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# Revisiting the Stealth Trading Hypothesis: Does Time-Varying Liquidity Explain The Size-Effect?\*

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## Abstract

Large trades have a smaller price impact per share than medium-sized trades. So far, the literature has attributed this effect to the informational content of trades. In this paper, we show that this effect can arise from strategic order placement. We introduce the concept of a liquidity elasticity, measuring the responsiveness of liquidity demand with respect to changes in liquidity supply, as a major driver for a declining price impact per share. Empirical evidence based on Nasdaq stocks strongly supports theoretical predictions and shows that the aspect of liquidity coordination is an important complement to rationales based on asymmetric information.

**JEL Codes:** G02, G10, G23

**Keywords:** stealth trading; price impact; liquidity elasticity; limit order book

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# 1 Introduction

Information on financial markets is aggregated through trading. A key aspect in market microstructure analysis and financial practice is to understand which trades move prices and by how much. In their seminal paper, Barclay and Warner (1993) show that 92% of a stock's cumulative price change over a given period takes place on medium-size trades. Since these trades, however, contribute significantly less to total trading volume and the total number of shares, their relative price impact (per share) is significantly higher than for small-size and large-size trades. This gave rise to the so-called *stealth trading hypothesis* that privately informed traders concentrate on medium-size trades. This hypothesis is in line with classical information-based theory (e.g., Kyle (1985)) suggesting that (large) informed traders slice their orders into smaller pieces in order to avoid revealing too much information to other traders.

Over the past two decades, the stealth trading hypothesis has received much attention due to a large body of confirming empirical evidence. Chakravarty (2001), Blau et al. (2009) and Ascioglu et al. (2011) replicate the methodology of Barclay and Warner (1993) and confirm the price-size regularity based on data from the New York Stock Exchange and Tokyo Stock Exchange.<sup>1</sup> While the separation between small-size and medium-size trades is not necessarily clear-cut on all markets, one central finding is common to basically all studies: the per-share trade-to-trade price change of small-size or medium-size trades is significantly larger than that of large-size trades. This result is strikingly robust across markets, assets and methodologies, and is also confirmed by studies focusing on price effects over longer intervals.<sup>2</sup>

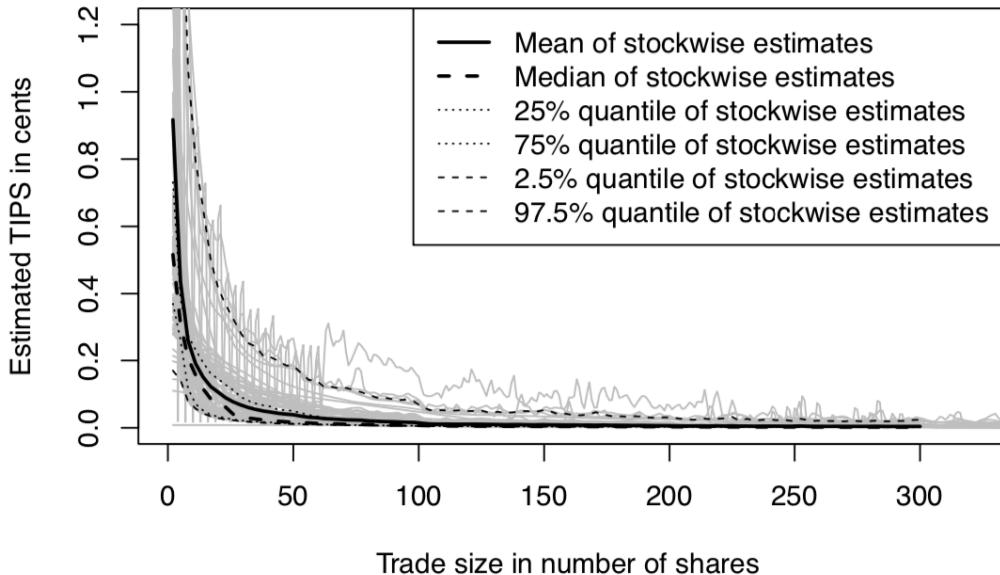
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<sup>1</sup>For instance, Chakravarty (2001) find that medium-sized trades account for roughly 47% of trade volume with their contribution to overall price changes being around 78%. In contrast, large orders account for about 51% of trade volume, but their contribution to overall price changes is just 25%.

<sup>2</sup>Employing error-correction methodology for changes in log ask and bid quotes at NYSE, Engle and Patton (2004) come to the same conclusion. Alexander and Peterson (2007) use 5-minute price changes, Frino et al. (2010) employ returns over intervals covering several trades, and Menkhoff and Schmeling (2010) focus on the long-run impact, estimated based on a structural VAR process. Anand and Chakravarty (2007) report similar effects based on options data by quantifying information shares

One way of formalizing the relatively larger impact of medium-size trades compared to large-size trades is to study the *trade impact per share* (henceforth TIPS), which – according to the stealth trading hypothesis – should be a declining function in the trade size. Recent evidence from Nasdaq trading reveals that this negative relationship is true even *instantaneously*. Figure 1 shows estimates of the average TIPS, computed as the per-share difference between the mid-quote *instantaneously* before and after a trade, as a function of trade size for all 100 stocks in the Nasdaq 100 composite index through the first quarter of 2014. As shown in this paper, this is a common regularity, which strongly holds for a wide range of stocks across the Nasdaq universe. The empirical evidence in

**Figure 1: Nonparametric estimates of the trade impact per share (TIPS) for all stocks in the Nasdaq 100 index.** The grey lines correspond to nonparametric estimates of the trade impact per share for the individual stocks as presented in more detail in Section 4.3. The solid line is the pointwise cross-sectional median of the stockwise estimates. The dashed lines are the cross-sectional 2.5% and 97.5% pointwise quantiles of the stockwise estimates.



support of the stealth trading hypothesis, however, is not fully conclusive. The underlying according to Hasbrouck (1995).

economic rationale typically builds on the theory of investors' trade-size choices in the spirit of Kyle (1985) and Admati and Pfleiderer (1988). In equilibrium, market makers infer from the size of a trade on the potential informational content and accordingly adjust their quotes. Such a framework is natural to explain the relationship between trade size and price changes over longer time spans (as in Alexander and Peterson (2007), Frino et al. (2010) and Menkhoff and Schmeling (2010)) or in classical market maker markets with comparably low trading frequencies as analyzed by Barclay and Warner (1993). Trade-to-trade price changes in liquid markets (as, e.g., analyzed by Ascioglu et al. (2011)) or even *instantaneous* price changes around trades in electronic order driven markets markets as illustrated by Figure 1, however, are not easily explained by the information content of trade sizes.

In this paper, we provide an alternative explanation for the price-size regularities observed in more recent data. We theoretically and empirically show that time-varying liquidity is a major driver of the observed effect. Our findings contribute to the ongoing literature by adding a perspective, which in the given context has been widely ignored so far and complements the reasoning beyond the argument of stealth trading. The intuition for the underlying mechanism builds on two effects. First, traders face liquidity costs in public limit order book markets and therefore strategically select the timing and location of their trades. This strategic selection ensures that market orders are submitted at times of high liquidity supply. The price impact of strategically placed orders is therefore lower than for randomly placed orders. Secondly, such a strategic placement of orders is more important for large orders than for smaller orders as large orders face higher downside costs when they are traded in illiquid periods. Consequently, large market orders have a lower price impact per share on prices than smaller orders. We therefore argue that stealth trading is not necessarily a pre-condition for the price-size regularity observed in Barclay and Warner (1993). It rather (or additionally) arises from the interaction of

time-varying market liquidity and strategic order placement.

Our theoretical setting is based on a simple one-period limit order book model, where traders arrive randomly at the market, but have discretion over the timing and placing of their orders. We assume that traders are pre-committed to trading in the sense that they are supposed to liquidate a given volume over a given time horizon. Their trading costs are determined by the limit order book depth determining the expected price of a market order. Assessing prevailing order book liquidity, traders can either submit market orders or can defer their trade decision to the future. This strategic placement is particularly important for large market orders, which generally cause significant imbalances between the demand and supply side of liquidity and thus induce non-trivial price reactions. In our theory, however, large imbalances do not occur since rational traders avoid submitting large orders in illiquid periods, but tend to synchronize their order placement with prevailing market liquidity.

Strategic order placement thus induces a link between time-varying liquidity supply and order placement decisions. To quantify the strength of this relation, we introduce the concept of a so-called *liquidity elasticity*. It measures the responsiveness of the demand side of liquidity with respect to changes in the underlying supply side (i.e., order book depth) due to strategic order placement. We show that the liquidity elasticity is a key concept for understanding the relationship between trade sizes and price impact. A high liquidity elasticity indicates a strong strategic placement of (large) orders, i.e., (large) orders tend to cluster at times when order book liquidity is high. Conversely, a small liquidity elasticity reveals that decisions on order placements are (widely) random. We empirically and theoretically show that a sufficiently high market liquidity elasticity induces a downward sloping TIPS profile as displayed by Figure 1, causing larger market orders to have a proportionally smaller price impact.

We show, moreover, that there is a second channel driving the price-size effect, which

arises from the shape of the prevailing limit order book. When the depth is comparably small on top of the book, but is more concentrated deeper in the book, larger orders cause a relatively smaller price impact than smaller orders since marginal shifts in the order book due to liquidity retraction decline with order size. Thus, a downward sloping TIPS may also arise in a market where trade decisions are purely random, as long as the depth per price level is increasing if one moves away from the best quotes.

Our theory provides a structural condition for the link between the monotonic decline of the TIPS profile and the combined effect of both channels, i.e., the liquidity elasticity and the order book shape. Under flexible parametric assumptions for the distribution of limit order book depth across price levels and the functional form of the liquidity elasticity, this structural condition boils down to a non-linear restriction on parameters in price impact regressions and regressions of order book depth on trading volumes. Nonparametric regression analysis based on message-level data from the Nasdaq100 composite index from January to April 2014 provides support for the imposed functional forms and the validity of underlying parametric (log-linear) regressions. We find strong empirical support for our theoretical predictions and the implied monotonicity of declining TIPS profiles. We particularly show that limit order book depth is a key determinant of strategic order placement and thus is an attractor of liquidity demand. Consequently, it is a major driving force behind the monotonicity of TIPS, explaining the price-size regularity as observed in Barclay and Warner (1993).

To identify which types of stocks are mostly affected by the two channels governing a declining TIPS, we analyze the cross-sectional variation of the estimated liquidity elasticity, the order book shape and the tightness of the TIPS monotonicity condition. We relate them to stock-specific liquidity characteristics, such as the quoted bid-ask spread, the market depth and the cumulative trading volume. We show that the liquidity elasticity and therefore a downward sloping TIPS profile is more pronounced in liquid markets.

This confirms the findings in Barclay and Warner (1993) and Chakravarty (2001), who argue that the price-size effect is more prevalent in markets with institutional investors. Our theory, however, suggests a different rationale. While Barclay and Warner (1993) interpret the pronounced size effect in liquid markets as evidence for more informed trades from institutional investors, our model shows that these effects can alternatively (or additionally) arise from liquidity coordination effects.

Several other theoretical papers have studied the role of market liquidity. These studies, however, mainly focus on the effect of the market design on market liquidity (see Parlour and Seppi (2003), Hendershott and Mendelson (2002), Foucault and Menkveld (2008) and Foucault et al. (2013)). In contrast, we analyze how liquidity itself affects the price formation process without making assumptions on possible asymmetric information.

This enables us to provide a complementary perspective to the information-driven arguments by Barclay and Warner (1993). While the latter effects certainly do play an important role in price formation, market liquidity, nevertheless, governs a wide spectrum of trading decisions, which in turn additionally affect the price formation process. This is evidently so, as most trading decisions, nowadays, are channeled to markets via dedicated brokerage services, irrespective of a trader's underlying motif, informed or not. Broker services are typically specialized in trading orders so as to minimize their client's transaction costs. Thus, at this stage, the process of trading eventually becomes influenced by liquidity considerations, which ultimately leads to distinct price effects. Hence, by linking the observed price-size regularities in high-frequency data to rather mechanical effects in a liquid electronic limit order book market, we provide an additional perspective complementing the empirical asset pricing literature and showing that market liquidity is a key aspect in explaining hitherto unresolved price regularities, as reported in, e.g., Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Stoll (2000), Chordia et al. (2001), Holmström and Tirole (2001), Pastor and Stambaugh (2003), O'Hara (2003),

Acharya and Pedersen (2005), Avramov et al. (2006), Sadka (2006) and Bekaert et al. (2007).<sup>3</sup>

Our paper is structured as follows. In Section 2, we introduce the trade-impact-per-share, TIPS, and outline the underlying framework for the modeling of price changes in a limit order book market with noise traders. In this market, we will show that a decreasing TIPS can solely arise by virtue of the order book shape. In Section 3, we extend the theory to include traders with strategic order submissions and analyze how liquidity and order book shape are capable of determining a decreasing TIPS. Section 4 empirically investigates the TIPS and the role of the order book shape and liquidity in determining its shape using tick data from the Nasdaq 100 universe. We formulate testable predictions that can be taken to the data and provide nonparametric evidence for the models used to formally test the predictions. The results indicate that there is strong evidence for a decreasing TIPS, which is mainly driven by liquidity considerations. Finally, Section 5 concludes.

