

# Microscopic Analysis of Thermodynamic Parameters from 160 MeV/n - 160 GeV/n\*

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## Abstract

Microscopic calculations of central collisions between heavy nuclei are used to study fragment production and the creation of collective flow. It is shown that the final phase space distributions are compatible with the expectations from a thermally equilibrated source, which in addition exhibits a collective transverse expansion. However, the microscopic analyses of the transient states in the reaction stages of highest density and during the expansion show that the system does not reach global equilibrium. Even if a considerable amount of equilibration is assumed, the connection of the measurable final state to the macroscopic parameters, e.g. the temperature, of the transient "equilibrium" state remains ambiguous.

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# 1 Motivation

Experimental information on yields of fragments and hadrons from high energy nuclear reactions has long been used to determine the thermodynamic state of the droplet of hot, dense matter before decay/freeze-out [1, 2, 3, 4, 5, 6, 9].

Recently, the spectra of light and medium mass fragments in the energy region from 150 MeV/n - 2 GeV/n [12] and hadrons have also been included into the description and the large discrepancy between the yield ratios and the "thermal" slopes became apparent.

At much higher energies, 10-15 GeV/n & 160-200 GeV/n, both, the yields [14, 15] and spectra [16] have been analyzed. The thermal parameters ( $T$ ,  $\mu_B$ ,  $\mu_s$ ), as well as dynamic flow, both in longitudinal and transverse direction, have been obtained. Ambiguous conclusions have been reached, e.g. Braun-Munzinger et al.[16] claim to extract temperatures of about 130MeV at AGS energies, while Cleymans et al.[7] find only 110MeV. At 200 GeV/n, Cleymans et al. find  $T \approx 190$ -200 MeV and Braun-Munzinger et al. sees 160-170MeV.

Here we present - based on our newly developed microscopic UrQMD model - a critical discussion of the reliability of the theoretical input and macroscopic model assumptions used in most of the above analysis:

## 2 Hadrons from a Thermal Source

The idea behind the thermal scenario is as follows: one assumes a thermalized system with a constant density  $\rho(r)$  (box profile), a constant temperature  $T(r)$  and a linear radial and longitudinal flow velocity profile  $\beta_{\perp}(r)$ ,  $\beta_{\parallel}(r)$ . These parameters are assumed to be the same for all hadrons/fragments. At some time  $t^{\text{break-up}}$  and density  $\rho^{\text{break-up}}$ , the system decouples as a whole and the particles are emitted instantaneously from the whole volume of the thermal source. A complete loss of memory results, due to thermalization - the emitted particles carry no information about the evolution of the source.

It is of utmost importance to take care of several corrections if one wants to use

- the inverse slope parameter  $T$  as thermometer[8],(since the feeding from  $\Delta$ 's, as well as the radial flow needs to be incorporated into the analysis),
- d/p,  $\pi$ /p resp., as an entropymeter[4](here even more feeding effects from unstable states has to be considered).

The hadron densities from a thermal source are given by

$$\rho_i = \frac{g_i}{2\pi^2} \int_0^\infty dp \frac{p^2}{\exp[(E_i - \mu_i)/T] \pm 1} \quad . \quad (1)$$

This yield is the nascent (primordial) distribution. On top of this rates, feeding contributions, as mentioned above, from other particles are taken into account by

$$\rho_\pi^{\text{final}}(\mu, T) = \rho_\pi(\mu, T) + \rho_\Delta(\mu, T)N^\pi(\Delta) + \dots \quad . \quad (2)$$

In addition, the Hagedorn volume correction can be applied to incorporate changes due to the finite volume of the hadrons.

It seems that a two parameter fit ( $\mu_q$ ,  $T$ ,  $\mu_s$  is fixed by strangeness conservation) to the hadronic freeze-out data describes the experimental results well, if feeding is included (cf. Fig.1)[16, 17]. Does this compatibility with a thermal source proof volume emission from a globally equilibrated source?

