersion is lower but the cross-sectional excess kurtosis is typically very high. The correlation of stocks over time was also studied, and can be captured by the variable s which is defined below in Equation 3. The empirical results show that during times of panic, correlations rise dramatically.

It is desirable to have a model that can explain all of these findings, namely, to preserve the fat-tailed time series properties of stocks, but which gives rise to the remarkable reduction in kurtosis and an increase in correlations cross-sectionally, which are characteristic of market panic. In such times, dispersion is high yet kurtosis is low, which implies that the data are more Gaussian in times of panic. This can be explained partially by the fact that the volatilities of the individual stocks are

16 See Tobias Preis/Dror Y. Kenett/H. Eugene Stanley/Dirk Helbing/Eshel Ben-Jacob: "Quantifying the Behavior of Stock Correlations under Market Stress", in: *Nature* (2012), http://www.nature.com/srep/2012/121018/srep00752/full/srep00752.html (accessed March 2015). Fabrizio Lillo/Rosario N. Mantegna: "Variety and volatility in Financial Markets", in: *Physical Review E* 62 (2000), pp. 6126–6134. Didier Sornette: *Why Stock Markets Crash: Critical Events in Complex Financial Systems*, Princeton, NJ: Princeton University Press 2002. Taisai Kaizoji: "Power Laws and Market Crashes – Empirical Laws on Bursting Bubbles", in: *Progress of theoretical physics*, Supplement 162 (2006), pp. 165–172. Giacomo Raffaelli/Matteo Marsili: "Dynamic Instability in a Phenomenological Mode of Correlated Assets", in: *Journal of Statistical Mechanics* 8 (2006), http://iopscience.iop.org/1742-5468/2006/08/L08001/pdf/1742-5468_2006_08_L08001.pdf (accessed March 2015).

17 See Lisa Borland: "Statistical Signatures in Times of Panic: Markets as a Self-Organizing System", in: *Quantitative Finance* 12 (2012), pp. 1367–1379. Lisa Borland/Yoan Hassid: "Market Panic on Different Time-Scales", in: *ArXiv e-prints* 1010.4917 (2010), http://arxiv.org/abs/1010.4917 (accessed March 2015).

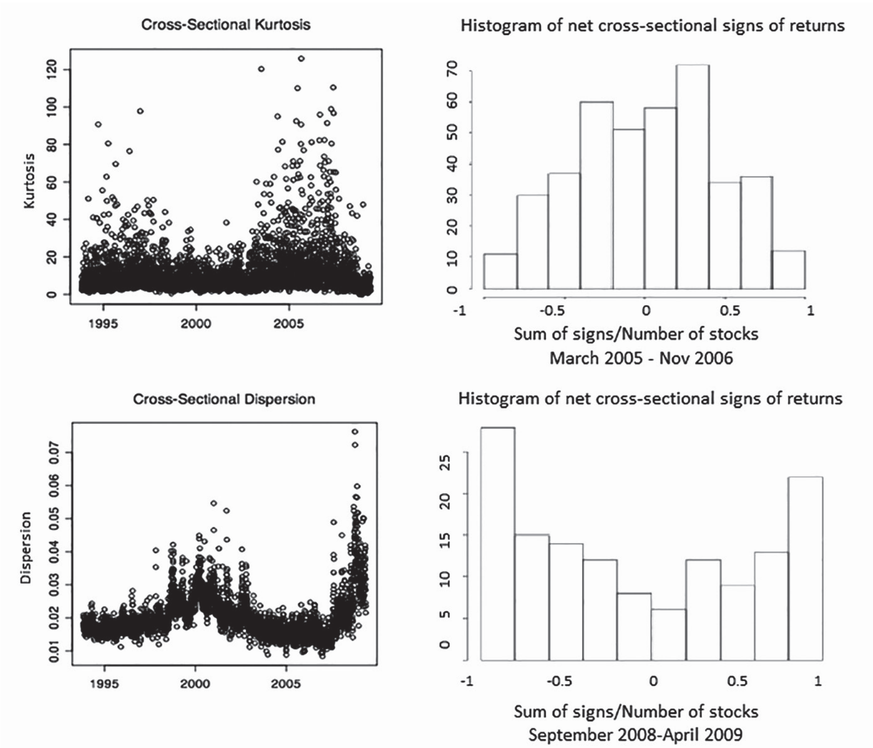

Figure 4: Left: Across the universe of stocks, the standard deviation (called dispersion) and the kurtosis are plotted over time. In times of perceived panic, dispersion is high while cross-sectional kurtosis is very low. The two quantities show strong negative correlation. Right: The empirical histogram of the quantity s (the ratio of the number of stocks that go up minus the number of stocks that go down, over the total number of stocks) is unimodal in normal times and bimodal in times of panic.

higher yet more alike in times of panic, which might be one contributing effect. However, it appears that the behavior of cross-sectional correlations is what drives the statistical signatures of markets in times of panic.