## 2 Trade Impact in Limit Order Book Markets

### 2.1 Trade Impact per Share

Let  $t_i$  and  $x_i > 0$  ( $i \in \mathbb{N}$ ) denote the random times and sizes of trades, respectively. For convenience and without loss of generality, throughout this paper, we focus on *buy* market orders. Implications for sell market orders are derived analogously.

Let  $p_i$  and  $p_{i+1}$  denote the price immediately before and after a trade execution. We

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<sup>3</sup>Further evidence is reported in Lakonishok et al. (1992), Eleswarapu and Reinganum (1993), Datar et al. (1998), Vayanos (1998), Vayanos and Vila (1999), Chan and Fong (2000), Chordia et al. (2002), Amihud (2002), Huang (2003), Chordia et al. (2008), Narayan and Zheng (2010), Chordia et al. (2014).

define the total price impact of a trade of size  $x_i$  by

$$\Delta(x_i) := p_{i+1}(x_i) - p_i, \quad (1)$$

where the notation  $p_{i+1}(x_i)$  highlights the dependence of the price in  $t_{i+1}$  on the trade size  $x_i$  chosen in  $t_i$ . We assume that for a given trade size  $x$ ,  $\Delta(x)$  is a random variable. The (conditionally) expected trade impact per share (TIPS) is then defined as

$$\delta(x) := \mathbb{E}\left[\frac{1}{x_i} \Delta(x_i) \middle| x_i = x\right] = \frac{1}{x} \mathbb{E}[\Delta(x)] \quad (2)$$

for a given trade size  $x$ . The price-size regularity, as observed by Barclay and Warner (1993), implies that, on a per-share basis, large orders have a lower price impact than smaller orders, i.e.,

$$\delta(x_S) > \delta(x_L), \quad x_S < x_L, \quad (3)$$

where  $x_S$  denotes a medium or small trade size, and  $x_L$  denotes a large trade size. Hence, the observed price effect providing basis for the stealth trading hypothesis is linked to the monotonicity of  $\delta(x)$  in the trade size  $x$ . Henceforth, we assume that  $\delta(x)$  is differentiable in  $x$ . We then define a market exhibiting a decreasing TIPS if  $\delta(x)$  is monotonically decreasing for sufficiently large trades, i.e., if

$$\delta'(x) < 0$$

for  $x > 0$ .

## 2.2 A Limit Order Book Model

The key feature of a limit order book market is the central order book, which aggregates outstanding limit orders according to a price-time priority convention. Let  $d_i \in \mathbb{R}^+$  denote

the cumulative order book depth at  $t_i$  and  $F_i$  the cumulative distribution function of sell-side market depth, characterizing how depth is distributed across price levels. Finally, let  $a_i$  denote the prevailing best ask price.

In order to make the analysis tractable, we make the following assumptions. First, the cumulative density function (c.d.f.) of market depth is time-invariant, i.e.,  $F_i = F$  for all  $i \in \mathbb{N}$ . Second, price levels take on continuous values in  $\mathbb{R}^+$ .<sup>4</sup> Third,  $F$  possesses a continuous density  $f \geq 0$ , such that

$$F(\Delta) = \int_0^\Delta f(p)dp, \quad (4)$$

and  $F$  has a unique inverse function  $F^{-1}$ .

The function  $F(\Delta)$  represents the proportion of order volume standing between the price level  $a_i$  and  $a_i + \Delta$ , instantaneously before the arrival of the  $i^{\text{th}}$  market order. Hence, if  $d_i$  is the total order volume on the sell side, then the total number of shares standing between prices  $a_i$  and  $a_i + \Delta$  is given by

$$d_i^\Delta := d_i F(\Delta) = d_i \int_0^\Delta f(p)dp. \quad (5)$$

Note that  $d_i = \lim_{\Delta \rightarrow \infty} d_i^\Delta$  because  $\lim_{\Delta \rightarrow \infty} F(\Delta) = 1$ .

Equation (5) gives the central link between the trade impact and the trade size. Accordingly, a buy market order of  $x_i = d_i^\Delta$  shares shifts the prevailing best ask price by  $\Delta$ . Therefore, in a limit order book market with order book shape distribution  $F$  and total depth  $d_i$ , we obtain the trade impact by inverting (5), i.e.,

$$\Delta(x_i) = F^{-1}\left(\frac{x_i}{d_i}\right). \quad (6)$$

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<sup>4</sup>This assumption should especially hold for stocks that are traded at high price levels, where the relative price variation (or relative tick size) is small.

Thus, the price impact of a trade of size  $x_i$  is just the value of the quantile function of the limit order distribution function,  $F^{-1}$ , evaluated at the ratio of the executed volume to the overall standing order volume,  $x_i/d_i$ . Hence, together with Equation (2), the TIPS can be written as

$$\delta(x) = \frac{1}{x} \mathbb{E} \left[ F^{-1} \left( \frac{x_i}{d_i} \right) \middle| x_i = x \right] = \frac{1}{x} \mathbb{E}_x \left[ F^{-1} \left( \frac{x_i}{d_i} \right) \right], \quad (7)$$

where  $\mathbb{E}_x[\cdot]$  is shorthand notation for the conditional expectation  $\mathbb{E}[\cdot|x_i = x]$ .

### 2.3 Power-Law Shaped Order Books

To further characterize the TIPS, we assume a power-law for the order book shape. The power-law family is sufficiently flexible to capture a wide range of realistic order book shapes while allowing for a convenient and parsimonious representation of the key quantities of interest.

Let  $\bar{\Delta}$  denote the highest ask price prevailing in the book. Then, the power-law specification of the sell-side order book satisfies

$$f(\Delta) = \begin{cases} A\Delta^{\frac{1}{\alpha}-1} & \text{if } \Delta < \bar{\Delta}, \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

with  $\alpha > 0$  and  $A$  denoting a normalization constant. The normalization restriction  $\lim_{\Delta \rightarrow \bar{\Delta}} F(\Delta) = 1$  implies  $A = 1/(\alpha \sqrt[\alpha]{\bar{\Delta}})$ .

In this setting, the trade impact  $\Delta(x_i)$  can be easily derived. Integrating  $f(\Delta)$  with respect to  $\Delta$ , we obtain  $F(\Delta) = \alpha A \Delta^{\frac{1}{\alpha}}$  for  $0 \leq \Delta \leq \bar{\Delta}$ . Using (6), we obtain the resulting trade impact as

$$\Delta(x_i) = \bar{\Delta} \left( \frac{x_i}{d_i} \right)^{\alpha}. \quad (9)$$

Hence, power-law shaped order books generate power-law trade impact functions, which have been extensively studied in the literature. For instance, many studies suggest that price impact functionals follow a square-root law.<sup>5</sup>

Exploiting (2) and (9), we can calculate the TIPS as

$$\delta(x) = \frac{\bar{\Delta}}{x} \cdot \mathbb{E}_x \left[ \left( \frac{x_i}{d_i} \right)^\alpha \right] = x^{\alpha-1} \underbrace{\bar{\Delta} \cdot \mathbb{E}_x \left[ \left( \frac{1}{d_i} \right)^\alpha \right]}_{=:K(x)>0} =: x^{\alpha-1} K(x), \quad (10)$$

where  $K(x) := \bar{\Delta} \mathbb{E}_x[d_i^{-\alpha}]$  with  $K(x) > 0$  since  $d_i > 0$ . Accordingly, a decreasing TIPS, i.e.,  $\delta(x)' < 0$ , arises if and only if

$$(\alpha - 1)K(x) + xK'(x) < 0 \quad (11)$$

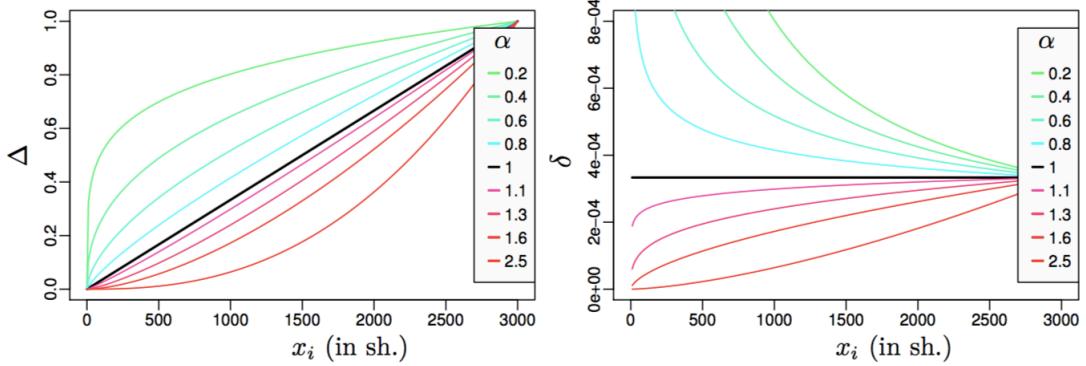
for all  $x \geq 0$ . Hence, the monotonicity of TIPS and of the order book shape are linked and depend on  $\alpha$ . Condition (11), however, also depends on the interaction between the market liquidity  $d_i$  and the trade volume  $x_i$  as captured by the term  $K'(x)$ . The latter captures the sensitivity of  $\mathbb{E}_x[d_i^{-\alpha}]$  with respect to changes of the trade size  $x$ . This term vanishes as long as market depth  $d_i$  and trade volume  $x_i$  are conditionally independent, implying  $K'(x) = 0$ . This benchmark case arises if the market order traders are noise traders who submit their orders randomly and independently from market prices and liquidity. In this case, trade sizes  $x_i$  and liquidity  $d_i$  are independent and therefore

$$\mathbb{E}_x \left[ \left( \frac{1}{d_i} \right)^\alpha \right] = \mathbb{E} \left[ \left( \frac{1}{d_i} \right)^\alpha \right] = K, \quad (12)$$

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<sup>5</sup>A linear price impact ( $\alpha = 1$ ) is assumed in a range of studies focusing on optimal trade execution strategies, see, e.g., Bertsimas and Lo (1998), Almgren (2001), Almgren and Chriss (1999), Almgren (2003a), Alfonsi et al. (2010), Alfonsi and Schied (2010) or Obizhaeva and Wang (2013). Various other theoretical and empirical studies suggest non-linear power laws for the price impact such as the square-root law ( $\alpha = 1/2$ ), see, e.g., Lillo et al. (2003), Gabaix et al. (2003), Almgren (2003b) and Bershova and Rakhlin (2013).

**Figure 2: Absolute trade impact and TIPS functional in power-law order books.** For a given trade size  $x_i$ , the figure illustrates the absolute trade impact  $\Delta(x_i)$  as of Equation (9) and the per-share trade impact (TIPS)  $\delta$  as of Equation (2) under a power-law book specification as of Equation (8). We set  $\bar{\Delta} = 1\$$  and  $d_i = 3000$  shares, corresponding to a realistic scenario for stocks that trade above 80\$. The trade size  $x_i$  is given in number of shares, whereas  $\Delta(x_i)$  is given in Dollars and  $\delta(x_i)$  (TIPS) is given in Dollar/share.



implying  $K'(x) = 0$ .

In this case, Equation (11) simplifies and the shape of TIPS is entirely driven by  $\alpha$ .

This is summarized in the following lemma:

**Lemma 1** (Decreasing TIPS under Noise Trading). *If market order traders are noise traders implying  $K'(x) = 0$ , the TIPS is decreasing in  $x$ , i.e.,  $\delta'(x) < 0$ , if and only if*

$$\alpha < 1. \tag{13}$$

Accordingly,  $\alpha < 1$  implies an upward sloping order book shape. Hence, if the order book shape is monotonically increasing, the per-share price impact of a trade is monotonically declining. In this case, a decreasing TIPS arises from a simple mechanical effect of liquidity consumption as illustrated in Figure 2.

Thus, even in this very simple setting without information asymmetry or strategic order submission behavior, we obtain a decreasing TIPS solely by virtue of the order book shape.

### 3 TIPS under Strategic Order Submission

The assumption of pure noise trading may be unrealistic as traders generally act strategically when they submit their orders, even when they are uninformed. In fact, brokers carefully choose the timing and placement of their orders in order to guarantee best execution.<sup>6</sup> Therefore, it is more realistic to assume that market liquidity  $d_i$  and trade volume  $x_i$  are generally interrelated, implying that  $K'(x) \neq 0$ .

We moreover assume that traders are pre-committed, that is, they need to trade an order of a specific size (due to informational or other reasons). The trader, however, has the strategic freedom about the timing of the trade, i.e., he may postpone his trade when trading is expected to be costly.

#### 3.1 The Traders' Trading Decision

We assume that at  $t_i$ , a risk-neutral trader enters the market and needs to buy  $n_i$  shares. We denote the trader's decision variable by  $\sigma$ . The trader can trade at  $t_i$  at the price provided by the limit order book, i.e.,  $\sigma = \text{trade}$ , and thus his trading volume equals  $x_i = n_i$ . In this case, he faces transaction costs  $\Pi^{\text{prime}}(n_i, d_i)$  implied by his trading volume  $n_i$  and the depth  $d_i$  currently prevailing on the (primary) market. Alternatively, he can defer his trade to the future, i.e.,  $\sigma = \text{defer}$ , with  $x_i = 0$ . Postponing the trade implies that the trader is neither explicit about the timing nor about the trading platform chosen for his (future) transaction. While he rules out to trade at  $t_i$  on the primary market, he may also decide to choose another trading platform, e.g., an upstairs market or brokerage network. We generically denote the resulting (expected) trading costs by

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<sup>6</sup>Brokerage businesses that operate client orders from institutional investors or hedge funds are specialized in trading strategies that guarantee best execution. Most of these trading strategies internalize the liquidity of the market, when they process client orders.

