Research Report

Housing as an Explanation for low Stock Market Participation – a Simulation Approach

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Introduction

Owner-occupied housing is the most important asset in many investors' portfolios. This article investigates optimal portfolios including real estate ownership and examines the resulting optimal intensity of stock market participation. Therefore, this paper explores whether low or no stock market participation can be rational.

The first sophisticated papers treating the issue of portfolio optimization appeared in the 1960's [Sharpe (1964), Mossin (1968), Merton (1969), Samuelson (1969)]. They all assume complete markets and therefore hardly address the issue of background risk, because in this framework background risk can be priced and capitalized into wealth. Under those conditions the statistical properties of background risk do not influence the optimal portfolio allocation to risky and risk-free assets.

In recent years the assumption of complete markets was exposed to increasing criticism and new theoretical as well as practical interest in the portfolio optimization problem arose. In contrast to early portfolio theory, current research assigns transaction costs and information asymmetry to significantly impact economic decisions, and therefore affects asset allocation [Leland (1985), Odean (1998), Barber and Odean (2001), Statman et al. (2006)].

Besides, a switch in the assumptions from complete to incomplete markets also entails the existence of background risks, which affect the asset allocation rules for an optimal portfolio. As a result, investors modify their portfolios' consistency to avoid non-market risk exposures [Mayers (1974), Duffie et al. (1997)]. The most frequently named sources of background risks are proprietary income and fluctuations in labor income and housing [Guiso et al. (1996), Fratantoni (1998), Heaton and Lucas (2000), Shum and Faig (2006)].

I use a model, based on Markowitz's (1952) theory of portfolio optimization, to integrate the ownership of real estate into households' portfolios as an additional restriction and derive the mean-variance optimal portfolio. First I calculate the optimal portfolio shares invested into Stocks and Bonds and T-Bills with respect to mortgage and house value. Those results are then compared to asset shares in simulated and optimized portfolios. The simulations show that low or no stock market participation can be rational with regard to house ownership as one source of background risk. In addition, the simulations reveal differences in the optimal portfolios according to investors' house-to-net worth ratios and their personal risk attitude. Finally, I calculate the optimal portfolio structure for several West German federal states and unfold that they match the simulation results.

Model

I focus on the issue how owner-occupied houses (instead of real estate investment trusts) impact German households' optimal shares in risky assets. One major country specific difference lies in the volatility of residential house prices. For example, US house prices are almost as volatile as risky assets, whereas West German house prices are nearly stable over time. From those facts I conclude that the shares of Stocks and Bonds and T-Bills vary significantly for German household portfolios compared to the US, all other things being equal.

The objective is to find optimal portfolios for all households that own a home as well as households that rent and to answer the question why German households hardly participate in stock markets. In allusion to Flavin and Yamashita (2002) I abstract from labor income or human capital and assume that wealth is held in any of n risky investments and owner-occupied houses. Further, the household can borrow up to the value of the house. All other financial investments have to be of positive value. I stick to Grossman and Laroque (1990), who assume that the house size cannot be adjusted after being purchased.

I calculate the optimal shares of risky and housing assets under the restriction that the house was purchased before the optimization process and its value is exogenously determined. Consequently, only the shares of the risky assets (x_t) can be adjusted to determine the efficient frontier, whereas the Housing-to Net-Worth ratio (h_t) is fix. The calculations include an investor who is unrestrained in dividing up his wealth among any risky asset class, but does not purchase a house (h_t=0.0).

Data

The mean returns and standard deviation and covariance matrix (Ω) are presented in Table 1. As expected, Stocks have the highest returns out of the five assets, whereas Houses show a much smaller return. Those results are significantly different from US data, which show a much stronger return for houses at a much higher volatility. I accent the House index's Sharpe-Ratio (SR=0.65) to be the highest in this data set, followed by Stocks (SR=0.62). It is important to acknowledge that I do not include taxes in my calculations and therefore probably overstate asset returns.

