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Alessandro Di Nola | Leo Kaas | Haomin Wang

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Leibniz Institute for Financial Research SAFE
Sustainable Architecture for Finance in Europe
info@safe-frankfurt.de | www.safe-frankfurt.de

Rescue Policies for Small Businesses in the COVID-19 Recession*

Alessandro Di Nola[†] Leo Kaas[‡] Haomin Wang[§]

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Abstract

While the COVID-19 pandemic had a large and asymmetric impact on firms, many countries quickly enacted massive business rescue programs which are specifically targeted to smaller firms. Little is known about the effects of such policies on business entry and exit, factor reallocation, and macroeconomic outcomes. This paper builds a general equilibrium model with heterogeneous and financially constrained firms in order to evaluate the short- and long-term consequences of small firm rescue programs in a pandemic recession. We calibrate the stationary equilibrium and the pandemic shock to the U.S. economy, taking into account the factual Paycheck Protection Program (PPP) as a specific grant policy. We find that the policy has only a small impact on aggregate employment because (i) jobs are saved predominately in less productive firms that account for a small share of employment and (ii) the grant induces a reallocation of resources away from larger and less impacted firms. Much of this reallocation happens in the aftermath of the pandemic episode. While a universal grant reduces the firm exit rate substantially, a targeted policy is not only more cost-effective, it also largely prevents the creation of "zombie firms" whose survival is socially inefficient.

JEL classification: E22, E65, G38, H25

Keywords: COVID-19, Heterogeneous Firms, Business Subsidies, Paycheck Protection Program

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[†]University of Konstanz, alessandro.di-nola@uni-konstanz.de

[‡]Goethe University Frankfurt and SAFE, kaas@wiwi.uni-frankfurt.de

[§]University of Konstanz, haomin.wang@uni-konstanz.de

1 Introduction

The 2020-21 recession induced by the COVID-19 pandemic differs from regular business-cycle downturns in important ways. Government-mandated shutdown policies, individual demand adjustments and disruptions of global production chains had a large and asymmetric impact on private businesses. In particular, the magnitude of output and employment declines were larger for smaller firms (see Bloom et al., 2021; Cajner et al., 2020).[1] Furthermore, business closures in the U.S. have increased sharply, again with much variation by firm size (Crane et al., 2021; Chetty et al., 2020). To stabilize income losses in the short-term and to prevent a severe and long-lasting impact on production capacities, governments in many countries swiftly implemented small business rescue programs in the form of grants or conditional loans. For instance, in March 2020 the U.S. enacted the Coronavirus Aid, Relief, and Economic Security (CARES) Act that allocated over $600 billion for the Paycheck Protection Program (PPP). In 2020, over three quarters of U.S. small businesses received the PPP loan and most are eventually forgiven (Autor et al., 2022).

Little is known about the effects of such a large-scale small business rescue policy from a macroeconomic perspective. On the one hand, offering liquidity to small businesses can prevent productive firms from permanently shutting down, impeding inefficient capital liquidation and facilitating a quicker economic recovery once the pandemic terminates. On the other hand, such rescue plans can inadvertently prolong the lives of unproductive ("zombie") firms, thus hampering efficient capital reallocation. Furthermore, the PPP program was designed to favor timeliness over targeting (Autor et al., 2022), resulting in an unprecedented fiscal cost as firms that are not impacted or at risk of liquidation also received the forgivable loan. Given that targeting financial aid to impacted firms requires a greater administrative burden, it is important to understand the cost-effectiveness of a targeted rescue policy compared to the rather universal PPP program.

The goal of this paper is to quantify the short- and long-run macroeconomic effects of the small business rescue policy enacted in the COVID-19 pandemic, and to evaluate a counterfactual targeted rescue grant. To this end, we build a general equilibrium model with firms that differ in productivity, the level of fixed capital, and financial assets or debt. Firms face financial constraints and capital adjustment frictions due to partial irreversibility of fixed in-

[1] Using anonymized administrative data provided by ADP (a large private provider of payroll services), Cajner et al. (2020) show that businesses with fewer than 50 employees reduced employment by more than 25 percent from March to April 2020, whereas larger firms saw declines of 15-20 percent over the same time period.

vestments. We interpret these entities as "small firms" in a non-corporate sector that can only borrow against collateral, have no access to capital markets and cannot easily liquidate a portion of their capital. Small firms incur overhead expenses including the maintenance cost of capital and payroll expenses. Our model also includes a corporate sector where firms are not financially constrained. The pandemic has an asymmetric impact on firms in the two sectors. Although the corporate sector is not the center of our analysis, it is important for factor reallocation and hence for macroeconomic adjustment following the pandemic shock.

We first calibrate our model such that the stationary equilibrium matches relevant aggregate and firm-level moments of the pre-pandemic U.S. economy. We draw data from various sources including semi-aggregate tables of the Statistics of U.S. Businesses (SUSB), the Business Dynamics Survey (BDS), as well as micro data from the Kaufman Firm Survey (KFS) to inform us about the balance sheets of small firms in the U.S. Our model closely replicates the heavily skewed firm size distribution observed in the SUSB, the pattern that the firm exit rate decreases in firm size observed in the BDS, and the debt-asset ratio and the share of indebted firms observed in the KFS. In our calibrated model, potential entrants with low productivity would not enter, while continuing firms exit if their productivity is too low or debt is too high.

We model the pandemic shock with four components: A shut-down shock that affects a fraction of small firms, a TFP shock on the corporate sector, a demand shock affecting the marginal utility of consumption, and a labor supply shock affecting the marginal utility of leisure. The shut-down shock captures the impact of government mandated lock-down policies that forced businesses offering "social" goods and services to temporarily close at the beginning of the pandemic. The demand shock is important for explaining the observed sharp drop in consumption, and the labor supply shock captures the drop in employment, possibly due to health risks of in-person working. We calibrate the pandemic shock to match the changes of U.S. output, consumption, aggregate employment and employment in small firms, while taking into account the PPP policy and their take-up rates by small firms.

Based on our calibrated model, we compare the PPP to the laissez-faire economy, a counterfactual scenario with no government intervention. We find that the PPP prevents 35% of small business exits at the onset of the pandemic. Despite being successful at preventing many businesses from permanently shutting down, the PPP is mostly ineffective in improving aggregate output and employment. The reason is twofold: *(i)* The PPP induces inefficient reallocation of resources towards the more impacted small-firm sector, away from larger and less impacted firms in the corporate sector. Further, within the small-firm sector, there is also a reallocation toward less productive firms. *(ii)* The PPP prevents business exits in less productive firms that account for only a small fraction of total employment. The lack of aggregate impact echoes previous findings in the literature (see Crouzet and Mehrotra, 2020) showing that although smaller firms are more exposed to aggregate volatility, the difference has only modest implications on aggregate fluctuations. Therefore, policies aiming to stabilize smaller firms should not be expected to have large macroeconomic consequences.

Next, we simulate a counterfactual policy that is similar to the PPP but only gives financial aids to impacted firms. Since the share of impacted firms is calibrated to be 11%, the fiscal cost of the targeted policy is only one seventh of the cost of the PPP. The targeted policy leads to a smaller employment improvement compared to the PPP over a 10 year period, but it is more cost-effective. Specifically, we compare the cost of an average employment increase by 1% over a 10-year period under the two policies and find that the cost of the targeted policy is 63% of the cost of the PPP.

Targeting rescue aids to impacted firms has long-run implications. We decompose the effects of the PPP and the targeted policy over ten years into the short-run (the first two quarters), the medium run (quarter 3 to the end of year 3), and the long run (years 4–10). We find that the PPP has not only a short-term, but also a highly persistent effect in reducing firm exit because it improves the balance sheet of its recipients. As a result, there is a persistent reallocation of resources from the corporate sector to small firms. By contrast, the targeted policy largely eliminates the short-run increase in the exit rate without generating persistent sectoral reallocation effects. Further, the targeted policy drastically reduces the emergence of "zombie firms" whose survival is socially inefficient.

Related literature Our work relates to different strands of the macroeconomic literature. Several studies analyze the impact of health policies in the COVID-19 pandemic by integrating epidemiological dynamics into macroeconomic general-equilibrium models (e.g. Eichenbaum et al., 2020; Glover et al., 2020) or demand and supply spillovers in multi-sector models (e.g. Guerrieri et al., 2020; Baqaee and Farhi, 2020). By focusing on the effects of business rescue policies, our model features a representative household and treats the pandemic shock as an exogenous event which impacts both the productivity of firms and household preferences for consumption and leisure in order to generate the factual employment, consumption and investment responses during the 2020 recession. While simplifying our model analysis, this modeling choice obviously rules out potential feedback effects of business rescue policies on the health sector.

Other recent work evaluates the macroeconomic and distributional impacts of fiscal policy in the COVID-19 recession. Bayer et al. (2020) and Bigio et al. (2020) analyze conditional and unconditional transfers to heterogeneous households. Faria-e-Castro (2021) studies the effectiveness of different types of fiscal policy, including transfers to firms, without considering firm heterogeneity. Complementary to these studies, our work focuses on the macroeconomic and welfare consequences of business rescue policies, while abstracting from distributional implications.