$\Pi^{other}$ . Consequently, the trader's transaction costs  $\Pi$  depending on  $\sigma$  can be written as

$$\Pi(\sigma, d_i, n_i) = \begin{cases} \Pi^{prime}(n_i, d_i) & \text{if } \sigma = trade, \\ \Pi^{other}(n_i) & \text{if } \sigma = defer. \end{cases} \quad (14)$$

We define  $\Pi$  as the so-called shortfall costs, i.e., the achieved price in excess of the best ask  $a_i$ . Thus, when the trader submits a buy market order of  $n_i$  shares, the transaction costs  $\Pi^{prime}$  are given by

$$\begin{aligned} \Pi^{prime}(n_i, d_i) &= d_i \int_0^{\Delta(n_i)} f(p)pdp = d_i \int_0^{F^{-1}(n_i/d_i)} f(p)pdp \\ &= \frac{\bar{\Delta} \cdot d_i}{\alpha + 1} \left( \frac{n_i}{d_i} \right)^{\alpha+1}. \end{aligned} \quad (15)$$

The upper bound  $\Delta(n_i)$  corresponds to the shift of the best ask price when  $n_i$  shares of market depth are matched by the order, which, according to Equation (9) equals  $\Delta(n_i) = F^{-1}(n_i/d_i) = \bar{\Delta}(n_i/d_i)^\alpha$ . Note that at time  $t_i$ ,  $\Pi^{prime}$  is known and thus deterministic, because the order book is observed.

In case the trader decides to defer his trading decision (or to move to another trading platform), we assume that he has an expectation of the future per-share transaction costs,  $\mu$ . Accordingly, the expected costs of trading at another point of time or on an alternative market are given by

$$\Pi^{other}(n_i) = (\mu + \xi)n_i, \quad (16)$$

where  $\xi$  is a zero-mean random variable representing uncertainty in commissions or costs associated with off-exchange trading. Note that Equation (16) is a linear pricing rule, which is generally considered to be consistent with the trading practice on trading venues, such as dark pools or upstairs brokerage markets, see, e.g., Keim and Madhavan (1996),

Booth et al. (2002) or Harris (2003).

We assume that the trader's objective is to choose the trading venue (or trade timing) that minimizes his expected transaction costs. Thus, his optimal choice is

$$\sigma_i^* = \arg \min_{\sigma \in \Sigma} \mathbb{E}_i[\Pi(\sigma, d_i, n_i)], \quad (17)$$

with  $\Sigma := \{\text{trade}, \text{defer}\}$  and  $\mathbb{E}_i$  denoting the expectation taken at time  $t_i$ . The trader chooses to trade on the primary limit order book market at time  $t_i$  if and only if  $\Pi^{\text{prime}} < \mathbb{E}_i[\Pi^{\text{other}}]$ . This decision depends on the market liquidity  $d_i$ , off-exchange per-share trading costs  $\mu$  and the trade size  $n_i$ . The following lemma summarizes the optimal decision rule and the resulting executed trade volume  $x_i$  in the primary market.

**Lemma 2** (Optimal Trade Decision). *The trader's optimal trading strategy  $\sigma_i^*$  at time  $t_i$  satisfies*

$$\sigma_i^* = \begin{cases} \text{trade} & \text{if } \mu > \Phi\left(\frac{n_i}{d_i}\right), \\ \text{defer} & \text{else,} \end{cases} \Leftrightarrow x_i = \begin{cases} n_i & \text{if } \mu > \Phi\left(\frac{n_i}{d_i}\right), \\ 0 & \text{else,} \end{cases} \quad (18)$$

with  $\Phi(y) = \frac{1}{y} \int_0^{F^{-1}(y)} f(p)pdp$  being the per-share transaction costs of a trade in the primary market that consumes a proportion of  $y$  of overall liquidity in the book.

**Proof:** See A.

Hence, if the expected per-share transaction costs for deferring the trade,  $\mu$ , are large relative to the current per-share transaction costs,  $\Phi(n_i/d_i)$ , the trader prefers to defer his trade to the future. The market liquidity  $d_i$  thus affects the trade size decision  $x_i$  through its impact on the per-share trading costs  $\Phi(n_i/d_i)$ . A higher depth  $d_i$  reduces the per-share trading costs and thus makes it more likely to trade at  $t_i$  in the primary market. This is easily seen by the fact that  $\Phi$  is a monotonously increasing function of  $n_i/d_i$ . Hence, a higher depth  $d_i$  attracts more market order submissions by virtue of reduced trading costs. This makes  $d_i$  and  $x_i$  positively correlated.

As a convenient characterization for the interdependence between  $d_i$  and  $x_i$ , we introduce the concept of a *liquidity elasticity*, defined as

$$\mathcal{E}_L(x) := x \cdot \frac{g'(x)}{g(x)}, \quad (19)$$

where  $g(x) := \mathbb{E}[d_i|x_i = x]$  denotes the expected depth when a trade size  $x$  is executed at  $t_i$ . A high liquidity elasticity indicates that liquidity supply strongly attracts liquidity demand. We can therefore conclude that in markets with strategic order submission, the liquidity elasticity is positive, i.e.

$$\mathcal{E}_L(x) > 0 \text{ for all } x. \quad (20)$$

In the following section, we will show that the liquidity elasticity  $\mathcal{E}_L(x)$  is a key determinant of a decreasing TIPS, which in turn is a major driver of the price-size regularities according to Barclay and Warner (1993).

### 3.2 Liquidity Elasticity and Decreasing TIPS

By defining the conditional expectation of the total price impact of a trade  $\Delta(x_i) = p_{i+1}(x_i) - p_i$  as

$$\tilde{\Delta}(x) := \mathbb{E}[\Delta(x_i)|x_i = x] = \mathbb{E}_x[\Delta(x_i)],$$

we can re-write the definition of the TIPS according to Equation (2) as  $\delta(x) = \tilde{\Delta}(x)/x$ . Its first-order derivative  $\delta'(x)$  is then obtained by

$$\delta'(x) = \frac{1}{x} \left( \tilde{\Delta}'(x) - \frac{\tilde{\Delta}(x)}{x} \right) = \frac{1}{x} \left( \tilde{\Delta}'(x) - \delta(x) \right). \quad (21)$$

To conveniently express the condition for a monotonically decreasing TIPS,  $\delta'(x) < 0$ ,

in terms of  $\mathcal{E}_L$ , we decompose  $d_i$  into its conditional expectation  $g(x_i) := \mathbb{E}[d_i|x_i]$  and an error term,

$$d_i = g(x_i) + \epsilon_i, \quad (22)$$

with  $\epsilon_i := d_i - g(x_i)$ , and  $\mathbb{E}[\epsilon_i|x_i] = 0$ . Inserting Equation (22) into Equation (6) and applying a Taylor expansion in  $\epsilon_i$  around  $\epsilon_i = 0$  yields

$$\begin{aligned} \Delta(x_i) &= F^{-1}\left(\frac{x_i}{g(x_i) + \epsilon_i}\right) \\ &= F^{-1}\left(\frac{x_i}{g(x_i)}\right) - \frac{1}{f\left(F^{-1}\left(\frac{x_i}{g(x_i)}\right)\right)} \frac{x_i}{g(x_i)^2} \epsilon_i + R, \end{aligned} \quad (23)$$

where  $R$  denotes the remainder term. Under the assumption of the error being small, i.e.,  $\epsilon_i \approx 0$ , we have  $R \approx 0$  and can show that

$$\tilde{\Delta}(x) := \mathbb{E}_x[\Delta(x_i)] \approx F^{-1}\left(\frac{x}{g(x)}\right) \quad (24)$$

and

$$\begin{aligned} \tilde{\Delta}'(x) &\approx \left[F^{-1}\left(\frac{x}{g(x)}\right)\right]' \\ &= \frac{1}{g(x)f(F^{-1}(x/g(x)))} \left(1 - x \frac{g'(x)}{g(x)}\right) \\ &= \frac{1}{g(x)f(F^{-1}(x/g(x)))} \left(1 - \mathcal{E}_L(x)\right). \end{aligned} \quad (25)$$

The assumption  $\epsilon_i \approx 0$  implies that the transaction volume executed at  $t_i$  is a relatively precise indicator for the prevailing depth, i.e., traders strategically choose their trade size

in accordance with the depth in the market. Under this assumption, we define

$$\Psi_{g,f}(x) := \frac{g(x)}{x} f(F^{-1}(x/g(x))) F^{-1}(x/g(x)). \quad (26)$$

Then, exploiting the approximations Equation (24) and Equation (25), we can formulate a condition for a decreasing TIPS as provided in the following proposition.

**Proposition 1** (Decreasing TIPS). *Let  $f$  be the density of the limit order book shape distribution  $F$  given by Equation (4) and  $x$  be the trade size. Then, TIPS is monotonically decreasing, i.e.,  $\delta'(x) < 0$ , if and only if*

$$1 - \Psi_{g,f}(x) < \mathcal{E}_L(x) \quad \forall x > 0. \quad (27)$$

**Proof:** See A.

The inequality (27) is determined by the two factors  $\mathcal{E}_L(x)$  and  $\Psi_{g,f}(x)$ . Note that the liquidity elasticity  $\mathcal{E}_L(x)$  solely depends on  $g(x)$  and measures the degree of liquidity coordination in the market, whereas  $\Psi_{g,f}(x)$  depends on  $g(x)$  and the order book shape via  $f(x)$ . Thus, a decreasing TIPS arises in two scenarios (or combinations thereof): In the first scenario, the liquidity elasticity  $\mathcal{E}_L(x)$  is sufficiently high, so that large orders are subject to a systematic self-selection mechanism and are dis-proportionally strongly absorbed by a thick order book (so-called "liquidity attraction"). In the second scenario,  $\Psi_{g,f}(x)$  is sufficiently high and large orders are strategically posted in periods of high market depth.

In the power-law case, it is easy to show that  $\Psi_{g,f}(x) = 1/\alpha$ , i.e.,  $\Psi_{g,f}(x)$  does not depend on the trade size  $x$  and is inversely related to the order book shape parameter  $\alpha$ . In this case, the TIPS monotonicity condition (27) is given by

$$1 - \frac{1}{\alpha} < \mathcal{E}_L(x). \quad (28)$$

In this case, the positivity of  $\mathcal{E}_L(x)$  is satisfied if  $\alpha < 1$ , corresponding to an upward-sloping order book.

Note that the approximations (24) and (25), resting on the assumption  $\epsilon_i \approx 0$  and leading to Equation (27), are only heuristic. In B, we establish testable conditions for the case of a power law shaped limit order book under which the inequality (28) is sufficient for  $\delta'(x) < 0$ , i.e., a monotonously decreasing TIPS. These conditions rest on an upper bound for the relative prediction error  $\epsilon_i/g(x_i)$  and a restriction on  $\alpha$  and  $\mathcal{E}_L(x)$ . The corresponding conditions are summarized in Proposition 2 in B.

## 4 Empirical Evidence

In this section, we empirically test whether our underlying theory is consistent with data from Nasdaq trading. We will proceed as follows. In Section 4.1, we formulate testable hypotheses by making use of flexible parametric assumptions on the order book shape and the relationship between depth and trade size. These parameterizations operationalize the relationships derived in the previous sections and yield testable hypotheses. In Sections 4.2 and 4.3, we present the underlying data and provide an initial nonparametric analysis yielding data-driven support for the choice of our parametric specifications. The latter are then used in Section 4.4 to conduct parametric statistical inference. Section 4.5 shows that the liquidity elasticity is the major driver of a decreasing TIPS. Finally, Section 4.6 explores in how far the stockwise variation of the driving forces behind a decreasing TIPS condition can be explained by certain stock-specific characteristics.

### 4.1 Testable Model Predictions

To test the validity of the condition in Proposition 1 based on minimal assumptions, one would need nonparametric estimates of  $\Psi_{g,f}$  and  $\mathcal{E}_L$ , which in turn depend on the unknown

functions  $g$  and  $f$ , capturing the relationship between depth and trade size and the order book shape, respectively. Given estimates for  $g$  and  $f$  and thus also for  $g'$  and  $F^{-1}$ , one could obtain plug-in estimates of  $\Psi_{g,f}$  and  $\mathcal{E}_L$  by replacing the unknown quantities with their estimates. The estimates for  $\Psi_{g,f}$  and  $\mathcal{E}_L$  could then be used to construct a test for the condition in Proposition 1. However, any such nonparametric procedure suffers from numerous severe drawbacks. Firstly, the complicated dependence of  $\Psi_{f,g}$  and  $\mathcal{E}_L$  on  $g$  and  $f$  makes the task of deriving distributional results extremely daunting. Secondly, even if one were able to derive the asymptotic distribution of the estimates for  $\Psi_{g,f}$  and  $\mathcal{E}_L$ , there would still remain the issue of selecting several smoothing parameters, which is further complicated by the absence of theoretical statistical guidance on how to optimally do this for testing purposes.