Results

Optimal portfolio allocation for real data in case of West Germany

	Stocks	Bonds	T-Bills	House	Mortgage
Mean (arith.)	0,088	0,020	-0,007	0,021	0,000
Std.	0,153	0,069	0,037	0,038	0,031
Correlation Matrix					
Stocks	1,000				
Bonds	0,011	1,000			
T-Bills	0,015	-0,759	1,000		
House	-0,004	-0,074	0,093	1,000	
Mortgage	0,177	-0,645	0,929	0,214	1,000
Covariance Matrix					
Stocks	0,02330				
Bonds	0,00012	0,00476			
T-Bills	0,00009	-0,00196	0,00140		
House	-0,00002	-0,00020	0,00013	0,00146	
Mortgage	0,00085	-0,00140	0,00109	0,00026	0,00099

Table 1: Return Matrix and Covariance Matrix for West Germany; Data from Bloomberg and Bureau of Statistics and IFS (1990-2007)

The Housing-to-Net-Worth (NW) ratio (h_t) plays a major role in my model due to its constraining effect on the mortgage ratio and consequently the mean-variance optimal portfolio. As mentioned in the introduction, households' wealth, saving rates and asset compilation, as well as their risk preferences (λ) vary dramatically over the life cycle. Therefore, I investigate the optimal asset allocation for different h_t, starting at 4.0 for young households with high leverage decreasing to a ratio of 0.5 for older households. Note that those ratios do not violate the restrictions on lending ($0 \geq X_{n,t} \geq -h_t$). In order to descry the mean-variance efficient portfolios under the housing constraint with respect to the various risk preferences I employ quadratic programming to calculate the optimal vector of financial assets, X. In Table 2, the results from the optimization are stated. Note that Stocks and Bonds and T-bills represent the percentage-shares of those three assets among the risky assets, excluding House and Mortgage. Consequently, those three assets must sum up to one in Table 2. Mortgage represents the leverage in accordance to the House value. A Mortgage of -0.8 indicates an 80%-debt financing of the House.

Optimal asset allocation in several German states

At last, I examine optimized portfolios for the states of NRW, Hessen, Rheinland-Pfalz, Bavaria, and Saarland to validate the simulation results. In essence each of those examples reveals special characteristic. NRW is the most populated German state, Hessen contributes the most to the national GDP, Rheinland-Pfalz is a very rural area, Bavaria exhibits the fastest growing economy and lowest unemployment rate, Saarland is suffering from the loss of its coal mining industry, Bremen is a city state with one of Germany's largest harbors and a very low rate of education, on average. Nevertheless, any state is part of Germany and therefore the only variable varying in the model is the House price development and its correlation with other assets. According to standard theory, the optimal portfolios should look quite similar, but I demonstrate that the change of one variable (in this case House) has a major impact on the optimal portfolio allocation. Basically, those results illustrate a scenario analysis for changes in the correlation matrix for the housing variable while all other variables are stable.

The outcome clearly identifies areas where very low stock market participation is fully rational. In case of Bremen and NRW picking $\lambda=10$ and $h_t=3.5$, respectively $h_t=4.0$ the optimal equity share in the portfolios are as low as 0.00 and 0.11. On the other hand, Rheinland-Pfalz and Saarland disclose an optimal equity share of 0.47 and 0.57 and simultaneously exhibit very low correlations between the house variable and other assets, reducing σ^2, for $\lambda=10$ and $h_t=4.0$. Although most papers on this topic assume an increase in the Stocks share with rising x_t, I find evidence that this is not necessarily true but strongly depends on the interaction between the single variables represented by Ω In addition, the optimal portfolios for each of the states are in accordance with the findings from the simulations. For example, a strong positive Stocks/House-covariance factor insinuates a strong decrease in the equity share for $\lambda=10$ and $h_t=4.0$. This finding is affirmed in the scenario analysis. Therefore, I conclude a general assumption for the optimal portfolio allocation according to λ and h_t is inaccurate if the interactions between all other influencing variables are neglected. One of the few findings to be valid in general is to optimally invest 100% of the portfolio's assets into Stocks in case of very low risk-aversion ($\lambda=2$). In addition, the scenario analysis unfolds a tendency towards reducing the Mortgage ratio for decreasing h_t and increasing λ.

