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High-Frequency Trading During Flash Crashes: Walk of Fame or Hall of Shame?

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High-Frequency Trading During Flash Crashes: Walk of Fame or Hall of Shame?*

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Abstract

We show that High Frequency Traders (HFTs) are not beneficial to the stock market during flash crashes. They actually consume liquidity when it is most needed, even when they are rewarded by the exchange to provide immediacy. The behavior of HFTs exacerbate the transient price impact, unrelated to fundamentals, typically observed during a flash crash. Slow traders provide liquidity instead of HFTs, taking advantage of the discounted price. We thus uncover a trade-off between the greater liquidity and efficiency provided by HFTs in normal times, and the disruptive consequences of their trading activity during distressed times.

JEL Classification: G10, G14.

Keywords: flash crashes; high-frequency traders (HFTs); liquidity provision; market making.

1 Introduction

The existing academic literature on high-frequency trading intermediation suggests, almost unanimously, that High Frequency Traders (HFTs) are beneficial to market liquidity, see, e.g., Hendershott, Jones, and Menkveld (2011) and Jones (2013), as well as to market efficiency, see, e.g., Chaboud, Chiquoine, Hjalmarsson, and Vega (2014). This paper shows that their role reverses during flash crashes, which HFTs contribute to either cause or magnify. They indeed consume liquidity during flash crashes, which is instead provided by "traditional" slow traders, and generate a transitory price impact which is unrelated to the permanent impact. Even HFT *designated* market makers[1] do not provide enough liquidity to avoid flash crashes when they happen in isolated stocks, and they actually significantly contribute to the over-reaction leading to the crash when it affects several stocks simultaneously. Their behavior can be explained by the fact that, in exceptional situations, the fear of trading against informed traders overcomes the foreseen compensation for liquidity provision.

The very existence of flash crashes casts doubts on the orderliness of the financial market architecture. The data, however, support the presence of a substantial number of such events. The importance of flash crashes in the financial literature exploded after the infamous event of May 6, 2010 (Easley, de Prado, and O'Hara, 2011; Madhavan, 2012; Andersen and Bondarenko, 2014; Andersen, Bondarenko, Kyle, and Obizhaeva, 2015; Menkveld and Yueshen, 2019). However, this was not an isolated event at all. Commenting on the Sterling flash crash of October 7, 2016, Bank for International Settlements (2017) writes: *"This event does not represent a new phenomenon but rather a new data point in what appears to be a series of flash events occurring in a broader range of fast, electronic markets than was previously the case in the post-crisis era, including those markets whose size and liquidity used to provide some protection against such events."* But how can we define a flash crash?

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[1]We use the term "designated market makers" in this framework to emphasize the fact that such traders enter into a written agreement with the exchange, although their exact role in the market, and the details of such agreements may vary across time.

According to Bank of England (2019), "*Flash episodes are large and rapid changes in the price of an asset that do not coincide with – or in some cases substantially overshoot – changes in economic fundamentals, before typically retracing those moves shortly afterwards.*". These events are sometimes called "mini flash crashes" (Biais and Foucault, 2014). Based on a formal implementation of this definition, Christensen, Oomen, and Renò (2017) provide evidence of frequent flash crashes in the futures markets on S&P 500, gold, oil, EUR/USD, Treasury notes, and corn, a result also echoed in Golub, Keane, and Poon (2017) for the US equity market. Thus, even supposedly liquid markets are subject to these occurrences. Invariably, automated trading is indicated as a potential culprit for these events, even if the evidence is, at most, anedoctical.

We use the methodology proposed by Christensen et al. (2017) to detect flash crashes in order to investigate in detail the behavior of HFTs during these events. The peculiar feature of the methodology is its power to detect flash crashes from transaction prices. This allows the collection of a large sample of genuine flash crash events, which can then be scrutinized. We apply this methodology to a unique data-set, obtained from the BEDOFIH database,[2] of tick-by-tick order-level data on 37 liquid French stocks belonging to the CAC40 index and traded on NYSE-Euronext Paris in 2013. Order and trade data come with a flag indicating the trader class (slow or high-frequency trader[3]), determined by the French market authority (AMF, 2017), and the trader account (owner, client or market maker), determined by NYSE-Euronext. This categorization allows us to uncover the role of different market participants during distressed events. In particular, the behavior of designated market makers is not what is expected by the market authority. Our policy implication is thus that the compensation scheme offered by the exchange is not a sufficient incentive for them to prevent, halt or even attenuate flash crashes.

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[2]www.eurofidai.org/en/high-frequency-data-bedofih.

[3]In the literature, there is often a distinction between algorithmic trading and high-frequency trading. For example, HFTs of investment banks and their clients could be considered algorithmic traders. While the distinction makes perfect sense from an economic standpoint, in this paper we stick to the taxonomy adopted by the AMF and described in Section 2.

Our paper relates to the literature analyzing the behavior of HFTs during distressed events. Kirilenko, Kyle, Samadi, and Tuzun (2017) and Menkveld and Yueshen (2019) investigate in detail the flash crash of May 6, 2010. Hautsch, Noè, and Zhang (2017) study news-driven events, while Megarbane, Saliba, Lehalle, and Rosenbaum (2018) focus on the releases of macroeconomic information on the same market we analyze (stocks belonging to CAC40). Brogaard, Carrion, Moyaert, Riordan, Shkilko, and Sokolov (2018) look at extreme price movements. We instead use flash crashes, identified from transaction data, as a measure of market distress. There are several advantages in doing so. First, flash crashes are, by construction, periods during which the price moves suddenly and largely, often absent any news on fundamentals. The highly directional and sustained price trends experienced during these events cannot be ascribed to large market volatility or jumps, since the latter interpretation merely implies a wider price dispersion, but not a *directional* move. Second, flash crashes are not compatible with "normal" market behavior, and are typically attributed to market frictions, such as large trading imbalances with low market depth (Grossman and Miller, 1988), asymmetric information (Barlevy and Veronesi, 2003), costly market presence for market makers (Huang and Wang, 2009), or predatory trading (Brunnermeier and Pedersen, 2005). Third, using flash crashes allows to identify a large sample of distressed events in a relatively short time period that did not trigger any trading halt. In our sample, this amounts to 148 flash crashes. This feature is common to the extreme price movements of Brogaard et al. (2018). However, as we show in Section 3 and Appendix A, extreme price movements are a largely different distressed sample with respect to flash crashes. So, our results can be considered complementary to their approach. Fourth, our procedure identifies the exact crash peak time and disentangles different phases of a flash crash, such as the beginning of the price drop and the subsequent recovery.

The granularity of our data set, which allows to discriminate the trading styles of different groups, allows to identify the sources of the crash, as well as of the transient price impact. We indeed show that flash crashes are mostly originated by HFT trading from investment

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banks and their clients, and that this trading is informed. The crash is then magnified by the reaction of market makers, especially when crashes are systematic, that is when they affect several stocks simultaneously. Market makers of investment banks start selling during the crash, fearing to supply liquidity to informed sellers. Market makers of pure HFT firms mostly contribute to the price decline with their quoting activity. These results are largely different with respect to the previous literature. For example, Kirilenko et al. (2017) find that the trading pattern of HFTs did not change when prices of the E-mini S&P 500 stock index futures fell sharply during the Flash Crash on May 6, 2010. On our data, we show the opposite. In particular, HFT marker makers are found to strongly enhance the crash with their trading and quoting activity, especially during systematic events. Brogaard et al. (2018) find that HFTs provide liquidity during extreme price movements, on average, even if they switch to the demand side if several stocks experience a simultaneous swing. However, they have neither evidence of HFTs causing price crashes, nor of the ambiguous role played by designated liquidity providers which are documented here. They indeed conclude that HFTs do not appear to cause extreme price movements; we instead show that HFTs are responsible for originating and exacerbating flash crashes. Our findings are finally broadly consistent with the results in van Kervel and Menkveld (2019), who document a similar "leaning-with-the-wind" behavior of HFTs during institutional orders, which are also proved to be mostly informed. With respect to their contribution, we provide at least three additions: first, we document this behavior without having to resort to orders of "institutional" type; second, given the granularity of our dataset, we show that this behavior is mainly due to investment bank HFTs trading on their own account. Third, we show that this mostly happens during systematic flash crashes. Finally, we show that market makers of pure HFT firms also contribute to the crash with their quoting activity.

The rest of the paper is organized as follows. Section 2 describes the data and presents the classification of market participants. Section 3 is dedicated to the methodology of flash crash detection and identification. The empirical results are presented in Section 4. Section

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[5](navigation) concludes. An Appendix contains additional results and comparisons.

2 Data description

2.1 Institutional structure

The Euronext stock market operates as an order-driven market with a limit order book. Euronext Paris is the division of the exchange that includes all the French instruments, including equities and derivatives. The daily schedule for the most liquid stocks is divided into different segments. The trading session starts at 7:15 a.m with a pre-opening phase, followed by an auction at 9:00 a.m. The main trading phase, where most of the trading activity takes place, starts at 9:00 a.m. and ends at 5:30 p.m. (this is the trading period we analyze). The daily schedule is then followed by a closing auction and a further trading session called "trading-at-last," where additional trades can take place at the closing price. The focus of this study is on the main trading phase, thus opening and closing activity are excluded from the analysis.

According to the Rule 4403/2 of Rulebook I (Euronext, 2014), during the continuous trading, Euronext has in place a set of trading safeguards that prevents price movements outside certain thresholds. Specifically, traded prices are constrained into a "collar," defined by a reference price plus/minus a percentage price change. If the execution of an order causes the breach of the collar, two outcomes are possible: 1) the order is partially executed inside the collar, without halting the continuous trading; 2) the trading process is halted, and the market is put in "reservation mode." Continuous trading resumes after an auction. None of the detected events in our sample triggers any of these measures. Thus, the collars are ineffective against preventing the occurrence of flash crashes.

The market model of Euronext Paris relies on the provision of liquidity by electronic market makers. Since 2011, NYSE Euronext have in place a program, called *Supplemental Liquidity Provision* (SLP), where electronic traders agree to post two-sided quotes during

the day and to provide a minimum passive execution volume. Liquidity provision is rewarded with a rebate, whereas aggressive executions benefit from a reduction in the trading fee for SLP members.[4] The orders sent by SLP members have to be electronic, using only their own funds and excluding the customer orders.

2.2 Data

The database is provided by the *Base Européenne de Données Financières à Haute Fréquence* (BEDOFIH), and it is composed of tick-by-tick data for 37 liquid stocks that are included in the CAC40 Index in 2013.[5] We can track the entire history of the orders, from the initial submission to the execution or the cancellation, with a timestamp at the microsecond level. One interesting feature of the database is that the data from the stock exchange are complemented by an identification flag, provided by the French stock market regulator (the *Autorité des Marchés Financiers*, AMF), that categorizes each trader into three groups: pure HFTs companies (PURE-HFT) such as Citadel or Virtu; Investment Banks with HFT activity (IB-HFT) such as Goldman Sachs, and all the remaining traders (NON-HFT). This classification is revised yearly, and the groups are mutually exclusive.[6] In addition, each trader is required to flag every order, in compliance with the Rulebook, according to the following list of possible accounts (see NYSE-Euronext, 2012): own account or own account for client facilitation (OWN); own account of an affiliate, or when operating from a parent company of the stock (PARENT); account of a third party, or client account (CLIENT); orders submitted pursuant to a liquidity provision agreement (MM); orders submitted for retail liquidity provider (RLP) or retail matching facility (RMO). Finally, each

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[4]The details of the scheme are available in NYSE-Euronext (2012). Bellia (2017) provides a detailed description of the SLP scheme and the role of electronic market makers in the NYSE Euronext.

[5]The three stocks of the CAC40 that are not included in our analysis are Arcelor Mittal, Gemalto, and Solvay. We exclude them from consideration, since their main trading venue is not the Paris branch of Euronext.

[6]See AMF (2017) for a description of the methodology applied to identify the traders. The identification algorithm is based on the median lifetime of an order (including both modifications and cancellations), plus a threshold based on the total number of cancellations. A further check is carried out by the AMF, taking into account the identity of the trader.

trade struck during the main trading phase has a flag that indicates the trade initiator, allowing to unambiguously identify on the one hand which trader-account is trading aggressively (demanding liquidity) and on the other hand which trader-account is trading passively (supplying liquidity).

Figure 1 shows important features of the trading activity for each aggregated trader category. The left panel shows how many times the inventory of a particular trader, on average for each stock-day, crosses the value of zero. We expect this number to be higher for high-frequency traders, and smaller for traders taking positions in the market. The figure shows, indeed, that PURE-HFTs manage inventories more often close to zero, as expected for typical market making activity. The center panel shows the percentage of trades done by a particular trader type. The absolute majority of trades (61%) are conducted by IB-HFTs, while 21% and 18% of trades are due to PURE-HFTs and NON-HFTs respectively. Hence, IB-HFTs is the most active category in this market. The right panel shows the cancellation ratio that is defined as the percentage of the total number of submitted orders that are cancelled prior to execution. The average number of cancellations is high for both PURE-HFTs and IB-HFTs (94% and 93% respectively), while it is relatively low for NON-HFTs (43%).

<center>INSERT FIGURE 1 HERE</center>

Note that IB-HFTs may be classified as HFTs with regards to the number of messages or cancellations, but not regarding the inventory position, for instance. The common data-driven methodologies to identify HFT activity (see Hasbrouck and Saar, 2013) or the labeling provided by the exchange usually identify only PURE-HFTs (Brogaard et al., 2018). This highlights a distinctive advantage of our data set, since it allows to track also the behavior of the investment banks with HFT activity and to discriminate them from the PURE-HFT group, since their behavior differs in terms of strategies and capital constraints.

According to Megarbane et al. (2018), who have the regulatory database with traders' identities, all members of the SLP program are pure High Frequency Trading companies

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or Investment Banks with HFT activities. No market making activity is carried out by Non-High Frequency traders in our sample.

3 Identification of flash crashes

In this section, we describe how we build our flash crashes database. We detect flash crashes using a novel econometric approach, proposed by Christensen, Oomen, and Renò (2017), which supports the notion that a flash crash is (in relative terms) a large downtick in the price over a short time horizon. As we will show, this is typically accompanied by a reversion in the price.

INSERT FIGURE 2 HERE

Before explaining the mechanics of the procedure in detail, we look at an example of a distressed event, reported in Figure 2. The top panel of the figure shows the evolution of the price of Technip on June 25, 2013. Around 11:50am the price starts to decline rapidly until it reaches its minimum a few seconds after 12:05. The total return over the 15-minutes is -2.35 %. After the crash, the price partially recovers. This is an example of what we consider a "flash crash": a large price drop in a short time followed by a partial recovery. The intermediate panel shows the novel test statistics we use to detect the flash crash. As it can be seen, the peak of the test statistics coincides with the peak of the crash.

Figure 2 highlights the difference between our flash crash detection technique and those based on volatility (or jumps) such as, for example, Extreme Price Movements (EPMs) considered by Brogaard et al. (2018). Indeed, the basic method of EPMs detection consists of simply labelling all 10-second intervals that belong to the 99.9th percentile of 10-second absolute midpoint returns for a stock as EPMs. However, despite of the cumulated price drop being large, the individual high-frequency (10-second) returns observed during the crash, displayed in the bottom panel of Figure 2, are actually compatible with the overall volatility of that day. In fact, the largest negative 10-second return of the day (-0.25%)

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occurred inside the volatility cluster before 11:00am. Notwithstanding with being volatile, the price level did not change significantly in the 30-minutes around that time. Hence, EPMs identify the intervals of high volatility (or jumps), e.g., as shown by the pink shadow area in Figure 2. In this case, they would not identify the crash.

To lay down our flash crash identification procedure precisely, we need a minimal amount of notation. Let $(p_t)_{t\geq 0}$ be the log-price process of an asset, which is defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. We assume p_t evolves according to the model:

$$dp_t = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t + \mathrm{d}J_t, \tag{1}$$

where μ_t is the instantaneous drift, σ_t is the associated spot volatility, W_t is a standard Brownian motion, and J_t is a jump process. The log-price p_t is observed on $[0, T]$ at irregular time points $0 = t_0 < t_1 < \ldots < t_n = T$, such that $\max_i(t_i - t_{i-1}) \to 0$ as $n \to \infty$. The discretely sampled log-return is defined as:

$$r_{t_i} = p_{t_i} - p_{t_{i-1}}, \quad i = 1, \ldots, n. \tag{2}$$

In model (1), the price changes, dp_t, include three components: $\sigma_t \mathrm{d}W_t$ and $\mathrm{d}J_t$ are responsible for volatility clusters and jumps, while the drift term $\mu_t \mathrm{d}t$ represents the local trend. As discussed, our methodology relies on the drift term. A natural estimator of μ_t is the kernel estimator:

$$\hat{\mu}_t^n = \frac{1}{h_n} \sum_{i=1}^n K\left(\frac{t_{i-1} - t}{h_n}\right) r_{t_i}, \tag{3}$$

where K is a kernel (a localizing function) and h_n is a bandwidth (approximatively, the localization window). The estimator (3) is asymptotically unbiased, however, its variance does not decrease as $h_n \to 0$, hence the estimator is inconsistent (Bandi, 2002; Kristensen, 2010) .

To circumvent this issue, it is sufficient to rescale the drift estimator properly, as shown

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by Christensen et al. (2017) who propose the following test statistics:

$$T_t^n = \sqrt{\frac{h_n}{K_2}} \frac{\hat{\mu}_t^n}{\hat{\sigma}_t^n},$$ (4)

where

$$\hat{\sigma}_t^n = \sqrt{\frac{1}{h_n} \sum_{i=1}^{n} K\left(\frac{t_{i-1} - t}{h_n}\right) r_{t_i}^2}$$ (5)

is a consistent estimator the spot volatility and $K_2 = \int_{\mathbb{R}} K(x)^2 \mathrm{d}x$ is a kernel-specific constant.

The measure $\hat{\mu}_t^n / \hat{\sigma}_t^n$ can be interpreted as the current velocity of the market. Under "normal" market conditions (in particular, if the instantaneous drift is locally bounded) the ratio is small (see, again, Figure 2). If the price is moving fast relative to the volatility (i.e., price changes are directional), the ratio is large. Christensen et al. (2017) formalize the later condition by assuming that there exists a "drift burst" time point τ_{db}, where $\mu_t \to \pm\infty$ as $t \to \tau_{\mathrm{db}}$. They prove that, in such points, $|T_t^n| \to \infty$ as $t \to \tau_{\mathrm{db}}$. In points in which the drift μ_t is bounded, the test statistic is instead standard normal. Therefore, the test statistic T_t^n can be used for identifying a flash crash around time t by rejecting the "normal" market conditions in favour of the presence of a drift burst, when $|T_t^n|$ is larger than a quantile of standard Normal distribution.

The drift burst test statistic is robust to compound Poisson jumps, infinite activity small jumps, autocorrelated market microstructure noise and pre-announced jumps which occur at a pre-determined time, e.g. as a consequence of dividends or macro announcements. Further, the test is robust to volatility explosions. Thus, our distress measure is not picking up neither jumps nor large volatility episodes, but just genuine exploding "trends".

How good is this test in identifying the typical V-shape behavior of a flash crash? Christensen et al. (2017) show that the test is effective along several dimensions. First, the test statistics is able to easily detect all the flash crashes that have been popularized in the press (such as the Treasury flash crash of October 15, 2014 or the Twitter flash crash of April

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23, 2013), and many more. Second, extensive Monte Carlo simulations show that the test is correctly sized, and that for events with a sufficiently large t-statistics, the probability of contamination by false positives is essentially zero. We refer the reader to Christensen et al. (2017) for details.

To implement the test, we compute the estimator of the drift and volatility based on a kernel-weighted average of observations in the vicinity of t, as defined in Eq. (5). The bandwidth h_n determines how fast we down-weight observations further away from t, with weights decided by the kernel function. We set $K(x) = \exp(-|x|)\mathbb{1}(x \leq 0)$, such that $\hat{\mu}_t^n$ and $\hat{\sigma}_t^n$ are computed with a left-sided exponential moving average (based on backward-looking data) to avoid look ahead bias. We employ a 5-minute bandwidth for the mean and a 25-minute bandwidth for the volatility. This means that, by construction, we are interested in flash crashes which develop on a time span of roughly 10 minutes. Transaction prices are pre-averaged (Jacod, Li, Mykland, Podolskij, and Vetter, 2009) to soften the impact of market microstructure noise, and the volatility estimator in (5) is robustified with an HAC correction. This setup is consistent with Christensen et al. (2017) to which the reader is again referred for details.