Further contributions examine the role of firms in the pandemic recession. Bilbiie and Melitz (2020) show how price rigidity amplifies the entry and exit dynamics, and Elenev et al. (2020) study the impact of different firm bailout policies, focusing on the linkages between the financial intermediation and production sectors. Gourinchas et al. (2022) calibrate a static model with heterogeneous firms and find that government grants were quite effective in reducing business exits but also costly due to the lack of targeting. These papers do not allow for persistent firm heterogeneity by productivity or financial assets and thus they do not examine the reallocation of production factors across firms and over time.

Most closely related to our work are Buera et al. (2021a) and Jo et al. (2021). Buera et al. (2021a) examine the impact of a pandemic shock on heterogeneous firms facing financial frictions, also including occupational choice and labor market frictions. Jo et al. (2021) use a model setting similar to ours which additionally features households with different health status (and hence endogenous pandemic dynamics). Different from our work, both papers do not analyze the role of government grants to small firms for firm selection and macroeconomic dynamics, and their models do not feature the partial irreversibility of capital investments that is central for our study.

Finally, we build on a large literature that incorporates heterogeneous, financially constrained firms into macroeconomic models. Our model is based on Khan and Thomas (2013), where we simplify their setup by featuring fixed, partially irreversible capital investments. As in Khan et al. (2016), entry and exit are endogenous, yet all debt is secured by collateral so that default does not occur. The long-term macroeconomic impact of credit-subsidy policies on heterogeneous firms has been studied by Buera et al. (2013), Buera et al. (2021b) and Jo and Senga (2019). While they focus on stationary environments, the reallocation effects via extensive (entry and exit) and intensive (factor intensity) margins are common to our work.

The rest of this paper is organized as follows. Section 2 briefly reviews the response of the U.S. economy in the COVID-19 recession and the PPP policy. Section 3 presents the model and Section 4 the calibration. Section 5 shows findings of the paper, and Section 6 concludes.

2 COVID-19 Pandemic in the U.S. and Rescue Policies

The recession induced by the COVID-19 pandemic differs from past recessions in important ways. The pandemic shock to the macroeconomy is deep but short-lived. Figure 1 shows the macroeconomic impact of the COVID-19 pandemic on the U.S. economy. Compared to the first quarter of 2020, the aggregate economy took a dramatic downturn in the second quarter of 2020: total non-farm output fell by 10.9%, employment by 12.9%, consumption by 9.7% and private domestic investment by 15.4%.

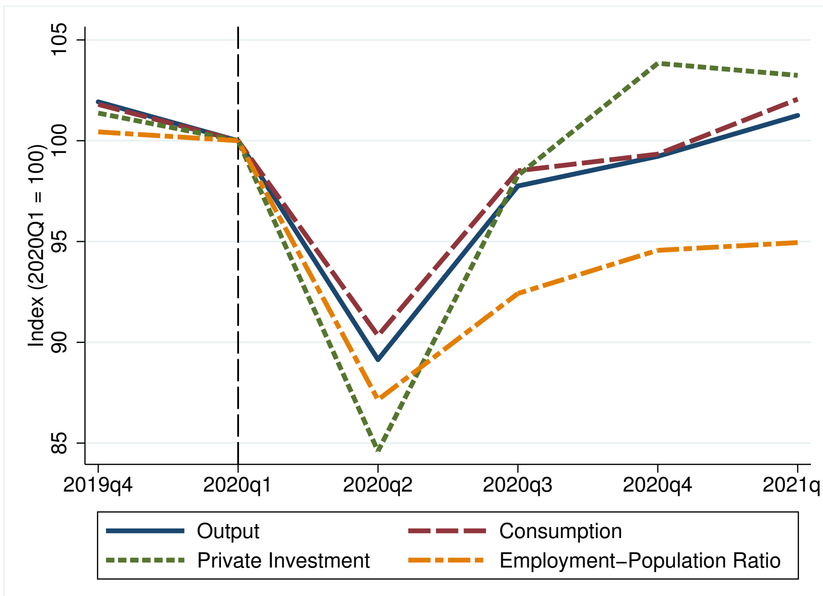


Figure 1: Macroeconomic Impacts of the COVID-19 Pandemic

To mitigate the spread of the coronavirus, many governments imposed strict shutdown and social-distancing policies at the beginning of the pandemic. The economic impact of the pandemic was felt disproportionately by small businesses as they face tighter borrowing constraints and may experience greater difficulties to liquidate their fixed capital. Bartik et al. (2020) report that small businesses are more likely to experience closures (temporary or permanent) than larger businesses. Bloom et al. (2021) show that small firms experience a larger drop in sales. Based on data from the ADP Research Institute, Cajner et al. (2020) show that firms with fewer employees experience greater employment losses compared to larger firms. Based on the their data, employment in firms with fewer than 500 employees drops by 23% relative to the pre-pandemic level, whereas employment in larger firms drops by only 18%.

Businesses offering "social goods" are particularly vulnerable. Bartik et al. (2020) report that industries with an above-average business closure rate include personal services (86%), arts and entertainment (71%), tourism (61.5%), restaurants (54%) and retail (except grocery, 52%).

In response to the potentially devastating impacts of the pandemic on small businesses, the U.S. allocated over $600 billion for the Paycheck Protection Program (PPP) starting in April 2020.[2] The program offers forgivable loans up to 2.5 times the average monthly payroll to small businesses with up to 500 employees. PPP loans feature an attractive interest rate of 1% p.a. and can be forgiven when certain requirements are met. Appendix A provides details on the terms of PPP loans and forgiveness requirements. According to the U.S. Small Business Administration, as of November 2021, 92% of all PPP loans issued in 2020 have been fully or partially forgiven.

Despite the high degree of heterogeneity in the pandemic impact on businesses, the distribution of PPP loans is largely untargeted and prioritizes speedy loan disbursement (Autor et al., 2022). According to Autor et al. (2020), initially about 70% of eligible firms applied for PPP loans, and Borawski and Schweitzer (2021) report that by the end of 2020 PPP loans had been taken up by 76% of U.S. small businesses.[3] It is not clear what the effect of targeting rescue policies to impacted firms is, and more generally, how effective small-business rescue policies are for macroeconomic outcomes.

A few empirical studies estimate the initial employment effect of the PPP program. Utilizing the PPP eligibility size threshold to differentiate treatment and control groups, Autor et al. (2020, 2022) find that the policy increased employment at eligible firms by about three percent at a high cost of around $200,000 per job-year. Using different data sources, Chetty et al. (2020) and Hubbard and Strain (2020) find somewhat smaller employment effects in the range of 1-2%. Bartlett III and Morse (2020) and Hubbard and Strain (2020) also find that the PPP had a significant impact on the survival probability of smaller businesses. Finally, Kurmann et al. (2021) use real-time establishment data and find that pandemic policies, such as the PPP, are effective in mitigating the negative employment effect on small businesses by relaxing financial constraints.

To study the impact of a business rescue policies in the short- and long-run, including spillover effects and reallocation of production factors in general equilibrium, we consider a model in which financially constrained small firms and unconstrained large firms are differentially impacted by a pandemic shock.

[2]In this paper, we only focus on the small firm rescue program issued in the year 2020.
[3]The total take-up rate of PPP loans in both 2020 and 2021 is around 94% (Autor et al., 2022).

3 The Model

We consider a discrete-time general equilibrium model of a closed economy. A single consumption/investment good is produced by firms that belong either to the corporate or to the non-corporate sector.[4] While firms in the corporate sector face no financial frictions, non-corporate firms (termed "small firms" in the following) can only borrow against the collateral value of their capital and they cannot raise equity from outside investors. Entry and exit into the non-corporate sector are endogenous outcomes. In particular, small firms face idiosyncratic productivity risk and may decide to liquidate their firm if they are unable or unwilling to continue operating.

While there is no aggregate risk, we consider the economy's response to a one-time unexpected shock and the transition path back to the unique steady-state equilibrium. To simplify notation, we index several variables by the time index t, indicating dependence on the aggregate state vector which is either constant (in steady state) or converges deterministically after the one-time shock.[5] The one-time shock includes a shut-down shock on a fraction of small firms, a TFP shock on the corporate sector, and shocks to household preferences over consumption and leisure.

3.1 Environment

3.1.1 Small Firms

Small firms operate a fixed amount of capital κ which is drawn from a finite set $\mathbb{K}$ upon entry and constant over time.[6] Capital is partly irreversible: upon exit only the fraction $\theta < 1$ of capital can be liquidated. Every period, the fraction δ of capital depreciates and is replaced by the firm's owner. Small firms further incur fixed operating cost $c^f(\kappa)$ which capture general and administrative expenses.

A small firm with capital κ hires labor services ℓ to produce output $xf(\kappa, \ell)$ where f is a strictly increasing, strictly concave, and decreasing returns to scale production function and x

[4]To keep our model reasonably simple, we do not differentiate between the goods produced for social and non-social consumption. Given that many social goods have close non-social substitutes (e.g. restaurants vs food at home, health clubs vs home gyms, or movie theaters vs streaming services), this seems a reasonable short-cut.

[5]Further simplifying notation, the time index is dropped from the firms' state and decision variables in which case we use the prime superscript to denote next period values of these variables.