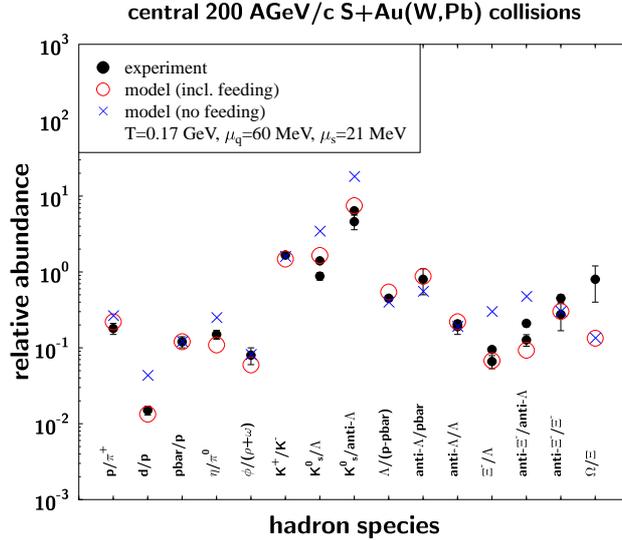


Figure 1: Hadron production from a thermal source.  $T = 170$  MeV,  $\mu_q = 60$  MeV,  $\mu_s = 21$  MeV. Total strangeness is conserved[16, 17].

The same quality of fit to the hadron yields is found if the evolution from an initial quark gluon plasma via a 1<sup>st</sup>order phase transition to hadronic matter is assumed, even with non-equilibrium evaporation of particles during the time evolution of the system[17].

### 3 Microscopic Model - Macroscopic Analysis

Let us start by calculating some macroscopic quantities within the framework of a microscopic model. In order to get information on the thermal collective energy sharing, we restrict our analysis for the moment to the transverse momentum spectra only. This procedure is motivated by the fact that transverse momenta are newly created and are not directly connected to the initial (longitudinal) beam momentum. A possible collective expansion in longitudinal direction cannot unambiguously be related to the properties of the hot and dense reaction zone because the system may have memory about its history, in particular the incident momentum of the beam.

The collective expansion velocities exhibit a decrease with increasing hadron/fragment mass, which is displayed (for 150 AMeV) in Fig. 2. The results on the averaged collective transverse momentum and temperature from the QMD calculation (full symbols) compare reasonably well to a experimental compilation of available data on central Au+Au collisions [10], which have been analysed with the very same method. Similar collective flow velocities and temperatures have also been reported from the EOS collaboration [11].

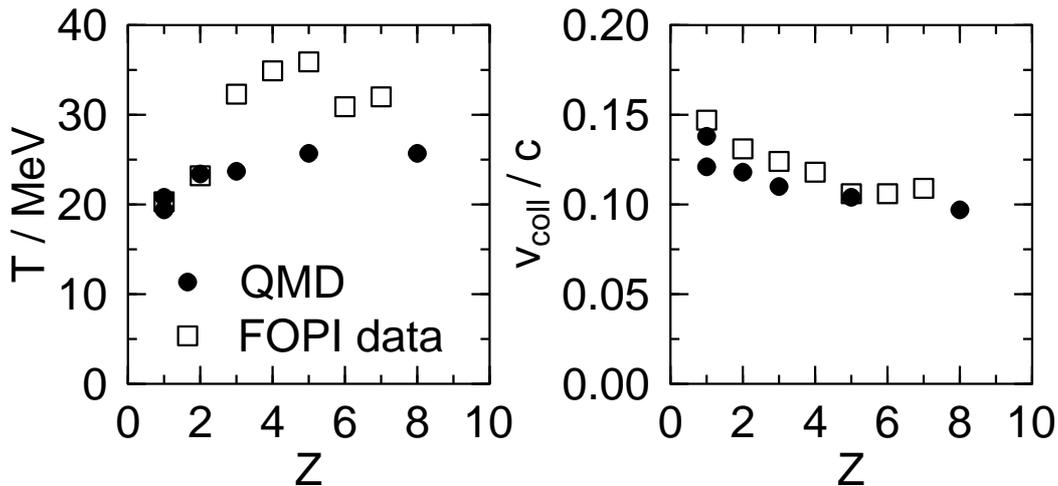


Figure 2: Temperature and flow velocities for fragments. The QMD results (closed circles) deduced from the fits to the transverse momentum spectra in Fig. 3 are compared to data of the FOPI-collaboration [10] (open squares).