We compute the above test statistic every second during the course of a trading session. To account for multiple testing, we exploit the simulation-based algorithm from Christensen et al. (2017) to set an appropriate critical value. We use a 99.9% confidence interval. The average threshold value is -4.9: values of the drift burst test statistics below this threshold are labelled as flash crashes. We only consider crashes after 9:30am. Appendix A is devoted to the comparison of the events detected by the drift burst test statistics with those detected with the EPM methodology of Brogaard et al. (2018), showing that the flash crashes we detect and EPMs are a largely different sample with limited overlap.

4 Empirical results

4.1 Anatomy of flash crashes

Table 1 reports summary statistics of events identified by the procedure explained in Section 3. The average crash duration is 9.5 minutes. An average price drop during a flash crash is -1.35%. During the largest crash, which occurred in ST Microelectronics on March 12, 2013, the price fell by -5.18%, while during the smallest one, which occurred in Pernod Ricard on October 2, 2013, the price declined by -0.37%. Table 1 also shows that while the crash duration represents only 1.87% of the duration of the trading day, on average, the crash accounts for 5.43% of daily trading volume (in days with a flash crash), nearly 6% of the number of trades, and roughly 21% of total selling volume in that day.

INSERT TABLE 1 HERE

A detailed summary of all 148 identified flash crashes is provided in Table F.1 and F.2 in the Internet Appendix. Additionally, Table F.3 in the Internet Appendix reports a summary of the detected flash crashes grouped according to each stock. It shows that in our sample flash crashes occur in 34 different stocks. For one stock, flash crashes occur only once. The largest number of crashes per year (10) corresponds to Alstom (ISIN FR0010220475). There is no visible relation between the number of flash crash occurrences and stocks' market capitalization, volatility and average return.

A pictorial representation of the temporal distribution of the 148 crashes over the year and the trading hour during the day is provided in Figure 3. The figure shows that crash events are scattered across the year and the time of the day uniformly, without a clear pattern emerging, for example around a specific time of the day. This rules out the hypothesis that crashes are due, for example, to macroeconomic announcements, which tend to happen at specific times of the day. In two prominent cases, clearly visible in the figure, we have crashes affecting multiple stocks simultaneously: on April 17, 2013 (with 14 stocks involved) and

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on September 3, 2013 (with 13 stocks involved).[7] From now on, we label these 27 events as "systematic". The remaining 121 events will be labelled as "non-systematic". The behavior of HFTs is very different for these two groups.

INSERT FIGURE 3 HERE

For each event, we denote by t_{crash} the point in time associated with the lowest value of the t-statistics after crossing the significance threshold. The beginning of the crash, labelled t_{start} is identified by the first crossing time of the t-statistics with -1 before t_{crash}. The difference $\tau = t_{\text{crash}} - t_{\text{start}}$ is the crash duration. The end of the recovery period is identified by the time $t_{\text{end}} = t_{\text{crash}} + 3\tau$. We also consider a pre-crash period, starting at the time $t_{\text{pre}} = t_{\text{start}} - 2\tau$. The analysis in this paper is based on price and order information, for each crash, from time t_{pre} to time t_{end}. In order to harmonize information coming from crashes with different duration, we use harmonized time units in which the crash duration is used as the time unit for each event.

INSERT FIGURE 4 HERE

Figure 4 shows the average cumulative return for systematic and non-systematic crashes, together with 10% and 90% quantiles for all events. This figure clearly illustrates the output of our identification strategy, described in Section 3, and also allows for a qualitative description of the average price process. The overall pattern of the detected events is the typical "skewed V" displayed by a flash crash. While on average the market price moves towards a new price level, it substantially over-reacts and declines to a price which is lower than the new fundamental level. This picture is consistent with informed trading conveying new

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[7]The collective flash crashes of April 17, 2013 are likely due to the announcement, that morning, of new austerity budget measures due to the pessimistic revision of growth figures by the French government, see e.g. https://lexpansion.lexpress.fr/actualite-economique/ce-qu-il-faut-retenir-du-nouveau-plan-budgetaire-de-la-france_1404490.html; the event of September 3 cannot instead be associated to any news, to the best of our knowledge. However, flash crashes in several stocks with no news can be explained by the phenomenon of liquidity spillovers, see e.g. Cespa and Foucault (2014). The behavior of the 37 stocks around the two crashes is shown in Figure B.1 in the Appendix B.

information into a permanent price impact (from the beginning to the end of the recovery period). The V-shape is deeper and more pronounced for systematic events, indicating more over-reaction than for non-systematic events on average. We can define formally, for each event, the permanent price impact (PPI) as:

$$\text{PPI} = \log p_{t_{\text{end}}} - \log p_{t_{\text{pre}}},$$

that is the logarithmic return from beginning to recovery[8], while the crash price impact is defined as:

$$\text{CPI} = \log p_{t_{\text{crash}}} - \log p_{t_{pre}},$$

and the transient price impact is defined as:

$$\text{TPI} = \text{CPI} - \text{PPI} = \log p_{t_{\text{crash}}} - \log p_{t_{\text{end}}}.$$

To better understand who is contributing to the transient and permanent components of the crash return, we further subdivide the crash period (from t_{start} to t_{end}) in three stages with equal duration: early, intermediate and late crash, as shown in Figure 4.

Figure 5 shows the relation between PPI and TPI, as well as the relation between CPI and crash duration. From Panel A, we can see that both the average PPI and the average TPI are highly significant in our detected sample, and this is true for both systematic and non-systematic events. The fact that PPI is mostly negative (with few exceptions) is a clear indication of informed trading. The fact the TPI is always negative (with one exception) is a clear indication of over-reaction. In particular, the transient component of systematic events is -0.78% on average, half of the average permanent component, and it is still -0.33% (and significantly negative) for non-systematic events, roughly one third of the permanent

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[8]Our choice of using the price after, on average 27 minutes, is in line with the typical approach of the market microstructure literature when estimating effective spreads, see e.g. Glosten (1987). Alternatively, we could use the price observed at the end of the day, or in the next day, as in van Kervel and Menkveld (2019). No differences would emerge in the subsequent analysis.

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component in this case. Panel B shows that the typical duration of the crash event ranges from a few minutes to 45 minutes, with an average duration of 9.5 minutes. The longer the crash, the more negative the CPI.

<div align="center">INSERT FIGURE 5 HERE</div>

4.2 Liquidity during a flash crash

Supplied liquidity evaporates during flash crashes. We report, in Figure 6, average volume, market depth, bid-ask spread and executed order age for the 121 non-systematic flash crashes. The corresponding figures for systematic events (which are qualitatively very similar) are reported in Figure B.2 in Appendix B. Market depth is the total number of shares offered across the first ten price levels on the bid and the ask side. The bid-ask spread is calculated as the logarithmic difference between the one minute average of the best bid and the best ask price. The executed order age is the time between trade and the time stamp at which the order was posted (or last modified). The mean cumulative return shown in Figure 4 is superimposed in all the figures to be used as a visual landmark of the average crash development.

<div align="center">INSERT FIGURE 6 HERE</div>

Panel A of Figure 6 shows that the crash is associated with a large selling pressure, as expected, and that recovery starts after selling stops. The fact that selling produces a permanent price change is consistent with informed trading.

Panel B shows that the bid-ask spread steadily increases during a flash crash, indicating increasing cost of transacting during these events, as well as the increased uncertainty on market fundamentals which is indeed typically accompanied by a widening of the bid-ask spread. Panel C shows that market depth, that is the ability of the market to absorb orders, is strongly reduced during a crash, recovering slower than the price itself, in line

<div align="center">15</div>

with the finding in Kirilenko et al. (2017) during the Flash Crash of May 6, 2010. This is again consistent with Grossman and Miller (1988), since the price change in their model is predicted to be deeper in illiquid market conditions. Panel D shows that average order duration increases along selling, since on its way down the price is hitting orders which were posted before the crash started. Overall, Figure 6 suggests that a flash crash occurs when selling pressure increases and not enough liquidity is provided to the sellers.

Figure 7 shows the average euro volume traded per minute, divided in buyer-initiated (panel A) and seller-initiated (panel B) trades for the non-systematic events (Figure B.3 in Appendix B reports the average for systematic events, which is qualitatively the same). The two patterns are similar even if the trading intensity of selling is much higher. We observe a sudden peak in average trading activity clearly associated with the beginning of the crash. This behavior in correspondence of the initial price drop is compatible with a large selling order which is executed in a series of child trades. Then trading slows down, to strongly accelerate again before the end of the crash. During recovery, the intensity of trading reverts back to normal activity. Figure 7 suggests that there are two distinct phases of the crash: an initial phase, triggered by sudden selling pressure, and a final phase, which occurs in a more illiquid market and precipitates the price much more. The observed trading volume in the first part of the crash is consistent with standard economic theory of immediacy demand, as in Grossman and Miller (1988), which predicts a V-shaped behavior for the price process when market makers accommodate the need for immediacy of a large seller. However, the second peak is not predicted by this theory, and could be due to the strategic intermediation of market makers facing the usual adverse selection problem, that is trying to disentangle whether selling is information-based or liquidity-based. We will turn to this fundamental point later, showing that the second peak generates the transient price impact.

INSERT FIGURE 7 HERE

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4.3 Trade analysis

We now turn to the analysis of the trading behavior of different trader groups during a flash crash, answering the following questions: who is responsible for the permanent price impact? Who is responsible for the transient price impact? And what motivates the trading behavior of different traders?

We analyze net trade imbalances (or net inventories) of each trader group. We split imbalances in initiating and liquidity supplying trades as follows. For each trade, denote by i the category which is initiating the trade with its demand, and by j the category who is accepting the offer (i and j may coincide). The quantity of money exchanged in the trade t is $Q_t \cdot P_t$, where Q_t is the number of stocks exchanged and P_t is the stock price. Thus, for category i, the imbalance on initiating trades on a given period is computed as:

$$\mathcal{I}_{\text{period}}^{(i),\text{init}} = \sum_{t \in \text{period}} s_t \cdot Q_t \cdot P_t \cdot I_{\{\text{t initiated by } (i)\}},$$

where $s_t = +1$ for buy orders, and $s_t = -1$ for sell orders, while $I_{\{.\}}$ is the indicator function. The imbalance on liquidity supplying trades for category j in a given period is similarly:

$$\mathcal{I}_{\text{period}}^{(j),\text{liq}} = \sum_{t \in \text{period}} s_t \cdot Q_t \cdot P_t \cdot I_{\{\text{t accepted by } (j)\}}.$$

The monetary net imbalance for category i in a given period is computed as:

$$\mathcal{I}_{\text{period}}^{(j)} = \mathcal{I}_{\text{period}}^{(i),\text{init}} + \mathcal{I}_{\text{period}}^{(j),\text{liq}}. \tag{6}$$

This measure is similar to that used by Brogaard et al. (2018), except for the fact that, in their paper, they use number of traded shares instead of euro volume.

INSERT FIGURE 8 HERE

Figure 8 reports average net imbalance, computed as in Eq. (6), and prices for systematic

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flash crashes (in Panel A) and non-systematic flash crashes (in Panel B). In both cases it is clear that IB-HFT OWN (followed by IB-HFT CLIENT for non-systematic events) initiate the crashes by acting as large sellers. This is informed selling since it moves the price toward a new fundamental level. By market design, the order imbalance generated by sellers should be absorbed by market makers, which are all HFTs (PURE-HFT and IB-HFT) in this market. Figure 8 focuses on IB-HFT MM only since the total inventory of PURE HFT MM is almost zero at all the crash stages (even if they affect the price with their quoting activity, see Section 4.4). For non-systematic events, IB-HFT MM do provide liquidity during the crash, somehow backed by NON-HFT traders, even if not sufficient to avoid price over-reaction. Then, already long the stock, they need to restore zero inventory so they do not help with the recovery, which is left to NON-HFT. This behavior is roughly consistent with standard theories of financial intermediation, as in the theory of immediacy provision of Grossman and Miller (1988), and their role as liquidity providers. Nevertheless, when the flash crash affects several stocks simultaneously (systematic events), they do not provide liquidity on average. Actually, they sell during the crash, and even more intensely than IB-HFT OWN in the late phase of the crash. The order imbalance of systematic flash crashes is actually absorbed, especially in the late phase of the crash, by "slow" NON-HFT traders, who buy at a discount price mostly through limit orders posted far before the crash started. Not only these traders, who do not use extensively algorithmic trading strategies, absorb the selling generated by "sellers" and market makers, but they also mostly contribute to the partial recovery of the price. Thus, HFTs generate the crash (through informed selling, mostly from OWN and CLIENT), and, when crashes are systematic, they precipitate it in its later stages because of additional selling by HFT market makers. Figure 8 displays averages. A statistical assessment of these results, which is possible given the large number of flash crashes we analyze, is contained in Appendix C, Table C.1. It shows that the pattern in Figure 8 is statistically significant.

Figure 9 shows the average trading imbalance changes per minute for the two most

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important trading categories (all trading categories are shown in Appendix B, Figures B.5 and B.6), that is IB-HFT MM and IB-HFT OWN. The intensity of imbalance change is further dissected into initiated buy, initiated sell, supplying buy and supplying sell. A positive value of the average means that the net imbalance is increasing; a negative value means that net imbalance is decreasing.

<div align="center">INSERT FIGURE 9 HERE</div>

Panel B and D show that the IB-HFT OWN are, on average, originating crashes, both systematic and non-systematic ones. Their increase in the intensity of selling is strongly correlated with the decline in price, and their selling accelerates till the peak of the crash. They also provide some liquidity (of course, each group consists of several individual traders), but their net effect is strongly negative, especially at the beginning and the end of the crash.

Panel A and C point again at the ambivalent behavior of IB-HFT MM. For non-systematic events, they provide liquidity either at the beginning or at the end of the crash. However, a fraction of them starts selling intensely just before the peak of the crash, as indicated in Panel A, thus consuming liquidity instead of providing it. For systematic events, there is no average liquidity provision at the beginning of the crash. Even if some liquidity is provided at the end of the crash, as shown in Panel C, the net effect is strongly negative, and almost as intense as that produced by IB-HFT OWN. As for Figure 8, a statistical assessment of Figure 9 is the subject of Appendix C, Tables C.2 and C.3, showing that the described patterns are statistically significant.

<div align="center">INSERT FIGURE 10 HERE</div>

To further investigate this issue, Figure 10 shows the distribution, across events, of trading imbalances changes of IB-HFT MM and IB-HFT OWN in the different phases of the crash and during recovery (Figure B.7 in Appendix B shows the same for non-systematic events). The Figure makes clear that, while IB-HFT OWN mostly sell at the beginning of the crash,

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IB-HFT MM start selling immediately after. In particular, they are always selling in the late phase of the crash.

Why do IB-HFT MM sell when flash crashes affect several stocks, instead of providing liquidity (when it is needed the most)? Their change in the trading strategy, clearly visible in Panel A of Figure 9, could be motivated by a series of rationales. First, they need to buy to restore zero inventory. Grossman and Miller (1988), however, predict that they should buy after price recovery. A most likely explanation provided by economic theories is that these traders face the problem that selling could not be motivated by pure immediacy demand, but could be informed (as it actually is, on average). Thus, they could start selling opportunistically to profit from this information, as postulated e.g. by the back-running theory of Yang and Zhu (2019). Another opportunistic reason to sell is predatory trading (Brunnermeier and Pedersen, 2005): realizing that the seller is in distress, they could push the price even more downward to buy later at an even lower price (and also restore zero inventory). Indeed, we do observe a surge in initiated buying from this category immediately before the crash peak, which is consistent with both back-running and predatory trading. Indeed, a mixed behavior has to be expected since market makers do not know if the price is going to recover or not (see the discussion in van Kervel and Menkveld, 2019).

INSERT TABLE 2 HERE

Support to the conjecture that IB-HFT MM sell during the crash fearing the initial selling they observe is informed is provided by Table 2, which reports the average profit (in €) during flash crashes for each category, together with standard errors. It is clear that IB-HFT MM lose a significant amount of money, on average 2799.72 euros, by providing liquidity during non-systematic crashes. They transfer this money, as expected, to IB-HFT OWN, who trade on information by selling at a higher price. By a simple back-of-the-envelope calculation, the average monetary profit they would obtain by providing all the required liquidity to the sellers in the French market would be just 28 euros,[9] therefore not enough to cover the

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[9]This rough estimate is obtained by multiplying the total liquidity need, taken from Panel A in Figure 6

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losses. One may argue that they still do make profits on average, since providing liquidity in tranquil times should be enough to cover these losses. However, if they have the option to still satisfy the requirements as liquidity providers without facing large losses in these cases, it is perfectly rational not to provide liquidity during distress. That is exactly what they do, since Table 2 show that IB-HFT MM are able to avoid losses during systematic events, passing the "hot potato" to slow traders. Our results thus question the effectiveness of the compensation scheme adopted by AMF.

The final aspect that we investigate is whether HFTs change their trading behavior during flash crashes. We perform this analysis using the methodology of Kirilenko et al. (2017), applied to the 148 flash crashes that we have in the sample. The full analysis is reported in Appendix D. Analyzing multiple crash events, we find that HFTs modify substantially their behavior, contrary to what found on the single May 6, 2010 event.

Summarizing, our results on trades point out the limited willingness of IB-HFT MM to provide liquidity during a flash crash. They provide some liquidity during non-systematic crashes, but during systematic crashes all IB-HFT MM are selling in the late phase of the crash. Thus, the role of market makers loses effectiveness exactly when the market mostly needs it, which exacerbates the crash. The role of liquidity providers is instead played by slow traders, especially those classified as "clients", through passive trades that they posted before the flash crash took place and which were not repositioned fast enough in the market.

4.4   Quoting activity analysis

We finally evaluate the impact of order submissions and cancelations on the price changes, in line with the recent literature suggesting that quotes play a relevant role for price discovery (Brogaard, Hendershott, and Riordan, 2019). We start by looking, in Figure 11, at the

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to be 1.4 millions of Euro, times the compensation per Euro, taken to be 0.20 bps per Euro (in 2013, this number changed from 0.20 to 0.22 to be back to 0.20). This estimate does not include the standard gain from market making activity, i.e. the bid-ask spread, which is included in the net monetary profit in Table 2.

average volume of cancelled orders (Panel A) and the average volume of new orders (Panel B), both being proxies of the level of activity of the traders, especially for HFTs. The Figure is for non-systematic events, the corresponding Figure for systematic events is reported in Appendix B, Figure B.4. Cancellations and new orders follow an almost identical pattern, indicating that limit orders are heavily used to change the positioning of traders in the book. The pattern displays the same two-peaks structure of volume intensity in Figure 7. The first peak corresponds to the beginning of the crash. Then the cancellation volume declines, to spike up again at the end of the crash. It then slowly reverts back to normal levels. Quote revision is almost exclusively used by HFTs. In particular, PURE-HFT MM are cancelling and revising most of the orders. This pattern also supports the presence of two distinct phases for a flash crash: an initial one, in which selling pressure is accommodated with a price drop, and an accelerating one which exacerbates the price decline. Results are confirmed also for systematic events, as documented in Appendix B.

INSERT FIGURE 11 HERE

This suggests orders are a primary channel to impact the price for HFTs. To assess the impact of quoting activity on the price, we estimate the impulse responses of mid-price changes on unit shocks in "trader specific mid-prices", computed as the average between best bid and best ask of each trader group. The impulse-response functions (IRFs) are estimated for each flash crash event in four periods: pre-crash, crash, late crash and recovery. The IRFs are estimated with the local projection methodology developed by Jordà (2005), which constitutes a robust alternative to standard VAR models. The methodology is briefly reviewed in Appendix E. Figure 12 shows estimated IRFs, averaged across systematic and non-systematic events, of the changes of mid-price on unit shocks in the changes of the average between best bid and best ask of different trader groups. The results are compelling in showing that market makers (both PURE-HFT and IB-HFT) play a dominant role in determining, through their quote revisions, mid-price changes during crash and recovery. Their role is particularly strong during systematic crashes, and this is especially true for

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PURE-HFT MM. This suggests market makers are actively moving the market through their quote activity instead of transactions. The role played by other market participants is not visible in the average impact. Further, the impact of PURE-HFT MM is stronger in the late phase of the crash and for systematic events, signaling that they are mostly responsible for the transient price impact with respect to quoting activity.

<div align="center">INSERT FIGURE 12 HERE</div>

Altogether, the evidence presented in this section about quoting activity provides statistical support to the main themes of this paper. First, quoting activity presents the same pattern of trading volume. Second, quoting activity has an impact on prices, most prominently impacted by market makers, in particular PURE-HFT MM. Again, their impact is stronger for systematic events than from non-systematic ones. And, again, the pattern can explain the over-reaction causing the transient price impact.