[6]As common with the putty-clay literature (e.g. Gilchrist and Williams, 2000; Gourio, 2011; Sorkin, 2015), this assumption limits capital-labor substitutability in the short-run, while capital reallocations occur via the business entry and exit margin.

is exogenous, idiosyncratic productivity which follows a monotone Markov chain on finite set $\mathbb{X}$ with transition probabilities $g(x'|x)$. Write

$$\pi_t(x,\kappa) \equiv \max_{\ell \geq 0} x f(\kappa,\ell) - w_t \ell - \delta\kappa - c^f(\kappa)$$

for the operating profit of a small firm with capital κ and with productivity x in period t, where w_t is the real wage.

Small firms can save and borrow in the capital market at safe gross interest rate $1/q_t$. Borrowing is secured by the collateral value of capital;[7] hence the debt issued in any period is constrained by the liquidation value of capital,

$$b' \leq \theta\kappa \ . \tag{1}$$

Furthermore, small firms cannot raise equity which implies that the dividend income in period t must be non-negative:

$$\pi_t(x,\kappa) - b + q_t b' \geq 0 \ , \tag{2}$$

where b is the debt due in period t (or the negative value of savings when $b < 0$).

At the beginning of every period, a firm that was active in the previous period may exit due to two distinct events. First, low-productivity and high-debt firms may not be able to fulfill the two financial constraints (1) and (2); these *illiquid* firms are then forced to exit. Second, the firm may voluntarily decide to exit. Regardless of the cause of exit, the firm's capital is liquidated and the owner retains the liquidation value net of debt repayment (or financial savings when $b < 0$) which is $\theta\kappa - b$.

Also at the beginning of every period, a mass M of potential entrants draw initial productivity, debt levels, and capital (x, b, κ) from the joint distribution $\Phi(x, b, \kappa)$ and decide to enter whenever the value of the new firm exceeds the installation cost net of debt, $\kappa - b$.

3.1.2 Corporate Firms

Firms in the corporate sector use capital K^c and labor services L^c to produce output $F(K^c, L^c)$ with strictly increasing, concave, constant returns production function F. Write $\Pi_t(K^c) = \max_L F(K^c, L^c) - w_t L^c$ for operating profits in the corporate sector in period t. Capital in the

[7]The price of collateral in our one-sector economy is identical to the price of final output, and hence does not respond to aggregate shocks. As in Kiyotaki and Moore (1997), a fall in the price of collateral would amplify a recessionary shock through a tightening of financial constraints. While commercial property prices declined during the COVID-19 recession, the drop was not nearly as severe as during the Great Recession 2008-09.

corporate sector also depreciates at rate δ.

3.1.3 Households

There is a unit mass of households with unit time endowment who derive period utility $u(C, 1-L)$ which is strictly increasing in consumption C and leisure $1-L$, with L denoting labor supply. Future utility is discounted with factor $\beta < 1$ per period. Households own all firms.

Households can perfectly insure against all idiosyncratic business risks. As a result, their consumption, labor supply and investment decisions are described by the utility maximization problem of a representative household. In period t, the representative household takes as given the aggregate state vector $X_t = (K_t^c, A_t, \mu_t^0)$ where K_t^c is the capital stock in the corporate sector, A_t are financial assets, and μ_t^0 is the measure of small firms over idiosyncratic states (x, b, κ) prior to entry and exit. The household decides about consumption C_t, labor supply L_t, gross investment I_t^c in the corporate sector, and financial assets for the next period A_{t+1} which are traded at price q_t. Further, the household decides the entry of small firms $d_t^e(x, b, \kappa)$ which equals one when potential firm (x, b, κ) enters and zero otherwise, liquidation of small firms $d_t^l(x, b, \kappa)$ which equals one when firm (x, b, κ) is liquidated and zero otherwise, and borrowing/savings decisions $b_t'(x, b, \kappa)$ of active firms. In recursive notation, the representative household's problem is

$$V_t(X_t) = \max \; u(C_t, 1-L_t) + \beta V_{t+1}(X_{t+1}) \; , \tag{3}$$

subject to the budget constraint

$$\begin{aligned} C_t + I_t^c + q_t A_{t+1} + M \int [\kappa - b] d_t^e(x, b, \kappa) \; d\Phi(x, b, \kappa) \leq \; & w_t L_t + A_t + \Pi_t(K_t^c) \\ & + \int \pi_t(x, \kappa) - b + q_t b_t'(x, b, \kappa) \; d\mu_t(x, b, \kappa) \\ & + \int (\theta\kappa - b) d_t^l(x, b, \kappa) \; d\mu_t^0(x, b, \kappa) \; , \end{aligned} \tag{4}$$

where

$$\mu_t = (1 - d_t^l)\mu_t^0 + M d_t^e \Phi \tag{5}$$

is the measure of active firms in period t (i.e., incumbent firms that are not liquidated plus entrants), the accumulation equation for capital in corporate firms,

$$K^c_{t+1} = (1-\delta)K^c_t + I^c_t \ , \tag{6}$$

financial constraints for continuing small firms,

$$b'_t(x,b,\kappa) \leq \theta\kappa, \qquad \pi_t(x,\kappa) - b + q_t b'_t(x,b,\kappa) \geq 0 \ , \tag{7}$$

and the dynamic evolution of the distribution measure of small firms which requires that the measure of small firms at the beginning of the next period satisfies

$$\mu^0_{t+1}(A) = \int \mathbb{I}_{(x',b'_t(x,b,\kappa),\kappa)\in A}\, g(x'|x)\ d\mu_t(x,b,\kappa) \ , \tag{8}$$

for all Borel sets $A \subset \mathbb{X} \times \mathbb{R} \times \mathbb{K}$.

The budget constraint (4) says that the household's expenditures for consumption, investment in corporate firms, financial assets and the investment expenditures of entrant firms (left-hand side) do not exceed the sum of labor income, the stock of financial assets at the beginning of the period, profit incomes of corporate and small firms, and the liquidation values of exiting small firms (right-hand side).

3.2 Competitive Equilibrium

3.2.1 Definition

Given an initial state (K^c_0, A_0, μ^0_0) with $A_0 = \int b \ d\mu^0_0(x,b,\kappa)$, a *competitive equilibrium* is a sequence of market prices (w_t, q_t), consumption, labor supply and investment decisions $(C_t, L_t, I^c_t, K^c_{t+1}, A_{t+1})$, entry, exit and borrowing decisions (d^e_t, d^l_t, b'_t) and distribution measures μ^0_{t+1}, μ_t of small firms, for all $t \geq 0$, such that

(i) The representative household solves the utility maximization problem (3)–(8).

(ii) The markets for labor and financial assets clear, i.e. in all periods t,

$$L_t = \int \ell_t(x,\kappa)\ d\mu_t(x,b,\kappa) + L^c_t \ ,$$
$$A_{t+1} = \int b'_t(x,b,\kappa)\ d\mu_t(x,b,\kappa) \ ,$$

with labor demand $\ell_t(x,\kappa) = \text{argmax}_\ell\, xf(\kappa,\ell) - w_t\ell$ and $L^c_t = \text{argmax}_{L^c}\, F(K^c_t, L^c) - w_t L^c$.

A *stationary competitive equilibrium* is a competitive equilibrium with constant state vector (K^c, A, μ^0) and constant market prices (w, q).

The goods market is in equilibrium because of Walras's law: the binding budget constraint together with the other market-clearing conditions implies that consumption plus investment equals aggregate output, $C_t + I_t = F(K_t^c, L_t^c) + \int [xf(\kappa, \ell_t(x)) - c^f(\kappa)] \, d\mu_t(x, b, \kappa)$, where

$$I_t = I_t^c + M \int \kappa d_t^e(x, b, \kappa) \, d\Phi(x, b, \kappa) - \int \theta \kappa d_t^l(x, b, \kappa) \, d\mu_t^0(x, b, \kappa) + \delta \int \kappa \, d\mu_t(x, b, \kappa)$$

is aggregate gross investment in corporate firms and in entrant firms. The accumulation equation for aggregate capital, denoted by $K_t = K_t^c + \int \kappa \, d\mu_t^0(x, b, \kappa)$, is

$$K_{t+1} = K_t - D_t + I_t \ ,$$

where

$$D_t = \delta K_t^c + \delta \int \kappa \, d\mu_t(x, b, \kappa) + (1 - \theta) \int \kappa d_t^l(x, b, \kappa) \, d\mu_t^0(x, b, \kappa)$$

is aggregate depreciation. Note that $(1 - \theta)\kappa$ is the loss of capital when a firm is liquidated.

