

Online Appendix

A Reduced Form Regressions

We directly investigate the impact of psychological and cognitive measure on the biases of survival beliefs, which we define as the difference between subjective and objective probabilities, $SSB_{i,h,m(h)} - OSP_{i,h,m(h)}$. Without imposing any structural assumptions from theory, we estimate the following linear model

$$SSB_{i,h,m(h)} - OSP_{i,h,m(h)} = \beta_0 + \beta_1 c_{i,h-2} + \beta_2 o_{i,h-2} + \beta \mathbf{X}_i + \epsilon_{i,h,m(h)}.$$

As controls, we include objective survival probabilities, $OSP_{i,h,m(h)}$, interaction terms $c_{i,h-2} \cdot OSP_{i,h,m(h)}$, $o_{i,h-2} \cdot OSP_{i,h,m(h)}$, and $c_{i,h-2} \cdot o_{i,h-2}$, as well as the additional control variables, \mathbf{X}_i , used in our Model 3 of the main robustness checks in the main text. We expect $\beta_2 > 0$ reflecting overestimation by optimists. To investigate the effect of cognition, we use the *absolute* error $|SSB_{i,h,m(h)} - OSP_{i,h,m(h)}|$ as the dependent variable and thus run the regression

$$|SSB_{i,h,m(h)} - OSP_{i,h,m(h)}| = \beta_0 + \beta_1 c_{i,h-2} + \beta_2 o_{i,h-2} + \beta \mathbf{X}_i + \epsilon_{i,h,m(h)}.$$

Since according to our theory increasing lack of cognition leads to a clockwise tilting of the PWF, an increase in the lack of cognition increases the imprecision of the SSB compared to the OSP . Hence, in this setting we expect $\beta_1 > 0$. Our results on these additional robustness checks confirming our main findings are summarized in Table 10.

B Quantile Regressions

We now investigate the robustness of our main findings through quantile regressions. This allows us to detect relationships that are not captured by mean effects. In our quantile

Table 10: OLS Estimates

	SSB-OSP (Relevant for Optimism)			SSB-OSP (Relevant for Cognition)		
Optimism _{t-2}	0.111*** (7.08)	0.172*** (11.77)	0.099*** (3.62)	-0.019* (-1.84)	-0.008 (-0.72)	-0.007 (-0.69)
Cognitive weakness _{t-2}	0.459*** (19.35)	0.081*** (2.88)	0.082*** (2.89)	0.206*** (13.57)	0.114*** (5.70)	0.081*** (2.58)
OSP _t		-0.576*** (-15.04)	-0.700*** (-12.80)		-0.101*** (-3.72)	-0.126*** (-3.82)
Optimism _{t-2} x OSP _t			0.171*** (3.17)			
Cognitive weak _{t-2} x OSP _t						0.079 (1.35)
Constant	-0.210*** (-13.45)	-21.45** (-2.50)	-21.79** (-2.54)	0.185*** (18.49)	13.10** (2.16)	13.29** (2.19)
Control Variables	No	Yes	Yes	No	Yes	Yes
Observations	11,954	11,954	11,954	11,954	11,954	11,954
Adjusted R ²	0.031	0.211	0.212	0.017	0.032	0.033

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Columns 2-4 are relevant for the association between optimism and the difference $SSB - OSP$, adding control variables and an interaction term one at a time. Columns 5-7 is relevant for the association between cognitive weakness and the absolute difference $|SSB - OSP|$ between the subjective and objective survival probability.

regressions, we take the difference between SSBs and OSPs as a dependent variable. Additionally, we include the level of the objective survival probability in our set of explanatory variables because the interval of our dependent variable is directly linked to the level of the OSP. We analyze every decile and estimate the results for all deciles jointly. As previously, standard errors are bootstrapped. Our regression specification including control variables is

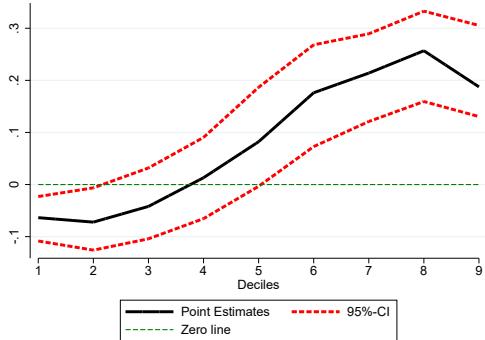
$$SSB_{i,h,m(h)} - OSP_{i,h,m(h)} = \beta_0 + \beta_1 OSP_{i,h,m(h)} + \beta_2 c_{i,h} + \beta_3 o_{i,h-2} + \mathbf{x}'_i \beta + \epsilon_{i,h,m(h)}. \quad (15)$$