Fig. 3 shows transverse momentum spectra of various charged fragments obtained with QMD for the system Au (150 MeV/nucleon,  $b=0$ ) + Au (symbols) together with fits to these calculated data, which are based on the assumptions from above. In fact, the corresponding count rates have been fitted, rather than the invariant distributions,

which are displayed. All spectra are compatible with temperatures between 20 and 25 MeV and averaged collective flow velocities of 0.1–0.13  $c$ .

On the contrary these rather high temperatures are in variance with the expectations ( $T \approx 8$  MeV at 150 MeV/nucleon) from a quantumstatistical analysis of the large number of intermediate mass fragments observed in the very same system [12]. This seems to support the idea of collective radial flow in addition. To illustrate this radial flow we take a look at the coordinate space distribution of the expanding fireball. Fig. 4 shows a typical snapshot of the expansion phase of a heavy ion reaction. Density contours are shown together with local collective expansion velocities. A strong correlation between configuration and velocity space is observed. Inside any of these cells the velocity distribution exhibits a gaussian shape.

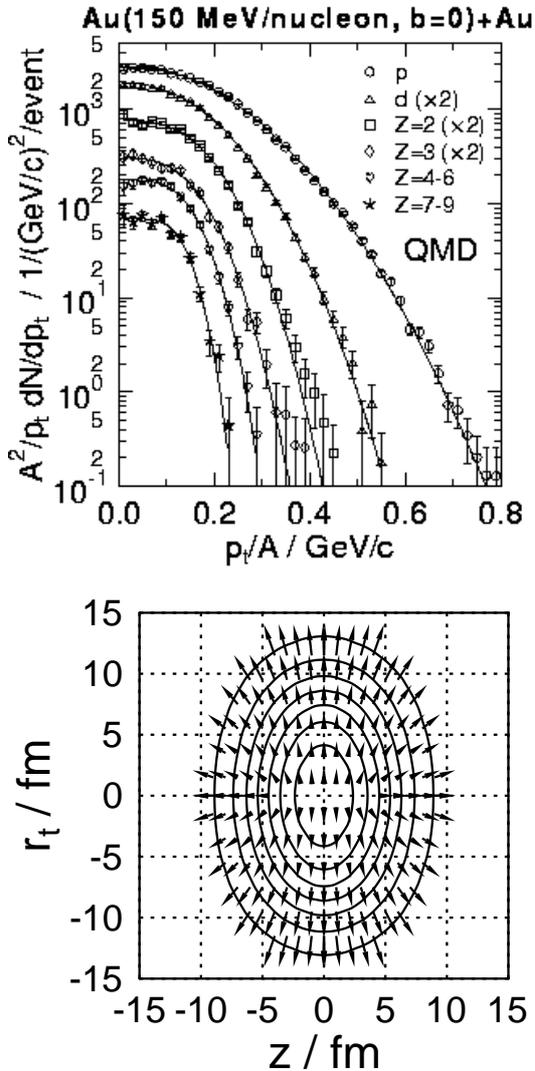


Figure 3: Invariant transverse velocity spectra of various reaction products of Au (150 MeV/nucleon) + Au at vanishing impact parameter. The predictions of the Quantum Molecular Dynamics model (symbols) have been fitted with a thermally equilibrated source, which expands azimuthally symmetric towards the transverse direction (lines). Note, all spectra are compatible with temperatures between 20 and 25 MeV and an averaged transverse collective velocity of 0.11–0.13  $c$ .