3.2.2 Equilibrium Characterization

The first-order condition of the household's financial savings problem implies that the asset price is

$$q_t = \beta \frac{u_C(C_{t+1}, 1 - L_{t+1})}{u_C(C_t, 1 - L_t)} \ . \tag{9}$$

Let $v_t^0(x, b, \kappa)$ be the value of a small firm before entry and exit decisions and let $v_t(x, b, \kappa)$ be the value after these decisions. These values represent the marginal contributions of the small firm to the household's utility, measured in units of the period-t consumption/investment good. That is, payments accruing in the next period $t+1$ are priced with the financial discount factor q_t. Therefore, the value of a continuing small firm satisfies the Bellman equation

$$v_t(x, b, \kappa) = \max_{b'} \pi_t(x, \kappa) - b + q_t b' + q_t \mathbb{E}_{x'|x} v_{t+1}^0(x', b', \kappa) \ , \tag{10}$$

$$\text{s.t. } b' \leq \theta \kappa \quad \text{and} \quad \pi_t(x, \kappa) - b + q_t b' \geq 0 \ .$$

At the beginning of the next period, the continuation value is $\mathbb{E}_{x'|x} v_{t+1}^0(x', b', \kappa)$ where the expectation is taken over realizations of next period's productivity x' conditional on current productivity x.

The value of a firm at the beginning of period t and prior to exit decisions, $v_t^0(x, b, \kappa)$, equals $\theta\kappa - b$ when the firm is illiquid, i.e. if $\pi_t(x, \kappa) - b + q\theta\kappa < 0$, or when $v_t(x, b, \kappa) < \theta\kappa - b$ in which case the firm is voluntarily liquidated. Otherwise the firm remains active. This implies that

$$v_t^0(x, b, \kappa) = \begin{cases} \theta\kappa - b & \text{if } \pi_t(x, \kappa) - b + q\theta\kappa < 0 \text{ or } v_t(x, b, \kappa) < \theta\kappa - b \ , \\ v_t(x, b, \kappa) & \text{else} \ , \end{cases} \tag{11}$$

with liquidation policy function

$$d_t^l(x, b, \kappa) = \begin{cases} 1 & \text{if } \pi_t(x, \kappa) - b + q\theta\kappa < 0 \text{ or } v_t(x, b, \kappa) < \theta\kappa - b \ , \\ 0 & \text{else.} \end{cases} \tag{12}$$

Regarding entry decisions, it is optimal to invest into a new firm (x, b, κ) in period t if the firm value is greater than the installation cost net of debt, which leads to the entry policy function

$$d_t^e(x, b, \kappa) = \begin{cases} 1 & \text{if } v_t(x, b, \kappa) \geq \kappa - b \ , \\ 0 & \text{else.} \end{cases} \tag{13}$$

Finally, the first-order conditions for labor supply and investment in corporate firms are

$$0 = \ u_C(C_t, 1 - L_t)w_t - u_{1-L}(C_t, 1 - L_t) \ , \tag{14}$$

$$1 = \ q_t \left[1 - \delta + F_K(K_{t+1}^c, L_{t+1}^c)\right] \ . \tag{15}$$

3.2.3 Firm Policies in Stationary Equilibrium

The borrowing and savings policies of small firms can be characterized as in Khan and Thomas (2013). Firms with sufficiently high savings (low debt) are not threatened by illiquidity in any future state. These *unconstrained firms* are able to pay positive dividends while keeping the buffer stock of financial savings sufficiently high. On the other hand, if financial savings are low (or debt is high), a firm may expect a future state of illiquidity (and hence forced exit) with positive probability. These *constrained firms* value retained earnings higher than dividends; therefore they pay no dividends until they build up enough savings and become unconstrained, unless they exit before.

We describe the savings policies and value functions of both firm types in a stationary equilibrium. The same logic applies for the transitional dynamics and is described in Appendix E. In stationary equilibrium, the financial discount factor is $q = \beta$ and the time index is dropped from all variables. Consider first the Bellman equations of unconstrained firms:

$$v(x,b,\kappa) = \ \pi(x,\kappa) - b + qb'(x,b,\kappa) + q\mathbb{E}_{x'|x} v^0(x', b'(x,b,\kappa), \kappa) \ , \tag{16}$$

$$v^0(x,b,\kappa) = \ \max[\theta\kappa - b, v(x,b,\kappa)] \ , \tag{17}$$

where the policy function $b'(x,b,\kappa)$ satisfies the two financial constraints (7) and ensures that the firm remains unconstrained in the next period (as verified below).[8] These two equations demonstrate that value functions of unconstrained firms take the form $v^0(x,b,\kappa) = V^0(x,\kappa) - b$ and $v(x,b,\kappa) = V(x,\kappa) - b$ (before and after exit). In words, the marginal value of financial assets held by the firm equals one. This is because a marginal payout today has the same value for the household owner as keeping these funds in the firm and receiving the payout later on. It further follows from the first Bellman equation that unconstrained firms are indifferent regarding the level of savings $-b'(x,b,\kappa)$, as long as they ensure that the firm remains unconstrained in the future. Dropping financial savings from the Bellman equation obtains

$$V(x,\kappa) = \pi(x,\kappa) + q\mathbb{E}_{x'|x} \max[\theta\kappa, V(x',\kappa)] \ . \tag{18}$$

The unique solution of this equation is strictly increasing in x (because π is strictly increasing in x and since the Markov process for x is monotone). This defines a cutoff productivity level

$$\tilde{x}(\kappa) = \min\{x \in \mathbb{X} | V(x,\kappa) \geq \theta\kappa\}$$

such that all unconstrained firms with capital κ and productivity below $\tilde{x}(\kappa)$ liquidate the firm, while all others stay.

To become and remain unconstrained, a firm needs to reduce financial debt below

$$\tilde{b}(\kappa) = \frac{\pi(\tilde{x}(\kappa), \kappa)}{1-q}$$

(or build savings exceeding $-\tilde{b}(\kappa)$). To see this, consider a firm with capital κ entering the period with $b \leq \tilde{b}(\kappa)$. If this firm's productivity is $x \geq \tilde{x}(\kappa)$, the firm can enter the next period with debt $b' = \tilde{b}(\kappa)$ and pay non-negative dividends $\pi(x,\kappa) - b + q\tilde{b}(\kappa) \geq 0$. If the firm's productivity is $x < \tilde{x}(\kappa)$, the firm is voluntarily liquidated.[9] In any case, the firm is

[8] Equation (17) permits voluntary exit. By definition, unconstrained firms never become illiquid and hence are never forced to exit.

[9] For unconstrained firms, this follows from the above arguments. For constrained firms, $v(x,b,\kappa) \leq V(x,\kappa) - b < \theta\kappa - b$, and hence liquidation is also optimal if $x < \tilde{x}(\kappa)$.

either voluntarily liquidated or the firm remains unconstrained in the next period (because of $b' = \tilde{b}(\kappa)$).

If a firm's debt is larger than $\tilde{b}(\kappa)$ (or savings are smaller than $-\tilde{b}(\kappa)$), the firm may become illiquid in the future with positive probability. To see this, suppose that productivity remains at $\tilde{x}(\kappa)$ for a sufficiently long time. Then the zero-dividend borrowing policy $b' = \frac{1}{q}(b - \pi(\tilde{x}(\kappa), \kappa))$ will over time lead to exploding levels of debt and eventually violate the borrowing constraint (as would any other policy with positive dividends). For such constrained firms it is optimal to pay zero dividends until savings exceed $-\tilde{b}(\kappa)$ because the value of retained earnings exceeds the one for a dividend payout. These considerations imply that the value and policy functions of constrained firms are

$$v(x, b, \kappa) = \max[0, \pi(x, \kappa) - b + q\tilde{b}(\kappa)] + q\mathbb{E}_{x'|x} v^0(x', b'(x, b, \kappa), \kappa) \ , \tag{19}$$

$$v^0(x, b, \kappa) = \max[\theta\kappa - b, v(x, b, \kappa)] \ , \tag{20}$$

$$b'(x, b, \kappa) = \max\left[\tilde{b}(\kappa), \frac{1}{q}(b - \pi(x, \kappa))\right] \ , \tag{21}$$

if $\pi(x, \kappa) - b + q\theta\kappa \geq 0$, and $v(x, b, \kappa) = v^0(x, b, \kappa) = \theta\kappa - b$ otherwise.

3.3 Pandemic Shock and Rescue Policies

Suppose that the economy is in stationary equilibrium and that the COVID-19 pandemic shock hits in period $t = 0$. The shock is a one-time unexpected event that fades out over time.[10] We assume that the pandemic not only affects productivity, but also the demand for goods and services and the willingness of households to participate in the labor market.

Specifically, the shock has four components. First, a TFP shock ν_t^c on the corporate sector such that the corporate production function becomes $(1 + \nu_t^c)F(K_t^c, L_t^c)$. Second, a shut-down shock ν_t^n on a fraction η_i of impacted small firms such that the firm productivity of impacted firms becomes $(1+\nu_t^n)x$. We interpret the impacted firms as those small firms producing social goods and services that are particularly vulnerable to government-induced lockdown measures.

Third, a demand shock ν_t^d such that marginal utility of consumption becomes $(1 + \nu_t^d)u_C(C_t, 1 - L_t)$. Such a preference shock captures the observed drop in aggregate consumption at the onset of the pandemic in response to stay-at-home orders and increased health risks of consuming social goods or services. Finally, a labor supply shock ν_t^ℓ such that the marginal utility of leisure becomes $(1 + \nu_t^\ell)u_{1-L}(C_t, 1 - L_t)$. This reflects employment

[10]Since the shock is completely unforeseen, it has no impact on households and firms' behavior ex-ante.

adjustments possibly due to increased heath risks associated with in-person work.