By including the OSP on the right-hand-side of the regression, we control for biases induced by truncation and censoring, as underestimators cannot report SSBs less than zero and overestimators cannot report SSBs above one. The clockwise tilting of the PWF from increasing cognitive weakness we identified earlier is consistent with negative estimates of β_2 in lower percentiles and positive estimates in upper percentiles. This means that increasing cognitive weakness leads to a more pronounced underestimation for underestimators (who, on average, have high OSPs) and a more pronounced overestimation for overestimators (who, on average, have low OSPs). Irrespective of the percentiles, we also expect that $\beta_3 > 0$ because optimism leads to overestimation and $\beta_4 < 0$ because higher OSPs decrease the distance between SSBs and OSPs.

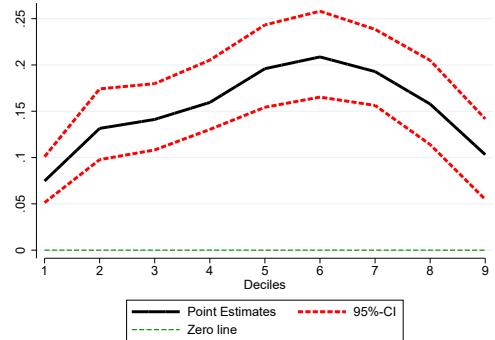
We report our results in Figure 13, thereby confirming our hypotheses. Interestingly, we also find that the effects of optimism are strongest for the intermediate percentiles. This is consistent with the non-linear probability weighting function: in the lowest percentiles, we have individuals with, on average, high OSPs, where the structure of the non-linear PWF forces subjective beliefs to converge to 1, cf. Figure 2. Likewise, in the highest percentile, individuals have, on average, low OSPs, which forces subjective beliefs to converge to 0. Thus, under a non-linear PWF, there is less room for motivational variables to impact the formation of SSBs at extreme OSPs of 0 and 1.

Figure 13: Quantile Regression: Coefficient Estimates

(a) Δ Cogn. Weakn.



(b) Optimism



Notes: Coefficient estimates of equation (15) by deciles of underestimation and the respective bounds of the 95%-confidence intervals, which are calculated with the percentile method (1,000 replications). *Source:* Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

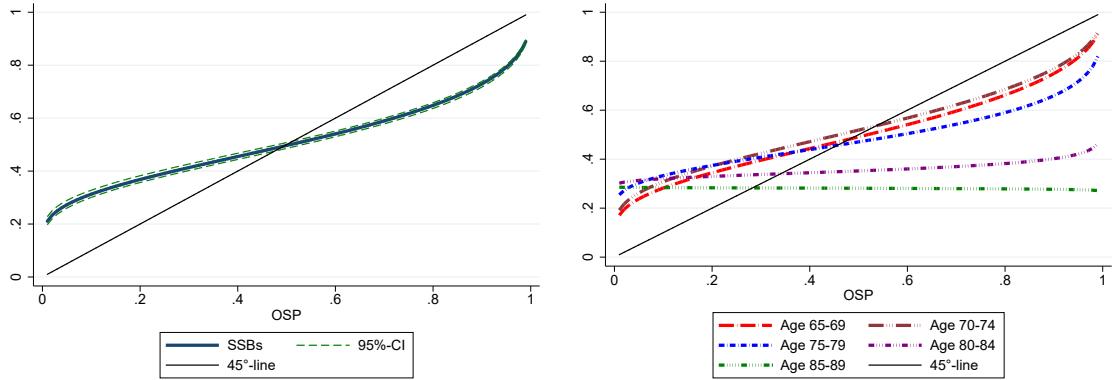
C Focal Point Answers

To investigate the sensitivity of our results with respect to focal point answers, we repeat the estimation of non-linear PWFs by excluding observations with focal point answers at SSBs of 0%, 50% and 100%. Results are presented in Figure 14. In contrast to the corresponding Figure 4, probability weighting functions for the highest target age group are now downward sloping. Since we regard upward sloping PWFs as plausible, this finding is another indication (beyond the histograms shown in Appendix A) that focal point answers do have information content, which justifies including all these observations in our main analyses.