5 Conclusions

In this paper we investigate the role of high frequency trading during flash crashes, and we show that HFTs do not play a beneficial role to market efficiency and liquidity during periods of pronounced market distress. Using a novel econometric methodology proposed by Christensen et al. (2017) we detect 148 flash crashes in one year (2013) of blue-chip French stocks. The granularity of our database allow us to distinguish HFTs vs non-HFTs, as well as different trading groups. Importantly, PURE-HFT firms have been largely investigated in the literature, while IB-HFT to a much lesser extent. Our analysis shows that HFTs, and in particular IB-HFTs, do play a significant role in causing flash crashes. IB-HFT Owners push the price down with informed selling at the beginning of the crash; IB-HFT Clients follow to profit opportunistically on this information, especially during non-systematic crashes; IB-HFT Market Makers also follow, especially when the crash is systematic; PURE-HFT Market Makers use intense quote revision which brings the price down during crashes, contributing to

the over-reaction in the late phase. The behavior of market makers in a market that became already illiquid creates overshooting and a transitory price impact. Even if IB-HFT Market Makers during systemic crash contribute to overshooting, their behavior is still rational. When crashes affect several stocks, they sell increasingly as the crash develops to avoid big losses against informed traders. This behavior is very different from what they are usually doing in "tranquil" phases. The main category that stops crash and supports the recovery is the NON-HFT, who receive a compensation from buying at discount.

Our conclusion, based on a large set of crashes, is that none of the trader categories considered here fits the hall of shame or the walk of fame completely. For example, even if IB-HFT MM, which should provide liquidity on the market under a designated market making agreement, actually intensify systematic crashes considerably, they do so just to avoid big losses against informed trading. Our paper documents that in diverse market situations of distress, traders react rationally but unfortunately not necessarily in the direction of efficient market functioning. Thus, our empirical findings can be informative for market design, to provide the right incentives or a different market structure (see, e.g., the discussion in Budish, Cramton, and Shim, 2015).

Our paper has also important policy implications. Electronic liquidity providers can indeed recover zero net inventory and provide liquidity on average, even if not enough to prevent flash crashes to happen. Our study recommends a deeper evaluation of the recent MiFID II regulation. In fact, MiFiD II recognized algorithmic liquidity provision as pivotal to the sound functioning of financial markets. The new regulation specifically endorses the automatic liquidity provision by electronic market makers, imposing specific binding agreements between the exchange and the trading firms. What our analysis shows is that this rule, already in place at the NYSE Euronext Paris stock exchange, is not sufficient to prevent flash crashes, and could be revised in light of this objective. Possible solutions to this problem are: change in their compensation scheme; or a change in the mechanism of trading halts, which, as it is, does not work with flash crashes since these cannot be associated with

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excess volatility.

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Table 1 Summary statistics of Flash Crash events

	mean	std	min	max	quantile(0.1)	quantile(0.5)	quantile(0.9)
return	-1.35	0.80	-5.18	-0.37	-2.42	-1.11	-0.62
duration (min)	9.53	8.45	0.07	41.60	2.12	7.32	19.34
duration (%)	1.87	1.66	0.01	8.16	0.42	1.44	3.81
N. trades during crash	582.10	442.32	89.00	2612.00	182.90	452.50	1191.70
N. trades daily	10337.07	5710.67	2501.00	44264.00	5258.50	8823.00	19025.90
N. trades during crash (%)	5.99	4.29	1.07	35.98	2.46	5.01	9.66
Signed volume during crash	-1276.37	1347.86	-9291.14	1722.17	-2942.18	-1049.13	-19.78
Signed volume daily	-4848.60	15259.88	-128877.88	42409.70	-16296.97	-3218.63	4980.44
Signed volume during crash (%)	21.07	182.75	-667.12	1592.26	-58.15	9.19	64.44
Absolute volume during crash	4598.60	3789.11	537.53	18268.03	1210.75	3521.45	9530.02
Absolute volume daily	93943.52	69076.44	16946.98	435185.60	34809.38	75966.21	183647.23
Absolute volume during crash (%)	5.43	4.06	0.71	34.71	2.02	4.50	9.03

Note. This table reports summary statistics of the price drop during Flash Crashes, flash crash duration and the characteristics of the stocks in which flash crashes occur: returns during the crash (per cent), duration of crashes in minutes and as a fraction of length of the trading session, number of trades, signed and absolute trading volume (in thousands of euros) over the whole trading session and during the crash period in the same units and relative to the whole trading day (per cent). The full database is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data, with trader group and account flags, are from BEDOFIH.

Table 2 Net monetary profit (€) during flash crashes.

	Systematic	Non-Systematic
PURE-HFT CLIENT	24.36	11.07
	(25.94)	(124.21)
PURE-HFT MM	−377.78	−164.09
	(541.03)	(444.22)
PURE-HFT OWN	−60.17	−27.52
	(108.69)	(312.32)
IB-HFT CLIENT	−1024.37	241.62
	(889.47)	(774.40)
IB-HFT MM	−75.61	−2799.72***
	(959.95)	(706.58)
IB-HFT OWN	5396.13**	2239.13*
	(2713.46)	(1204.43)
IB-HFT PARENT	−208.34	−423.07*
	(484.79)	(238.88)
NON-HFT CLIENT	−3164.53	−438.11
	(2250.18)	(996.79)
NON-HFT OWN	−765.22	1375.07**
	(1457.17)	(690.90)

Note. This table presents the average monetary profit during flash crashes, divided into systematic and non-systematic events. Standard errors are in parenthesis. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Figure 1. Trading activity by category

Panel A

Inventory Cross Zero

Panel B

Trading Activity

Panel C

Cancellation ratio
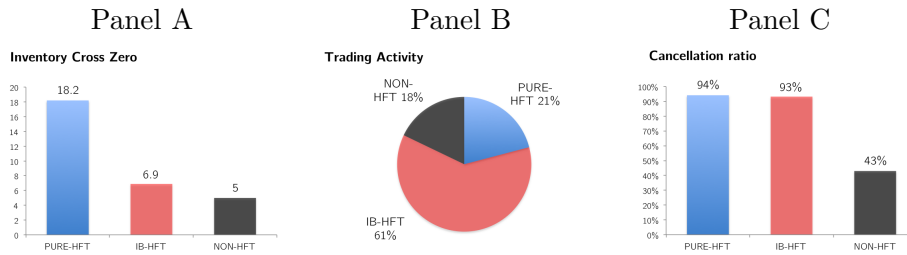
Note. This figure presents the average number of times per stock day when the inventory of a trader crosses the zero value (panel A), trading activity (panel B) and the cancellation ratio (panel C). The full database is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the entire year 2013. Order flow and trade data, with trader group and account flags, are from BEDOFIH.

Figure 2. An example of a crash event
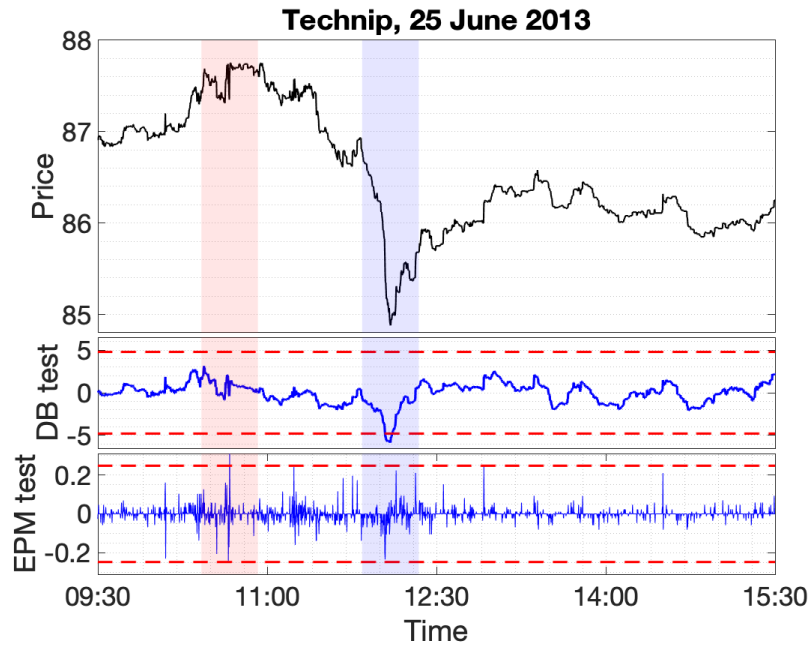
Note. This figure reports the evolution of the price of Technip over a flash crash events on June 25, 2013, which is detected using the methodology of Christensen, Oomen, and Renò (2017), but not detected by the extreme price movement approach of Brogaard et al. (2018). The upper panel reports the time-series traded price sampled at the 10-seconds grid. The middle panel reports the drift burst (DB) test statistics we use to detect flash crashes in our sample. The lower panel shows the basic test used to detect extreme price movement (EPM), that is 10-second returns. Dashed-red lines represent 99.9% confidence bands.

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Figure 3. Temporal distribution of crash events in the data
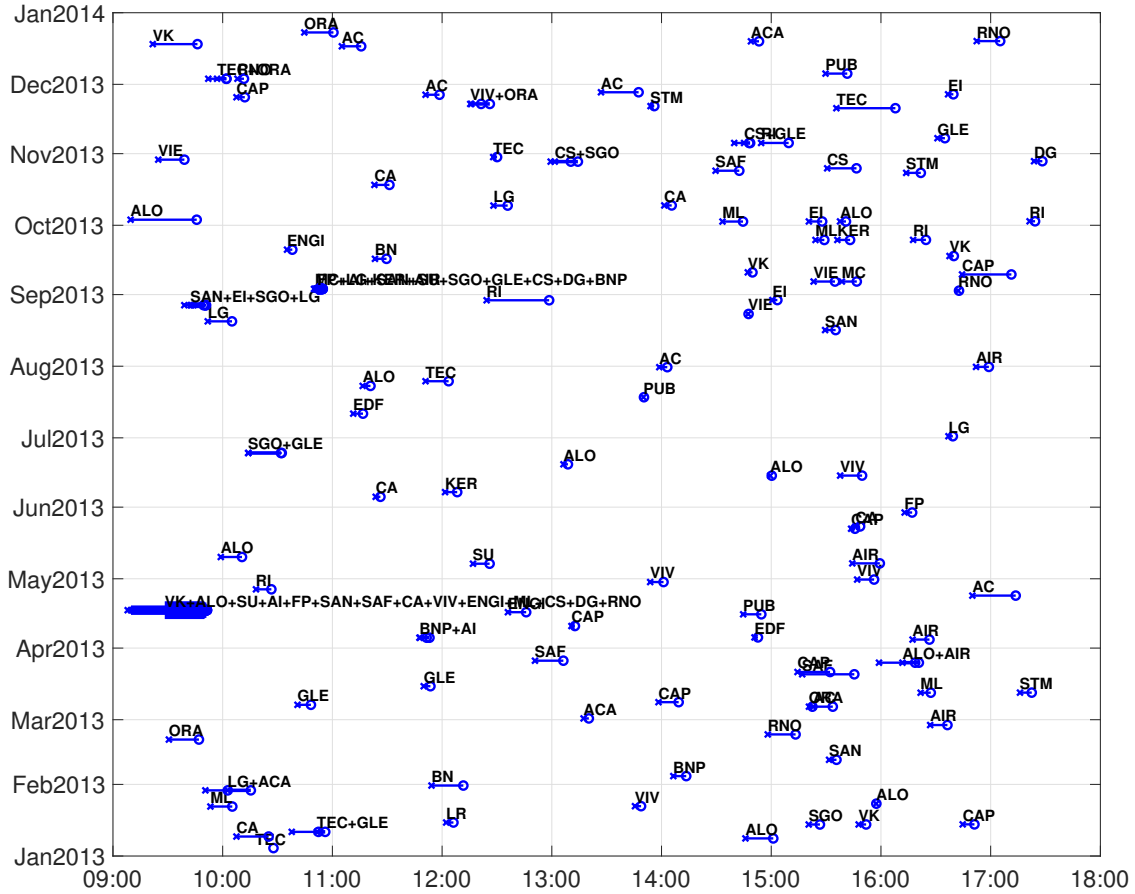
Note. This figure presents, for each of the 148 crash events which are analyzed in the empirical application, on the x axis, the time of the crash start t_0 (with a cross) and the time of the crash end t_1 (with a circle) connected by a segment. The y axis report the day of the year. For each crash, the ticker of the corresponding stock is also reported. Tickers are connected by a "+" sign when the crash is simultaneous. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure 4. Average cumulative return dynamics during a crash event
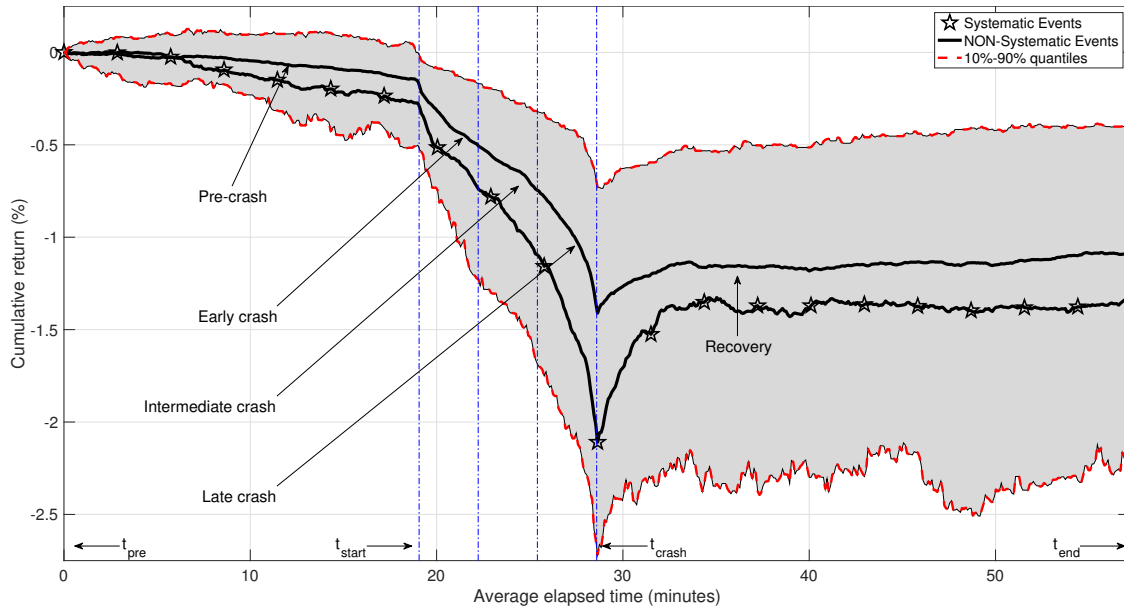
Note. This figure reports the average of the price evolution over systematic and non-systematic flash crash events, as well as the 10-90% quantiles for all the 148 events. The vertical lines separate the different periods in the evolution of the crash as labelled in the figure and described in the text. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Figure 5. Average cumulative return dynamics during a Flash Crash

Panel A: Permanent vs. Transitory price impact   Panel B: Crash price impact vs crash duration
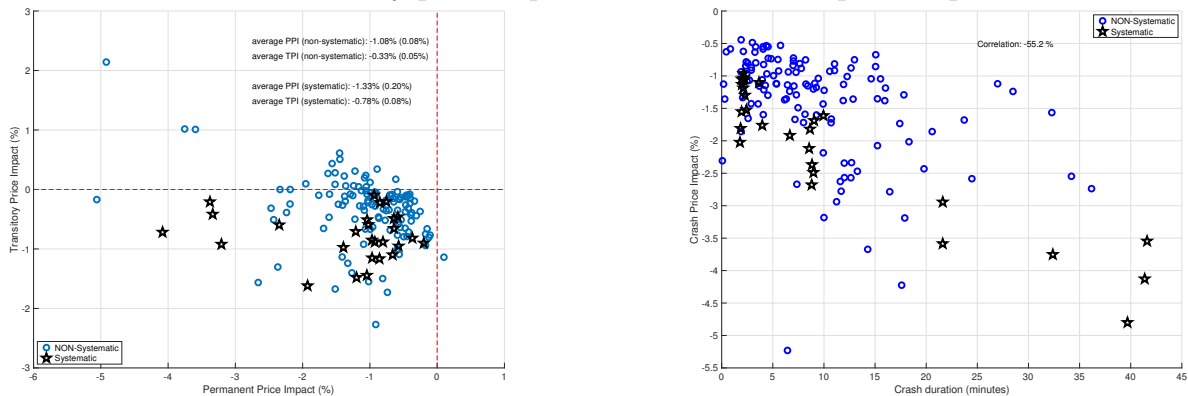
Note. This figure presents the permanent and transitory price impact. Panel A reports the scatter plot of the Transient Price Impact (negative log-return from peak to end) with the Permanent Price Impact (log-return from beginning to end). Means and standard deviations (in parenthesis) are reported for both systematic and non-systematic events. Panel B reports the scatter plot of the Crash Price Impact (log-return from beginning to peak) with the duration of the crash. A longer duration implies a deeper crash. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure 6. Liquidity measures during non-systematic flash crashes

Panel A: Signed volume.

Panel B: Bid-ask spread.

Panel C: Market depth.

Panel D: Executed order age.

Note. This figure reports four measures of liquidity averaged across the 148 flash crash events considered in this study. Panel A: cumulative signed monetary volume (negative volume = sell). Panel B: bid-ask spread. Panel C: market depth (difference from beginning). Panel D: the age of the executed orders (we exclude orders with age less than 0.1 seconds and those coming from the previous day). On each panel, we superimpose the average price evolution for visual comparison. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure 7. Volume per minute for initiated buyer and initiated seller trades (non-systematic)

Panel A: buyer-initiated volume

Panel B: seller-initiated volume
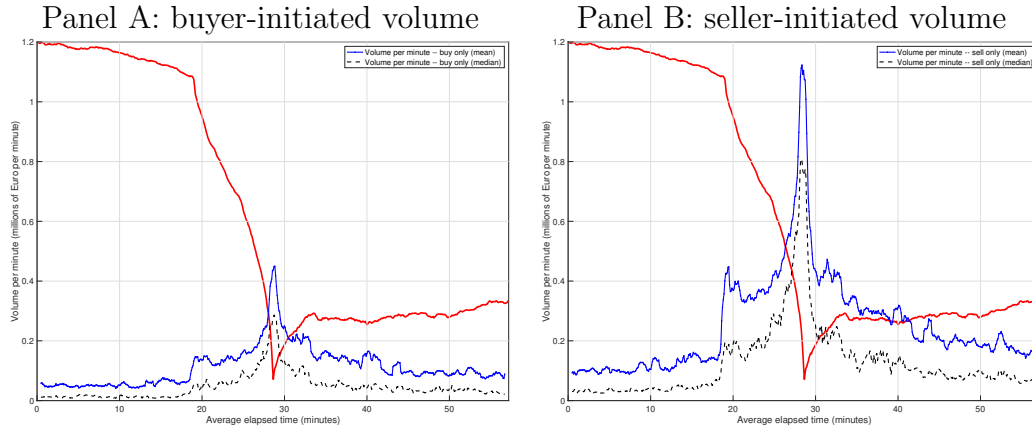
Note. This figure depicts the average euro volume traded per minute during crash events, separated in buyer-initiated trades (Panel A) and seller-initiated trades (Panel B). On each panel, we superimpose the average price evolution for visual comparison The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Figure 8. Average trading imbalance of different trading categories during a flash crash