Now, in the full spirit of Haken's synergetics, a proxy for the collective behavior of all stocks in the market is defined as

$$s_{up} = \frac{s_{up} - s_{down}}{s_{up} + s_{down}} \qquad \text{Equation 3}$$

where s_{up} is the number of stocks that have positive returns over a given interval, and s_{down} is the number of stocks that have negative moves on that same interval (for example, over the course of one day). If s = 0, then roughly the same number of stocks have moved up as have moved down, and the assumption is that the stocks had little co-movement and therefore were uncorrelated. If, however, all stocks move together, either up or down, the value of s will be +1 or -1, and the stocks will have high correlation. By consequence, if s = 0, there is no correlation, and we are in a disordered state. But if s ≠ 0, then there is correlation, and things are in an ordered state. Let us now make a leap and connect to some terminology from physics. We shall call s the order parameter. It is a macroscopic parameter that tells us whether there is order and correlation in a system. In physics, particularly in the field of non-equilibrium thermodynamics and synergetics, the concept of the order parameter is often used to describe systems that exhibit spontaneous self-organization. Examples range from chemical kinetics to laser dynamics, from fluid dynamics to biological systems, from collective behavior in both the animal and human world to cloud formation. To illustrate the concept, let us look at an example which should be familiar and intuitive to most, namely, magnetism.

In a ferromagnetic system, the total magnetic moment depends on the orientation of the individual magnetic spins comprising the system. It is proportional to the quantity

$$m_{up} = \frac{m_{up} - m_{down}}{m_{up} + m_{down}} \qquad \text{Equation 4}$$

where m_{up} and m_{down} denote the number of spins lined up and down respectively. Depending on the value of T, the magnetic system will either be in an ordered state (all spins lined up) or a disordered state (spins in random directions). To describe the dynamics of the magnetic moment m, the framework of synergetics yields the following expression, the Langevin equation

$$\frac{dm}{dt} = -am - bm^3 + F_t \qquad \text{Equation 5}$$

where F_t is thermal noise. The coefficient a can be written as a = $\alpha(T - T_c)$ where T_c is the so-called critical temperature. One can envision these dynamics as motion in a potential well V.. If $T > T_c$, then the only minimum is the trivial one at m = 0 (spins in random directions). But for $T < T_c$, two real roots appear, yielding non-zero values of m. Clearly, m can be positive or negative, depending on which minima are reached by the system (spins all up or all down). This is referred to as symmetry breaking. Due to the noise, the dynamics can also drive m from one minimum to the other. Because the value of T determines whether the system is in the disordered state (m = 0) or the ordered state (m ≠ 0), it is called the control parameter. The probability distribution of the system in the disordered state will be a unimodal one, while the probability distribution of m in the ordered state will be bimodal around +m and -m. As T passes from above to below T_c, or vice-versa, there is clearly a phase transition: the state of the system is drastically altered. In this type of symmetric system, we are dealing with a second order phase transition.

In the current setting, we make an analogy between the variable s and the magnetic moment m. There are rather drastic changes in the cross-sectional distribution of stocks in times of panic versus more normal market conditions. Histograms of s in both periods show that in normal times, s is unimodal, centered around zero, and in panic times, a bimodal distribution is observed, consistent with the framework of a phase transition leading to self-organization in panic times (see figure 4).

The dynamics of s are modeled the same as the magnetic moment, namely

$$\frac{ds}{dt} = -as - bs^3 + F_t \qquad \text{Equation 6}$$

with a = $\sigma_c - \sigma_0$, where Ft is a Gaussian noise term and σ_0 corresponds to the baseline volatility level of stocks. This volatility is assumed constant across all instruments and essentially measures the general uncertainty in the environment, in this sense acting much as the temperature in the magnetic system. Note that it is the feedback effects in the system which induce stock-specific variations in volatility over time and can largely explain most of the excess volatility observed in stock time series, whereas the parameter σ_0 is not driving the stock-specific dynamics, but simply describes a 'global' level of risk. The quantity σ_c would correspond to a critical level of uncertainty, below which the market is in a normal phase, and above which we have the onset of panic. Much as in the case of ferromagnetism, where the control parameter T can be tuned externally above or below the critical temperature, in this model the uncertainty level σ_0 captures the external environment. In a sense, it represents the general perception of risk in the public mind.