We also take into account the PPP policy in the calibration. Because of the high forgiveness rate of PPP loans (see Section 2), we model the rescue policy as a grant rather than a loan. We assume that an exogenous $\eta = 0.76$ share of firms receive the grant in period $t = 0$ and that the probability of receiving the grant is independent of whether the firm is impacted, thus capturing the lack of targeting of the PPP policy.[11] Moreover, the grant is unconditional and does not need to be repaid. As a robustness check, we show in Appendix C that our main results hold when the grant is conditional on payroll spending.

Let $b_p(x_0, \kappa)$ be the amount of the grant offered to a firm with capital κ and whose absent-of-shock productivity is x_0 in the impact period ($t = 0$). In line with the actual PPP policy, we assume that the grant amount is equal to 10 weeks of payroll as follows:

$$b_p(x_0, \kappa) = X_p w^* \ell^*(x_0, \kappa) \ , \tag{22}$$

where $X_p = 2.5/3$ (our model period is a quarter), w^* is the wage rate in the steady state, and $\ell^*(x_0, \kappa)$ is labor demand in the steady state.

To model the grant, we only need to modify the profit of small firms on the transition path. In $t = 0$, the profit function reads

$$\pi_0(x, \kappa, \iota, s) = \max_{\ell \geq 0} (1 + \nu_0^n \iota) x f(\kappa, \ell) + s b_p(x, \kappa) - w_0 \ell - \delta\kappa - c^f(\kappa) \ , \tag{23}$$

where ι is a dummy variable indicating whether the firm is impacted by the pandemic and s is a dummy variable for receipt of the grant. In $t \geq 1$, we have

$$\pi_t(x, \kappa, \iota) = \max_{\ell \geq 0} (1 + \nu_t^n \iota) x f(\kappa, \ell) - w_t \ell - \delta\kappa - c^f(\kappa) \ .$$

Note that it is always optimal for firms to choose to take up the maximum grant because grants prevent liquidations (for constrained firms) or raise dividends (for unconstrained firms). We provide further details and describe how we solve the model on the transition path in Appendix E.2.

The government finances the rescue grant by imposing a lump-sum tax on households. Since households own small firms who benefit from the rescue grant and since households are perfectly insured, the fiscal cost of the grant has no direct bearing on household consumption

[11] Using data from the Small Business Administration, Borawski and Schweitzer (2021) document that 76% of U.S. small businesses received the PPP loan in 2020. In addition, PPP loans reached small businesses in all industries without substantial variation.

and labor supply decisions apart from the effects on equilibrium prices. In addition, since the representative household is financially unconstrained and there is no distortionary taxation, the timing of lump-sum taxation is irrelevant as Ricardian equivalence applies in our model.[12]

4 Calibration

4.1 Steady-State Calibration

We calibrate the model in steady state to the U.S. economy prior to the COVID-19 pandemic. Each period in our model corresponds to a quarter of a calendar year. We define small firms as businesses with fewer than 500 employees, which is the threshold for eligibility of the PPP loans.

4.1.1 Functional Form Assumptions

In the corporate sector, the production function is

$$F(K^c, L^c) = A(K^c)^{\alpha}(L^c)^{1-\alpha} \ .$$

In the non-corporate sector, each firm has the production technology

$$f(\kappa, \ell) = A(\kappa^{\gamma_1}\ell^{1-\gamma_1})^{\gamma_2} \ ,$$

where $\gamma_1 \in (0,1)$ is the capital share and $\gamma_2 \in (0,1)$ is a span-of-control parameter. The production function exhibits decreasing returns to scale, possibly due to diminishing returns of managerial supervision as in Lucas (1978). The log productivity $\ln(x)$ follows an AR(1) process with mean $\ln(\bar{x})$, standard deviation ε_x and autocorrelation ρ_x.

Among potential firm entrants, the distribution of initial productivity is independent from the initial distribution of debt and capital. We assume that, conditional on κ, the initial debt distribution is such that $\theta\kappa - b$ is exponentially distributed with λ for all $\kappa \in \mathbb{K}$.

We assume that the initial productivity is drawn from a log-normal distribution such that $\ln(x_0\bar{x})$ and $\varepsilon_x^2/(1-\rho_x^2)$ are, respectively, the mean and variance of $\ln(x_0\bar{x})$. $x_0 < 1$ is a shift parameter that captures a smaller size of startups compared to incumbent firms.

[12]Of course, Ricardian equivalence can fail for many reasons, such as distortionary taxation. While studying the timing of taxation and the implications of rising public debt during the COVID-19 pandemic is an interesting issue, its analysis is beyond the scope of this paper.

The utility function of the representative household is

$$U(C, 1-L) = \frac{C^{1-\sigma}}{1-\sigma} + \zeta(1-L) \ .$$

4.1.2 Calibration Strategy and Data

Table 1 shows the values of parameters that are determined outside of the model. The resale value of capital θ is taken from Lanteri and Rampini (2021), which is in the range of estimated values in the literature (Li et al., 2016). Following Jo and Senga (2019), we assume that $\gamma_1 = 0.3182$ and $\gamma_2 = 0.88$.

Parameter	Description	Value	Source
β	Subjective discount factor	0.989	Annual interest rate of 4%
α	Capital Share corporate sector	0.300	Standard
δ	Capital depreciation rate	0.015	Annual depreciation rate of 6%
γ_1	Capital Share small firms	0.318	Jo and Senga (2019)
γ_2	Span of control	0.880	Jo and Senga (2019)
A	TFP shifter	0.250	Normalization
θ	Resale value of capital	0.500	Lanteri and Rampini (2021)
$\bar{x}$	Mean of $\ln(x)$	1.000	Normalization
σ	CRRA utility coefficient	2.000	Standard

Table 1: External Parameters

Table 2 lists parameters calibrated internally and the main data targets that help identifying each parameter. It is well understood that all these parameters jointly take an impact on various model statistics, but we can nonetheless determine which parameter mostly affects which target. To calibrate the parameters, we compute the model counterparts of these targets and choose parameter values to minimize the sum of squared percentage distances between model and data moments. Appendix F provides additional information on how some of the moments are calculated.

Most data targets are obtained from publicly available tables from the Statistics of U.S. Businesses (SUSB) and the Business Dynamics Statistics (BDS) in 2010-2018. We use the confidential data from the Kauffman Firm Survey (KFS) to calculate statistics involving firms' balance sheet information and firm dynamics. The KFS is a single-cohort longitudinal dataset. The first survey of KFS was conducted in 2004 on a representative sample of new firms. A follow-up survey is conducted every year until 2011. We obtain fixed expenses from an analysis by Sageworks based on Census 2007 data.

Parameter	Description	Value	Main Target	Data Source
Preferences				
ζ	Marginal utility of leisure	27.719	Time spent in market work	Standard
AR(1) of idiosyncratic productivity x				
ε_x	Standard deviation	0.146	Small firm exit rate	BDS
ρ_x	Autocorrelation	0.969	Autocorrelation of employment in continuing small firms	KFS
Entry and exit				
M	Mass of potential entrants	43.490	Small firm share of employment	SUSB
x_0	Productivity shifter of entrants	0.101	Average employment in entrants	BDS
λ	Initial debt distribution	0.314	Debt to asset ratio, share of firms with debt	KFS
Small firm technology				
$\kappa_1; \kappa_2$	Capital levels	$15.3; 215.2$	Average employment in small firms	SUSB
			Employment share by firm size	SUSB
Φ_κ	Prob. of κ_1	0.242	Firm size distribution	SUSB
$c^f(\kappa_1); c^f(\kappa_2)$	Fixed costs	$0.43; 4.92$	Fixed expense to revenue ratio	Sageworks
			Firm exit rate by firm size	BDS

Notes: Our sources include the Business Dynamics Statistics (BDS), Statistics of U.S. Businesses (SUSB), Kauffman Firm Survey (KFS) and data from Sageworks.

Table 2: Internal Parameters and Calibration Targets

4.1.3 Parameters and Model Fit

Table 2 shows the calibrated parameter values, and Table 3 and Figure 2 show the fit of targeted moments. Our model fits the data well. Although we use only two levels of fixed capital, we generate variation of firm shares, employment shares and exit rates across four firm-size classes. In particular, we match the fact that the distribution of small businesses is heavily skewed towards firms with fewer employees (see Figure 2.a): Around 80% of small businesses have less than 10 employees and only a small fraction, 1.7%, has more than 100 employees.[13] We also match relatively well the employment shares by firm size bins, which are more evenly distributed across the four bins. In addition, the exit rate in our model decreases in firm size, as the data indicates. We further replicate the pattern that entering firms have fewer employees.