Hence, in order to formally test the validity of the decreasing TIPS condition, we additionally impose flexible parametric forms on the order book shape and the relationship between depth and trade size. In doing so, we are able to state testable hypotheses in terms of parameters in (log) linear regression models. Denote the instantaneous price impact induced by a trade by  $\Delta_i$ . Then, we assume that  $\Delta_i$  can be modeled as  $\Delta_i = \Delta(x_i)\tilde{\epsilon}_i^\Delta$  with  $\tilde{\epsilon}_i^\Delta$  being a multiplicative error term satisfying  $\mathbb{E}[\tilde{\epsilon}_i^\Delta|x_i] = 1$  and  $\Delta(x_i)$  following a power-law function of  $x_i/d_i$  as given by Equation (9). Such multiplicative specifications provide a natural framework for the modelling of positive-valued random variables, see, e.g., Hautsch (2012) for an overview.

In a similar vein, we assume that the relationship between depth and trade size can be modelled using a power law relationship. This implies a multiplicative specification  $d_i = g(x_i)\tilde{\epsilon}_i^d$  with

$$g(x_i) := \mathbb{E}[d_i|x_i] = Bx_i^q, \quad \tilde{c} > 0, \quad (29)$$

where  $\tilde{\epsilon}_i^d$  denotes an error term with  $\mathbb{E}[\tilde{\epsilon}_i^d|x_i] = 1$ ,  $q > 0$  is the power law parameter, and

$B$  is a constant. Such a specification implies  $\mathcal{E}_L = q$ .

Using these flexible functional form assumptions allow us to restate the main model prediction in terms of the parameters  $\alpha$  and  $q$ :

**Testable Prediction 1** (Decreasing TIPS under power law assumptions).

*Assume a trade size  $x^c \geq 1$  such that  $g(x) = Bx^q$  for all  $x \geq x^c$  and a power-law trade impact as given by Equation (9). Then, the necessary and sufficient condition for a decreasing TIPS is*

$$1 - \frac{1}{\alpha} < q.$$

Taking logarithms of the multiplicative specifications  $d_i = g(x_i)\tilde{\epsilon}_i^d$  and  $\Delta_i = \Delta(x_i)\tilde{\epsilon}_i^\Delta$  results in the log-linear specifications

$$\ln(d_i) = c + q \ln(x_i) + \epsilon_i^d, \quad (30)$$

$$\ln(\Delta_i) = b + \alpha \ln(x_i/d_i) + \epsilon_i^\Delta, \quad (31)$$

with  $\epsilon_i^d = \ln \tilde{\epsilon}_i^d - \mathbb{E}[\ln \tilde{\epsilon}_i^d]$ ,  $c = \ln B + \mathbb{E}[\ln \tilde{\epsilon}_i^d]$ , and  $\epsilon_i^\Delta = \ln \tilde{\epsilon}_i^\Delta - \mathbb{E}[\ln \tilde{\epsilon}_i^\Delta]$  and  $b = \ln(\bar{\Delta}) + \mathbb{E}[\ln \tilde{\epsilon}_i^\Delta]$ .

Additionally, a necessary condition for the adequacy of the power law shaped limit order book model is that the shape parameter is positive:

**Testable Prediction 2** (Power law limit order book).

*The power-law limit order book model is appropriate if*

$$\alpha > 0.$$

## 4.2 Data

We use order message data from the platform LOBSTER processing Nasdaq TotalView data.<sup>7</sup> The data contains submissions, cancellations and executions of any limit order. Moreover, it provides a precise identification on trade directions. We employ data of the Nasdaq 100 composite index constituents from the first three months of 2014. For the  $i^{\text{th}}$  trade , we record the trade size  $x_i$  and compute the *instantaneous* price impact  $\Delta_i$  as the *signed* difference between the mid-quote immediately before and after the trade. This ensures that price changes are only due to trades and are all non-negative. For each trade, we record the cumulative order book depth from level 1 up to 5, denoted by  $d_{j,i}$ ,  $j = 1, \dots, 5$ , respectively. These quantities yield alternative measures for  $d_i$ . By neglecting depth beyond the 5th level, we presume that this part of the order queue is widely irrelevant for most trading decisions.

The upper panel of Table 1 contains cross-sectional summary statistics of stockwise time averages of the number of shares per trade, the cumulative depth up until the 5<sup>th</sup> level, the bid-ask spread, the midquote price, and the daily realized variance, computed as the daily sum of squared price changes. Given our research objective, it is natural to focus only on trades with non-zero impact. As reported in the last row of Table 1, this reduces the data set by about 50%. In the second panel of the table, we report the relative proportions of trades with trade sizes exceeding the cumulative depth on a given level. These quantitites indicate how often the respective order book levels are depleted by the execution of a single trade. We observe that for the vast amount of trades, the trade size equals the number of shares on the first level of the order book. Conversely, the percentage of trade sizes exceeding the cumulative depth on higher levels tails off rapidly. Finally, there are hardly any trades, whose size exceeds the 5<sup>th</sup> level of cumulative depth  $d_5$ , thus indicating that basically no information is lost by not considering order

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<sup>7</sup>See <https://lobsterdata.com/>.

book levels beyond the 5<sup>th</sup> level. We shall see later that our parametric assumptions are mostly plausible for trades exceeding a critical size of 100 shares. This size seems to be a typical minimal trading volume which is undercut in just 3% of all transactions (see the penultimate row of Table 1).

**Table 1: Cross-sectional summary statistics of stock and trading characteristics.** The table provides cross-sectional summary statistics of time averages of the number of shares per trade, the cumulative depth in terms of number of shares up until level 5, the bid-ask spread measured in cents, the price computed as the midquote in dollars, and the daily sum of squared impacts (in squared cents). The averages are computed for all buy side trades with a positive impact. We moreover report the relative proportions of trades with trade sizes exceeding the cumulative depth on up to five levels and trades with trade sizes greater than 100 shares. *Zero impacts* gives the percentage of zero impacts in the original data set, which has to be dropped for the analysis.

	Mean	St. Dev.	Min.	10th	Median	90th	Max.
Trade Size $x$	729	3490	60	100	185	804	34807
$d_1$	775	3847	54	96	184	869	38386
$d_2$	4258	25809	121	230	658	4644	258045
$d_3$	8078	48425	177	358	1271	9045	484062
$d_4$	12220	71890	232	489	1996	14443	718613
$d_5$	16220	93539	294	607	2819	19910	935068
Spread (cents)	2.65	3.92	1.00	1.02	1.23	5.20	28.56
Midquote (Dollar)	111.13	179.95	3.54	25.00	63.31	184.45	1236.71
Daily Tick RV (cents <sup>2</sup> )	18675	62507	9	163	1136	34014	494258
Observations	64359	48527	2217	23638	51910	106543	354090
$x = d_1$ (%)	95.3	2.3	83.8	93.0	96.2	97.3	97.8
$x > d_1$ (%)	2.85	2.44	0.55	0.98	1.86	6.19	11.87
$x > d_2$ (%)	0.516	0.768	0.000	0.060	0.205	1.285	3.889
$x > d_3$ (%)	0.178	0.290	0.000	0.015	0.075	0.440	1.592
$x > d_4$ (%)	0.069	0.117	0.000	0.006	0.031	0.158	0.678
$x > d_5$ (%)	0.030	0.048	0.000	0.002	0.014	0.063	0.274
$x < 100$ (%)	3.09	1.75	0.00	0.67	2.99	5.30	9.47
Zero impact (%)	49.33	13.07	31.03	33.89	46.59	69.52	90.42

### 4.3 Nonparametric (Pre-)Analysis

To provide support for the functional form assumptions made above, we perform a nonparametric analysis. In particular, we investigate the validity of the log-linear representations

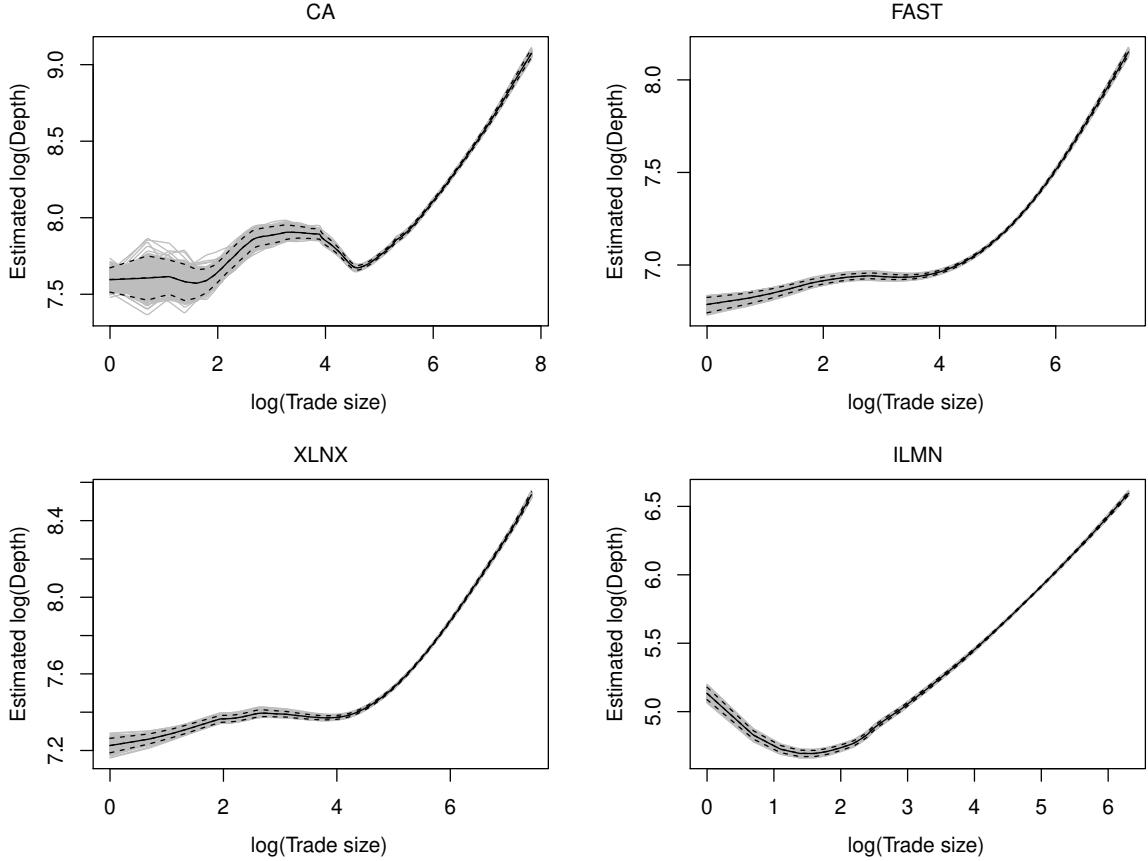
in (30) and (31) by estimating the following nonparametric regression model counterparts using the cumulative three-level depth as a measure for market depth  $d_i$ ,

$$\ln(d_{3,i}) = m^d(\ln(x_i)) + v_i^d, \quad (32)$$

$$\ln(\Delta_i) = m^\Delta(\ln(x_i/d_{3,i})) + v_i^\Delta, \quad (33)$$

where  $m^d(\ln(x_i))$  and  $m^\Delta(\ln(x_i/d_{3,i}))$  denote the conditional mean of  $\ln(d_{3,i})$  and  $\ln(\Delta_i)$ , respectively, and  $v_i^d$  and  $v_i^\Delta$  are corresponding error terms. By treating  $m^d$  and  $m^\Delta$  as unknown functions of the underlying regressors  $\ln(x_i)$  and  $\ln(x_i/d_{3,i})$ , we let the data “speak” on how the regressor in each equation is related to the corresponding dependent variable. This allows us to assess how reasonable the linear relationships postulated in (30) and (31) are. The estimation of  $m^d$  and  $m^\Delta$  is done using local linear kernel regression with an optimal bandwidth. As an optimality criterion, we choose the prediction accuracy of the dependent variable in the final month of our sample. The local linear kernel regression estimator is chosen as it automatically corrects for boundary bias issues in nonparametric regression problems, when the support of the data is bounded, which naturally arises here due to the positivity of trade sizes. Furthermore, the behaviour of the local linear regression estimator is well understood. In particular, it is well known that under suitable regularity conditions, the estimators are consistent and asymptotically normal, see, for instance, Fan and Gijbels (1996) or Li and Racine (2007). We now provide details of the procedure for the estimation of  $m^d$ . The procedures for estimating  $m^\Delta$  are analogous, any deviations will be mentioned below. By construction, for  $\mathbb{E}[v_i^d | \ln(x_i)] = 0$ , the regression function  $m^d$  is the conditional expectation of log depth given the (log) trade size. Let the sample of observations of log depth and log trade sizes be given by  $\{(\ln(d_{3,i}), \ln(x_i))\}_{i=1}^N$ .

**Figure 3: Local linear estimates of  $m^d$  in Equation (32) for CA, FAST, ILMN and XLNX.** The solid black line corresponds to the estimate. The grey lines are estimates for 500 bootstrap replications of the estimator. The dashed lines are the pointwise 2.5% and 97.5% quantiles over the bootstrap estimates. The estimation range for each stock covers trade sizes up until the 99<sup>th</sup> sample percentile.