Figure 4: Snapshot of the expansion after 50 fm/c starting from a 5 fm separation of projectile and target surfaces. Density contours are at 0.1–0.7  $\rho_0$ . The arrows indicate the direction and the absolute velocity (proportional to the length) of the collective motion of the individual fluid cells.

These “local temperatures” vary only slightly over large volumes in real space which supports a common analysis of a larger number of cells. For this purpose the central reaction volume is defined as the sum of all cells in which the local density exceeds half the maximum density at this instant, implying a time-dependent volume. In Fig. 5 the properties of the excited nuclear matter inside this volume are displayed as a function of time. Even in the late stages of the expansion this zone still contains  $\approx 1/3$  of the mass of the entire Au+Au system. The evolution of the spreads of the local velocity distributions suggests a rapid cooling, which goes in line with a strong density decrease. The different temperatures tend to converge in the late stages only.

At densities around  $0.1\text{--}0.5 \rho_0$ , where the freeze-out of fragments is supposed to happen, the corresponding temperatures have dropped below 10 MeV. This is no longer in vast disagreement with the chemical temperatures needed for the understanding of the large intermediate mass fragment multiplicities [12].

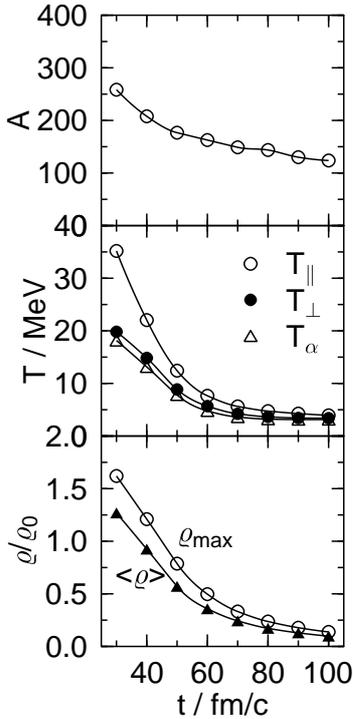


Figure 5: Thermodynamics in the central reaction zone, i.e. the volume where the local density is at least half of the maximum value. Mass content a), spreads of the local velocity distributions b), and maximum as well as averaged density c) are displayed as a function of time.

## 4 Analysing the Source Microscopically

We have shown in the two previous sections that the final state spectra, although they seem to be perfectly described by an equilibrated, expanding source, suggest too high

temperatures even if collective flow effects are taken into account. Locally much lower temperatures are observed in the very same class of events.

In turn the question arises what is wrong with the assumptions which underlied the global fit. Let us analyse the density  $\rho(r)$ , and the radial flow at "break-up time"  $t \approx 50 \text{ fm}/c$ . From Fig. 6 we conclude that

- the radial flow rises linearly with the transverse distance  $\beta(r) \approx a \cdot r$ , as assumed in many simplified macroscopic models,
- the temperature  $T$  is roughly constant over the radius at freeze-out (not shown in the Figure)[13],
- the density distribution  $\rho(r)$  falls of smoothly. This, however is in strong contrast to the box profiles used in the macroscopic analysis!

How can we understand this difference? We know from textbooks that  $PV = NRT$ . Dividing by  $V$  (using  $R = 1$ ) we get

$$P = \rho T \quad . \quad (3)$$

However, in hydrodynamics the relation

$$\frac{d}{dt}\vec{p} = -\nabla P(\rho, T) \quad (4)$$

holds. This means, the gradients of the pressure  $P$  lead to flow, starting with the (isentropic) expansion into the vacuum at the surface. Even if there were an initial box profile, it is changed into a complex final shape  $\rho(r)$ .