Moment	Data	Model
Average employment in small firms	9.2516	9.5054
Small firm share of employment	0.4895	0.5836
Small firm exit rate	0.0198	0.0244
Average employment in entrants	5.2935	5.6323
Fixed expense to revenue ratio	0.2448	0.1821
Autocorr. employment	0.9667	0.9294
Debt to asset ratio	0.0820	0.0831
Time spent in market work	0.3300	0.2947
Share of firms with debt	0.3288	0.2732
Exit rate by employment size bins		
0 to 9	0.0246	0.0297
10 to 19	0.0048	0.0040
20 to 99	0.0033	0.0031
100 to 499	0.0016	0.0000

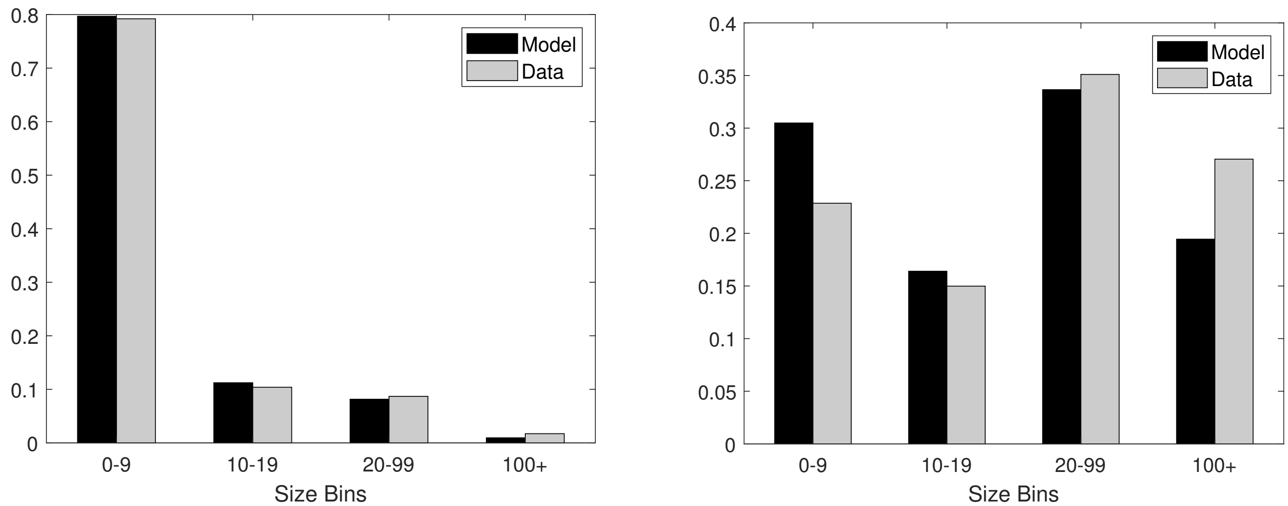
Notes: The table shows some model statistics of the benchmark economy and the empirical counterparts based on data from BDS, SUSB and KFS. Firm and employment shares by employment size are shown in Figure 2.

Table 3: Model Fit

To illustrate the dependence of exit and entry decisions on productivity and debt, we show the optimal policy for the small firms with $\kappa = \kappa_2$ in Figure 3.[14] Incumbent firms with high debt and low productivity are forced to liquidate because they are unable to pay a positive dividend. Remaining low productivity firms voluntarily liquidate because the value of the

[13]Note that we abstract from non-employer firms.

[14]The policies are qualitatively similar for κ_1. We omit them for the sake of conciseness.



(a) Firm Size Distribution (b) Employment Distribution

Figure 2: Model Fit. Firm Size and Employment Distributions in the Small Firms Sector

(a) Exit Decision (b) Entry Decision

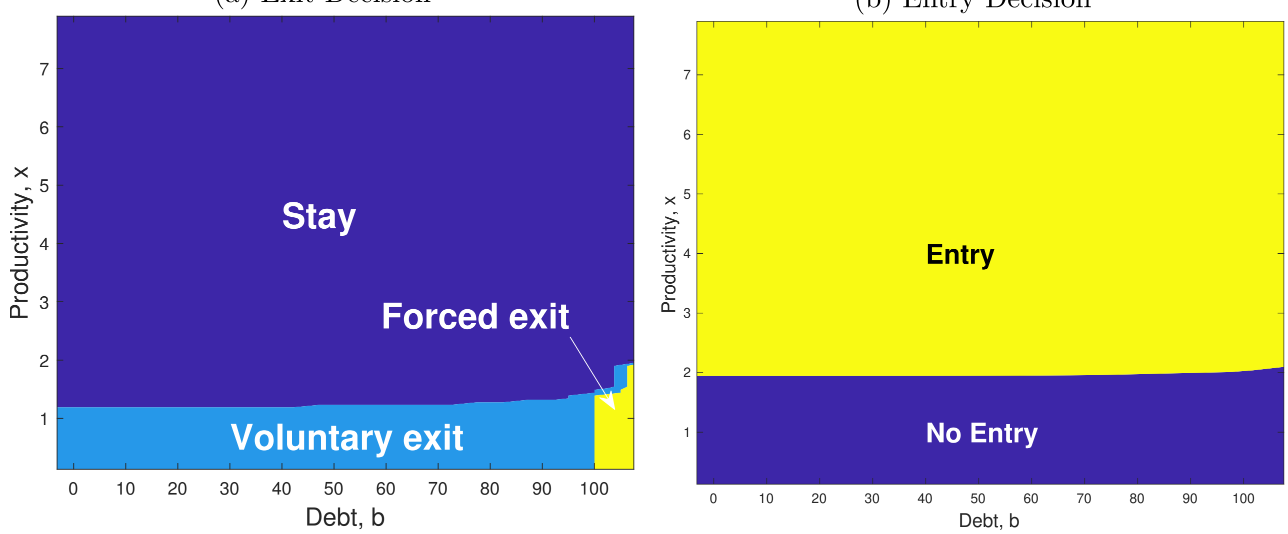


Figure 3: Small Firm Exit and Entry ($\kappa = \kappa_2$)

firm is below the liquidation value. The entry decision depends mainly on firm productivity. Because of the partial irreversibility of capital investment, the productivity threshold for entry is higher compared to the productivity thresholds for staying.

4.2 Calibrating the COVID-19 Shock

The calibration of pandemic shocks is based on the scenario with the PPP grant (see Section 3.3). We assume that the shock decays exponentially with parameter $\rho \in (0, 1)$. That is, for each component of the shock $s \in \{c, n, d, \ell\}$,

$$\nu_t^s = \rho^t \nu^s \text{ for all } t = 0, 1, ...$$

Since some small firms had to shut down under lockdown policies at the beginning of the pandemic, we assume that the shock to impacted small firms is $\nu^n = -1$. We calibrate the initial magnitudes of the other pandemic shocks (ν^c, ν^d, ν^ℓ) and the share of impacted small firms η_i to match the magnitudes of impacts of the COVID-19 pandemic on total output, consumption, employment and employment in small firms in the second quarter of 2020 (which corresponds to the first period of the transition). We calibrate the decay parameter ρ to match the change in total output in the third quarter of 2020.

Table 4 shows the values of calibrated shock parameters and Table 5 compares the pandemic impact in the data to those in the model. Our calibration reveals that 11% of small firms are impacted, so that the TFP shock on the corporate sector is smaller than the average productivity shock in the non-corporate sector. There is a sizable preference shock such that the marginal utility of consumption drops and the marginal disutility of working increases upon impact. The baseline grant economy tracks the observed pandemic impact closely, including the change in private investment and output in small firms, which we do not target in the calibration procedure.

Parameter	Description	Value
η_i	Fraction of impacted small firms	0.110
ν^c	Productivity shock the corporate sector	-0.007
ν^d	Preference shock	-0.184
ν^l	Labor supply shock	0.250
ρ	Autocorrelation	0.112

Table 4: Calibrated Pandemic Shock Parameters

Figure 4 shows the paths of calibrated shocks and aggregate variables in our model. The most salient feature of Figure 4b is the speed of the recovery. With a calibrated persistence $\rho = 0.11$, output bounces back to just 2.2% below the initial steady state level in the second period of the pandemic after falling by almost 11% on impact. The figure also shows that our

Description	Data	Grant (Baseline)
Targeted		
Output, 2020Q2:	-10.857	-11.351
Output, 2020Q3:	-2.246	-3.114
Consumption, 2020Q2:	-9.667	-9.781
Total employment, 2020Q2:	-12.850	-11.760
Employment small, 2020Q2:	-16.021	-14.600
Untargeted		
Private investment, 2020Q2:	-15.398	-19.953
Small firm output, 2020Q2 :	-15.650	-14.938

Notes: The pandemic shocks are calibrated so that the "Grant baseline" economy matches the data. *Data sources:* GDP, consumption, investment and aggregate employment are taken from FRED (fred.stlouisfed.org). Employment by firm size comes from data provided by Cajner et al. (2020), who compute employment changes by firm size using data from ADP. Small firm output is from Bloom et al. (2021).