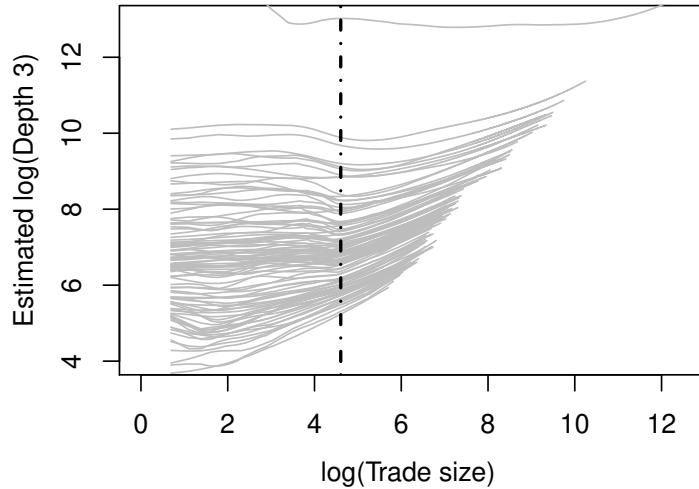
Then, the local linear estimator solves the locally weighted least squares problem

$$\sum_{i=1}^N (\ln(d_{3,i}) - a_u - b_u \ln(x_i))^2 K\left(\frac{\ln(x_i) - u}{h}\right) \quad (34)$$

with respect to the parameters  $(a_u, b_u)'$ . The function  $K(x)$  is a kernel weight function with the bandwidth given by the parameter  $h$ . For each value  $u$ , the minimizer  $\hat{a}_u$  is the estimator of the regression function at the point  $u$ , i.e.,  $\hat{a}_u = \hat{m}^d(u)$ . Under suitable regularity conditions, it can be shown that  $\hat{m}^d$  is a (pointwise) consistent estimator of  $m^d$ . Under

additional conditions it can be shown to be (pointwise) asymptotically normally distributed. We implement the estimator using the Epanechnikov kernel  $K(x) = \frac{3}{4}(1 - x^2)\mathbf{1}_{|x| \leq 1}$ . The estimates are computed based on an equidistant grid of the regressor values between one share and the 99<sup>th</sup> sample percentile of the trade size distribution. There are too few observations to be able to estimate the function well in the extreme tail of the distribution. For the same reason, we estimate  $m^\Delta$  on a grid between the 0.5<sup>th</sup> and the 99.5<sup>th</sup> sample percentiles of the observed trade size to depth ratio. Details on the (optimal) choice of the bandwidth are provided in C. To quantify the estimation uncertainty, we utilize a bootstrap procedure. Corresponding details are given in D. Figure 3 displays the estimation results

**Figure 4: Local linear estimates of  $m^d$  in Equation (32) for all 100 stocks.** The grey lines correspond to the estimates for the individual stocks. The solid line is the pointwise cross-sectional median of the stockwise estimates. The dotted dashed vertical line is at  $\ln(100)$ , i.e. the regressor value for a trade size equal to 100 shares. The dashed lines are the pointwise cross-sectional 2.5% and 97.5% quantiles of the stockwise estimates. By definition, the cross-sectional quantiles of the estimates can only be computed over the range of the regressor that is observed for all 100 stocks. The estimation range for each stock covers trade sizes up until the 99<sup>th</sup> sample percentile.



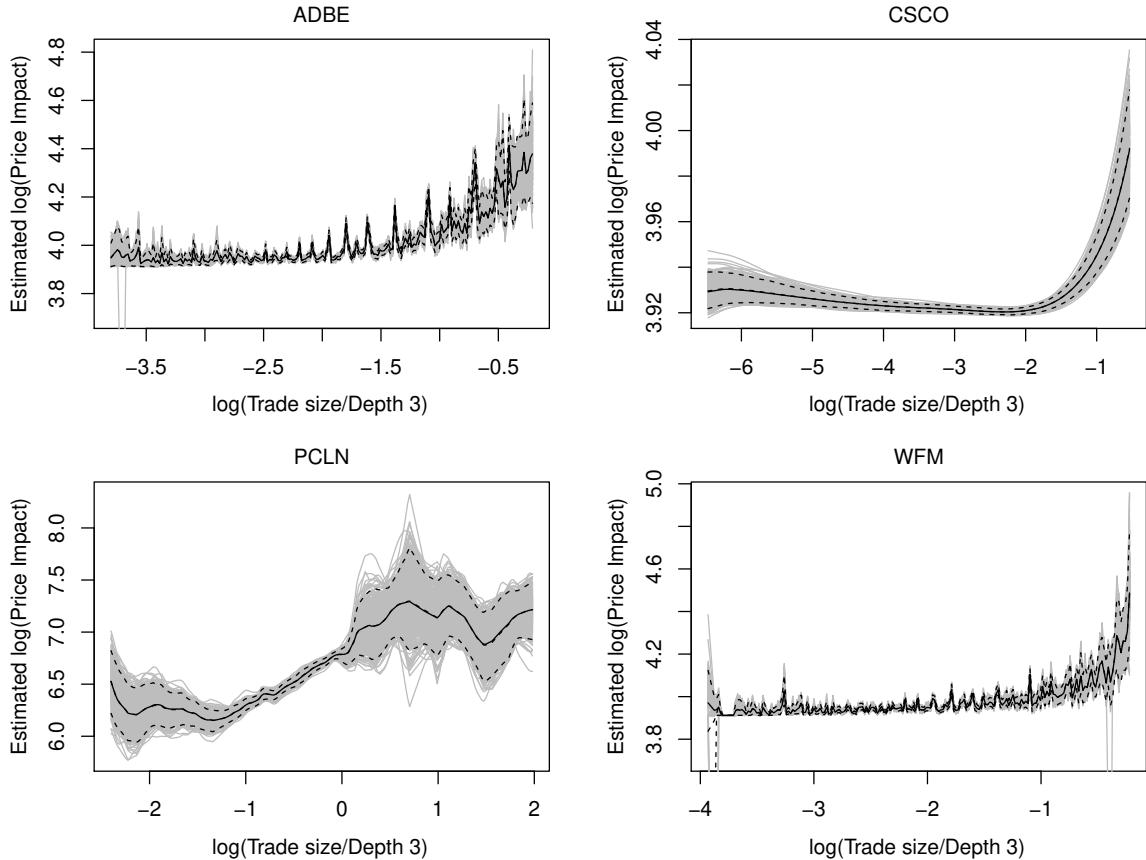
for  $m^d$  in Equation (32) for four randomly selected stocks. The solid black line is the estimate using the actual data. In order to gauge the uncertainty in the estimation results,

the grey lines provide estimates for 500 empirical bootstrap samples, whose pointwise 2.5% and 97.5% quantiles are given by the two dashed lines. Even though the estimation uncertainty for smaller trades is larger, one can still clearly see some stock-dependent nonlinear structure. Most importantly, however, the estimates are linear for sufficiently large trade sizes.

According to Figure 4, which plots the estimates for all 100 stocks together, this linearity is confirmed across the entire Nasdaq 100 universe. Each grey line represents the estimate from a stock, the solid line and the dashed lines are the cross-sectional median, and 2.5% and 97.5% quantiles, respectively (which are only computable for regressor values that are observed for all stocks). The estimation results for all 100 stocks reveal the same qualitative features: stock-specific (idiosyncratic) relationships for small trade sizes and linear relationships for large sizes. The linear behaviour is generally present for stocks with trade sizes above 100 shares (corresponding to 4.6 in logarithmic terms), which is true in approximatey 97% of all cases in our sample (see Table 1).

Corresponding estimates for  $m^\Delta$  in Equation (33) are slightly less easily interpreted. The inherent discreteness of the dependent variable in combination with bandwidths, which are comparably small in most cases, lead to relatively irregular fits as illustrated in the top left and lower right panels of Figure 5. The extreme spikes are due to a lack of observations for some bootstrap samples. Most smoother fits, such as in the top right figure, are basically constant and only increase for large regressor values. In contrast, the lower left panel provides an example of a fit with more variation. Again, the estimate is fairly constant with a linear increase for regressor values between about  $-1.5$  and  $0$ . The discreteness in the dependent variable leads to small bandwidths and to wigglier fits making the comparison of the estimates over all stocks more difficult. For this reason, we have chosen a slightly larger bandwidth in Figure 6 to illustrate the appropriateness of the linear approximation across all stocks.

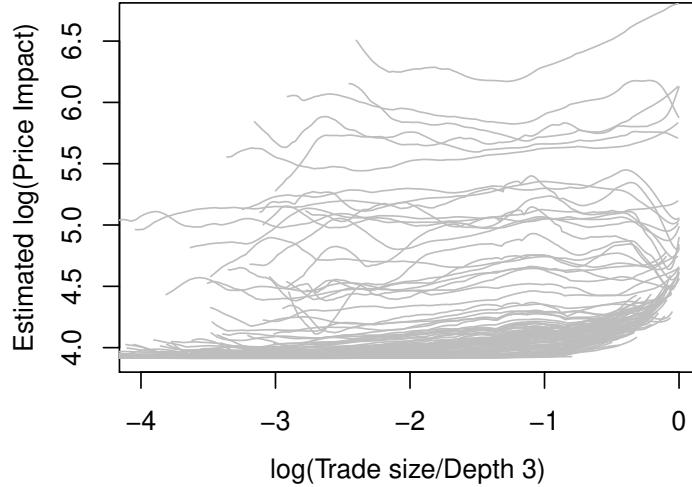
**Figure 5: Local linear estimates of  $m^\Delta$  in Equation (33) for ADBE, CSCO, PCLN and WFM.** Based on data with trades of size at least 100 shares. The solid black line corresponds to the estimate. The grey lines are estimates for 500 bootstrap replications of our estimator. The dashed lines are the 2.5% and 97.5% pointwise quantiles over the bootstraps estimates. The estimation range for each stock covers regressor values up until the 99<sup>th</sup> sample percentile and above the 0.5<sup>th</sup> sample percentile.



#### 4.4 Parametric Analysis

The nonparametric pre-analysis in Section 4.3 provides convincing support for the linearity assumption in  $m^d$  for trades of at least 100 shares. The evidence in favour of the linearity of  $m^\Delta$  is mixed in large part due to the discreteness in price changes. Nevertheless, since a linear approximation does not seem inconceivable and there is no obvious alternative parametric form, the log-linear specifications in Equations (30) and (31) can be viewed as

**Figure 6: Local linear estimates of  $m^\Delta$  in Equation (33) for all 100 stocks.** Based on data with trades of size at least 100 shares. The grey lines correspond to the estimates for the individual stocks. The bandwidths are set to 0.075 of the estimation range. The fits are plotted for regressor values between  $-4$  and  $0$ .



convenient data-driven approximations.

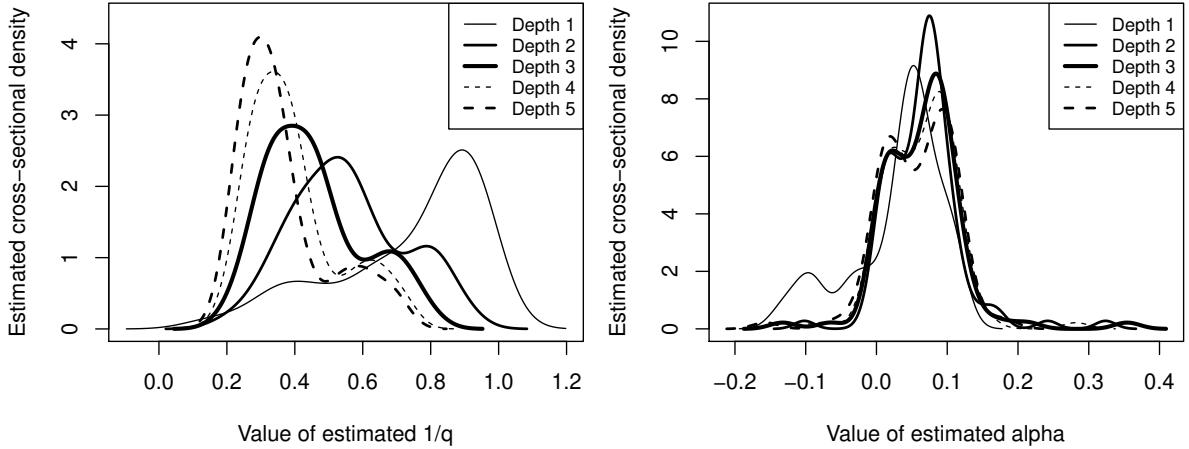
Note, however, that in case of Equation (30), as soon as we impose the linearity assumption, it no longer holds that the regressor and the error term are uncorrelated. Recall that we take the perspective of a liquidity demander who strategically posts his order in response to the currently *observed* liquidity supply. From an empirical viewpoint, this perspective is natural as market depth  $d_i$  is known *before* the trading volume  $x_i$  is placed. Consequently,  $d_i$  can be easily assumed to be (weakly) exogenous for  $x_i$  but not vice versa. While this reversed causality does not affect the validity of the nonparametric analysis in Section 4.3, it makes least squares estimates in regression (30) inconsistent. A simple procedure to produce consistent estimates of  $q$  is to run the reverse regression,

$$\ln(x_i) = k + 1/q \ln(d_i) + \epsilon_i^x, \quad (35)$$

whose linearity is obviously supported by the strong evidence for the linearity of  $m^d$ , while

the regressor  $d_i$  can be assumed to be weakly exogenous. Consequently, running regression (35) yields a consistent estimate of  $1/q$ , which in turn allows us to test the validity of the decreasing TIPS condition.