Panel A

Panel B
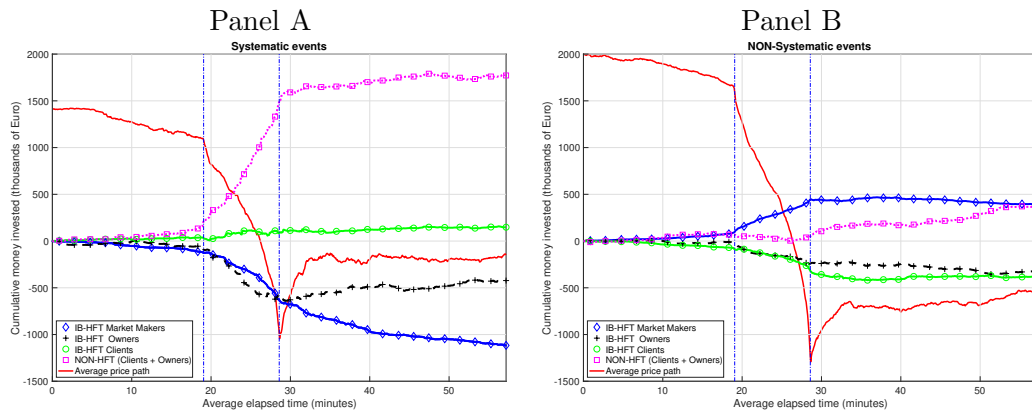
Note. The figure reports the average monetary net imbalance (negative corresponding to selling, positive corresponding to buying) of Investment Bank HFT Market Makers, the sum of IB-HFT CLIENT and IB-HFT OWN, and the sum of NON-HFT CLIENT and NON-HFT OWN. We superimpose the average cumulative return during the events for visual comparison. In Panel B, the average is applied to the 27 systematic events of April, 17 and September 3. In Panel A, the average is applied to the remaining 121 events. For NON-systematic events affecting single stocks individually, the behavior of HFT MM is roughly consistent with their designated role and with standard theory of market making, as e.g. in Grossman and Miller (1988), which implies buying as price declines and selling after price recovery. Later in the paper, we also provide some evidence of back-running (Yang and Zhu, 2019) for some traders in the IB-HFT group. For systematic events, IB-HFT market makers contribute to the selling originating the flash crash together with IB-HFT CLIENT and OWN. The role of liquidity providers for the systematic flash crashes is left to the NON-HFT traders. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.
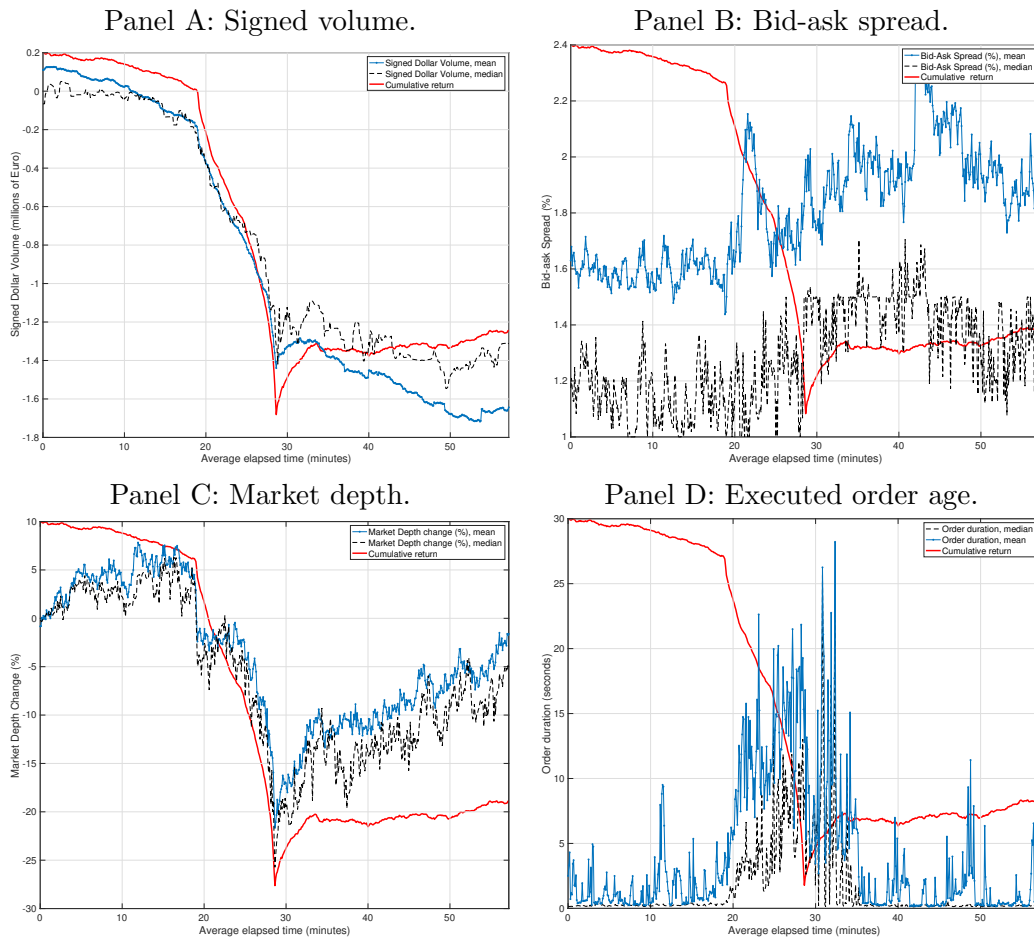
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Figure 9. Average trading imbalance changes per minute

Panel A: NON-systematic flash crashes.

Panel B: NON-systematic flash crashes.

Panel C: systematic flash crashes.

Panel D: systematic flash crashes.
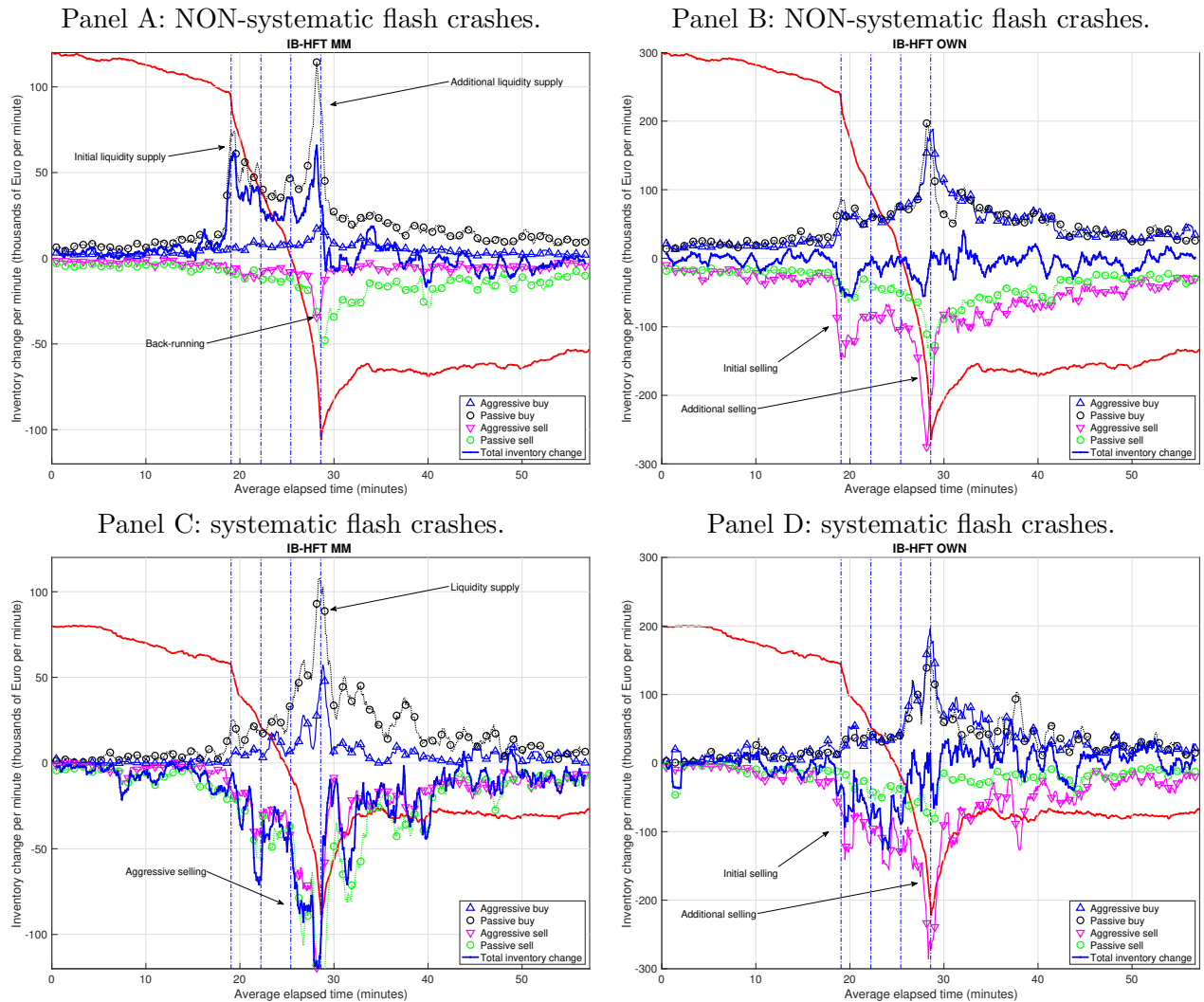
Note. The figure displays the trading imbalance change per minute for IB-HFT Market Makers and IB-HFT Owners. separated in buyer initiated, seller initiated, buyer passive and seller passive trades. Only non-systematic events are averaged. Panel A and B report averages for non-systematic flash crashes. Panel C and D report averages for systematic flash crashes. The y-units are not uniform across different trader categories. Back-running in Panel A refers to the theory of Yang and Zhu (2019). The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure 10. Monetary net imbalances changes per minute of systematic events

Column A: IB-HFT MM   Column B: IB-HFT OWN
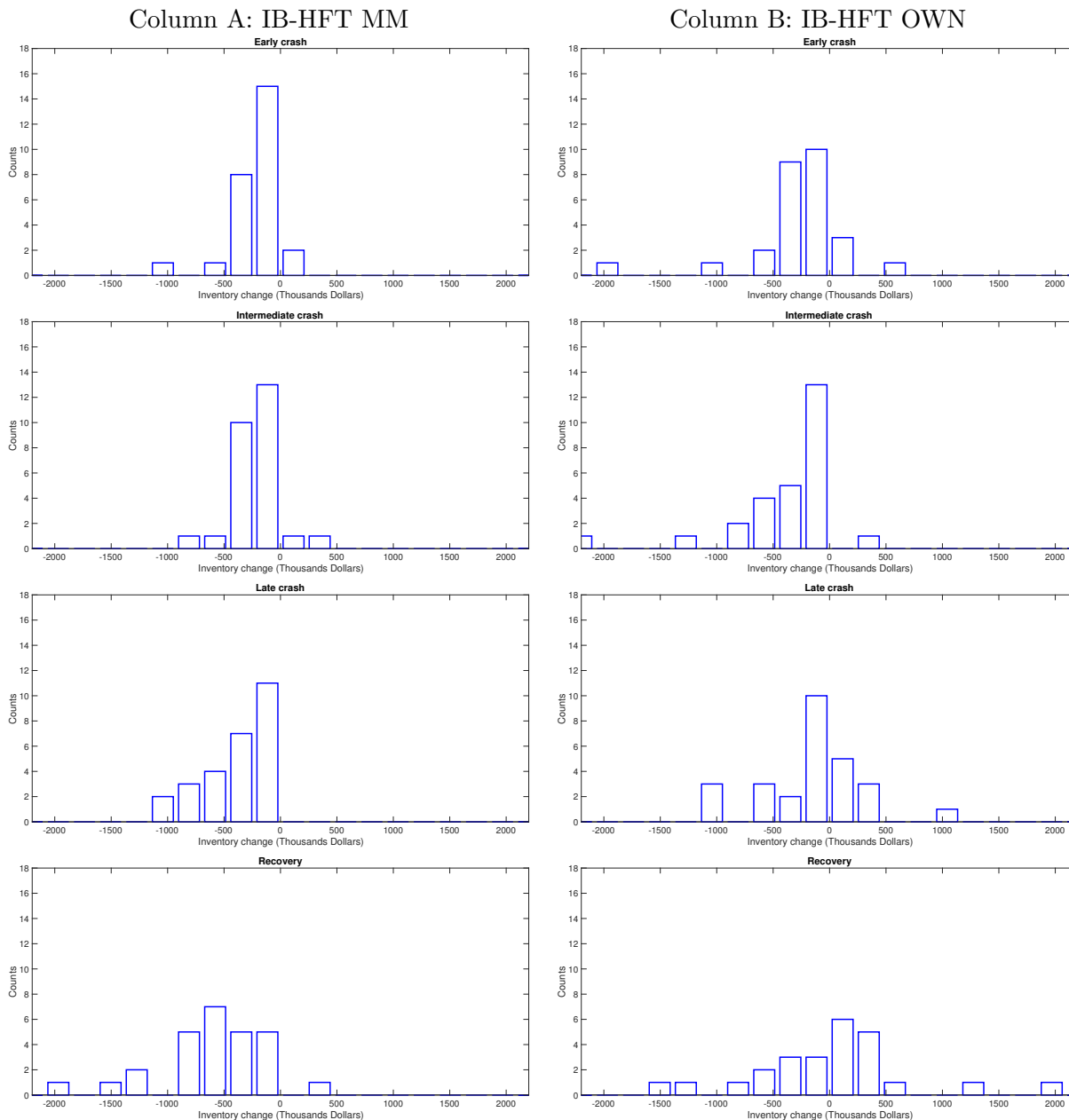
Note. The figure depicts the empirical distribution of monetary net imbalances changes for IB-HFT MM and IB-HFT OWN at different stages of the crash for systematic events. Column A is for IB-HFT MM. Column B is for IB-HFT OWN. We can see that IB-HFT MM sell more intensely as price declines, causing substantial over-reaction. In particular, during the late phase of the crash, they are always selling. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure 11. Cancellations and new orders volume by trader category (non-systematic)

Panel A: cancellations   Panel B: new orders
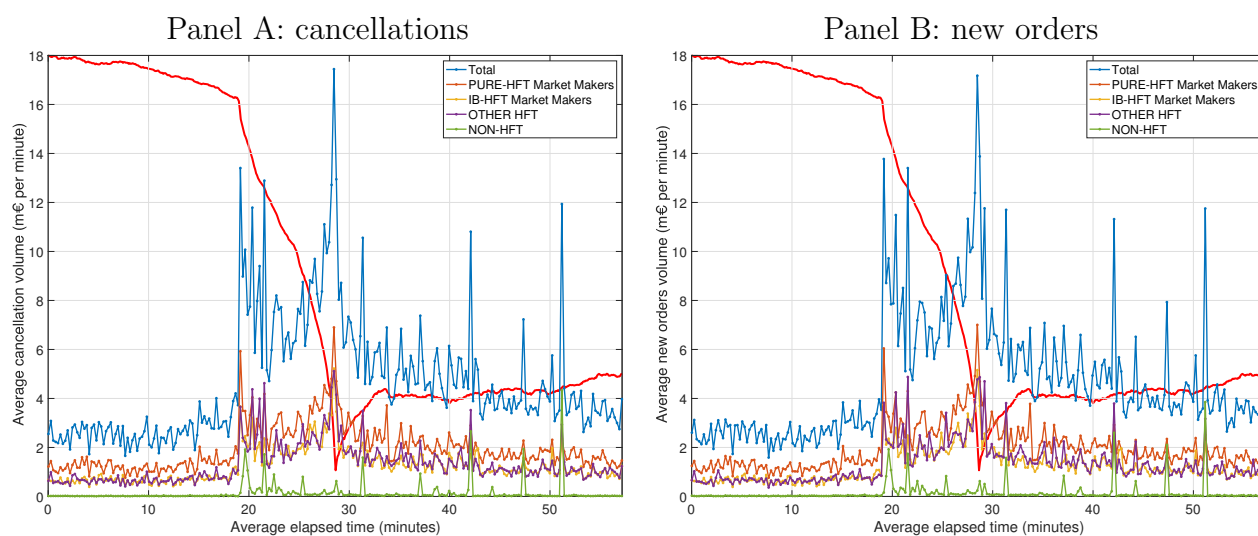
Note. This figure displays the average per minute euro volume of cancelled orders (Panel A) and new orders (Panel B). On each panel, we superimpose the average price evolution for visual comparison. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Figure 12. Average Estimated Impulse Response Functions

Column A: NON-systematic flash crashes   Column B: Systematic flash crashes.
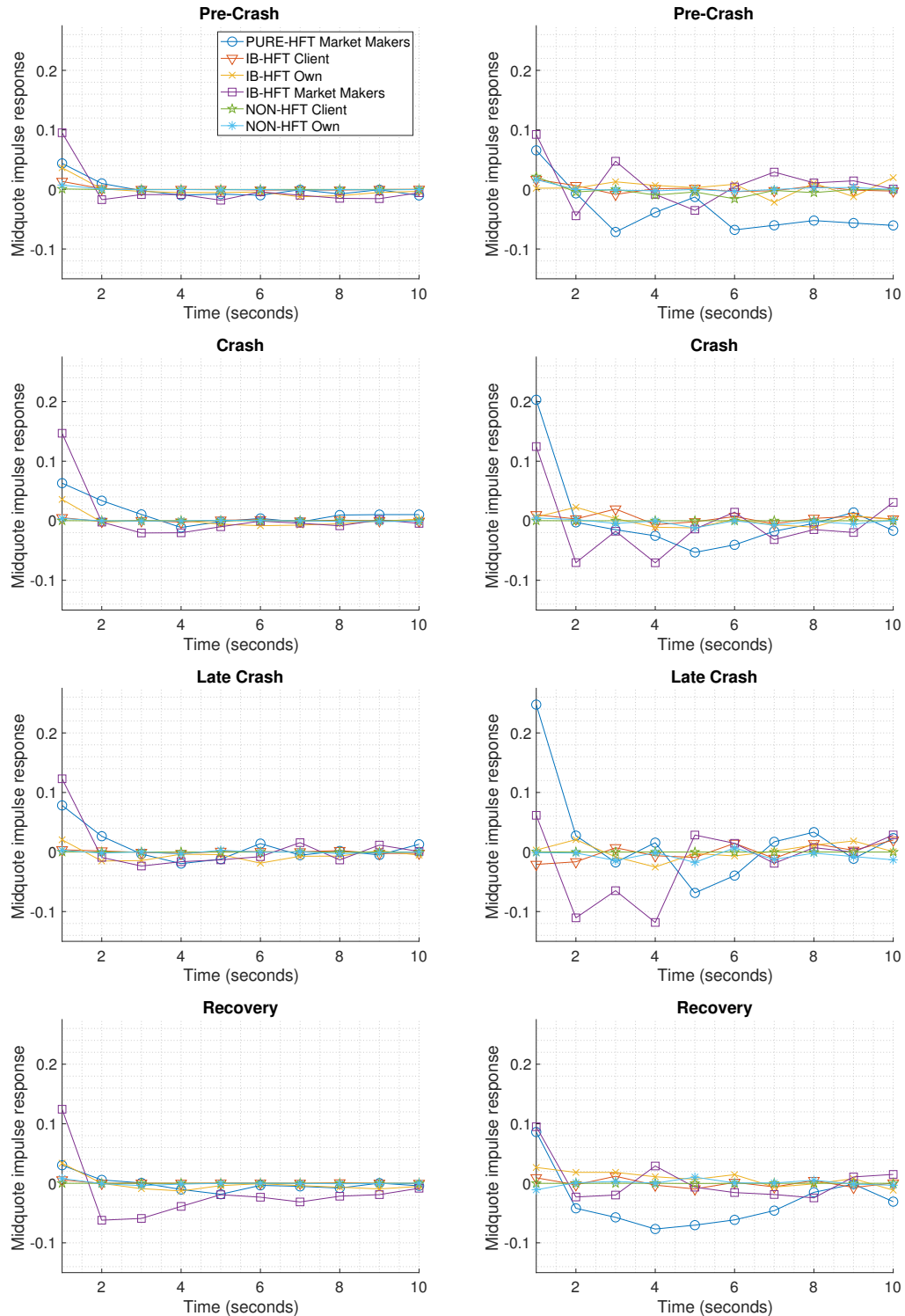
Note. The figure shows impulse response functions of the changes of mid-price on unit shocks in the changes of the average between best bid and best ask of different trader groups, for non-systematic (Column A) and systematic (Column B) events, in four different phases of the crash/recovery. Impulse response functions are estimated using the local projection methodology developed by Jordà (2005).

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A   Appendix: A comparison with the EPM method

In Brogaard et al. (2018), distress events are mainly identified using Extreme Price Movements (EPMs). EPMs are detected with one of the following methods. The first methods simply labels all 10-second intervals that belong to the 99.9th percentile of 10-second absolute midpoint returns for each stock as EPMs. The second method identifies EPMs based on the residuals from the return autoregression. We apply the same methodology to our data set.

Table A.1 compares the identified (negative) EPMs with the flash crashes detected using our methodology. On average, for each stock, both EPM detection methods label 0.05% of 10-seconds intervals as EPMs. However, only a little fraction of our flash crashes is identified as EPMs: the first method detects roughly 26.35% of flash crashes, while the second one – only 18.92%. The pure power of the EPM approach is rather natural, as the typical flash crash episodes are not exhausted by a few large price movements, but represent a "dense" series of different magnitude price changes leaning to the same direction. On the other hand, EPM approach exhibit a large number of false-positives due to oversampling periods of high volatility, as also admitted by the authors. Summarizing, we compute that the overlap between flash crashes and EPMs is limited, and thus that our distressed sample is different from that analyzed by Brogaard et al. (2018).