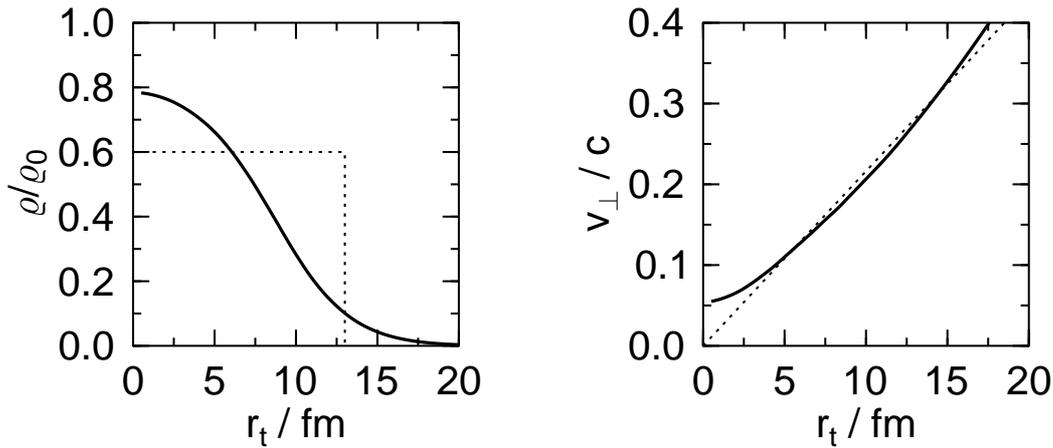


Figure 6: Local density and collective velocities as a function of the transverse distance in the  $z = 0$  plane. The Figure corresponds again to the situation after 50 fm/c. The calculated shapes (solid lines) are compared to the assumptions, which entered the global fit (dotted lines). The longitudinal as well as the azimuthal collective velocity vanish.  $v_{\perp}$  does not reach 0 for  $r_{\perp} \rightarrow 0$  due to the fact that the innermost cell is not at  $r_{\perp} = 0$ .

This may not be considered a drastic source of uncertainty. However, the problem comes from the assumption of a  $\beta^{\max}$  at  $R^{\text{break-up}}$ . As expressed in Fig. 6, the matter distribution has a long tail, which picks up high flow velocity components. In the box model such "cells" do not exist. Their contribution to the high energy tails of the spectra must then be "simulated" by an artificially enhanced "temperature" in the fit. The microscopic analysis suggests, on the contrary, the cells to be considerably cooler. The specific combinations of densities and temperatures, which are traversed in the course of the reaction, are important. The complete space-time evolution is needed to study the agreement with the expectations from the quantum statistical analysis of the fragment distributions in the final state [12].

## 5 Thermalisation at AGS/SPS ?

How does this picture change at higher beam energies, say 10-15 GeV/n, resp. 160-200 GeV/n? Here we also find late clustering of fragments, while high energy protons cool

the reaction zone. Mattiello[18], as well as Bleicher[19] have shown, that the break-up<sup>1</sup> density distributions  $\rho(r)$  are not at all box like. Even a suppression in  $\rho(r)$  for  $r \rightarrow 0$  fm was reported [18, 19].

Fig.7 shows the transverse velocity profiles of protons and tritons at freeze-out (top) and the transverse freeze-out densities of p's, d's, t's and He's (bottom) in central Au+Au (Pb+Pb, resp.) events at central rapidities.

In general, flow correlations arise for both p's and t's both at AGS to SPS (Fig.7, top). The radial velocity profiles are convex. The shapes of the profiles are nearly independent of beam energy and particle type (cluster). Note, however that unlike at low energies the radial velocities level off at about  $0.6c$ . They are not longer linear in  $r_{\perp}$ ! The radial density profiles (Fig.7, bottom) indicate, a wide range of distances  $r_{\perp}$  with high  $\beta_{\perp}$  values which contribute to the proton distribution. The expansion of the exploding hot hadronic matter is thus visible by the large apparent "temperatures" of the baryon spectra.