**Figure 7: Cross-sectional estimates of  $1/q$  in Equation (35) and  $\alpha$  in (31) for all 100 stocks.** Based on data with trades of size at least 100 shares. The left hand panel provides a kernel density estimate of the cross-sectional estimates of  $1/q$  in Equation (35). The right hand panel provides a corresponding kernel density estimate of the cross-sectional estimates of  $\alpha$  in Equation (31). *Depth k* refers to using the cumulative order book depth up until level  $k$  as the underlying depth measure. The kernel density estimates are constructed using the `density` function in **R** with the default rule of thumb bandwidth and a Gaussian kernel.



#### 4.4.1 Parameter estimates

For each stock in our sample, we obtain estimates of  $\alpha$  and  $1/q$  based on Equation (31) and Equation (35) using different order book levels as underlying depth measure. These estimates are graphically summarized in Figure 7. We observe that the cross-sectional density of  $1/q$  tends to be bi-modal and shifts to the left as more depth levels are utilized. In any case, the estimates of  $1/q$  are clearly positive and in nearly all cases below one, thus yielding estimates for  $q$  exceeding one. The same is true for a wide majority of the estimates of the order book shape parameter  $\alpha$ , as shown in the right-hand panel of

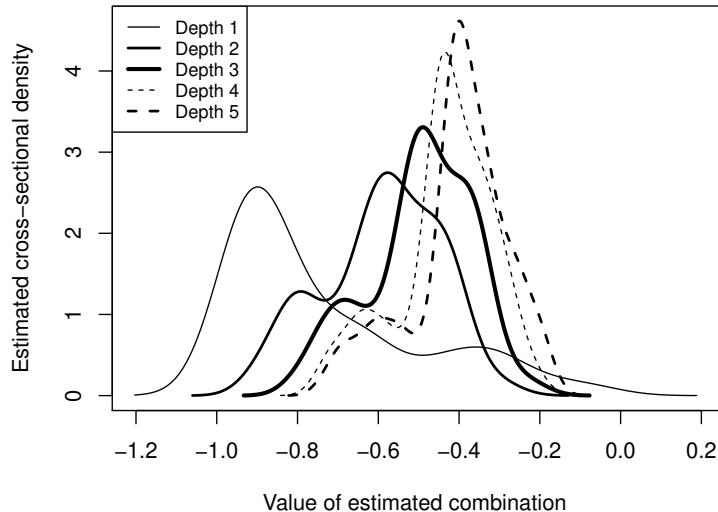
Figure 7. We find values of  $\alpha$  between 0 and 0.2 in most cases with the cross-sectional distribution of  $\alpha$  being essentially independent of the underlying depth measure. These findings are supported by stock-specific estimation results.<sup>8</sup>

Note that the condition for a decreasing TIPS according to Prediction 1, can be formulated in terms of  $1/q$  and  $\alpha$  as

$$\gamma := \gamma(1/q, \alpha) = \alpha/q - 1/q - \alpha < 0.$$

According to Figure 8, showing the cross-sectional density of  $\gamma(1/\hat{q}, \hat{\alpha})$ , we observe that the estimates are clearly negative, with modal values between  $-0.4$  and  $-0.6$ , thus strongly confirming Prediction 1.

**Figure 8: Cross-sectional estimates of  $\gamma$  for all 100 stocks.** Based on data with trades of size at least 100 shares. The lines refer to the cross-sectional densities of the respective estimates over all stocks. The lines *Depth k* refer to the estimates of  $\gamma := \gamma(1/q, \alpha) = \alpha/q - 1/q - \alpha$  using the estimates for the slope parameters from Equation (31) and Equation (35) based on cumulative order book depth up to level  $k$ . The kernel density estimates are constructed using the `density` function in **R** with the default rule of thumb bandwidth and a Gaussian kernel.




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<sup>8</sup> Available upon request from the authors.

The testable predictions 1 and 2 can thus be formally tested as

$$H_0^1 : \gamma \geq 0 \text{ vs. } H_a^1 : \gamma < 0$$

and

$$H_0^2 : \alpha \leq 0 \quad \text{vs.} \quad H_a^2 : \alpha > 0.$$

To test the decreasing TIPS condition  $\gamma < 0$ , we exploit the asymptotic normality of the estimates in Equation (31) and Equation (35) along with the delta method to establish

$$\sqrt{N}(\hat{\gamma} - \gamma) \xrightarrow{d} N(0, \sigma_\gamma^2),$$

with  $\sigma_\gamma^2 = (\alpha - 1)^2 \sigma_{1/q}^2 + (1/q - 1)^2 \sigma_\alpha^2$ , where  $\sigma_\alpha^2$  and  $\sigma_{1/q}^2$  denote the asymptotic variances of  $\hat{\alpha}$  and  $\widehat{1/q}$ , respectively. Denote by  $s_\alpha^2$  and  $s_{1/q}^2$  the heteroscedasticity and autocorrelation-consistent covariance estimates of  $\sigma_\alpha^2$  and  $\sigma_{1/q}^2$  according to Andrews (1991). Then,  $\sigma_\gamma^2$  can be consistently estimated by  $s_\gamma^2 = (\hat{\alpha} - 1)^2 s_{1/q}^2 + (\widehat{1/q} - 1)^2 s_\alpha^2$ .

Table 2 provides the number of stocks for which the null hypothesis is rejected. We obtain strong evidence in favour of a decreasing TIPS for nearly all Nasdaq 100 stocks based on all depth measures and various significance levels. The empirical support for the validity of a power law order book shape, however, is less strong but still convincing in favor of the underlying hypothesis. This is particularly true if the cumulative depth across more than two levels is used.<sup>9</sup>

We finalize our discussion by formally testing the validity of the predictions for the *entire* set of Nasdaq 100 stocks. Conducting tests for each stock individually, the significance level for the simultaneous test over *all* stocks is no longer guaranteed to correspond to the

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<sup>9</sup>Results for a third test on an upward sloping order book shape corresponding to  $H_0^3 : \alpha \geq 1$  vs.  $H_a^3 : \alpha < 1$  are not provided here. Given that the estimates of  $\alpha$  are considerably closer to zero than to one, there is very strong evidence supporting the hypothesis on all customary significance levels.

**Table 2: Rejections of  $H_0^1$  and  $H_1^1$ .** The table entries give the number of rejections of the underlying null hypotheses for all 100 stocks. We report the outcomes when carried out for each stock individually as well as those using the Bonferroni correction to control the family wise error rate at the given significance levels 0.1, 0.05, 0.01 and 0.001.

	Depth ( $d$ )	Individual test levels				Simultaneous test levels			
		0.1	0.05	0.01	0.001	0.1	0.05	0.01	0.001
$H_0^1$	$d_1$	100	100	100	100	100	100	99	99
	$d_2$	100	100	100	100	100	100	100	100
	$d_3$	100	100	100	100	100	100	100	100
	$d_4$	100	100	100	100	100	99	99	99
	$d_5$	100	100	99	99	99	99	99	99
$H_0^2$	$d_1$	75	75	74	72	72	72	72	67
	$d_2$	95	95	95	95	95	95	94	91
	$d_3$	94	94	93	92	92	92	92	87
	$d_4$	93	93	91	89	89	89	88	85
	$d_5$	90	90	89	83	83	83	83	79

significance level we provided for the individual tests.<sup>10</sup> In order to control for this effect, we also provide the results from our tests using the Bonferroni correction in the last four columns of Table 2. Although the Bonferroni correction is conservative in the sense that it suffers from low power to pick up the alternative hypothesis, we find that the number of rejections of  $H_0^1$  is basically unchanged. Hence, these results provide clear support for the validity of a decreasing TIPS and slightly lower but still convincing evidence for power-law shaped order books.

Finally, an alternative way to investigate the validity of the predictions for the entire set of stocks is by aggregating the estimates over the stocks and performing the corresponding tests on the aggregate level. Here, we make use of methods employed in random effects meta-analyses as can be found, for instance, in Higgins et al. (2009). The procedure essentially computes a weighted average of the stockwise estimates with the weights inversely depending on the stockwise variance estimates. The details of the procedure are given in E. The results, provided in Table 3, strongly support the validity of our testable

<sup>10</sup>It may be the case that we are picking up lots of false positives, which in our case would correspond to supporting evidence though the model is incorrect.

predictions.

**Table 3: Cross-sectional aggregates of the estimates of  $\alpha$ ,  $1/q$  and  $\gamma$ .** Stock-specific estimates of  $\alpha$ ,  $1/q$  and  $\gamma = \alpha 1/q - 1/q - \alpha$ , aggregated based on the randomized effect meta-analysis methods according to Higgins et al. (2009), see E. Standard errors given in parentheses.

Equation	Depth	const	$\widehat{1/q}$	$\widehat{\alpha}$	$\widehat{\gamma}$
(35)	$d = d_1$	1.3386 (0.0796)	0.7367 (0.0165)		-0.7337 (0.0150)
	$d = d_2$	1.5655 (0.1151)	0.5561 (0.0163)		-0.591 (0.0136)
	$d = d_3$	1.9717 (0.0858)	0.4546 (0.0118)		-0.494 (0.0010)
	$d = d_4$	2.2466 (0.0813)	0.3961 (0.0110)		-0.4368 (0.0093)
	$d = d_5$	2.3783 (0.0804)	0.3649 (0.0106)		-0.4046 (0.0092)
(31)	$d = d_1$	4.2902 (0.0519)		0.0306 (0.0067)	-0.7337 (0.0150)
	$d = d_2$	4.3521 (0.0404)		0.066 (0.0049)	-0.591 (0.0136)
	$d = d_3$	4.3862 (0.0178)		0.0631 (0.0033)	-0.494 (0.0010)
	$d = d_4$	4.4098 (0.0175)		0.0607 (0.0034)	-0.4368 (0.0093)
	$d = d_5$	4.4261 (0.0180)		0.058 (0.0034)	-0.4046 (0.0092)

## 4.5 Drivers of $\gamma$

According to our model, the TIPS monotonicity condition  $\gamma(1/q, \alpha) = \alpha/q - 1/q - \alpha < 0$  rests on a non-linear relationship of the order book shape  $\alpha$  and the liquidity elasticity  $q$ . To quantify the relative importance of these two channels for a decreasing TIPS, we analyze the relative importance of  $\widehat{\alpha}$  and  $\widehat{q}$  in a linearized relationship as captured by the cross-sectional regression model

$$\widehat{\gamma}_i = \beta_0 + \beta_q \widehat{q}_i + \beta_\alpha \widehat{\alpha}_i + u_i, \quad i = 1, \dots, 100, \quad (36)$$

where  $\hat{\gamma}_i$ ,  $\hat{q}_i$ , and  $\hat{\alpha}_i$  denote the stock-specific estimates of  $\gamma$ ,  $q$ , and  $\alpha$  and  $u_i$  denotes a zero mean error term uncorrelated with the regressors. To assess the relative importance of the two regressors, we consider three simple measures that are easily computed from least squares estimates of Equation (36) and are reported in Table 4.

The first importance measure is the absolute value of the estimated standardized regression coefficients in Equation (36), i.e., estimates that are standardized by the standard error (and are thus independent of the measurement scale) after the regressors have been centred. Such standardized regression coefficients are interpreted as the estimated change in the dependent variable measured in standard deviations due to a one standard deviation increase in the regressor, holding all else fixed. According to the second and 3<sup>rd</sup> column in Table 4, the standardized regression coefficients associated with liquidity,  $|\hat{\beta}_q|$ , are larger than that for the order book shape,  $|\hat{\beta}_\alpha|$ , particularly, when depth exceeding two levels is utilized.

The 4<sup>th</sup> and 5<sup>th</sup> column of Table 4 give the product of the estimated standardized regression coefficient and the sample correlation of the corresponding regressor with the dependent variable. This measure was originally introduced by Hoffman (1960) and formally derived by Pratt (1987). It can be interpreted as the fraction of the model  $R^2$  due to the respective variable. Again, the liquidity elasticity is found to be more important than the order book shape. The effect is most pronounced when using more than two depth levels. The main drawback of this methodology, however, is that in certain situations, such as here, due to correlation between the regressors, the measure may become negative (see, e.g., Thomas et al. (1998)). As a third measure of relative importance, we consider the increase in explanatory power in terms of  $R^2$  due to the inclusion of the respective regressor. Since each of the two regressors in Equation (36) can be added as the first or second variable, we compute the increase in  $R^2$  for both cases and report the corresponding average in the 6<sup>th</sup> and 7<sup>th</sup> column of Table 4. The idea of

averaging increases in  $R^2$  over all possible orderings has been introduced in Lindeman et al. (1980). Confirming the findings above, also this measure indicates that liquidity elasticity is relatively more important than the order book shape for a decreasing TIPS. We can thus conclude that the liquidity coordination is the major driver behind a downward-sloping TIPS profile.