An alternative perspective to take is to only exclude focal point answers at 0.5. Through this we directly address the concern that our results might be driven by a 50-50 answering heuristic that does not contain any information. Results shown in Figure 15 reveal that this is not a concern. Now, the PWFs look almost identical to those shown in the main text in Figure 4.

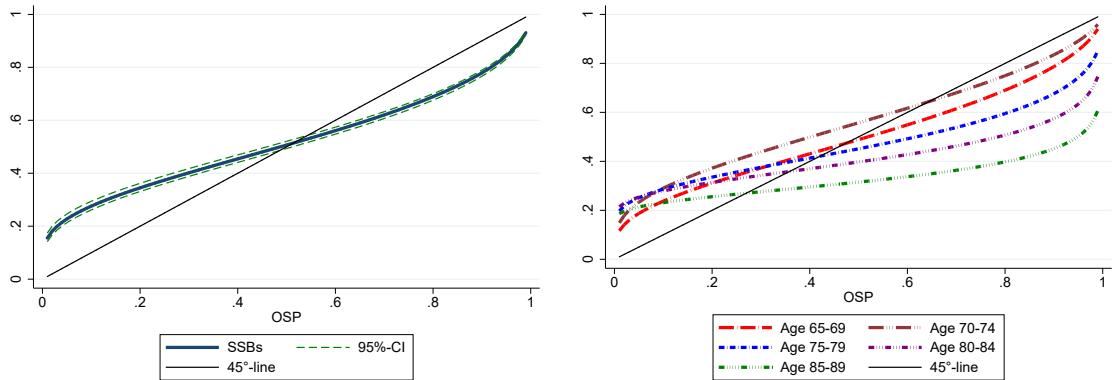
Based on this sample, we reestimate our baseline specification and conduct the decomposition analysis as in Figure 7. Results shown in Figure 16 are very similar to our main results. The main change concerns the effect of optimism, which now leads to an upward

Figure 14: Non-Linear PWFs: Excl. All Focal Point Answers



Notes: Estimated Prelec probability weighting functions when focal point answers at 0, 0.5, 1 are excluded. Parameters estimated with non-linear least squares. *Source:* Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

Figure 15: Non-Linear PWFs: Excl. Focal Point Answers at 0.5

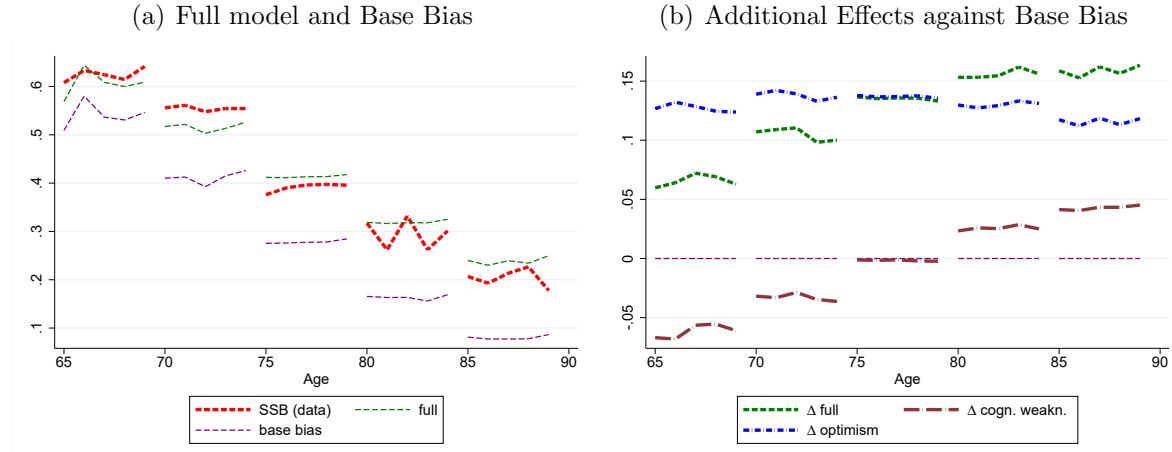


Notes: Estimated Prelec probability weighting functions when focal point answers at 0.5 are excluded. Parameters estimated with non-linear least squares. *Source:* Own calculations, Health and Retirement Study (HRS), Human Mortality Database (HMD).

bias of about 13% p compared to about 10% p in our baseline results.