Figure A.1 is the same as Figure 4, now computed with events identified with EPMs. While there is a slight evidence of extreme price movement, on average, it is clear that EPMs are not capturing the V-shapes of a flash crash. The Figure makes clear that EPMs look more at "volatility"-induced price changes, while our methodology looks more at "drift"-induced price changes. This explains why the two samples identified by EPMs and the flash crash test of Christensen et al. (2017) are substantially different.

Figure A.1. **Average cumulative return dynamics around an Extreme Price Movement**.
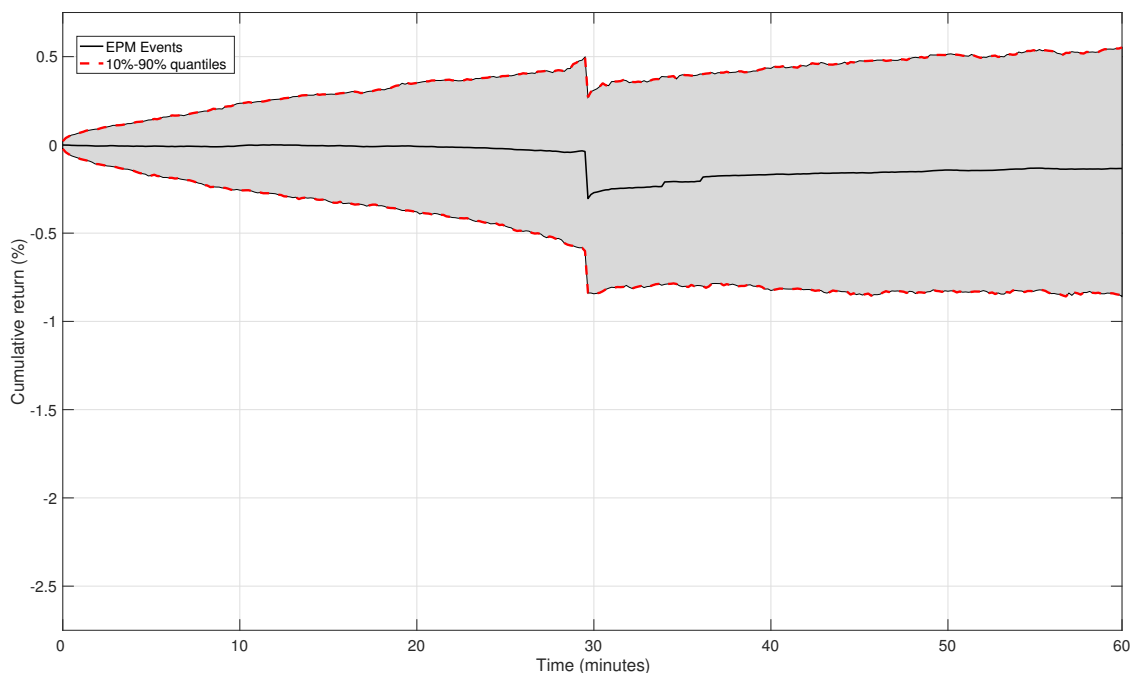
Note. This figure reports the average of the price evolution around extreme price movements (EMPs), as well as the 10-90% quantiles for all the events. The EPMs are detected detected by labelling the 10-second intervals that belong to the 99.9th percentile of 10-second absolute midpoint returns for each stock as EPMs.

Table A.1 Compares detected EPMs and flash crashes

	# of EPMs per stock	Detected flash crashes
First method	286.48 (0.05%)	39 (26.35%)
Second method	282.19 (0.05%)	28 (18.92%)

Note. This table reports in the first column the average (across stocks) number of EPMs detected in a single stock in absolute value and as a fraction (per cent) of the total number of the 10-second returns in our sample. The second column reports the number of our flash crashes episodes identified as EPMs according to the methodology presented by Brogaard et al. (2018). The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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B   Appendix: Additional figures

Figure B.1. Price evolution during systematic flash crashes.

Panel A: April, 17.   Panel B: September, 3.
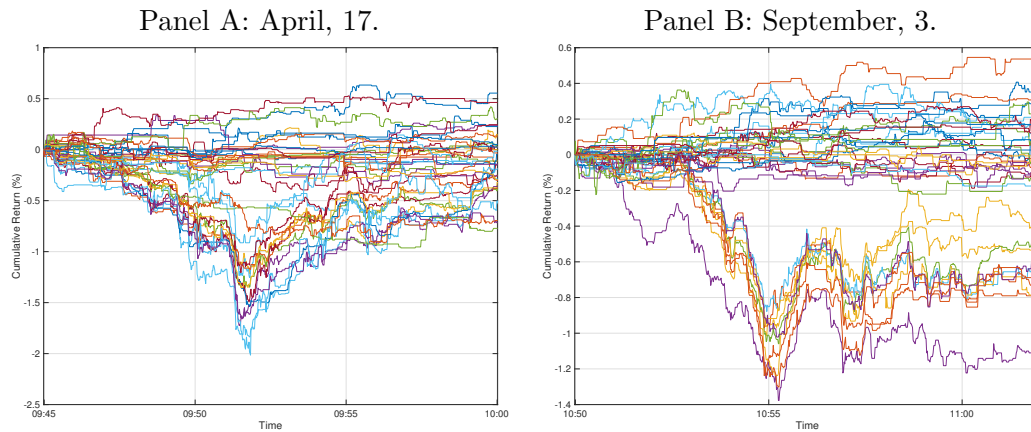
Note. This figure shows the price evolution of the 37 blue-chips French stocks during the systematic flash crashes of April 17, 2013 (Panel A) and September 3, 2013 (Panel B). 6 for systematic events. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure B.2. Liquidity measures during a flash crash (systematic).

Panel A: Signed volume.

Panel B: Bid-ask spread.

Panel C: Market depth.

Panel D: Executed order age.

Note. This figure reports four measures of liquidity averaged across the systematic flash crash events (compared to Figure 6 for non-systematic events). Panel A: cumulative signed monetary volume (negative volume = sell). Panel B: bid-ask spread. Panel C: market depth (difference from beginning). Panel D: the age of the executed orders (we exclude orders with age less than 0.1 seconds and those coming from the previous day). On each panel, we superimpose the average price evolution for visual comparison. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure B.3. Volume per minute for initiated buyer and initiated seller trades (systematic).

Panel A: buyer-initiated volume   Panel B: seller-initiated volume
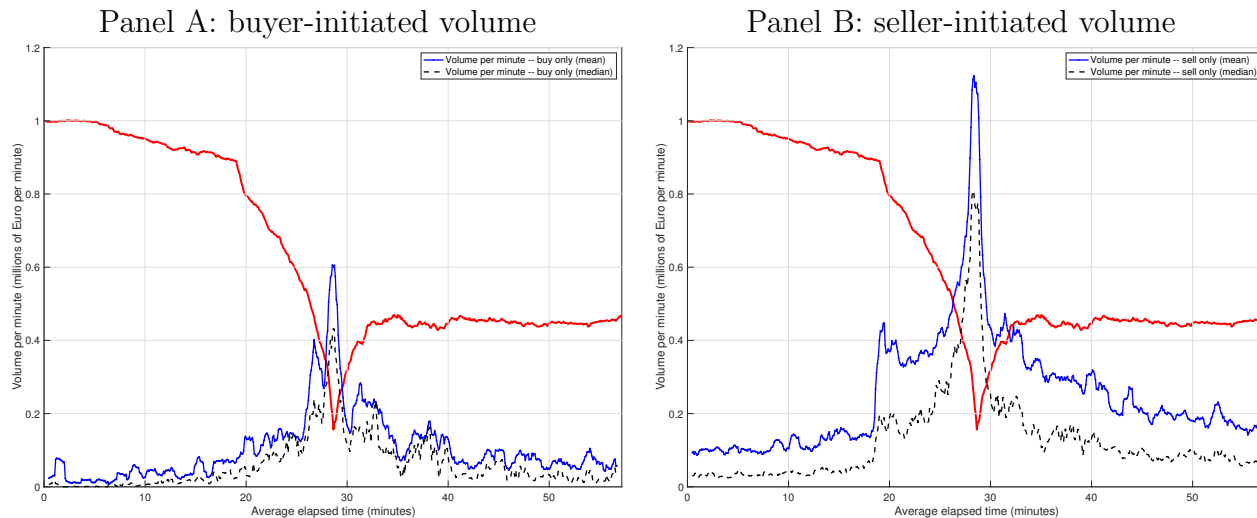
Note. This figure shows the average euro volume traded per minute during crash events, separated in buyer-initiated trades (Panel A) and seller-initiated trades (Panel B) only for systematic events (compared to Figure 7 for non-systematic events) The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Figure B.4. Cancellations and new orders volume by trader category (systematic).

Panel A: cancellations   Panel B: new orders
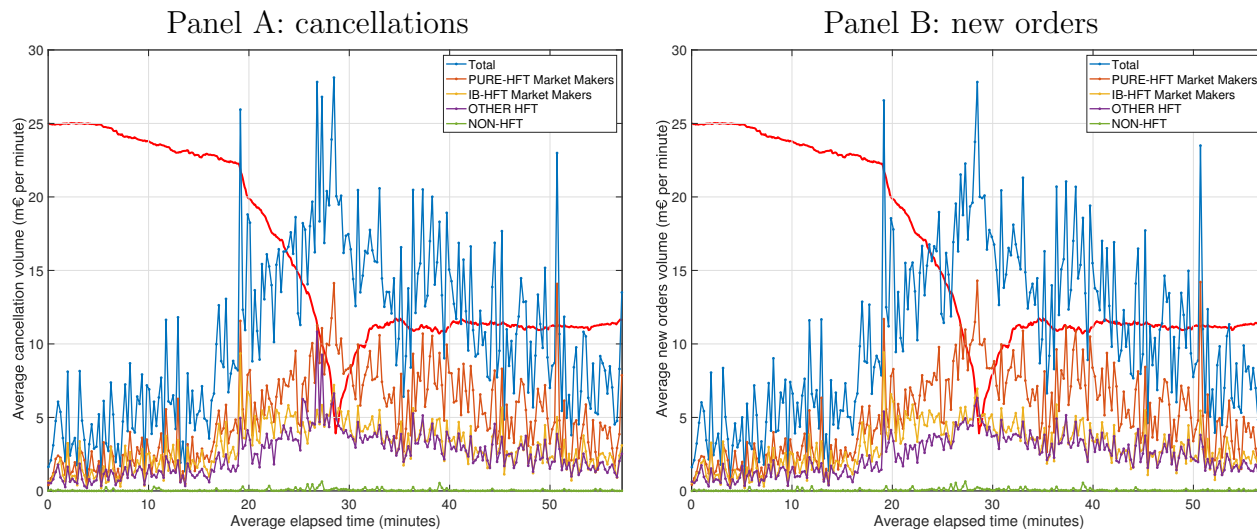
Note. This figure display the average per minute euro volume of cancelled orders (Panel A) and new orders (Panel B) only for systematic events. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure B.5. Cumulative net trading imbalances per minute of different trader groups (non-systematic).
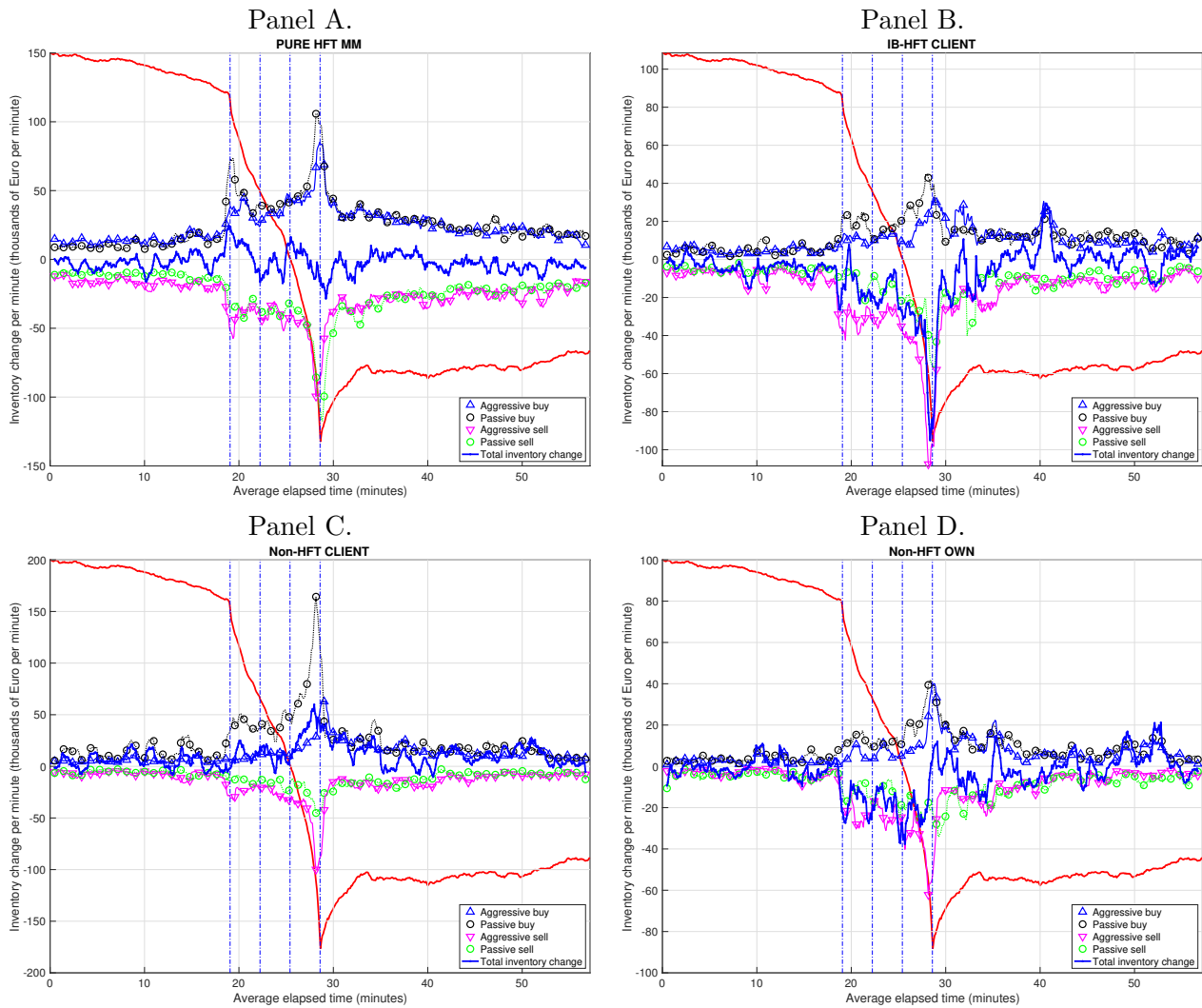
Note. This figure reports inventory change per minute for the most active trading categories not considered in Table 9 during non-systematic events, separated in buyer initiated, seller initiated, buyer passive and seller passive trades. The y-units are not uniform across panels. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Figure B.6. Cumulative net trading imbalances per minute of different trader groups (systematic).
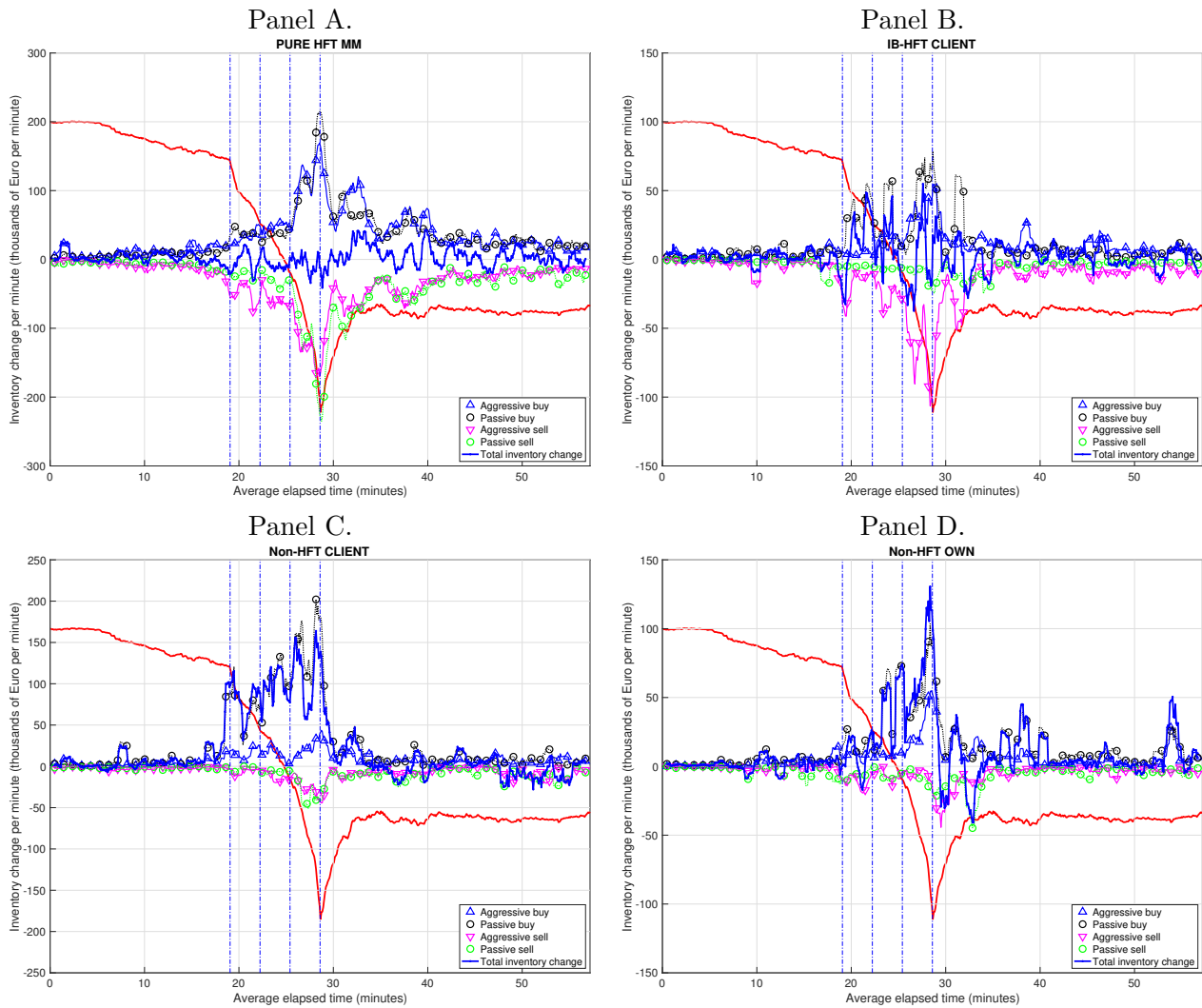
Panel A.

Panel B.

Panel C.

Panel D.