**Table 4: Relative importance of the order book shape and liquidity elasticity in explaining TIPS.** The table contains estimated measures of the relative importance of the estimated order book shape  $\hat{\alpha}$  and the estimated liquidity elasticity  $\hat{q} := (\widehat{1/q})^{-1}$  in explaining  $\hat{\gamma}$  over the cross-section. The 2<sup>nd</sup> and 3<sup>rd</sup> column give the absolute values of the estimated standardized regression coefficients. The 4<sup>th</sup> and 5<sup>th</sup> column contain estimates of the Pratt measure, defined as the product of the standardized regression coefficient and the sample correlation between the dependent variable and the respective regressor. The 6<sup>th</sup> and 7<sup>th</sup> column contain the average increase in  $R^2$  due to the inclusion of the variable in parenthesis. Finally, the last two columns provide the  $R^2$  of the fitted model in Equation (36) and the correlation between the two corresponding regressors.

Depth	$ \hat{\beta}_q $	$ \hat{\beta}_\alpha $	$\hat{\beta}_q \hat{\rho}_{\hat{q}}$	$\hat{\beta}_\alpha \hat{\rho}_{\hat{\alpha}}$	LMG( $\hat{q}$ )	LMG( $\hat{\alpha}$ )	$R^2$	$\hat{\rho}_{\hat{q}, \hat{\alpha}}$
$d_1$	0.56	0.46	0.56	0.44	0.74	0.68	0.86	-0.66
$d_2$	0.98	0.19	1.12	-0.12	0.71	0.22	0.73	0.68
$d_3$	1.00	0.26	1.09	-0.09	0.75	0.07	0.80	0.52
$d_4$	0.97	0.19	1.05	-0.05	0.80	0.04	0.83	0.41
$d_5$	0.95	0.13	1.02	-0.02	0.84	0.01	0.86	0.25

## 4.6 Cross-sectional Determinants

In this section, we analyze to which extent the cross-sectional variation of the estimates  $\hat{\alpha}$ ,  $\hat{q}$  and  $\hat{\gamma}$  can be explained by stock-specific characteristics, represented by the daily averages of the bid-ask spread and the depth as well as the daily realized variance and trading volume. We perform the following cross-sectional regressions,

$$\hat{\gamma}_i = c_\gamma + \beta_{\gamma, spr} spr_i + \beta_{\gamma, d_1} d_{1,i} + \beta_{\gamma, rv} rv_i + \beta_{\gamma, vol} vol_i + u_{\gamma,i} \quad (37)$$

$$\hat{q}_i = c_q + \beta_{q, spr} spr_i + \beta_{q, d_1} d_{1,i} + \beta_{q, rv} rv_i + \beta_{q, vol} vol_i + u_{q,i} \quad (38)$$

$$\hat{\alpha}_i = c_\alpha + \beta_{\alpha, spr} spr_i + \beta_{\alpha, d_1} d_{1,i} + \beta_{\alpha, rv} rv_i + \beta_{\alpha, vol} vol_i + u_{\alpha,i}, \quad (39)$$

where  $i = 1, \dots, 100$  indexes the stocks and  $spr_i$ , and  $d_{1,i}$  denote the time-average over stock  $i$ 's dollar bid-ask spread and the depth on top of the book immediately before a trade, respectively. The variables  $rv_i$  and  $vol_i$  represent the daily realized variance computed as the sum of the squared instantaneous price impacts over each day and the cumulative daily trading volume, respectively. The left-hand variables are the estimates of  $\alpha$ ,  $q$ , and  $\gamma$  resulting from Section 4.4. All regressors are centered and standardized (i.e., scaled by their respective sample standard deviation) in order to remove scale effects.

Table 5 reports the resulting least squares estimates. The results can be summarized as follows. The TIPS condition  $\hat{\gamma}$ , reflecting the "strength of evidence" of a declining TIPS significantly decreases with smaller spreads, larger volatility and larger volume. Hence, these results suggest that a declining TIPS profile becomes more likely for more liquid stocks. For the estimated liquidity elasticity  $\hat{q}$  we observe analogous relationships, confirming our previous evidence that the liquidity elasticity is the major driver of the TIPS profile monotonicity. Finally, we observe  $R^2$  values ranging roughly around 50% to 60%, indicating that a substantial part of the cross-sectional variation of the TIPS parameters can be indeed explained by liquidity characteristics and volatility.

## 5 Conclusions

The stealth trading hypothesis is one of the most intriguing observations of price returns. Barclay and Warner (1993) suggests that this effect arises from informed trading as large informed traders try to slice their trades into smaller orders. Thus, the informational impact of trades should only be seen in non-large orders.

In this paper, we provide an alternative non-informational rationale for the observed effect that alludes to the phenomenon of *liquidity begets liquidity*. We develop a simple but stylized model of trading in limit order book markets, where traders strategically choose whether to trade in the primary public limit order book based on their perception

**Table 5: Estimates of cross-sectional determinants of  $\hat{\gamma}$ ,  $\hat{q}$ , and  $\hat{\alpha}$ .** The table contains the OLS estimates of the models in Equations (37), (38) and (39). The superscript stars reflect significance on the 0.1, 0.05, and 0.001 level.

Model	Dep. Var.	Depth	spr	$d_1$	$rv$	$vol$	$R^2$
(37)	$\hat{\gamma} := \widehat{1/q(\hat{\alpha} - 1)} - \hat{\alpha}$	$d_1$	1.9*** (0.19)	-0.03 (0.05)	-1.22*** (0.19)	-0.17*** (0.06)	0.75
		$d_2$	1.62*** (0.26)	-0.16** (0.07)	-1.28*** (0.25)	-0.3*** (0.08)	0.53
		$d_3$	1.58*** (0.28)	-0.04 (0.08)	-1.34*** (0.28)	-0.33*** (0.08)	0.44
		$d_4$	1.53*** (0.28)	0.1 (0.08)	-1.27*** (0.28)	-0.36*** (0.08)	0.43
		$d_5$	1.52*** (0.28)	0.17** (0.08)	-1.19*** (0.27)	-0.36*** (0.08)	0.46
(38)	$\hat{q} := (\widehat{1/q})^{-1}$	$d_1$	1.67*** (0.14)	-0.002 (0.04)	-0.82*** (0.13)	-0.05 (0.04)	0.87
		$d_2$	1.82*** (0.17)	-0.06 (0.05)	-1.12*** (0.17)	-0.18*** (0.05)	0.79
		$d_3$	1.66*** (0.21)	-0.04 (0.06)	-1.05*** (0.21)	-0.25*** (0.06)	0.68
		$d_4$	1.42*** (0.24)	0.05 (0.07)	-0.86*** (0.24)	-0.3*** (0.07)	0.58
		$d_5$	1.24*** (0.26)	0.15** (0.07)	-0.7*** (0.25)	-0.32*** (0.08)	0.53
(39)	$\hat{\alpha}$	$d_1$	-1.87*** (0.23)	-0.12* (0.07)	1.24*** (0.23)	0.07 (0.07)	0.61
		$d_2$	-0.06 (0.26)	-0.09 (0.08)	0.66** (0.26)	-0.25*** (0.08)	0.5
		$d_3$	-0.66** (0.28)	-0.08 (0.08)	1.17*** (0.27)	-0.28*** (0.08)	0.45
		$d_4$	-1.26*** (0.3)	-0.09 (0.09)	1.53*** (0.3)	-0.35*** (0.09)	0.35
		$d_5$	-1.79*** (0.3)	-0.09 (0.09)	1.74*** (0.3)	-0.39*** (0.09)	0.34

of transaction costs. Under these assumptions, large traders prefer to trade in the limit order book market, when sufficient liquidity is provided. Therefore, observed large trades in these markets have a proportionally smaller price impact compared to smaller trades. Hence, the mutual attraction of market liquidity and trade volume can explain the presence of a decreasing price impact per share (TIPS).

From our model we derive certain testable predictions that are extensively investigated using high-frequency order-message data from the first quarter of 2014 for all constituents of the Nasdaq 100 index. We are capable of showing that a decreasing TIPS is virtually universally present in our data. Furthermore, we also show that liquidity coordination, as captured by our estimate of liquidity elasticity, seems to be the main driver behind this result. Finally, we show that the magnitude of the TIPS and thus the strength of the evidence for a decreasing TIPS depends on the liquidity of the respective stock.

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## Appendix

### A Proofs

*Proof of Lemma 2.* The trader decides to trade if and only if the costs of trading are less than the expected costs of deferring, thus when  $\Pi^{prime} < \mathbb{E}_i[\Pi^{other}]$ . Plugging in the expression after the second equality of Equation (15) for  $\Pi^{prime}$  and the expression for  $\Pi^{other}$  given in Equation (16) as well as using that  $\xi$  is a zero mean random variable, the condition for trading simplifies to

$$d_i \int_0^{F^{-1}(n_i/d_i)} f(p)pdp < \mu n_i,$$

which yields the result after dividing both sides by  $n_i$ . □

*Proof of Proposition 1.* From the discussion preceding the statement of the proposition, we have

$$\delta'(x) \approx \frac{1}{x} \left( \tilde{\Delta}'(x) - \delta(x) \right) \quad (40)$$

as well as

$$\tilde{\Delta}'(x) \approx \frac{1}{g(x)f(F^{-1}(x/g(x)))} \left( 1 - \mathcal{E}_L(x) \right), \quad (41)$$

$$\delta(x) \approx \frac{1}{x} F^{-1} \left( \frac{x}{g(x)} \right). \quad (42)$$

Thus, approximately up to first order, a decreasing TIPS can be characterized by

$$\begin{aligned} & \delta'(x) < 0 \\ & \Leftrightarrow \tilde{\Delta}'(x) - \delta(x) < 0 \\ & \Leftrightarrow \frac{1 - \mathcal{E}_L(x)}{g(x)f(F^{-1}(x/g(x)))} - \frac{1}{x} F^{-1} \left( \frac{x}{g(x)} \right) < 0 \\ & \Leftrightarrow 1 - \mathcal{E}_L(x) < \underbrace{\frac{g(x)}{x} f(F^{-1}(x/g(x))) F^{-1} \left( \frac{x}{g(x)} \right)}_{= \Psi_{f,g}(x)} \\ & \Leftrightarrow 1 - \Psi_{f,g}(x) < \mathcal{E}_L(x). \end{aligned}$$

In the power law order book case, we have  $f(p) = Ap^{1/\alpha-1} = \frac{1}{\alpha} \frac{F(p)}{p}$ . Thus,

$$f(F^{-1}(p)) = \frac{1}{\alpha} \frac{F(F^{-1}(p))}{F^{-1}(p)} = \frac{1}{\alpha} \frac{p}{F^{-1}(p)},$$

from which it follows that

$$\Psi_{f,g}(x) = \frac{g(x)}{x} \frac{1}{\alpha} \frac{x/g(x)}{F^{-1}(x/g(x))} F^{-1}(x/g(x)) = \frac{1}{\alpha}.$$

□

## B Sufficient Conditions for the TIPS Monotonicity Condition (28)

Using  $d_i = g(x_i) + \epsilon_i$ , we can write

$$\Delta(x_i) = F^{-1}\left(\frac{x_i}{g(x_i) + \epsilon_i}\right) =: \Psi(\epsilon_i).$$

In case of a power law shaped order book we have

$$\Psi(\epsilon_i) = \overline{\Delta}\left(\frac{x_i}{g(x_i) + \epsilon_i}\right)^\alpha,$$

with all derivatives of  $\Psi(\epsilon_i)$  existing and the  $k$ -th derivative given by

$$\Psi^{(k)}(\epsilon_i) = (-1)^k \prod_{j=0}^{k-1} (\alpha + j) \frac{1}{(g(x_i) + \epsilon_i)^k} \Psi(\epsilon_i).$$

Under the assumption of the prediction error relative to the prediction being less than 100%, i.e.,

$$\frac{|x_i|}{g(x_i)} \leq M$$

for some  $M < 1$ , and for  $\alpha \in [0, 1]$ , we have  $\prod_{j=0}^{k-1} (\alpha + j) \leq \alpha k!$ , from which it follows that

$$\frac{|\Psi^{(k)}(0)|}{k!} |\epsilon_i|^k \leq \alpha M < 1.$$

In this case, the Taylor expansion of  $\Psi(\epsilon_i)$  around 0 is given by

$$\Psi(\epsilon_i) = \Psi(0) \left[ 1 + \Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) \right]$$

with

$$\Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) = \left[ \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \prod_{j=0}^{k-1} (\alpha + j) \left( \frac{\epsilon_i}{g(x_i)} \right)^k \right].$$

Then, the approximation in (24) can be formalized as  $|\Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right)| \leq \delta$  for  $\delta = \alpha \frac{M}{1-M}$ , which, with an application of Fubini's theorem, yields

$$\tilde{\Delta}(x) := \mathbb{E}[\Psi(\epsilon_i) | x_i = x] = \Psi_x(0) \left[ 1 + \mathbb{E}[\Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) | x_i = x] \right] \in [\Psi_x(0)(1 - \delta), \Psi_x(0)(1 + \delta)],$$

where  $\Psi_x(0) := F^{-1}(x/g(x))$ .

Using the above approximation  $\tilde{\Delta}(x) = \Psi_x(0) \left[ 1 + \mathbb{E}[\Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) | x_i = x] \right]$ , we obtain

$$\delta'(x) = \Psi'_x(0) - \frac{\Psi_x(0)}{x} + R(\alpha, x)$$

with remainder term

$$R(\alpha, x) = \left[ \Psi'_x(0) - \frac{\Psi_x(0)}{x} \right] \mathbb{E} \left[ \Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) | x_i = x \right] + \Psi_x(0) \frac{d\mathbb{E}[\Upsilon \left( \alpha, \frac{\epsilon_i}{g(x_i)} \right) | x_i = x]}{dx}. \quad (43)$$

In the following we will establish  $|R(\alpha, x)| < |\Psi'_x(0) - \frac{\Psi_x(0)}{x}|$  ensuring that the condition

$$|\Psi'_x(0) - \frac{\Psi_x(0)}{x}| < 0$$

is sufficient for  $\delta'(x) < 0$ .