Finally, to underscore that focal point answers do contain information for our question at hand, we run logistic regressions of the probability of providing a focal point answer on our measures of cognitive weakness, optimism, the objective survival probability, with and without control variables. Results shown in Table 11 show that with increasing cognitive

Figure 16: Non-Linear PWF: Decomposition over Age, Excluding Focal Point Answers at 0.5



Notes: Sample averages of predicted subjective survival beliefs according to equations (6) and (7) by age, excluding focal point answers at 0.5; Panel (a): ‘full’: \widehat{SSB} ; ‘base bias’: \widehat{SSB}^b ; Panel (b): ‘ Δ full’: $\widehat{SSB} - \widehat{SSB}^b$; ‘ Δ cogn. weakn.’: $\widehat{SSB}^{bc} - \widehat{SSB}^b$; ‘ Δ optimism’: $\widehat{SSB} - \widehat{SSB}^{bc}$.

weakness respondents are more likely to provide focal point answers at 0 and 1, respectively. This is consistent with Hill et al. (2004) who show that uncertainty with respect to survival beliefs increases in cognitive weakness. With increasing optimism, the probability to provide a focal point answer of 0 decreases.

D Extending the Non-Linear PWF

We now relax our structural interpretation of parameters ξ_h and δ_h in terms of cognition and optimism by rewriting (5) as

$$\xi_{i,h} = \xi_0 + \xi_1 c_{i,h-2} + \xi_2 o_{i,h-2}, \quad (16a)$$

$$\theta_{i,h} = \theta_0 + \theta_1 o_{i,h-2} + \theta_2 c_{i,h-2}, \quad (16b)$$

and thus the nonlinear PWF becomes

$$SSB_{i,h,m(h)} = \left(\exp \left(- (-\ln (OSP_{i,h,m(h)}))^{\xi_0 + \xi_1 c_{i,h-2} + \xi_2 o_{i,h-2}} \right) \right)^{\theta_0 + \theta_1 o_{i,h-2} + \theta_2 c_{i,h-2}}. \quad (17)$$

Table 11: Focal Point Answers, Marginal Effects of Logit Regression

	$SSB = 0$		$SSB = 0.5$		$SSB = 1$	
	(1)	(2)	(3)	(4)	(5)	(6)
Cognitive weakness _{t-2}	0.132*** (5.85)	0.095*** (3.29)	-0.094*** (-2.67)	-0.032 (-0.73)	0.270*** (13.18)	0.175*** (6.97)
Optimism _{t-2}	-0.101*** (-6.75)	-0.083*** (-5.53)	-0.019 (-0.85)	-0.005 (-0.22)	0.046*** (3.47)	0.043*** (3.17)
OSP	-0.331*** (-25.35)	-0.279*** (-7.74)	0.092*** (5.56)	0.082 (1.36)	0.117*** (11.07)	0.141*** (3.91)
Control Variables	No	Yes	No	Yes	No	Yes
Observations	11,954	11,954	11,954	11,954	11,954	11,954
Pseudo R ²	0.137	0.172	0.004	0.014	0.039	0.068

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The dependent variable is a binary variable which is one if the respondent gave a focal point answer of zero shown in columns (1) and (2), of 0.5 shown in column (3) and (4) and of 1 shown in columns (5) and (6). The variable is zero for all other non-missing observations of SSB.

Table 12 compares our baseline results to those of estimating the non-structural non-linear model (17). We find that the estimates of the new coefficients ξ_2, θ_2 enter significantly, and that inclusion of these additional coefficients mainly affects the intercept term ξ_0 .

We interpret these results by help of the decomposition of the PWF in Figure 17. As for the baseline specification increasing cognitive weakness leads to a clockwise tilting of the PWF. The new additional effect of $\theta_2 < 0$ is that with increasing cognitive weakness the PWF is shifted up, similar to the effect of optimism. As a consequence the intersection point (OSP_0, SSB_0) moves up. Switching on optimism leads to an upward shift, as in our baseline results, and, since the new slope coefficient $\xi_2 > 0$ (and thus of opposite sign to ξ_1) it additionally leads to a counter-clockwise tilting.