Note. This figure reports inventory change per minute for the most active trading categories not considered in Table 9 during systematic events, separated in buyer initiated, seller initiated, buyer passive and seller passive trades. The y-units are not uniform across panels. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.
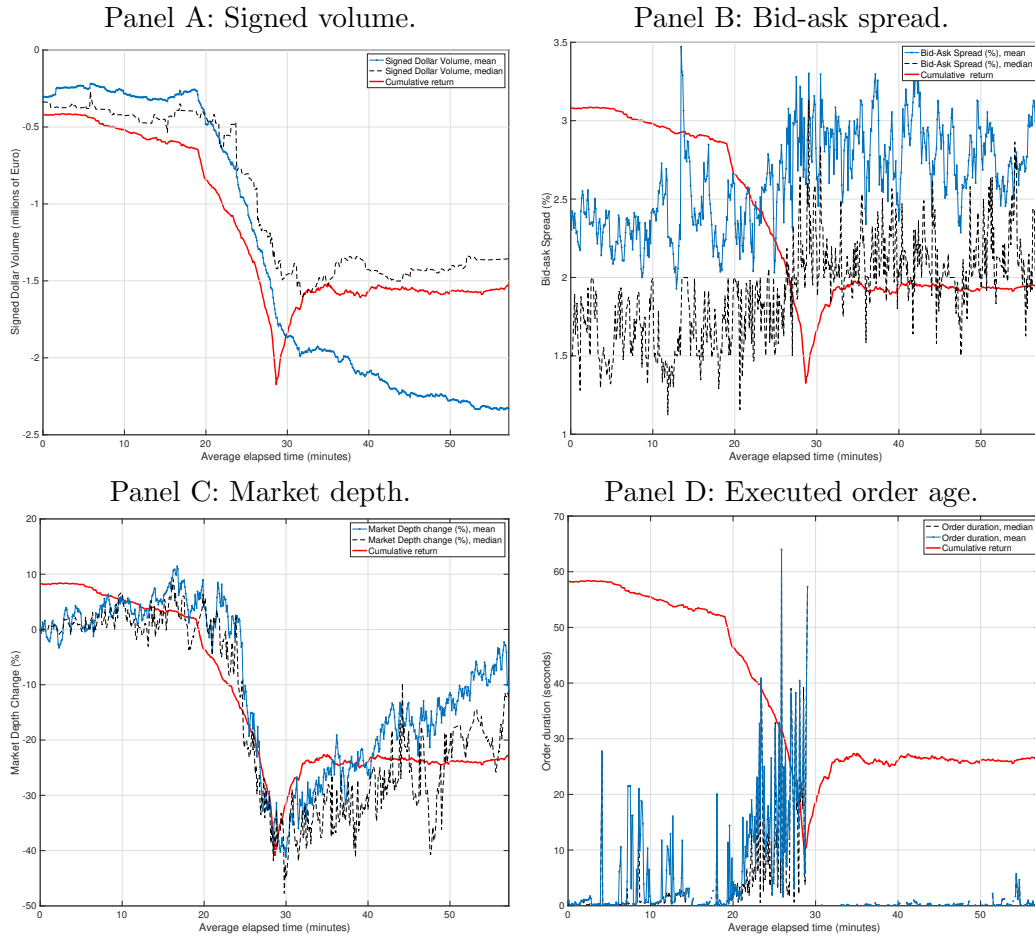
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Figure B.7. Monetary net imbalances per minute of non-systematic events.

Column A: IB-HFT MM   Column B: IB-HFT OWN
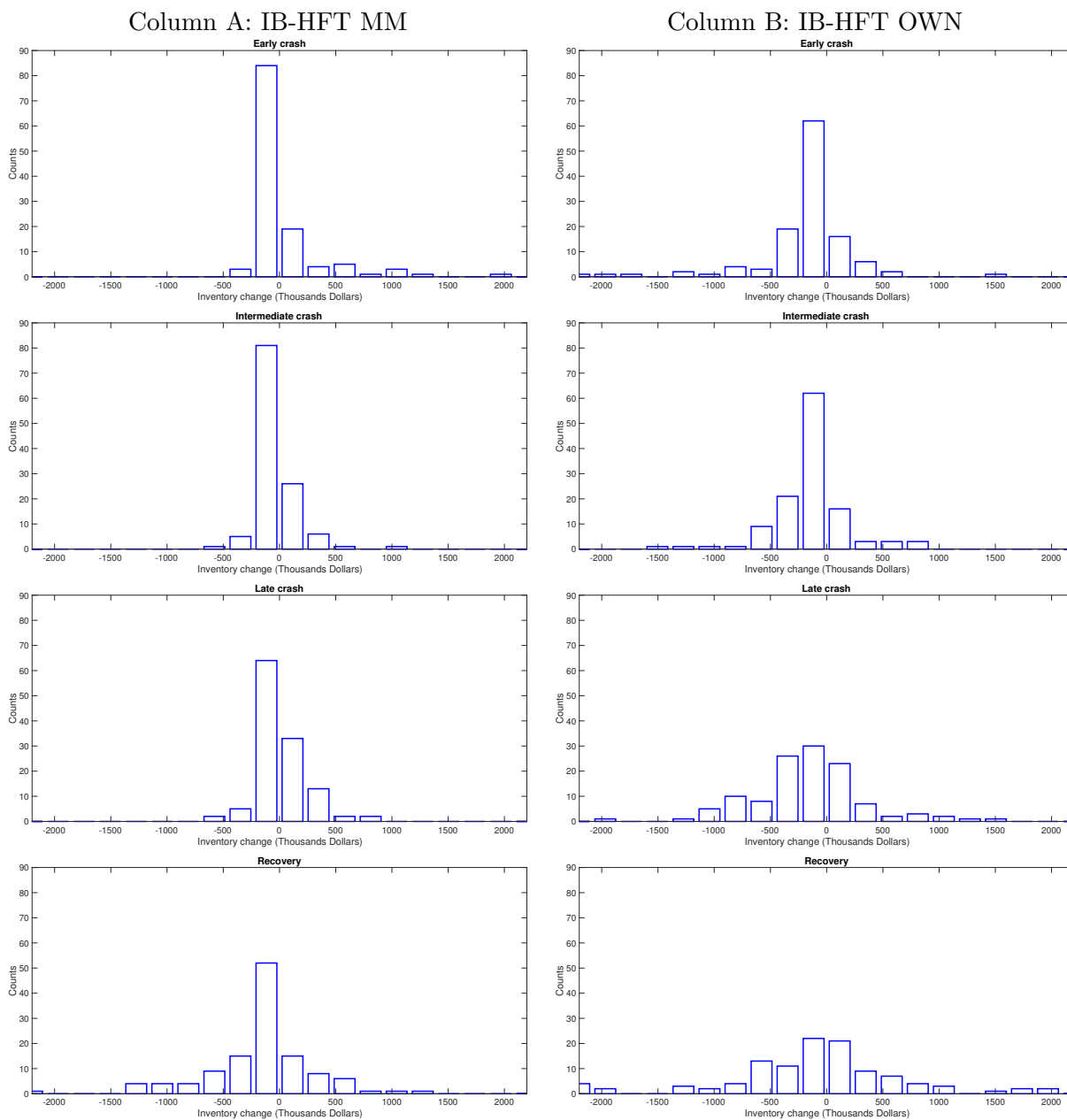
Note. Same as Figure 10 for non-systematic events.

C   Appendix: A statistical assessment

This section contains a statistical assessment of the trading behavior of different categories during a flash crash. Having 148 different crash events allows us to assess whether what we observe is statistically significant, i.e. that the observed trading behavior are precise trading strategies which are constantly implemented, and not the artifact of statistical fluctuations.

Table C.1 reports the cross-sectional average and standard deviation of net trade imbalances $\mathcal{I}_{\text{period}}^{(j)}$ in the five different periods of a flash crash, for the trading categories for which this number is non-negligible and divided into systematic and non-systematic flash crashes. The numbers are expressed in thousands of euros, and reported with their standard deviation below. We only report significantly active categories.

Substantial heterogeneity emerges in the behavior of trader groups, showing the statistical significance of the results illustrated in Figure 8. PURE-HFTs do not accumulate any significant inventory during the crash, which is consistent with their trading mandate. The only exception is the liquidity they provide to recovery for non-systematic events. However, they do not stop trading, as concluded in Kirilenko et al. (2017) by looking at the Flash Crash of May 6, 2010 only. This result also complements the findings in Bellia (2017), who instead shows that in "normal" market periods, PURE-HFT MM activity improves market liquidity significantly. This conclusion does not hold anymore in distressed times.

Regarding the remaining HFT categories, IB-HFTs, we observe more heterogeneity. The imbalance of IB-HFT OWN is largely negative across the whole crash, consistent with their role of big sellers, and selling is particularly massive and prolonged during non-systematic events. IB-HFT CLIENT are also net sellers, but they follow IB-HFT OWN and mostly sell in the late crash for non-systematic events, while their net inventory is non-significant during systematic events. This shows that informed trading seems to originate from the IB-HFT OWN, and then propagates to the IB-HFT CLIENT. The attempt of clients to profit from the information coming from owners generates the big part of the second wave of selling which, in a liquidity deprived market, contributes to the transient price impact crash.

Regarding IB-HFT MM, the table shows that they do provide significant liquidity during the price drop and (mildly) during price recovery, but only for non-systematic events. However, the liquidity they provide is not sufficient to prevent the transitory price impact. For systematic events, they are significant big sellers along all stages. They also increase

their net inventory as the IB-HFT OWN decrease their selling pressure. In the late phase of the crash and during recovery, they are the largest sellers. This is the main result of the paper and is shown here to be strongly statistically significant. IB-HFT Parents are also particularly active in becoming big sellers in the late phase of the crash. Analyzing the flash crash of May 6, 2010, Menkveld and Yueshen (2019) argue that it has been caused by large investors. However, they do not provide information on whether large investors employ HFTs or NON-HFTs for placing their orders, whether they split their orders or not, the reason why large investors sell, and whether large investors are the investment banks or the clients on their accounts, nor they clarify the role of market makers in providing liquidity during the crash. Our analysis supplies this information.

Importantly, Table C.1 also shows that NON-HFTs play an important role in reducing the negative effects of the crash and in helping the price recovery afterwards. The trade imbalance of NON-HFT CLIENT is positive and highly significant during both price drop and recovery, and especially for systematic events, where they play the role of liquidity providers in place of the market makers. Their net inventory becomes increasingly positive as the crash develops and then price recovers. NON-HFT OWN instead contribute mildly to the non-systematic crashes, while they also become significant net liquidity providers in the late phase of systematic crash.

Now, in order to separate liquidity provision from aggressive trading, we analyze the monetary imbalances, invested by a trader who is initiating a trade separately from the money invested when providing liquidity ($\mathcal{I}_{\text{period}}^{(i),\text{init}}$ and $\mathcal{I}_{\text{period}}^{(j),\text{liq}}$, respectively). The aim of this analysis is to investigate whether the net imbalance is due to a low activity (i.e. low initiated trading and low liquidity provision) or to a significant large activity (i.e. high initiated trading and high liquidity provision). To add information with respect to Table C.1, we now report net imbalance per minute, to measure the intensity of accumulated inventory. This also allows to compare fairly among periods with different duration.

Table C.2 reports average amounts of money (per minute) invested in initiating trades ($\mathcal{I}_{\text{period}}^{(i),\text{init}}$) by different trader categories at different stages of a flash crash, again divided for systematic and non-systematic crashes. The role of IB-HFT OWN and CLIENT as aggressive sellers is confirmed. In particular, owners start the process and remain aggressive across the whole crash, while clients become increasingly aggressive. In the late crash, we

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also see significant selling by NON-HFT CLIENT (for non-systematic events only); NON-HFT OWN for non-systematic events, while they are aggressively buying during the late phase of systematic crash; IB-HFT CLIENT; and IB-HFT MM, especially for systematic events where they are the largest aggressive sellers.

Table C.3 complements Table C.2 by showing the euro volume of passive trades per minute, which is the flip side of the aggressive order. NON-HFT Clients are the big liquidity suppliers, especially in the late crash phase and especially for systematic crashes. The fact that they provide liquidity is always significant. The second-best liquidity supplier during the late phase of systematic crashes are NON-HFT OWN. IB-HFT MM provide significant liquidity only for non-systematic events. They are the first liquidity providers in the initial phase of the crash, which is in line with their contractual role. As the crash develops, their relative role as liquidity providers declines, and in the late phase of non-systematic crash they are even surpassed by NON-HFT CLIENT. As said, they do not provide any liquidity through limit orders during systematic crashes. PURE-HFT MM are mildly significant as liquidity providers, but the net amount is small compared to other trader groups. For example, IB-HFT OWN also provide a large amount of liquidity. IB-HFT MM do provide significant liquidity at the beginning of the crash. Overall, Table C.3 confirms and complements the previous finding on categorized traders activity during flash crashes.

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Table C.1 Average trade imbalances (k€).

Panel A: Systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	−1.11	0.00	6.17	10.90	13.12
	(1.11)	(0.00)	(5.80)	(10.90)	(8.83)
PURE-HFT MM	44.55	−4.67	−26.43	−17.27	137.12
	(31.31)	(31.34)	(28.10)	(34.33)	(84.56)
PURE-HFT OWN	−20.02	3.34	18.98	4.46	−3.83
	(14.97)	(2.57)	(12.33)	(9.26)	(7.36)
IB-HFT CLIENT	28.58	52.73	27.81	−24.47	63.41
	(32.60)	(72.55)	(46.09)	(72.98)	(63.21)
IB-HFT MM	−127.32***	−105.36**	−117.98***	−274.55***	−491.03***
	(34.88)	(44.97)	(39.24)	(49.16)	(94.87)
IB-HFT OWN	−94.98	−168.84*	−265.59***	−59.06	165.76
	(67.02)	(88.80)	(90.44)	(85.93)	(206.97)
IB-HFT PARENT	−53.57**	−1.52	−86.08***	−224.09***	−197.01***
	(23.23)	(19.36)	(31.48)	(60.60)	(56.69)
NON-HFT CLIENT	195.74**	226.00***	308.02***	372.75***	74.32
	(91.39)	(74.38)	(72.49)	(83.01)	(85.02)
NON-HFT OWN	28.09	27.46	134.87*	209.58***	194.80**
	(39.45)	(35.85)	(74.83)	(65.23)	(97.68)

Panel B: NON-systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	5.93	−4.80	15.44**	4.38	−1.44
	(8.49)	(8.14)	(7.20)	(11.07)	(19.64)
PURE-HFT MM	−37.96	27.01	−12.64	−0.80	−108.51***
	(23.72)	(17.03)	(13.15)	(20.07)	(29.86)
PURE-HFT OWN	−28.41	2.55	5.27	4.91	27.89
	(30.68)	(5.31)	(4.20)	(4.62)	(18.84)
IB-HFT CLIENT	−84.77**	−39.52*	−58.51***	−137.66***	−57.33
	(35.12)	(22.36)	(20.48)	(36.75)	(88.78)
IB-HFT MM	93.29***	143.54***	85.04***	127.89***	−55.14
	(24.16)	(29.56)	(16.29)	(20.01)	(42.87)
IB-HFT OWN	−16.27	−117.70**	−31.21	−73.52	−86.56
	(53.84)	(49.69)	(30.65)	(52.83)	(132.57)
IB-HFT PARENT	3.79	13.85	19.17**	42.87***	−20.73
	(14.37)	(9.29)	(9.35)	(10.67)	(52.54)
NON-HFT CLIENT	87.81*	38.79**	31.60*	107.58***	309.31***
	(50.25)	(19.62)	(19.00)	(35.08)	(104.48)
NON-HFT OWN	−23.45	−63.92**	−54.14**	−73.83***	−1.02
	(25.18)	(28.09)	(27.34)	(28.41)	(58.19)

Note. This table reports the cross-sectional average, and standard errors in brackets, of the trade imbalances measured in k€, in the five different periods of the crash described in Figure 4. Panel A: reports averages on the 27 flash crashes which happen in several stocks on April 17 and September 3, 2013. Panel B: reports averages on all the other flash crashes.The significance of the mean is evaluated with a standard *t*-test. One star denotes 90% significance, two stars 95% significance and three stars 99% significance. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Table C.2 Average money invested in initiated trades (market orders) per minute (k€per minute).

Panel A: Systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	0.00	0.00	1.15	0.00	1.88
	(0.00)	(0.00)	(1.83)	(0.00)	(1.43)
PURE-HFT MM	8.80	−82.56**	−52.58	−49.17	31.46**
	(7.25)	(41.02)	(36.36)	(49.26)	(14.66)
PURE-HFT OWN	−3.71	5.45	12.65	8.01	−0.28
	(3.38)	(3.75)	(8.40)	(12.53)	(1.13)
IB-HFT CLIENT	−0.38	−62.09**	−57.53**	−177.58***	−5.77
	(6.56)	(28.53)	(24.10)	(63.58)	(7.40)
IB-HFT MM	−12.70**	−88.20**	−92.82***	−289.78***	−49.32***
	(5.08)	(37.27)	(33.81)	(50.58)	(10.72)
IB-HFT OWN	−16.68	−289.84**	−360.62***	−257.31***	−55.52**
	(14.79)	(128.60)	(119.34)	(97.71)	(26.85)
IB-HFT PARENT	−10.60**	−5.19	−122.89***	−251.31***	−21.82***
	(5.12)	(21.57)	(41.58)	(68.09)	(7.66)
NON-HFT CLIENT	6.57	45.53*	11.51	−10.16	−0.39
	(5.11)	(24.59)	(25.29)	(42.15)	(8.75)
NON-HFT OWN	4.42	−12.73	8.97	103.23***	2.80
	(3.46)	(38.32)	(23.04)	(38.24)	(7.44)

Panel B: NON-systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	−0.55	−10.96	4.49	−11.72	−0.42
	(1.30)	(10.17)	(3.15)	(8.72)	(3.15)
PURE-HFT MM	−11.47***	−26.61	−23.30	−59.86**	−8.27*
	(3.67)	(17.63)	(20.38)	(25.27)	(4.33)
PURE-HFT OWN	−2.89	−4.13	−3.42	−3.42	1.36
	(2.99)	(6.56)	(4.35)	(4.25)	(2.65)
IB-HFT CLIENT	−19.49***	−96.54***	−88.27***	−220.47***	−9.94
	(6.08)	(27.41)	(19.79)	(34.26)	(7.82)
IB-HFT MM	2.46	−12.27	1.80	−24.59*	−2.78
	(3.05)	(10.08)	(10.92)	(14.67)	(4.22)
IB-HFT OWN	−28.47***	−270.30***	−122.30***	−297.83***	−33.91**
	(10.20)	(63.15)	(37.85)	(67.28)	(14.64)
IB-HFT PARENT	−0.30	8.57	11.09	26.93**	−1.87
	(2.35)	(7.61)	(7.48)	(10.52)	(6.52)
NON-HFT CLIENT	−10.80***	−78.65***	−62.80***	−157.72***	17.75**
	(3.39)	(14.17)	(14.04)	(31.14)	(8.79)
NON-HFT OWN	−4.23	−87.91***	−77.53***	−131.56***	0.67
	(2.67)	(24.69)	(28.18)	(28.69)	(4.11)

Note. This table reports the cross-sectional average, and standard errors in brackets, of the money invested in initiating trades through market orders measured in k€, in the five different periods of the crash described in Figure 4. Panel A: reports averages on the 27 flash crashes which happen in several stocks on April 17 and September 3, 2013. Panel B: reports averages on all the other flash crashes. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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Table C.3 Average money invested in liquidity supplying trades (limit orders) per minute (k€per minute).

Panel A: Systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	−0.27	0.00	7.88	15.94	0.25
	(0.27)	(0.00)	(8.30)	(15.94)	(0.17)
PURE-HFT MM	2.07	75.72***	13.89	23.90	−9.16***
	(2.99)	(21.55)	(16.15)	(27.36)	(3.54)
PURE-HFT OWN	−1.17	−0.55	15.13	−1.49	−0.35
	(1.03)	(1.30)	(14.33)	(2.62)	(0.39)
IB-HFT CLIENT	7.35	139.26	98.22	141.77**	16.08
	(5.96)	(100.16)	(68.75)	(70.91)	(11.06)
IB-HFT MM	−18.35**	−66.00*	−79.84**	−112.01***	−30.52***
	(7.44)	(35.46)	(35.82)	(42.01)	(10.87)
IB-HFT OWN	−6.48	42.75	−28.04	170.89**	82.48**
	(15.51)	(48.06)	(55.44)	(84.86)	(33.57)
IB-HFT PARENT	−2.47	2.96	−3.07	−76.63**	−10.21***
	(1.52)	(10.22)	(10.90)	(31.17)	(3.02)
NON-HFT CLIENT	41.17**	285.20***	439.25***	555.66***	12.48
	(20.93)	(103.10)	(107.23)	(135.89)	(10.96)
NON-HFT OWN	2.43	52.91**	188.40*	203.47***	28.87**
	(8.21)	(23.33)	(103.49)	(75.10)	(12.41)

Panel B: NON-systematic flash crashes.

	pre-crash	early crash	intermediate crash	late crash	recovery
PURE-HFT CLIENT	1.99	3.95	18.11*	18.12	0.19
	(1.47)	(3.94)	(9.24)	(12.00)	(0.58)
PURE-HFT MM	2.22	66.13***	4.80	58.69***	−9.37***
	(3.73)	(16.80)	(10.82)	(17.41)	(3.25)
PURE-HFT OWN	−4.04	7.86***	11.13**	10.60**	3.17**
	(4.64)	(2.98)	(4.63)	(4.72)	(1.61)
IB-HFT CLIENT	−1.19	38.70	2.64	19.01	0.62
	(4.73)	(24.48)	(17.40)	(32.84)	(9.58)
IB-HFT MM	20.29***	222.33***	122.65***	211.75***	−6.18
	(4.65)	(40.89)	(19.48)	(24.99)	(6.18)
IB-HFT OWN	24.50**	98.06**	76.62*	190.24***	19.83
	(11.14)	(39.77)	(41.77)	(48.79)	(20.26)
IB-HFT PARENT	1.22	11.70	16.96**	35.80***	−1.50
	(1.62)	(8.06)	(7.84)	(7.95)	(3.03)
NON-HFT CLIENT	32.22***	135.42***	109.04***	315.15***	32.55***
	(11.03)	(26.45)	(29.04)	(49.38)	(10.10)
NON-HFT OWN	−1.49	−5.62	−1.70	23.52	−0.83
	(4.54)	(23.25)	(16.95)	(26.28)	(7.08)

Note. This table reports the cross-sectional average, and standard errors in brackets, of the money invested in liquidity supplying trades through limit orders measured in k€, in the five different periods of the crash described in Figure 4. Panel A: reports averages on the 27 flash crashes which happen in several stocks on April 17 and September 3, 2013. Panel B reports averages on all the other flash crashes. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

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D   Appendix: Change of trading behavior

This section addresses the question of whether HFTs change their trading behavior during flash crashes by estimating the Kirilenko et al. (2017) model for inventory changes of different trader groups. Kirilenko et al. (2017) answered in the negative by using data from the flash crash of May 6, 2010 only. We instead answer in the positive, and this is made possible by the fact that we analyze 148 crashes instead of a single one.