In contrast to the protons, there is a strong localisation of the cluster emission region to the surface of the reaction zone ( $r_t \approx 6$  fm). Cluster formation is attenuated for  $r_t \rightarrow 0$  fm, due to the highly uncorrelated momenta of the coalescing baryons and the small freeze-out probability due to further collisions with newly produced hadrons. For  $r_t > 6$  fm cluster formation is also suppressed, as the emission volume increases and density decreases, which leads to larger average distances between the baryons, thus decreasing the coalescence probability. Even the upcoming momentum correlations (i.e. radial flow) are not able to counter-balance this effect.

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<sup>1</sup>In high energy cascade calculations, the particle freeze-out is defined for each particle individually as the last point of strong interaction of that particle.

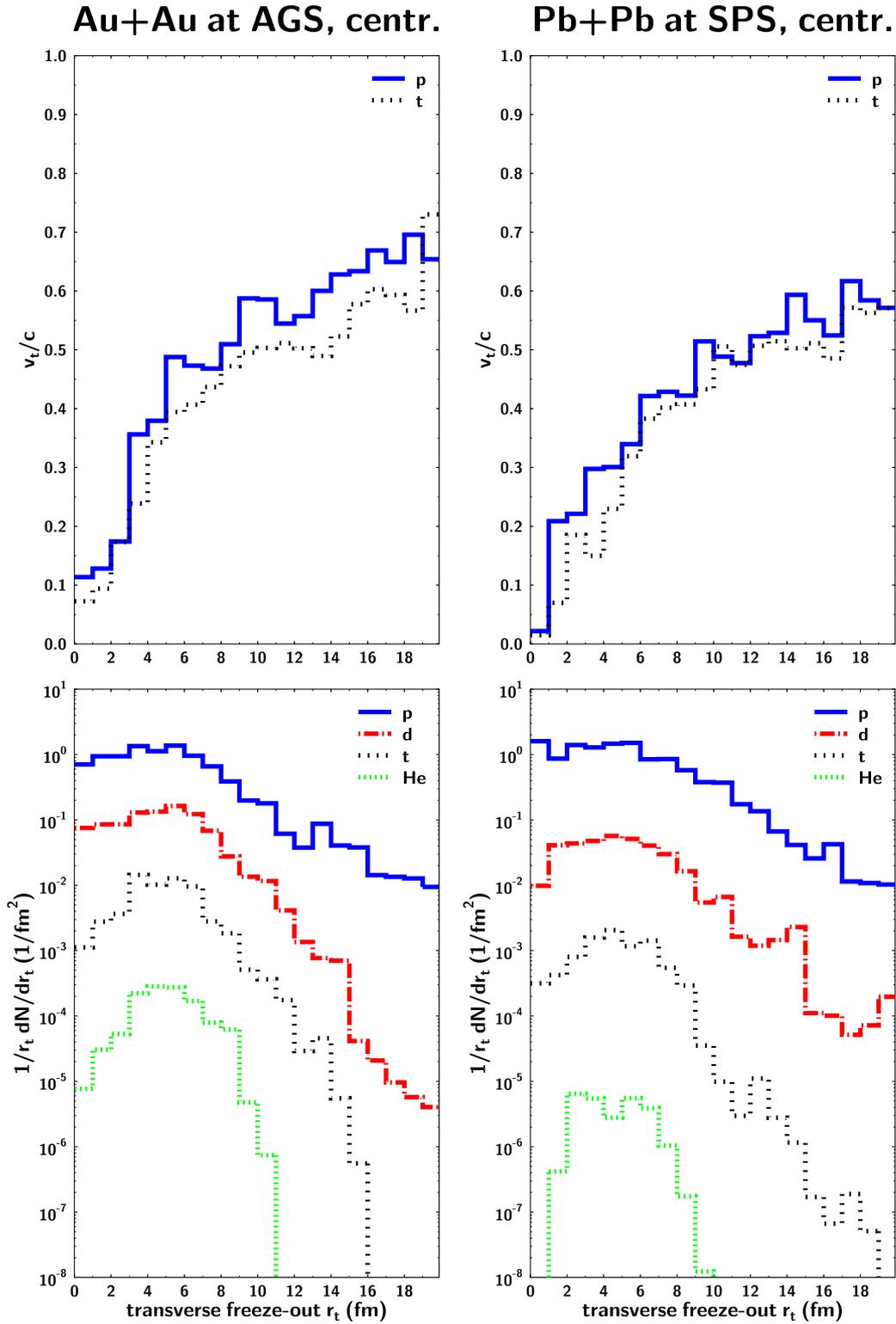


Figure 7: Left: Top: The reaction Au+Au at 10.7 AGeV, (central events and central rapidities selected). Transverse velocity profile of protons and tritons vs transverse freeze-out radius. Bottom: Radial freeze-out density of p, d, t, He. Right: The same for Pb+Pb at 160 AGeV.

In Fig.8 the proton (dotted) freeze-out radius and time is compared to the freeze-out of  $\pi$ 's (dashed) and kaons (dashed dotted, grey). The transverse freeze-out radius of the kaons is found smaller than those of the pions and protons. The freeze-out time of the kaons is much earlier than for the  $\pi$ 's and protons. Most probable freeze-out times (in fm/c) for K: $\pi$ :p are 9:15:23, which reflects the vastly different cross sections of these particles in