Conclusion

The paper's goal is to find a rational explanation for low stock market participation. Therefore I incorporate housing as a background risk into a portfolio and derive the mean-variance optimal portfolios for various levels of h_t and λ. The results from those analyses clearly identify the interactions between the different assets, represented in Ω, as the main driver for stock market participation. In addition I substantiate that low or even no stock market can be rational, depending on Ω. In the end I control for the validity of the simulation results by optimizing portfolios for different states of Germany.

Nevertheless, housing is only one of the most frequently named sources of background risk, whereas I do not include proprietary income and fluctuations in labor income in my analyses. The model is flexible enough to be extended to incorporate those issues and to examine how the optimal portfolio would deviate if Heaton and Lucas's (1999) findings concerning the amount of wealth is integrated. Another interesting fact would be to examine the optimal portfolios for entrepreneurs based on Gentry and Hubbard's (1989) findings which unfold a higher savings ratio for those investors. Incorporating all three sources of background risk into this model should display the optimal portfolios according to the very different input factors and allow for a comparison of the rationality of different investor groups according to their individual characteristics.

		Degree of risk-aversion, λ				
Housing-to-NW ratio	**Assets in Portfolio**	**2**	**4**	**6**	**8**	**10**
h=0.0	Stocks	1,000	0,778	0,574	0,466	0,374
	Bonds	0,000	0,222	0,426	0,507	0,472
	T-Bills	0,000	0,000	0,000	0,027	0,154
	Mortgage	0,000	0,000	0,000	0,000	0,000
h=0.5	Stocks	1,000	0,815	0,612	0,562	0,535
	Bonds	0,000	0,185	0,388	0,438	0,465
	T-Bills	0,000	0,000	0,000	0,000	0,000
	Mortgage	-1,000	-1,000	-1,000	-0,668	-0,400
h=1.0	Stocks	1,000	0,853	0,669	0,658	0,648
	Bonds	0,000	0,147	0,331	0,342	0,352
	T-Bills	0,000	0,000	0,000	0,000	0,000
	Mortgage	-1,000	-1,000	-0,953	-0,730	-0,596
h=1.5	Stocks	1,000	0,890	0,764	0,786	0,808
	Bonds	0,000	0,110	0,236	0,214	0,192
	T-Bills	0,000	0,000	0,000	0,000	0,000
	Mortgage	-1,000	-1,000	-0,899	-0,750	-0,661
h=2.0	Stocks	1,000	0,927	0,886	0,965	0,764
	Bonds	0,000	0,073	0,114	0,035	0,000
	T-Bills	0,000	0,000	0,000	0,000	0,236
	Mortgage	-1,000	-1,000	-0,872	-0,761	-0,770
h=2.5	Stocks	1,000	0,965	1,000	0,629	0,430
	Bonds	0,000	0,035	0,000	0,000	0,000
	T-Bills	0,000	0,000	0,000	0,371	0,570
	Mortgage	-1,000	-1,000	-0,868	-0,930	-1,000
h=3.0	Stocks	1,000	1,000	0,826	0,525	0,429
	Bonds	0,000	0,000	0,000	0,000	0,000
	T-Bills	0,000	0,000	0,174	0,475	0,571
	Mortgage	-1,000	0,999	-0,940	-1,000	-1,000
h=3.5	Stocks	1,000	1,000	0,684	0,523	0,427
	Bonds	0,000	0,000	0,000	0,000	0,000
	T-Bills	0,000	0,000	0,316	0,477	0,573
	Mortgage	-1,000	-1,000	-1,000	-1,000	-1,000
h=4.0	Stocks	1,000	1,000	0,682	0,521	0,425
	Bonds	0,000	0,000	0,000	0,000	0,000
	T-Bills	0,000	0,000	0,318	0,479	0,575
	Mortgage	-1,000	-1,000	-1,000	-1,000	-1,000

Table 2: Optimal Asset Allocation for West German Households with respect to Owner-Occupied Housing