The decomposition over age shown in Figure 17 shows that the differential effect of cognitive weakness is more or less as before in our baseline results, i.e., with increasing cognitive weakness individuals over the life-cycle overestimate their survival probabilities more. We now find that the overall effect of optimism is downward sloping because of the additional counter-clockwise tilting of the PWF. Thus, if anything, then the effect of

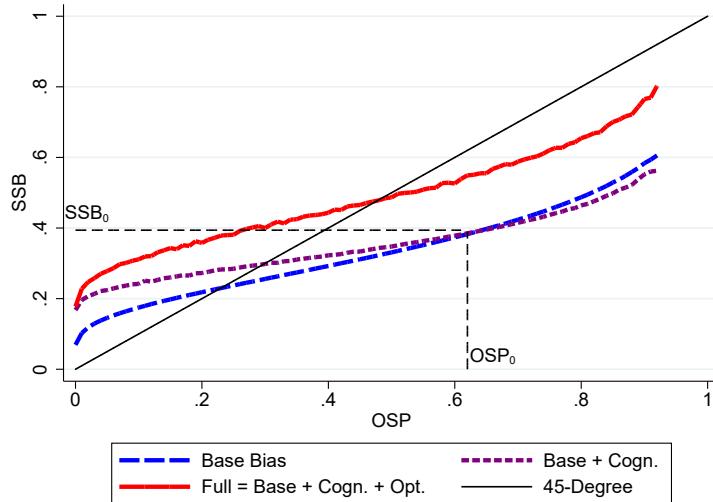
Table 12: OLS Regressions

	Baseline	Non-structural
ξ_0 Intercept Cognition	0.540*** (19.33)	0.380*** (7.77)
ξ_1 Slope Cognition	-0.399*** (-5.60)	-0.323*** (-4.45)
θ_0 Intercept Optimism	1.140*** (42.31)	1.270*** (34.59)
θ_1 Slope Optimism	-0.433*** (-12.14)	-0.454*** (-12.53)
ξ_2 New Slope Optimism		0.213*** (4.26)
θ_2 New Slope Cognition		-0.310*** (-5.59)
Observations	11,954	11,954

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

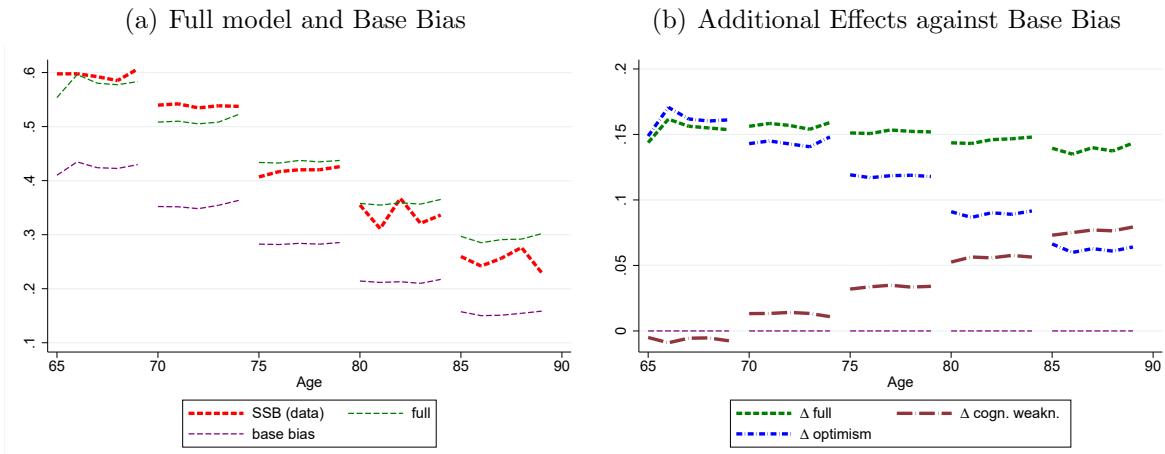
Figure 17: Decomposition of Non-linear PWFs with Equation (17)



Notes: Sample averages of predicted non-linear probability weighting functions according to equations (6) and (7); ‘base bias’: \widehat{SSB}^b ; ‘base + cogn. weakn.’: \widehat{SSB}^{bc} ; ‘full’: \widehat{SSB} .

optimism is decreasing with age. This reinforces our interpretation of our main results that increasing optimism is not the reason for the overestimation of old-age survival probabilities.

Figure 18: Non-Linear PWF with Equation (17)



Notes: Sample averages of predicted subjective survival beliefs according to equations (6) and (7) by age; Panel (a): ‘full’: \widehat{SSB} ; ‘base bias’: \widehat{SSB}^b ; Panel (b): ‘ Δ full’: $\widehat{SSB} - \widehat{SSB}^b$; ‘ Δ cogn. weakn.’: $\widehat{SSB}^{bc} - \widehat{SSB}^b$; ‘ Δ optimism’: $\widehat{SSB} - \widehat{SSB}^{bc}$.