Following the notions of synergetics, the hypothesis is that financial markets appear to exhibit a phase transition from the disordered to the ordered state after crossing a critical level of risk perception. Putting the dynamics together, one has the multi-timescale feedback process for each stock k, namely,

$$dy_i^k = \sigma_i^k \, d\omega_i^k \qquad \text{Equation 7}$$

with k = 1… N, and the volatility of each stock k given by equation 2.
The random variables ω_i^k are drawn from a Gaussian distribution, uncorrelated in time, yet among themselves at a given time point i across stocks k, they are correlated with correlation |s|. The macroscopic order parameter s is therefore just a signature of the cross-stock correlations whose dynamic behavior manifests itself in the order parameter equation of equation 6.[18]

What do we expect to see from this model? We have already seen that across time (see for example figure 1), it models many properties of real financial time series very well. Across stocks, if $\sigma_0 < \sigma_c$, correlations fluctuate around $s = 0$, and we expect to see a unimodal distribution of S. The cross-sectional kurtosis should be rather high

18 We impose that the coefficients must always be such that $|s| \leq 1$.

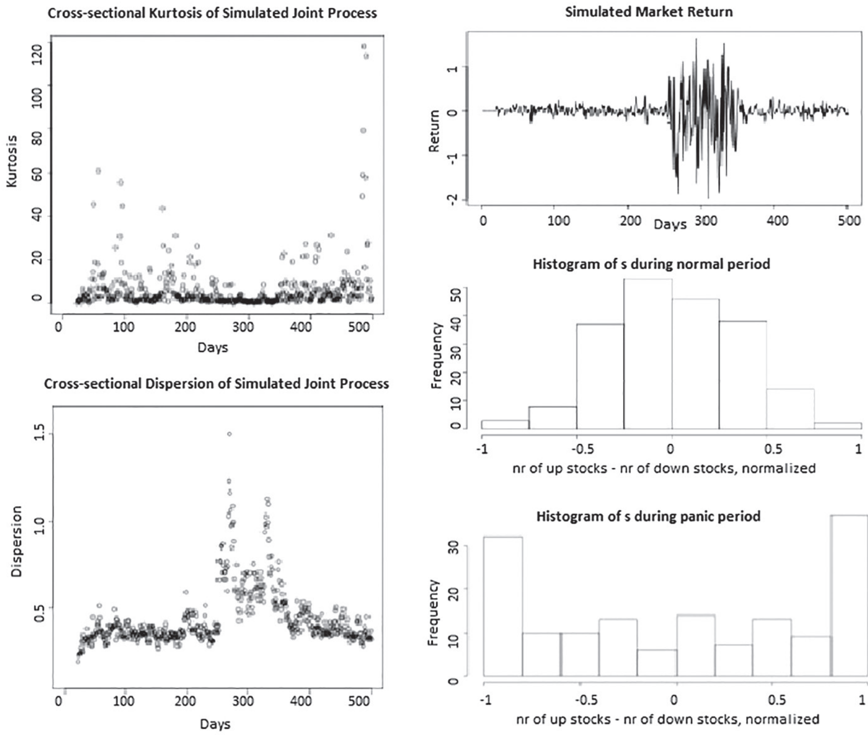

Figure 5: Results of a simulated synergetic model that captures the self-organizing behavior of the market in times of panic. Top right: A simple simulation of the aggregate market return based on a multi-timescale feedback model of the underlying stock returns. Middle and bottom right: The distribution of the order parameter s in normal times and during times of panic. Left: Cross-sectional kurtosis and dispersion of the simulated stock returns. The simulation reproduces the qualitative behavior of real markets.

since there is no mechanism to cause either stocks or stock volatilities to have any co-movement at all. Therefore, at each time point it is as if the cross-sectional returns are drawn from a Gaussian process with stochastic volatility, yielding a fat-tailed distribution as the superposition. Then, as the market crashes with $\sigma_0 > \sigma_c$, the system enters a phase transition. The order parameter s becomes $s \neq 0$ and the system enters the ordered phase with high co-movement. Because the random variables ω_i^k are now correlated across stocks, cross-sectional returns will be more similar and the distribution will have lower kurtosis. Additionally, due to the fact that the phase transition is triggered by an external shock in volatility, all stocks will tend to have higher volatilities and higher cross-sectional dispersion.