Table 5: Pandemic Impact and Rescue Policies (% Change from 2020Q1)

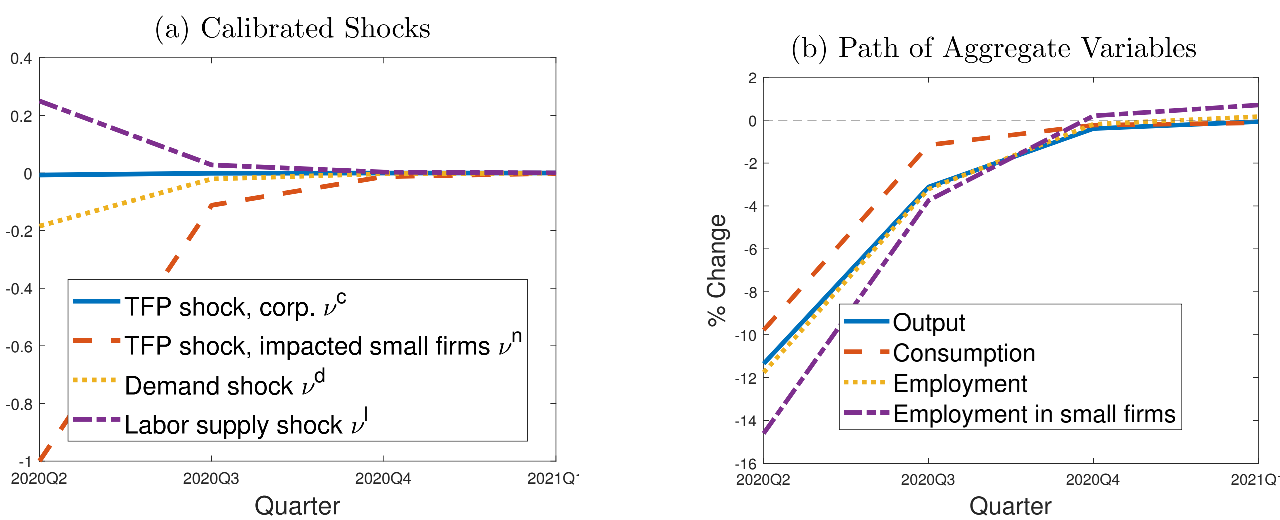
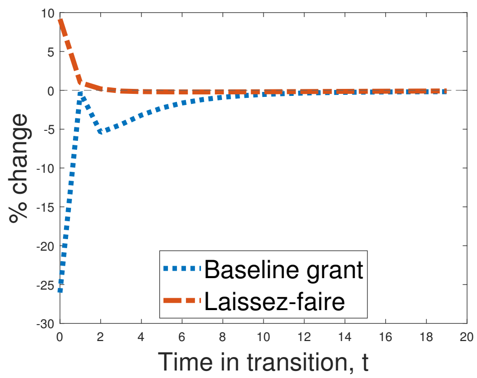
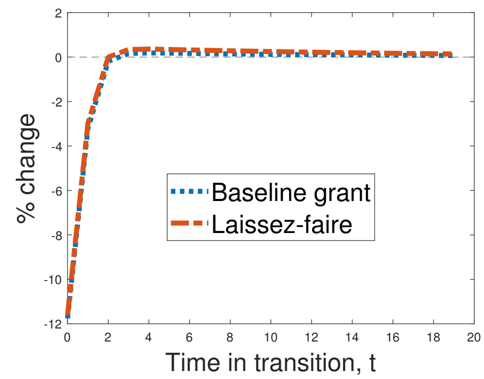


Figure 4: Pandemic Shock Calibration

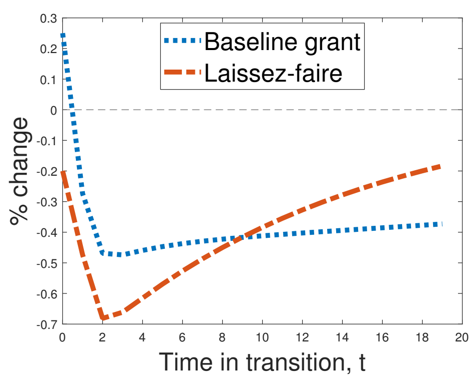
model captures the notable feature of the pandemic recession that consumption drops almost as much as output, in line with the factual dynamics shown in Figure 1.



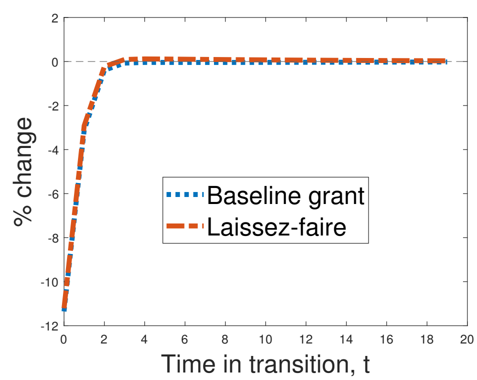
(a) Small Firm Exit Rate



(b) Aggregate Employment



(c) Aggregate Capital



(d) Aggregate Output

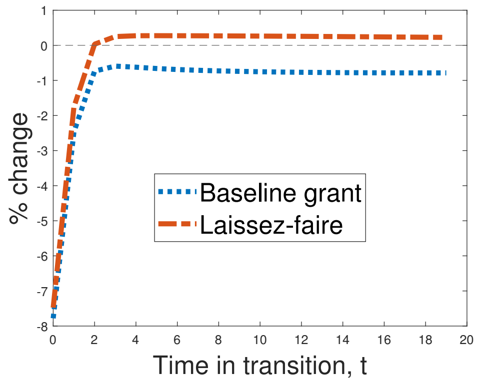
Figure 5: Impulse Response to the Pandemic Shock

5 Findings

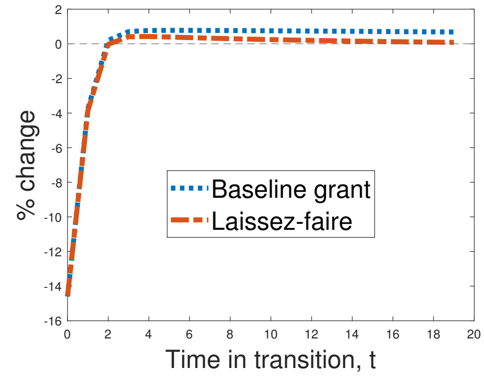
5.1 Impact of the Rescue Grant

To understand the macroeconomic impacts of the rescue grant, we consider a counterfactual laissez-faire economy which is absent of any government intervention. Figure 5 shows the impulse response of the aggregate economy under the baseline policy environment with the rescue grant and under the laissez-faire environment over a 20-quarter horizon.

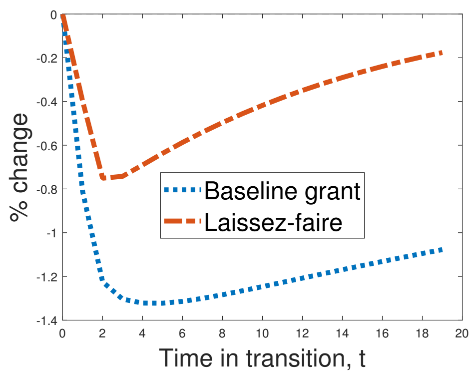
Based on our calibrated model, in the absence of the rescue grant, the pandemic shock induces an increase in the firm exit rate by almost 10% upon impact (Figure 5a). The baseline grant reduces the exit rate by 25% compared to the pre-pandemic period. Overall, the baseline grant is successful in preventing 35% of business exits. The reduction in business exits leads to an increase of aggregate capital on impact (Figure 5c). This is driven by an increase in capital in small firms, many of which decide to stay rather than exit, while the stock of corporate



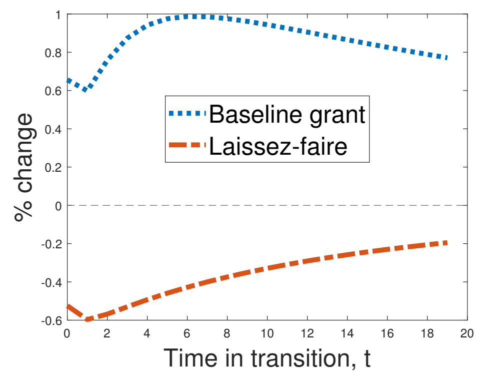
(a) Employment in the Corporate Sector



(b) Employment in Small Firms



(c) Capital in the Corporate Sector



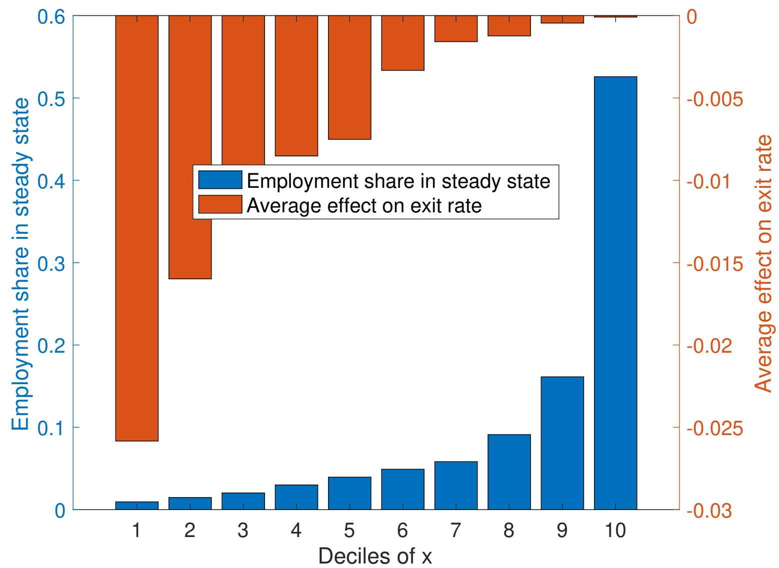
(d) Capital in Small Firms

Figure 6: Impulse Response to the Pandemic Shock by Sector

capital is predetermined and does not respond immediately to the pandemic shock. However, in subsequent quarters, aggregate capital falls below the pre-pandemic level and the recovery is notably slower compared to the laissez-faire scenario. This suggests that the grant has mostly a temporary benefit but has persistent adverse consequences on aggregate capital.

