Lattice-supersolids in bosonic quantum gases with Rydberg excitations

In the course of this thesis we discuss a certain kind of supersolid, the lattice-supersolid, which can be realized using quantum gases in an optical lattice trap. The lattice-supersolid, which simultaneously possesses off-diagonal and diagonal long-range order in its density matrix and also breaks the discrete translational symmetry of an underlying lattice, is induced by self-ordering of the gas due to strong long-range van der Waals interactions. In the considered scenario, the interactions are facilitated by the excitation of atomic Rydberg states, which exhibit enhanced van der Waals forces.

In the first part of this thesis (chapters 1-3), we review the relevant basics of quantum gases, Rydberg physics and introduce the extended Bose-Hubbard model. We start with the relevant methods and devices of the vast toolbox available in common quantum gas experiments, as well as consider the main concepts behind superfluidity and supersolidity. This is followed by an introduction of some basic concepts of Rydberg atoms in quantum many-body systems, with a focus on the facilitation of long-range interactions and the implementation in a theoretical model. Thereafter a brief introduction is given, on the realization of the Bose-Hubbard model in optical lattice systems and its extension to include Rydberg states, which concludes the introductory part of this thesis.

In the following part (chapters 4-6), we introduce the theoretical tools used to derive the results presented in the final part. First, an introduction to a real-space extension of bosonic dynamical mean-field theory (RB-DMFT) for bosonic systems with long-range interactions in the Hartree approximation is given. This method is based on the non-perturbative self-consistent evaluation of the lattice Green's function, which also incorporates the effect of nearest neighbor correlations due to the non-condensed particles. Then we focus on a quasiparticle expansion of the Bose-Hubbard model, which has its foundation in linearized fluctuations of a static mean-field ground-state, allowing for the prediction of a vast range of experimentally relevant observables. Lastly, we introduce an efficient truncation scheme for the local bosonic Fock-basis, which allows for the simulation of phases with high condensate density at a vastly reduced computational effort.

In the final part (chapters 7 and 8), we discuss the application of both methods to itinerant bosonic gases in two-dimensional optical lattices, in order to predict the equilibrium ground-state phases, as well as the signatures of supersolidity and its formation in spectral functions and the dynamic and static structure factor. Specifically, we focus on two limiting cases. Firstly, we consider a two-component gas, as realized by two hyperfine ground states, for example, of rubidium-87, where one component is off-resonantly excited to a Rydberg state, which generates a soft-core shaped interaction potential. Secondly, we discuss the opposing limit, using near-resonant excitations of Rydberg states, where the interacting component now directly corresponds to the Rydberg state, which interacts via a van der Waals potential. In both cases we discuss the rich variety of supersolid phases, which are found for a wide range of parameters. We also discuss how some of these phases can be realized in experiment.

In the subsequent appendices (A to D) we discuss some methodological details. Most notably, we consider the possible Fock-extension of the Hartree approximation (appendix A), introduced in the RB-DMFT treatment of the extended Bose-Hubbard model.

Kurzzusammenfassung (deutsch):

Diese Arbeit ist der Bestimmung des reichhaltigen Phasendiagramms im Vielteilchen-Grundzustand und der zugehörigen spektralen Eigenschaften eines ultrakalten Quantengases bosonischer Atome in einem zweidimensionalen optischen Gitter unter dem Einfluss kohärent angeregter Rydbergzustände gewidmet. Da diese atomaren Anregungen starke langreichweitige Van-der-Waals-Wechselwirkungen zwischen den sogenannten Rydbergatomen hervorrufen, können diese eine Symmetriebrechung der diskreten Translationssymmetrie des optischen Gitters verursachen. Im

Zuge dieser Arbeit betrachten wir den Fall einer abstoßende Van-der-Waals-Wechselwirkung. In isolierenden Phasen kommt es dann zur Formierung einer Treppe von Dichtewelle-Strukturen. Demgegenüber stehen Phasen bei denen ein makroskopischer Anteil der Teilchen als Gitter-Superfluid vorliegt. In diesen Phasen ist es ebenfalls möglich, dass die Van-der-Waals-Wechselwirkung der Rydberanregungen eine Dichtewelle bewirkt, sodass es in der Kombination zur Gitter-Supersolidität kommt. Gekennzeichnet ist diese durch eine langreichweitige Ordnung, welche sich sowohl über die Nebendiagonalelemente der Dichtematrix (Kennzeichen der Superfluidität) als auch über die Diagonale der Dichtematrix erstreckt und dabei die diskrete Translationssymmetrie des Gitters bricht (Kennzeichen der Dichtewelle).