Indeed, simulations of the model confirm these expectations. In figure 5, results are shown using the realistic assumption that the baseline volatility is roughly

σ = 20%, and at a certain time a volatility shock σ_{shock} = 60% (consistent with levels observed in the VIX volatility index[19] in late 2008) is applied to the system. This external fear induces a phase transition from the disordered state where correlation among stocks is relatively low (centered at around zero) to a highly ordered state where the correlations are significantly different from zero, namely $|s| \approx 80\%$. When the shock subsides, the system slowly decays to the disordered state again.

As expected, the market volatility rises when s is in the ordered state, which corresponds to the panic phase. In addition, the cross-sectional dispersion rises during the market panic, while the cross-sectional kurtosis drops close to zero. The correlation between the two quantities in this example is -17%, consistent with empirical observations that also showed a strong negative correlation as well. Histograms corresponding to the distribution of the order parameter s in the normal market phase as well as in the panic phase are in excellent agreement with the empirical observations of the real market data, namely, unimodal in the normal phase and bimodal during panic.

Other Agent Based Models and Network Approaches

So far, we have mainly discussed a modeling approach to the dynamics of financial time series based on the mesoscopic level, namely, by way of stochastic equations for the evolution of price and volatility. Other approaches involve modeling the market participants as independent agents, interacting with each other according to certain rules and utilities. Although much of this research does not explicitly make reference to synergetics as a field, it is easy to see that the basic notions could be cast in such a framework since they have all the necessary ingredients: They all display local nonlinear interactions that lead to higher order macroscopic dynamics, which in turn mimic many properties of financial observables. One example is the body of work concerned with minority games.[20] Another interesting class of models that does directly utilize tools from synergetics are the heterogeneous agent models.[21] Here, the market participants (or agents) are assumed to be in two main groups: fundamentalists and chartists. Fundamentalists believe that the asset price will revert back to its fundamental value; chartists on the other hand will either buy or sell depending on factors such as the prevailing price trend. Essentially, modeling the competition and interaction between these two groups of agents, these

19 Also known as the "fear index". For explanations and further literature, see http://en.wikipedia.org/wiki/VIX (accessed March 2015).

20 See Damien Challet/Matteo Marsili/Yi-Cheng Zhang: *Minority Games: Interacting Agents in Financial Markets*, Oxford: Oxford University Press 2005.

21 See Cars H. Hommes: "Heterogeneous Agent Models in Economics and Finance", in: Leigh Tesfatsion/Kenneth L. Judd (eds.): *Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics*, Amsterdam: Elsevier Science 2006, pp. 1109–1186.

models employ ideas from synergetics research and result in informative models of price formation that capture many of the realistic features observed in actual financial markets.

Conclusion

We have reviewed the anomalous features inherent in financial data across time and across stocks that reflect the very complex and nonlinear interactions leading to price formation in financial markets. The approach proposes models that intuitively capture potential dynamics at play in financial markets. These models have been verified by testing whether they can reproduce the observed statistical signatures and stylized facts within a variety of market settings, including derivative markets such as options.

The common denominator in the various studies reviewed here is the presence of cooperative effects, memory and nonlinear feedback. Therefore, many interesting techniques from the field of statistical physics and synergetics can be applied. In particular, we described a model for the joint dynamics of stock price formation over time and across stocks, where the onset of market panic was described as a self-organized phenomenon. The correlations take on the role of the order parameter of the system. It was shown that in times of panic, there appears to be a phase transition from the disordered to the ordered state driven by the volatility and uncertainty perceived in the market. It has also been pointed out that various other bodies of work have been done to model the more direct interactions of market participants or agents. Inevitably, cooperative phenomena are key ingredients in these works as well.

As a continuously evolving and changing field, finance will certainly continue to pose interesting challenges for scientists and practitioners alike for years to come. In addition, it seems inevitable that tools from physics, in particular from the fields of synergetics and non-equilibrium statistical physics, will continue to make valuable contributions to motivating models and help guide our understanding of financial markets.