Under the assumption  $\frac{|x_i|}{g(x_i)} \leq M$  and  $\alpha \in [0, 1]$ , the first term in (43) is bounded by  $|\Psi'_x(0) - \Psi_x(0)/x| \cdot \alpha M/(1 - M)$ .

In order to deal with the second term in (43), we write  $\mathbb{E}[\Upsilon(\alpha, \epsilon_i/g(x_i))|x_i = x] = \sum_{k=1}^{\infty} \Upsilon_k(\alpha, \epsilon_i/g(x_i))$  with

$$\Upsilon_k(\alpha, \epsilon_i/g(x_i)) = \frac{(-1)^k}{k!} \frac{\mathbb{E}[\epsilon_i^k|x_i = x]}{g^k(x)} \prod_{j=0}^{k-1} (\alpha + j),$$

from which we get

$$\Upsilon'_k(\alpha, \epsilon_i/g(x_i)) = \frac{(-1)^k}{k!} \frac{\frac{d\mathbb{E}[\epsilon_i^k|x_i=x]}{dx}}{g^k(x)} \prod_{j=0}^{k-1} (\alpha + j) - k \frac{g'(x)}{g(x)} \Upsilon_k(\alpha, \epsilon_i/g(x_i)).$$

Assuming  $\sup_x |g'(x)/g(x)| < \infty$  and  $\left| \frac{d\mathbb{E}[\epsilon_i^k|x_i=x]}{dx} / g^k(x) \right| < L^k$  with  $L < 1$  for all  $k$  and all  $x$ , then

$$|\Upsilon'_k(\alpha, \epsilon_i/g(x_i))| \leq \alpha L^k + k \sup_x \frac{|g'(x)|}{g(x)} \alpha M^k.$$

Consequently, we have  $\sum_{k=1}^{\infty} \sup_x |\Upsilon'_k(\alpha, \epsilon_i/g(x_i))| \leq \alpha L/(1-L) + \sup_x |g'(x)/g(x)| \alpha M / ((1-M)^2)$  establishing the uniform convergence of  $\sum_{k=1}^{\infty} \Upsilon'_k(\alpha, \epsilon_i/g(x_i))$ . Therefore, the second term of (43) is bounded by  $\Psi_x(0)\{\alpha L/(1-L) + \sup_x |g'(x)/g(x)| \alpha M / ((1-M)^2)\}$  yielding

$$|R(\alpha, x)| \leq |\Psi'_x(0) - \Psi_x(0)/x| \alpha \frac{M}{1-M} + \Psi_x(0) \left\{ \alpha \frac{L}{1-L} + \sup_x \frac{|g'(x)|}{g(x)} \alpha \frac{M}{(1-M)^2} \right\}.$$

Hence, a sufficient condition for  $|R(\alpha, x)| < |\Psi'_x(0) - \Psi_x(0)/x|$  is given by

$$\frac{\Psi_x(0)}{|\Psi'_x(0) - \frac{\Psi_x(0)}{x}|} \left\{ \sup_x \frac{|g'(x)|}{g(x)} \alpha \frac{M}{(1-M)^2} + \alpha \frac{L}{1-L} \right\} < 1 - \alpha \frac{M}{1-M} = \frac{1 - M - \alpha M}{1 - M}.$$

In the power law case  $\Psi_x(0) = \overline{\Delta} \left( \frac{x}{g(x)} \right)^\alpha$ , we have

$$\frac{\Psi_x(0)}{\Psi'_x(0) - \frac{\Psi_x(0)}{x}} \alpha \frac{g'(x)}{g(x)} = \frac{x g'(x)/g(x)}{1 - 1/\alpha - x g'(x)/g(x)} = \frac{\mathcal{E}_L(x)}{1 - 1/\alpha - \mathcal{E}_L(x)}.$$

Then, choosing a constant  $C > 0$  such that  $\alpha L/(1-L) + \sup_x |g'(x)/g(x)|\alpha M/((1-M)^2) < C \sup_x |g'(x)/g(x)|\alpha M/((1-M)^2)$  yields

$$\sup_x \frac{|\mathcal{E}_L(x)|}{|1 - 1/\alpha - \mathcal{E}_L(x)|} \leq \frac{1}{C} \frac{(1-M)(1-M-\alpha M)}{M} \quad (44)$$

as a sufficient condition to guarantee  $|R(\alpha, x)| < |\Psi'_x(0) - \frac{\Psi_x(0)}{x}|$ .

Under the assumption that the prediction error is independent of the trade size we have  $L = 0$  and can choose  $C = 1$ . We summarize the analysis in the following proposition.

**Proposition 2.** (*Sufficient conditions for decreasing TIPS*).

*In the case of power law shaped limit order book markets with  $\alpha \in (0, 1]$ , if*

- (a) *the prediction errors of traders are bounded in the sense that  $\frac{d_i - \mathbb{E}[d_i|x_i=x]}{\mathbb{E}[d_i|x_i=x]} =: \frac{\epsilon_i}{g(x)} \leq M < 1$  and  $\sup_x |g'(x)/g(x)| < \infty$ ,*

- (b) *the prediction error  $\epsilon_i$  is independent of the trade size  $x_i$ ,*

*then the following two conditions are jointly sufficient to ensure a decreasing TIPS:*

(i)  $1 - 1/\alpha - \mathcal{E}_L(x) < 0$ ,

(ii)  $\frac{|\mathcal{E}_L(x)|}{|1 - 1/\alpha - \mathcal{E}_L(x)|} < \frac{(1-M)(1-M-\alpha M)}{M}$ .  $\square$

These conditions are directly testable under the assumption that the liquidity elasticity is constant, i.e.,  $\mathcal{E}_L(x) = q$ . Then, given estimates of  $\alpha$  and  $q$  the first of these conditions is Testable Prediction 1 as formulated in Section 4.4. To test the second condition we can

compute  $M$  given estimates of  $q$  and  $\alpha$  and check whether  $M < 1$ . Solving

$$\frac{1}{\widehat{1/q} - 1/\widehat{\alpha}\widehat{1/q} - 1} = \frac{(1-M)^2}{M}$$

for  $M$  gives a solution that is less than one for all stocks irrespective of the depth measure utilized to estimate  $1/q$  and  $\alpha$  in Equations (31) and (35). This procedure yields a cross-sectional mean of  $M$  between 0.7 and 0.75 depending on the depth measure used.

## C Bandwidth Choice

The bandwidth is chosen such that the prediction error of the fit in the last month of our sample is minimized. Given the data  $\{(\ln(d_i), \ln(x_i))\}_{i=1}^N$ , the steps are as follows:

1. Split the sample in two parts: A training sample  $\{(\ln(d_i), \ln(x_i))\}_{i=1}^M\}$  consisting of the data in the first two months and a test sample  $\{(\ln(d_i), \ln(x_i))\}_{i=M+1}^N\}$  consisting of the data in the third month. The index  $M + 1$  is thus the index of the first trade in the third month.
2. Find the unique regressor values in the test sample that are within the estimation range and collect them to form  $\mathcal{X} = (\ln(x_1^{\text{test}}), \dots, \ln(x_q^{\text{test}}))'$ . Estimate the function  $m^d$  at each value in  $\mathcal{X}$  using a bandwidth  $h$  from a grid of bandwidth values  $\mathcal{H}$ . For each  $h \in \mathcal{H}$  we then get an estimator  $\hat{m}_h^d$ .
3. Choose  $h \in \mathcal{H}$  to minimize the squared prediction error for points in the test set within the estimation range

$$\hat{h} = \arg \min_{h \in \mathcal{H}} \sum_{i=M+1}^N [\ln(d_i) - \hat{m}_h^d(\ln(x_i))]^2 \mathbf{1}_{\ln(x_i) \in \mathcal{X}}.$$

4. The final estimate of  $m^d$  as presented in Section 4.3 is  $m_{\hat{h}}^d$ , the local linear kernel

regression estimator with bandwidth parameter  $\hat{h}$ , estimated using the entire sample  $\{(\ln(d_i), \ln(x_i))\}_{i=1}^N$ .

In Step 3, the grid of bandwidth values used should preferably be chosen so that  $\hat{h}$  is an interior solution, taking neither the largest nor the smallest value of the grid. The chosen bandwidth grids encompasses values in terms of fractions of the estimation range from 0.001 to 1. Whenever  $\mathcal{X}$  contains more than 1000 points (as in the case of  $m^\delta$ ), the estimator is computed on an equidistant grid of points spanning the range of  $\mathcal{X}$ . The fits needed in step 4 are then based on linearly interpolating the estimates.

## D Bootstrap Procedure

A simple bootstrap procedure is used to quantify the uncertainty of the nonparametric estimates  $\hat{m}^d$  and  $\hat{m}^\Delta$ . We only detail the procedure for  $\hat{m}^d$  as the steps for the other estimators are analogous. Recalling that  $\hat{m}^d$  is estimated based on the data  $\{(\ln(d_i), \ln(x_i))\}_{i=1}^N$ , the steps of the bootstrap procedure are as follows:

1. For  $b \in \{1, \dots, 500\}$ , draw with replacement  $N$  data points from the original sample to form the bootstrap sample  $\{(\ln(d_i^{[b]}), \ln(x_i^{[b]}))\}_{i=1}^N$ .
2. For each bootstrap sample, estimate  $m^d$  yielding  $B = 500$  estimates  $\{\hat{m}^{d,[b]}, b = 1, \dots, B\}$ . The bandwidth used in the estimation is taken to be  $\hat{h}$ , i.e., the one determined in the estimation of the orginal data.
3. The reported quantile curves are pointwise, i.e., for every  $u$ , we compute the desired quantile of  $\hat{m}^{d,[b]}(u)$ ,  $b = 1, \dots, B$ .

It is conceivable to include a bandwidth selection step as detailed in the previous subsection for each bootstrap estimate in Step 2. However, this greatly increases the computational burden of the procedure. On inspection for some selected stocks, we observe only little change in the selected bandwidth compared to  $\hat{h}$ .

## E Cross-sectional aggregation

In this section, we provide some details on the aggregation procedure used to obtain the estimates in Table 3. The procedure is based on random effects meta analysis as detailed in Higgins et al. (2009). In contrast to the related Bayes Hierarchical approach as in DuMouchel (1994), we do not have to specify prior distributions for the parameters of the distribution governing the overall effects. We give the details for the computation of the aggregate estimate of the slope parameter  $\gamma$ . For the other parameters we proceed analogously.

Let  $\hat{\gamma}_i$  be the estimate of  $\gamma$  for stock  $i$  denoted by  $\gamma_i$ . Similarly,  $s_i^2$  is the corresponding (heteroscedasticity and autocorrelation consistent) estimate of the (asymptotic) variance of  $\hat{\gamma}_i$ . The method assumes that the estimates for each stock are normally distributed,

$$\hat{\gamma}_i \sim N(\gamma_i, \sigma_i^2). \quad (45)$$

Furthermore, it is assumed that all realizations  $\gamma_i$  are i.i.d. draws from an unknown distribution, whose first two moments are given by  $\mathbb{E}[\gamma_i] = \gamma$  and  $\text{Var}[\gamma_i] = \sigma^2$ . The aim of the exercise is to obtain estimates of  $\gamma$  and  $\sigma^2$ . We will ignore any uncertainty in estimating the variance of the estimates, i.e., we replace the unknown  $\sigma_i^2$  by the estimates  $s_i^2$ . Then, estimates for  $\gamma$  and  $\sigma^2$  according to Higgins et al. (2009) (see also DerSimonian and Laird (1986) and Whitehead and Whitehead (1991)) are given by

$$\hat{\gamma} = \sum_{i=1}^{100} w_i \hat{\gamma}_i \quad \text{with weights} \quad w_i = \frac{(s_i^2 + \hat{\sigma}^2)^{-1}}{\sum_{j=1}^{100} (s_j^2 + \hat{\sigma}^2)^{-1}},$$

where

$$\hat{\sigma}^2 = \max \left\{ 0, \frac{Q - (100 - 1)}{\sum_{j=1}^{100} s_j^{-2} - \sum_{j=1}^{100} s_j^{-4} / \sum_{j=1}^{100} s_j^{-2}} \right\}$$

with

$$Q = \sum_{i=1}^{100} (\hat{\gamma}_i - \tilde{\gamma})^2 s_i^{-2} \quad \text{and} \quad \tilde{\gamma} = \frac{\sum_{i=1}^{100} \hat{\gamma}_i s_i^{-2}}{\sum_{i=1}^{100} s_i^{-2}}.$$

Table 3 reports  $\hat{\gamma}$  along with its standard error  $\widehat{\text{SE}}(\hat{\gamma}) = \sqrt{1/\sum_{i=1}^{100} w_i}$ . Note, that  $\hat{\gamma}$  is simply a weighted average of the stock specific estimates with weights  $w_i$  that are inversely related to  $s_i^2$ .

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