the nuclear medium ( $\sigma \approx 16, 25, 40$  mb) . The emission times and the positions of different particles are clearly distinguishable, in contrast to the simple volume breakup of the expanding fireball were one assumes a equal time freeze-out of all particles.

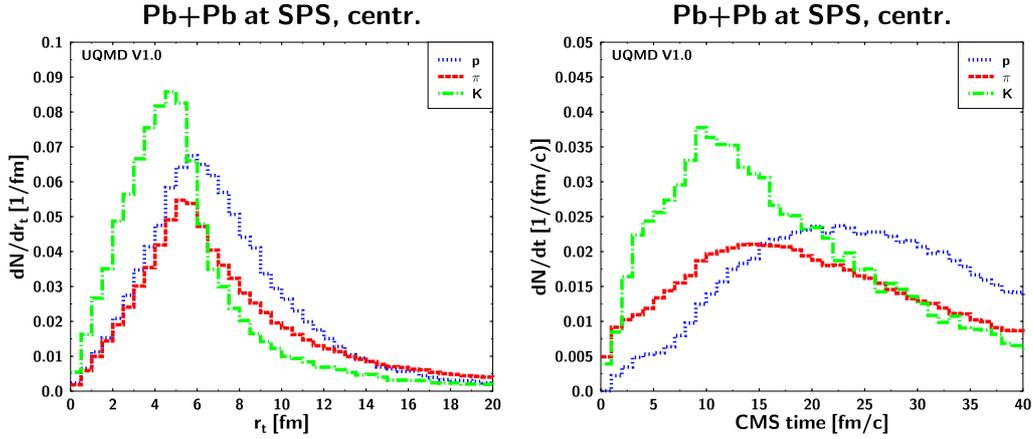


Figure 8: Left: Normalized radial freeze-out densities of kaons, pions and protons in the reaction Pb+Pb at 160 AGeV, centrally triggered. Right: Freeze-out times of kaons, pions and protons in the same reaction.

Switching off particle decay one finds smaller radii and earlier mean freeze-out times, for baryons as well as for mesons. The investigated freeze-out distribution can be checked via HBT analysis. The radial velocity profile rises linearly to  $0.5c$  and saturates.

## 6 Conclusion

We have presented UrQMD/QMD transport calculations for head on collisions of Au+Au (Pb+Pb). Temperatures were deduced by fitting final state particle spectra as well by analysis of local velocity distributions in the intermediate reaction stages. The latter are in agreement with the temperatures which are necessary to explain the large fragment abundances in terms of a chemical equilibrium.

In summary, any temperature obtained from particle spectra – even if collective flow has been taken into account – can only serve as a rough estimate for the complex space-time conditions at the individual freeze-out. In addition we found that the transverse flow varies for different particles and fragments. The freeze-out times and densities are not unique - the configuration space distributions are essentially unknown from experimental data.

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