The model reads:

$$
\begin{aligned}
\Delta y_t = &\alpha + \phi \Delta y_{t-1} + \delta y_{t-1} + \sum_{i=0}^{3} \beta_i (\Delta \log p)_{t-i} \\
&+ D^D \left(\alpha^D + \phi^D \Delta y_{t-1} + \delta^D y_{t-1} + \sum_{i=0}^{3} \beta_i^D (\Delta \log p)_{t-i} \right) \\
&+ D^U \left(\alpha^U + \phi^U \Delta y_{t-1} + \delta^U y_{t-1} + \sum_{i=0}^{3} \beta_i^U (\Delta \log p)_{t-i} \right) + \varepsilon_t,
\end{aligned}
\tag{7}
$$

where D^D is a dummy which is activated during the crash period (from t_{start} to t_{crash}, according to our definitions), D^U is a dummy which is activated during the recovery period (from t_{crash} to t_{end}), y_t is the net inventory of each category (measured in million euros and starting at the beginning of each day) at time t, and Δy_t is the change in inventory over the period from $t-1$ to t; $(\Delta \log p)_t$ is the logarithmic return of mid-prices from $t-1$ to t.[10] We estimate model (7) for each day in which one of the 148 crash events occurs, and for each trading category separately. The inventories and prices are recorded on grids of 10 seconds. Significance of the variables associated with the crash (recovery period) would imply a different trading behavior during that phase. Average coefficients, together with their associated t-statistic (robustified to account for heteroskedasticity using the White correction) are reported in Table D.1 for non-systematic flash crashes, and Table D.2 for systematic flash crashes.[11]

The first thing we immediately notice is that, in the crash and recovery periods, the

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[10]The model here is slightly modified with respect to Kirilenko et al. (2017). Specifically, we use 3 ten-seconds lags (instead of 20 one-second lags) and we regress on logarithmic returns instead of price changes, since we consider different stocks and days and we need to guarantee uniformity across different price levels.

[11]To compute the t-statistic on average coefficients, we assume that coefficient estimates are independent across different events.

Table D.1 Non-systematic flash crashes

	PURE-HFT MM	PURE-HFT OWN	IB-HFT MM	IB-HFT OWN	IB-HFT CLIENT	NONHFT CLIENT	NONHFT OWN
constant	-0.000***	0.000	0.000*	0.000	-0.000***	0.000	0.000
	(-4.62)	(0.73)	(1.74)	(1.09)	(-4.85)	(1.22)	(1.34)
Δy_{t-1}	0.004	0.029***	0.065***	0.091***	0.099***	0.081***	0.080***
	(1.15)	(3.73)	(15.01)	(19.59)	(13.81)	(14.70)	(8.35)
y_{t-1}	-0.002***	-0.001	0.000	0.000	-0.000	-0.001***	-0.000
	(-13.82)	(-1.37)	(0.91)	(1.12)	(-0.77)	(-7.44)	(-1.55)
Δp_t	0.181***	0.002	-0.227***	-0.064**	0.050	-0.107***	0.061***
	(11.34)	(0.13)	(-14.12)	(-2.37)	(1.60)	(-3.99)	(2.61)
Δp_{t-1}	-0.086***	-0.020**	0.030***	0.097***	0.018	-0.054***	0.020
	(-9.13)	(-2.22)	(3.82)	(5.24)	(1.39)	(-2.88)	(1.20)
Δp_{t-2}	-0.041***	-0.008	-0.011*	0.003	0.039***	-0.004	0.004
	(-4.79)	(-1.06)	(-1.75)	(0.13)	(3.81)	(-0.20)	(0.28)
Δp_{t-3}	-0.050***	-0.003	-0.011*	0.045***	0.025**	-0.012	-0.006
	(-5.84)	(-0.49)	(-1.74)	(2.93)	(2.34)	(-0.73)	(-0.30)
D^D*constant	-0.000	0.000	0.001***	0.001	0.003	0.006***	-0.000
	(-0.26)	(0.27)	(4.47)	(0.81)	(0.86)	(2.90)	(-0.21)
$D^D \Delta y_{t-1}$	-0.029*	-0.063**	-0.076***	-0.051***	-0.040*	-0.061***	-0.032
	(-1.87)	(-2.08)	(-4.37)	(-3.04)	(-1.85)	(-3.85)	(-1.37)
$D^D y_{t-1}$	-0.092***	-0.047***	-0.040***	-0.061***	-0.035***	-0.027***	-0.036***
	(-9.86)	(-3.22)	(-7.38)	(-7.68)	(-4.35)	(-3.73)	(-4.88)
$D^D \Delta p_t$	0.045	0.012	-0.676***	1.034***	0.076	-0.834***	0.205*
	(0.47)	(0.14)	(-7.47)	(7.04)	(0.63)	(-4.45)	(1.91)
$D^D \Delta p_{t-1}$	-0.063	0.068	-0.088	-0.282**	0.148	0.001	0.307***
	(-0.91)	(1.40)	(-1.40)	(-2.36)	(1.48)	(0.00)	(2.70)
$D^D \Delta p_{t-2}$	0.014	0.059	0.040	0.085	0.095	0.030	-0.084
	(0.25)	(1.49)	(0.79)	(0.80)	(1.06)	(0.28)	(-1.07)
$D^D \Delta p_{t-3}$	0.054	0.025	-0.113**	-0.164*	0.046	-0.045	0.127*
	(0.91)	(0.60)	(-2.20)	(-1.74)	(0.59)	(-0.48)	(1.73)
D^U*constant	-0.001*	-0.001*	0.003***	0.001	-0.003**	0.003***	-0.001***
	(-1.78)	(-1.94)	(6.70)	(0.82)	(-2.35)	(2.96)	(-3.68)
$D^U \Delta y_{t-1}$	-0.019	0.042*	-0.043***	-0.037***	-0.013	-0.022	-0.029
	(-1.60)	(1.66)	(-3.05)	(-2.83)	(-0.82)	(-1.59)	(-1.27)
$D^U y_{t-1}$	-0.052***	-0.071***	-0.054***	-0.047***	-0.033***	-0.036***	-0.037***
	(-12.37)	(-5.59)	(-9.18)	(-11.21)	(-9.89)	(-8.67)	(-6.37)
$D^U \Delta p_t$	-0.303***	0.226**	-0.222***	0.167*	-0.107	0.424***	-0.101
	(-4.99)	(2.10)	(-4.30)	(1.84)	(-1.40)	(3.67)	(-1.45)
$D^U \Delta p_{t-1}$	0.090**	0.023	-0.058*	-0.129*	-0.017	-0.024	0.006
	(2.31)	(0.24)	(-1.74)	(-1.85)	(-0.26)	(-0.37)	(0.12)
$D^U \Delta p_{t-2}$	0.010	0.205*	0.023	-0.061	0.123**	-0.052	-0.071
	(0.31)	(1.89)	(0.81)	(-1.10)	(2.14)	(-0.89)	(-1.45)
$D^U \Delta p_{t-3}$	0.086**	0.167	-0.017	0.015	-0.069	-0.004	0.027
	(2.02)	(1.29)	(-0.52)	(0.25)	(-1.32)	(-0.05)	(0.50)

Note. The significance of the mean is evaluated with a standard *t*-test, adjusted for heteroskedasticity. One star denotes 90% significance, two stars 95% significance and three stars 99% significance. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Table D.2 Impulse-response function on logarithmic returns during crash and recovery.

	PURE-HFT MM	PURE-HFT OWN	IB-HFT MM	IB-HFT OWN	IB-HFT CLIENT	NONHFT CLIENT	NONHFT OWN
constant	−0.000***	−0.000	0.000	0.000**	0.000	0.000	−0.000
	(−4.11)	(−1.11)	(0.46)	(2.17)	(1.51)	(1.18)	(−0.13)
Δy_{t-1}	−0.006	0.011	0.060***	0.083***	0.037*	0.058***	0.097***
	(−0.73)	(1.21)	(5.60)	(6.23)	(1.93)	(5.98)	(4.29)
y_{t-1}	−0.005***	0.000	0.000***	0.000	0.000	−0.000	0.000
	(−8.43)	(0.20)	(5.16)	(0.08)	(1.56)	(−1.06)	(1.34)
Δp_t	−0.036	0.006	−0.038*	0.021	0.081***	0.025	−0.072**
	(−1.19)	(0.69)	(−1.87)	(0.43)	(3.78)	(0.59)	(−2.35)
Δp_{t-1}	−0.067***	−0.004	0.040***	0.050**	0.010	−0.005	−0.030*
	(−4.35)	(−0.97)	(3.95)	(1.97)	(0.52)	(−0.46)	(−1.92)
Δp_{t-2}	−0.048***	−0.003	0.012	0.007	0.029***	0.019	0.003
	(−3.73)	(−1.17)	(1.29)	(0.34)	(2.64)	(1.64)	(0.14)
Δp_{t-3}	−0.038***	0.001	0.014	−0.006	0.022*	0.009	0.026
	(−2.82)	(0.23)	(1.47)	(−0.27)	(1.70)	(0.66)	(1.24)
D^D*constant	0.000	−0.000	0.000	0.000	−0.000*	0.001***	0.000
	(1.02)	(−0.99)	(0.19)	(0.06)	(−1.89)	(2.71)	(1.53)
$D^D \Delta y_{t-1}$	−0.039	−0.043	−0.075*	−0.020	0.045	−0.048	−0.010
	(−1.06)	(−1.19)	(−1.77)	(−0.64)	(1.17)	(−1.48)	(−0.19)
$D^D y_{t-1}$	−0.089***	−0.055***	0.006	−0.060***	−0.032***	−0.025***	−0.017
	(−5.14)	(−2.71)	(0.48)	(−3.25)	(−2.71)	(−2.67)	(−1.51)
$D^D \Delta p_t$	0.341***	0.036	0.119**	−0.019	0.029	−0.652***	−0.397*
	(3.50)	(0.90)	(2.05)	(−0.11)	(0.25)	(−4.45)	(−1.65)
$D^D \Delta p_{t-1}$	−0.095	0.016*	0.079*	0.128	−0.080	−0.138*	0.032
	(−1.27)	(1.76)	(1.88)	(1.07)	(−1.10)	(−1.88)	(0.24)
$D^D \Delta p_{t-2}$	−0.052	0.020	−0.008	0.016	0.050	−0.118	0.198
	(−0.79)	(1.22)	(−0.17)	(0.09)	(0.61)	(−1.24)	(1.28)
$D^D \Delta p_{t-3}$	−0.038	0.010	−0.061	0.211*	−0.033	−0.039	−0.037
	(−0.74)	(0.98)	(−1.44)	(1.85)	(−0.59)	(−0.56)	(−0.53)
D^U*constant	−0.001***	−0.000**	−0.001***	0.001	0.000	0.001	0.005***
	(−4.67)	(−2.45)	(−4.46)	(1.12)	(0.52)	(0.77)	(6.50)
$D^U \Delta y_{t-1}$	−0.001	−0.013	−0.068***	−0.021	0.002	−0.037	−0.037
	(−0.03)	(−0.52)	(−2.59)	(−0.81)	(0.05)	(−1.20)	(−1.14)
$D^U y_{t-1}$	−0.085***	−0.036***	−0.023***	−0.050***	−0.059***	−0.016**	−0.119***
	(−6.42)	(−3.91)	(−4.55)	(−4.61)	(−4.59)	(−2.09)	(−10.76)
$D^U \Delta p_t$	0.153***	−0.006	−0.052	−0.072	−0.089	−0.062	−0.010
	(2.70)	(−0.60)	(−1.27)	(−0.68)	(−1.47)	(−0.82)	(−0.14)
$D^U \Delta p_{t-1}$	−0.000	0.010*	0.078***	−0.055	0.007	0.082	−0.101
	(−0.00)	(1.90)	(2.64)	(−0.67)	(0.13)	(1.35)	(−1.18)
$D^U \Delta p_{t-2}$	0.073**	0.011**	0.016	0.012	0.007	0.014	−0.056
	(2.00)	(2.22)	(0.66)	(0.15)	(0.19)	(0.26)	(−1.18)
$D^U \Delta p_{t-3}$	0.089**	0.002	0.074***	−0.077	0.033	−0.004	−0.147**
	(2.38)	(0.38)	(2.62)	(−0.73)	(0.87)	(−0.08)	(−2.10)

Note. The significance of the mean is evaluated with a standard *t*-test, adjusted for heteroskedasticity. One star denotes 90% significance, two stars 95% significance and three stars 99% significance. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

regression model yields significant differences in many instances, for both systematic and non-systematic events. For example, Kirilenko et al. (2017) report a significant negative coefficient on the lagged inventory level in the pre-crash period (and interpret this as an indication of mean-reversion), but no significant changes in the crash and recovery period. We instead see that mean-reversion is much stronger for all categories during crash and recovery, with respect to the pre-crash and post-recovery period. There is only one exception to this regularity: the positive (but insignificant) coefficient of IB-HFT MM during the crash period (and normal periods, in this case significant) for systematic events, denoting lack of mean reversion for these traders which, instead, should hold a capacity limit. This is consistent with their reported behavior during systematic events, see Figure 8, panel A.

In Kirilenko et al. (2017), HFT inventory changes are positively related to contemporaneous and lagged price changes for the first few seconds, then they become negatively correlated, while Market Makers are negatively related to contemporaneous price changes and positively to lagged ones. We broadly confirm this finding, which has just to be specialized to our trader groups. For example, IB-HFT MM and IB-HFT OWN inventory changes behave like the Market Makers of Kirilenko et al. (2017), while PURE-HFT MM behave like the HFTs of Kirilenko et al. (2017). The main difference with their finding is the significant changes for the price/inventory coefficients during the crash and recovery period. For example, while IB-HFT Market Makers become more significantly negatively correlated to contemporaneous price changes during non-systematic crashes (signalling more inventory absorption), the opposite holds for systematic crashes, since they become net sellers (see again Figure 8). Slow traders (NON-HFT CLIENT and NON-HFT OWN) turn the correlation of their inventory changes with contemporaneous price changes to negative during systematic crashes, consistent with the fact they have been shown to be the main liquidity providers for these events.

Summarizing, the analysis of the regression model (7) yields two conclusions. The first one is to provide further statistical significance on the results on net inventories, by showing significant changes in inventory management of different traders during flash crashes at an alternative frequency and with a fully blown statistical model. The second conclusion is that the significant change in the management of net inventory is robust to the interaction with prices: when price changes are added to the model, the behavior of the traders is still

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different during flash crashes with respect to normal times, and in line with what shown in the previous sections. In particular, the estimated model parameters confirm that IB-HFT Market Makers consume liquidity during systematic crashes, and the role of liquidity providers is left to slow traders, who buy at a discount with respect to the fundamental price.

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E   Appendix: the local projection methodology

Let y_t be a (7×1)-vector with the first component being the mid-price at time t, and the other components representing the averages between best bid and best ask for the six trader groups: PURE HFT MM, IB-HFT CLIENT, IB-HFT OWN, IB-HFT MM, NON-HFT CLIENT and NON-HFT OWN. y_t is a non-stationary vector with cointegrated components, where cointegrating equation represent the long-run equilibrium between "mid-prices" of different trader group (the presence of cointegration is revealed using Johansen's methodology). Hence, Δy_t is a stationary process following a vector error correction model of the form:

$$\Delta y_{t+1} = \Psi_1 \Delta y_t + \dots \Psi_p \Delta y_{t-p+1} + \Psi_0 y_t + \epsilon_{t+1}, \tag{8}$$

where $\Psi_1, \dots, \Psi_p$ are the parameter matrices, $\Psi_0 = -\Phi(1)$, with $\Phi(1)$ being a reduced-rank (with rank 1 as we have single cointegrating relation) matrix which can be expressed as $\Phi(1) = BA'$, where B is an (7×1) parameter matrix and $z_t = A'y_t$ is a stationary variable representing cointegrating relationship. Equation (8) can be rewritten in the state space form:

$$
\begin{bmatrix}
z_{t+1} \\
\Delta y_{t+1} \\
\Delta y_t \\
\vdots \\
\Delta y_{t-p+1}
\end{bmatrix}
=
\begin{bmatrix}
I - A'B & A'\Psi_1 & \dots & A'\Psi_{p-2} & A'\Psi_{p-1} \\
-B & \Psi_1 & \dots & \Psi_{p-2} & \Psi_{p-1} \\
0 & I & \dots & 0 & 0 \\
\vdots & \vdots & \dots & \vdots & \vdots \\
0 & 0 & \dots & I & 0
\end{bmatrix}
\begin{bmatrix}
z_t \\
\Delta y_t \\
\Delta y_{t-1} \\
\vdots \\
\Delta y_{t-p+2}
\end{bmatrix}
+
\begin{bmatrix}
A'\epsilon_{t+1} \\
\epsilon_{t+1} \\
0 \\
\vdots \\
0
\end{bmatrix}, \tag{9}
$$

or more compactly, as

$$\xi_{t+1} = G\xi_t + \widetilde{\epsilon}_{t+1}. \tag{10}$$

Notice that the state-space representation implies that, for for $h = 1, \dots, H$, linear forecasts of Δy_{t+h} can be computed as:

$$
\begin{cases}
\Delta y_{t+h} &= G_{[2,1]}^h z_t + G_{[2,2]}^h \Delta y_t + \sum_{j=3}^{p-2} G_{[2,j]}^h \Delta y_{t-j+2} + v_{t+h}, \\
v_{t+h} &= \epsilon_{t+h} + C_1 \epsilon_{t+h-1} + \dots + C_{h-1} \epsilon_{t+1},
\end{cases} \tag{11}
$$

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where $G^h_{[i,j]}$ denotes the $[i,j]$ block of the matrix G raised to the h-th power and $C_1, \ldots C_{h-1}$ are (7×7) matrices from the Wold representation $\Delta y_t = \sum_{j=0}^{\infty} C_j \epsilon_{t-j}$. This implies that the impulse response at lag h of Δy_t on a shock $\epsilon_t = \delta$ can be calculated as:

$$IR(\Delta y_{t+h}, \delta) = \mathbf{E}(\Delta y_{t+h} | \epsilon_t = \delta; y_{t-1}, \ldots) - \mathbf{E}(\Delta y_{t+h} | \epsilon_t = 0; y_{t-1}, \ldots) = G^h_{[2,1]} A' \delta + G^h_{[2,2]} \delta. \tag{12}$$

Following Jordà (2005) and Chong, Jordà, and Taylor (2012) we first estimate the cointegrsting equation by OLS and then directly estimate the impulse responses as:

$$\widehat{IR}(\Delta y_{t+h}, \delta) = \widehat{G}^h_{[2,1]} A' \delta + \widehat{G}^h_{[2,2]} \delta, \tag{13}$$

with

$$\begin{pmatrix} \widehat{G}^1_{[2,1]} & \widehat{G}^1_{[2,2]} \\ \vdots & \vdots \\ \widehat{G}^H_{[2,1]} & \widehat{G}^H_{[2,1]} \end{pmatrix} = Y'_H M_W X (X' M_W X)^{-1}, \tag{14}$$

where the matrices Z_H, Y_H, X and W collect respectively the observations $\{z'_{t+1}, \ldots z'_{t+H}\}$, $\{\Delta y'_{t+1}, \ldots \Delta y'_{t+H}\}$, $\{z'_t, \Delta y'_t\}$ and $\{1, \Delta y'_{t-1}, \ldots \Delta y'_{t-p+2}\}$, with $t = p+1, \ldots, T - H$, and $M_W = I - W(W'W)^{-1}W$. We use $p = 2$ and take $\delta = \delta_j$ to be a vector with unit j-th element and the other elements being zero, for $j = 2, \ldots, 7$.

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Internet Appendix to
High-Frequency Trading During Flash Crashes:
Walk of Fame or Hall of Shame?

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March 2020

Abstract

This online appendix present additional tables complementing the main text of the paper. The tables provide the detailed list of detected flash crashes and report the summary of detected events groped according to each stock.

F   Additional tables

Tables F.1 and F.2 provides the detailed list of the 148 flash crash events in our sample. For each event it shows the date and time (of the crash beginning and the peak) of a flash crash occurrence, the duration of a flash crash and the name and isin code of a corresponding stock.

Table F.3 reports a summary of the detected flash crashes groped according to each stock. It shows that in our sample flash crashes occur in 30 different stocks. The number of flash crashes per year ranges from 1 to 5 with an average rate of 2.2 events per year. For 9 stocks flash crashes occur only once. The largest number of crashes per year (five events) correspond to Société Générale.

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Table F.1 Flash Crash Events constituting the distressed sample (i)

Date	Isin	Company	Time Begin	Time Peak	Duration (min:sec)	Date	Isin	Company	Time Begin	Time Peak	Duration (min:sec)
4-Jan	FR0000131708	Technip	10:19:27	10:29:29	0:10:01	10-Apr	FR0000125338	Cap Gemini	13:11:55	13:13:52	0:01:57
8-Jan	FR0010220475	Alstom	14:47:54	15:03:40	0:15:46	15-Apr	FR0000130577	Publicis Groupe SA	14:46:08	14:56:04	0:09:56
9-Jan	FR0000120172	Carrefour	10:07:42	10:25:18	0:17:36	16-Apr	FR0010208488	ENGIE	12:36:04	12:47:55	0:11:51
11-Jan	FR0000130809	Société Générale	10:53:06	10:57:35	0:04:30	17-Apr	FR0000131906	Renault	9:30:08	9:51:45	0:21:37
11-Jan	FR0000131708	Technip	10:39:28	10:54:05	0:14:37	17-Apr	FR0010220475	Alstom	9:30:01	9:51:26	0:21:24
14-Jan	FR0000125338	Cap G\'emini	16:45:52	16:52:20	0:06:28	17-Apr	FR0010208488	ENGIE	9:44:54	9:51:34	0:06:41
14-Jan	FR0000125007	Saint-Gobain	15:21:35	15:27:45	0:06:10	17-Apr	FR0000073272	Safran	9:29:55	9:51:56	0:22:00
14-Jan	FR0000120354	Vallourec	15:48:07	15:52:15	0:04:08	17-Apr	FR0000127771	Vivendi Universal	9:42:54	9:51:24	0:08:30
15-Jan	FR0010307819	Legrand	12:04:25	12:08:24	0:03:58	17-Apr	FR0000125486	Vinci	9:30:09	9:51:45	0:21:36
22-Jan	FR0000127771	Vivendi Universal	13:46:51	13:50:10	0:03:19	17-Apr	FR0000120628	Axa	9:42:50	9:51:45	0:08:56
22-Jan	FR0000121261	Michelin	9:54:07	10:06:05	0:11:58	17-Apr	FR0000120354	Vallourec	9:30:02	9:51:25	0:21:23
23-Jan	FR0010220475	Alstom	15:59:27	15:59:30	0:00:04	17-Apr	FR0000120578	Sanofi Synthelabo	9:42:55	9:51:30	0:08:35
29-Jan	FR0000120537	Lafarge	9:52:51	10:05:06	0:12:15	17-Apr	FR0000120172	Carrefour	9:42:34	9:51:29	0:08:55
29-Jan	FR0000045072	Credit Agricole	10:02:42	10:15:25	0:12:43	17-Apr	FR0000120073	Air Liquide	9:41:34	9:51:30	0:09:56
31-Jan	FR0000120644	Danone	11:54:52	12:12:21	0:17:28	17-Apr	FR0000121261	Michelin	9:42:50	9:51:40	0:08:50
4-Feb	FR0000131104	BNP	14:08:05	14:15:11	0:07:06	17-Apr	FR0000120271	Total	9:30:09	9:51:34	0:21:26
11-Feb	FR0000120578	Sanofi Synthelabo	15:31:45	15:36:10	0:04:25	17-Apr	FR0000121972	Schneider	9:30:00	9:51:29	0:21:29
20-Feb	FR0000133308	Orange	9:32:28	9:49:00	0:16:33	23-Apr	FR0000120404	Accor	16:50:24	17:14:07	0:23:43
22-Feb	FR0000131906	Renault	14:59:51	15:15:05	0:15:14	26-Apr	FR0000120693	Pernod Ricard	10:18:56	10:27:19	0:08:23
26-Feb	NL0000235190	EADS	16:29:03	16:38:35	0:09:32	29-Apr	FR0000127771	Vivendi Universal	13:55:11	14:02:29	0:07:19
1-Mar	FR0000045072	Credit Agricole	13:17:21	13:20:15	0:02:54	30-Apr	FR0000127771	Vivendi Universal	15:48:25	15:57:40	0:09:15
6-Mar	FR0000133308	Orange	15:22:29	15:24:25	0:01:56	7-May	NL0000235190	EADS	15:46:36	16:01:39	0:15:03
6-Mar	FR0000120404	Accor	15:23:22	15:34:03	0:10:41	7-May	FR0000121972	Schneider	12:17:56	12:27:00	0:09:04
7-Mar	FR0000130809	Société Générale	10:42:26	10:49:45	0:07:19	10-May	FR0010220475	Alstom	10:01:03	10:12:20	0:11:17
8-Mar	FR0000125338	Cap G\'emini	13:59:34	14:10:35	0:11:01	22-May	FR0000125338	Cap Gemini	15:44:54	15:47:05	0:02:11
12-Mar	NL0000226223	STMicroelectronics	17:18:25	17:24:55	0:06:30	23-May	FR0000120172	Carrefour	15:46:17	15:48:51	0:02:34
12-Mar	FR0000121261	Michelin	16:22:32	16:28:04	0:05:33	29-May	FR0000120271	Total	16:13:10	16:17:20	0:04:10
15-Mar	FR0000130809	Société Générale	11:51:19	11:55:10	0:03:52	5-Jun	FR0000120172	Carrefour	11:23:47	11:26:35	0:02:48
20-Mar	FR0000073272	Safran	15:17:00	15:45:28	0:28:28	7-Jun	FR0000121485	Kering	12:02:29	12:09:09	0:06:41
21-Mar	FR0000125338	Cap Gemini	15:15:37	15:33:26	0:17:49	14-Jun	FR0010220475	Alstom	15:01:31	15:02:23	0:00:52
25-Mar	FR0010220475	Alstom	16:00:55	16:20:45	0:19:49	14-Jun	FR0000127771	Vivendi Universal	15:39:05	15:51:09	0:12:05
25-Mar	NL0000235190	EADS	16:14:00	16:23:06	0:09:05	19-Jun	FR0010220475	Alstom	13:08:27	13:10:54	0:02:27
26-Mar	FR0000073272	Safran	12:50:52	13:06:22	0:15:30	24-Jun	FR0000130809	Soci\'et\'e G\'en\'erale	10:15:26	10:33:25	0:17:59
4-Apr	NL0000235190	EADS	16:19:32	16:29:26	0:09:54	24-Jun	FR0000125007	Saint-Gobain	10:15:02	10:33:25	0:18:23
5-Apr	FR0000131104	BNP	11:49:35	11:53:15	0:03:39	1-Jul	FR0000120537	Lafarge	16:39:15	16:41:45	0:02:30
5-Apr	FR0010242511	EDF	14:53:00	14:54:55	0:01:55	11-Jul	FR0010242511	EDF	11:13:35	11:18:55	0:05:20
5-Apr	FR0000120073	Air Liquide	11:47:43	11:53:20	0:05:37	18-Jul	FR0000130577	Publicis Groupe SA	13:51:35	13:51:56	0:00:21

Note. The table reports the detailed list of the 148 flash crash events in our sample. For each event it shows the date and time (of the crash beginning and the peak) of a flash crash occurrence, the duration of a flash crash and the name and isin code of a corresponding stock. The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH.

Table F.2 Flash Crash Events constituting the distressed sample (ii)

Date	Isin	Company	Time Begin	Time Peak	Duration (min:sec)	Date	Isin	Company	Time Begin	Time Peak	Duration (min:sec)
23-Jul	FR0010220475	Alstom	11:18:39	11:22:57	0:04:18	2-Oct	FR0000121261	Michelin	14:34:11	14:45:30	0:11:19
25-Jul	FR0000131708	Technip	11:52:35	12:05:16	0:12:41	2-Oct	FR0000120693	Pernod Ricard	17:21:58	17:25:05	0:03:08
31-Jul	FR0000120404	Accor	13:59:14	14:03:35	0:04:21	2-Oct	FR0010220475	Alstom	15:39:45	15:43:00	0:03:15
31-Jul	NL0000235190	EADS	16:54:15	17:01:15	0:07:00	2-Oct	FR0000121667	Essilor International	15:21:32	15:28:50	0:07:18
16-Aug	FR0000120578	Sanofi Synthelabo	15:30:00	15:35:45	0:05:45	3-Oct	FR0010220475	Alstom	9:29:40	9:48:00	0:18:20
20-Aug	FR0000120537	Lafarge	9:54:11	10:07:20	0:13:09	9-Oct	FR0000120172	Carrefour	14:01:46	14:05:45	0:04:00
23-Aug	FR0000124141	Veolia Environnement	14:48:18	14:48:39	0:00:21	9-Oct	FR0000120537	Lafarge	12:30:21	12:38:28	0:08:07
27-Aug	FR0000120578	Sanofi Synthelabo	9:39:30	9:51:30	0:12:00	18-Oct	FR0000120172	Carrefour	11:23:12	11:34:48	0:11:35
27-Aug	FR0000125007	Saint-Gobain	9:44:07	9:51:35	0:07:28	23-Oct	NL0000226223	STMicroelectronics	16:16:06	16:24:16	0:08:10
27-Aug	FR0000120537	Lafarge	9:44:47	9:51:56	0:07:09	24-Oct	FR0000073272	Safran	14:29:39	14:42:41	0:13:02
27-Aug	FR0000121667	Essilor International	9:44:59	9:51:29	0:06:30	25-Oct	FR0000120628	Axa	15:31:14	15:47:16	0:16:02
29-Aug	FR0000120693	Pernod Ricard	12:25:04	12:59:15	0:34:12	28-Oct	FR0000120628	Axa	13:00:01	13:11:11	0:11:11
29-Aug	FR0000121667	Essilor International	15:01:36	15:04:20	0:02:44	28-Oct	FR0000125486	Vinci	17:25:06	17:29:35	0:04:29
2-Sep	FR0000131906	Renault	16:44:04	16:44:31	0:00:26	28-Oct	FR0000125007	Saint-Gobain	13:02:45	13:15:25	0:12:39
3-Sep	FR0000121485	Kering	10:52:46	10:54:56	0:02:10	29-Oct	FR0000124141	Veolia Environnement	9:29:59	9:40:05	0:10:06
3-Sep	FR0000121014	Lvmh Moet	10:52:45	10:54:54	0:02:09	30-Oct	FR0000131708	Technip	12:29:41	12:31:48	0:02:07
3-Sep	FR0000131104	BNP	10:53:21	10:55:15	0:01:55	5-Nov	FR0000120693	Pernod Ricard	14:55:14	15:10:07	0:14:53
3-Sep	FR0000130809	Société Générale	10:53:18	10:55:10	0:01:52	5-Nov	FR0000130809	Société Générale	14:46:45	14:50:15	0:03:30
3-Sep	FR0000120628	Axa	10:53:09	10:55:10	0:02:01	5-Nov	FR0000120628	Axa	14:40:25	14:48:38	0:08:13
3-Sep	FR0000125486	Vinci	10:52:56	10:55:14	0:02:18	7-Nov	FR0000130809	Société Générale	16:32:20	16:36:35	0:04:15
3-Sep	FR0000120073	Air Liquide	10:51:15	10:54:55	0:03:40	20-Nov	FR0000131708	Technip	15:37:23	16:09:40	0:32:17
3-Sep	FR0000120578	Sanofi Synthelabo	10:52:54	10:54:55	0:02:01	21-Nov	NL0000226223	STMicroelectronics	13:56:25	13:58:30	0:02:06
3-Sep	FR0000120271	Total	10:53:00	10:54:55	0:01:55	22-Nov	FR0000133308	Orange	12:17:19	12:27:59	0:10:41
3-Sep	FR0000125007	Saint-Gobain	10:51:03	10:55:04	0:04:01	22-Nov	FR0000127771	Vivendi Universal	12:16:49	12:22:49	0:06:00
3-Sep	NL0000235190	EADS	10:52:56	10:54:55	0:01:59	25-Nov	FR0000125338	Cap Gemini	10:08:52	10:13:24	0:04:32
3-Sep	FR0000120537	Lafarge	10:52:40	10:54:59	0:02:19	26-Nov	FR0000120404	Accor	11:49:31	11:59:05	0:09:34
3-Sep	FR0000121972	Schneider	10:52:38	10:55:07	0:02:29	26-Nov	FR0000121667	Essilor International	16:37:43	16:41:02	0:03:18
6-Sep	FR0000121014	Lvmh Moet	15:39:26	15:47:35	0:08:09	27-Nov	FR0000120404	Accor	13:27:19	13:47:55	0:20:36
6-Sep	FR0000124141	Veolia Environnement	15:24:16	15:43:00	0:18:44	3-Dec	FR0000133308	Orange	9:58:45	10:03:55	0:05:10
9-Sep	FR0000125338	Cap Gemini	16:45:34	17:12:40	0:27:07	3-Dec	FR0000131906	Renault	10:09:54	10:13:25	0:03:30
10-Sep	FR0000120354	Vallourec	14:47:32	14:50:04	0:02:32	3-Dec	FR0000131708	Technip	9:53:52	10:03:48	0:09:56
16-Sep	FR0000120644	Danone	11:23:54	11:30:20	0:06:26	5-Dec	FR0000130577	Publicis Groupe SA	15:31:10	15:43:04	0:11:55
17-Sep	FR0000120354	Vallourec	16:37:51	16:40:15	0:02:24	17-Dec	FR0000120404	Accor	11:05:41	11:16:16	0:10:35
20-Sep	FR0010208488	ENGIE	10:37:10	10:40:05	0:02:55	18-Dec	FR0000120354	Vallourec	9:30:00	9:46:41	0:16:41
24-Sep	FR0000121485	Kering	15:37:05	15:44:04	0:06:58	19-Dec	FR0000045072	Credit Agricole	14:48:57	14:53:20	0:04:23
24-Sep	FR0000121261	Michelin	15:25:05	15:30:00	0:04:55	19-Dec	FR0000131906	Renault	16:54:09	17:07:10	0:13:01
24-Sep	FR0000120693	Pernod Ricard	16:18:15	16:25:20	0:07:05	23-Dec	FR0000133308	Orange	10:46:27	11:02:27	0:16:00

Note. The table reports the detailed list of the 148 flash crash events in our sample. For each event it shows the date and time (of the crash beginning and the peak) of a flash crash occurrence, the duration of a flash crash and the name and isin code of a corresponding stock.

Table F.3 Summary of Flash Crashes grouped by stocks

Isin	Name	Market Cap (M Euro)	Average daily trading (N. trades)	Average daily volume (M Euro)	No. DBs	Mean return	Median return	Std return	Mean Dura-tion	Median Dura-tion	Std Du-ration
FR0000045072	Credit Agricole	23'221	12'774	88	3	-1.13	-0.89	0.47	0:06:40	0:04:23	0:05:17
FR0000073272	Safran	21'064	8'569	60	4	-1.11	-0.98	0.53	0:19:45	0:18:45	0:06:56
FR0000120073	Air Liquide	32'047	12'821	128	3	-0.99	-0.99	0.30	0:06:24	0:05:37	0:03:12
FR0000120172	Carrefour	20'858	13'152	116	6	-1.53	-1.57	0.74	0:07:55	0:06:27	0:05:58
FR0000120271	Total	145'995	26'025	348	3	-1.23	-1.01	0.49	0:09:10	0:04:10	0:10:40
FR0000120354	Vallourec	5'035	9'902	51	5	-1.18	-0.78	0.60	0:09:26	0:04:08	0:08:57
FR0000120404	Accor	7'822	7'872	49	6	-1.10	-1.07	0.48	0:13:15	0:10:38	0:07:21
FR0000120537	Lafarge	15'652	11'512	76	6	-1.12	-0.97	0.40	0:07:35	0:07:38	0:04:37
FR0000120578	Sanofi Synthélabo	101'851	27'238	400	5	-0.94	-0.87	0.57	0:06:33	0:05:45	0:03:52
FR0000120628	Axa	48'784	19'042	200	5	-1.24	-1.02	0.71	0:09:16	0:08:56	0:05:05
FR0000120644	Danone	30'688	14'526	176	2	-1.40	-1.40	0.02	0:11:57	0:11:57	0:07:49
FR0000120693	Pernod Ricard	21'799	10'385	96	5	-0.88	-0.69	0.59	0:13:32	0:08:23	0:12:18
FR0000121014	LVMH Moet Henessy	66'353	13'133	200	2	-1.27	-1.27	0.33	0:05:09	0:05:09	0:04:14
FR0000121261	Michelin B	14'350	12'618	99	5	-1.50	-1.77	0.72	0:08:31	0:08:50	0:03:13
FR0000121485	Kering	19'395	6'899	83	3	-0.89	-0.84	0.29	0:05:16	0:06:41	0:02:42
FR0000121667	Essilor International	16'592	10'950	86	4	-0.98	-0.93	0.15	0:04:58	0:04:54	0:02:17
FR0000121972	Schneider	35'628	16'767	164	3	-1.79	-1.43	1.15	0:11:01	0:09:04	0:09:39
FR0000124141	Veolia Environnement	6'338	10'989	68	3	-2.46	-2.79	1.21	0:09:44	0:10:06	0:09:12
FR0000125007	Saint-Gobain	22'193	13'639	116	5	-1.36	-1.34	0.33	0:09:44	0:07:28	0:05:47
FR0000125338	Cap Gémini	7'876	9'677	61	7	-0.87	-0.87	0.28	0:10:09	0:06:28	0:09:20
FR0000125486	Vinci	28'713	14'361	124	3	-1.25	-1.18	0.79	0:09:28	0:04:29	0:10:34
FR0000127771	Vivendi Universal	25'660	13'320	143	6	-1.39	-1.34	0.66	0:07:45	0:07:54	0:02:59
FR0000121667	Essilor International	16'592	10'950	86	3	-1.52	-1.26	0.54	0:07:24	0:09:56	0:06:11
FR0000130809	Société Générale	33'722	32'204	317	7	-1.62	-1.52	0.83	0:06:11	0:04:15	0:05:27
FR0000131104	BNP	70'354	33'015	364	3	-1.14	-1.03	0.28	0:04:13	0:03:39	0:02:38
FR0000131708	Technip	7'942	10'665	75	6	-1.17	-1.08	0.48	0:13:36	0:11:21	0:10:06
FR0000131906	Renault	17'064	14'722	118	5	-1.57	-1.33	0.88	0:10:46	0:13:01	0:08:41
FR0000133308	Orange	23'630	21'114	167	5	-1.47	-1.25	0.82	0:10:04	0:10:41	0:06:28
FR0010208488	ENGIE	40'349	13'831	148	3	-2.02	-1.68	0.81	0:07:09	0:06:41	0:04:29
FR0010220475	Alstom	8'126	11'838	81	10	-1.45	-1.47	0.67	0:09:45	0:07:47	0:08:29
FR0010242511	EDF	47'729	9'368	64	2	-1.40	-1.40	0.67	0:03:37	0:03:37	0:02:25
FR0010307819	Legrand	10'633	6'387	45	1	-1.08	-1.08		0:03:58	0:03:58	
NL0000226223	STMicroelectronics	7'098	8'668	44	3	-2.60	-1.41	2.16	0:05:35	0:06:30	0:03:08
NL0000235190	EADS	43'550	20'886	212	6	-0.88	-0.96	0.25	0:08:46	0:09:19	0:04:15

Note. This table reports summary of Flash Crashes, which occurred in 30 different stocks during the year 2013. For each stock, which experienced a flash crash, the table reports average yearly market capitalization, average daily number of trades and trading volume, the number of detected events, mean, median and standard deviation of flash crash durations and of the price drop during the crash.The sample is composed of 37 stocks traded on NYSE Euronext Paris that belong to the CAC40 index, for the year 2013. Order flow and trade data are from BEDOFIH. Market capitalizations are from Bloomberg.

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