The reduction in business exits does not translate into significant improvements in aggregate output or employment (Figures 5b and 5d) for two reasons. First, the grant reallocates labor and capital away from the corporate sector. Figure 6 shows that, relative to the laissez-faire economy, the rescue grant leads to persistent declines of capital and employment in the corporate sector and persistent increases of both factors in the non-corporate sector. Thus, although the grant saves many small firms, it prevents the reallocation of production factors towards the less impacted corporate sector, which ultimately mutes improvements in aggregate output and employment.

The second reason for the modest responses of aggregate macro variables is that the grant rescues mainly low-productive firms which employ only few workers. Figure 7 shows the change



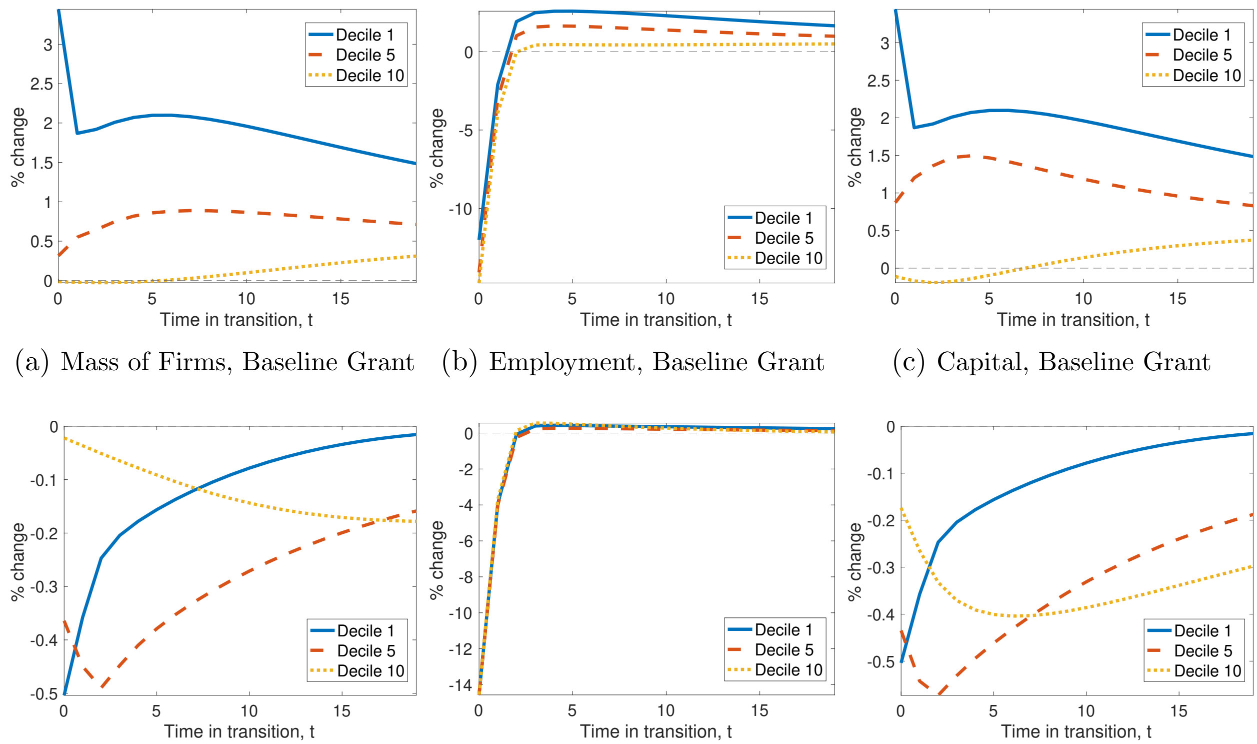
Notes: Average effect on the exit rate (top bars, inverted scale, right axis) measures the difference in the average quarterly exit rate between firms that receive the baseline grant and firms that do not receive the baseline grant in the impact period ($t = 0$) in the baseline grant environment. Deciles of x are determined according to the steady state distribution of all incumbent firms. Bottom bars show the employment shares in steady state (left axis).

Figure 7: Impact of the Grant on the Exit Rate and Employment Shares by Firm Productivity

in the exit rate induced by the grant (relative to laissez-faire) for each decile of firm productivity x and the corresponding share of total small firm employment in the steady state. While the rescue grant reduces the exit rate in all firms, the effect is much stronger for low-productivity firms that account for a disproportionately small share of aggregate employment. For example, the rescue grant reduces the exit rate of firms in the first productivity decile by 2.6 percentage points, but these firms account for only 1% of employment. As a result, the grant leads to a reallocation of employment and capital within the small firm sector in favor of less productive firms.

Figure 8 shows that small firms in the least productive decile gain the most in terms of employment and capital under the baseline grant environment (top panels) compared to the laissez-faire environment (bottom panels). As shown by the left panels, the grant policy moves the firm distribution towards the least productive firms. Much of this shift does not happen on impact but over several quarters and years following the pandemic shock. The other two panels demonstrate that this shift goes along with a long-term reallocation of production factors towards less productive businesses.

Another way to understand why saving firms has small aggregate consequences is that the



(a) Mass of Firms, Baseline Grant (b) Employment, Baseline Grant (c) Capital, Baseline Grant

(d) Mass of Firms, Laissez-Faire (e) Employment, Laissez-Faire (f) Capital, Laissez-Faire

Figure 8: Impulse Response by Decile of Small Firm Productivity x

pandemic impact on output is mainly channeled through the fall in firm productivity and labor demand in impacted firms, while firm exits play a minor role on output. To illustrate this, we decompose the output change in small firms in the first two years following the pandemic shock ($t = 0, ..., 7$), denoted Δ_Y, into three factors: (1) firm productivity change Δ_{TFP}, (2) labor demand adjustment Δ_L and (3) changes in firm exits and entry Δ_{Exit}. That is,

$$\Delta_Y = \Delta_{TFP} + \Delta_L + \Delta_{Exit} \ ,$$

where

$$\Delta_Y = \sum_{t=0}^{7} \left[\int [x_t(\iota) f(\kappa, \ell_t(x_t(\iota), \kappa)) - c^f(\kappa)] \, d\mu_t(x, b, \kappa, \iota) - \int [x f(\kappa, \ell(x, \kappa)) - c^f(\kappa)] \, d\mu(x, b, \kappa) \right] \ ,$$

$$\Delta_{TFP} = \sum_{t=0}^{7} \left[\int [\eta_i x_t(1) + (1 - \eta_i) x_t(0) - x] f(\kappa, \ell(x, \kappa)) \, d\mu(x, b, \kappa) \right] \ ,$$

$$\Delta_L = \sum_{t=0}^{7} \left[\int \eta_i x_t(1) [f(\kappa, \ell_t(x_t(1), \kappa)) - f(\kappa, \ell(x, \kappa))] \right.$$
$$\left. + (1 - \eta_i) x_t(0) [f(\kappa, \ell_t(x_t(0), \kappa)) - f(\kappa, \ell(x, \kappa))] \, d\mu(x, b, \kappa) \right] \ ,$$

$$\Delta_{Exit} = \sum_{t=0}^{7} \Bigg[\int [x_t(\iota) f(\kappa, \ell_t(x_t(\iota), \kappa)) - c^f(\kappa)] \; d\mu_t(x, b, \kappa, \iota) \\ - \int [\eta_i x_t(1) f(\kappa, \ell_t(x_t(1), \kappa)) + (1 - \eta_i) x_t(0) f(\kappa, \ell_t(x_t(0), \kappa)) - c^f(\kappa)] \; d\mu(x, b, \kappa) \Bigg] \; ,$$

with $x_t(\iota) \equiv (1 + \nu_t^n \iota)x$ defined as the productivity of firm x in period t on the transition path given the impact status ι. ℓ_t and μ_t are, respectively, labor demand and the firm distribution in period t on the transition path, whereas ℓ and μ are the steady-state counterparts. Δ_{TFP} is the effect of the shutdown shock (ν_t^n) on small-firm output while holding labor demand and the firm distribution at their steady state levels. Δ_L is the effect of labor demand adjustment while holding firm productivity at the pandemic levels and the firm distribution at the steady state level. Finally, Δ_{Exit} is the effect of changes in the firm distribution, which is in turn driven by changes in firm exits and entry, while holding firm productivity and labor demand at their pandemic levels.

We conduct the decomposition exercise in the laissez-faire environment and find that Δ_{TFP} and Δ_L account for 77% and 13% of total pandemic impact on small firm output, respectively. By contrast, Δ_{Exit} accounts for just below 10% of the total impact. This leaves little room for a pandemic rescue grant to improve small firm output because the main channel through which the grant impacts small firms is by reducing firm exits.[15]

In our baseline policy scenario, the grant is available without conditions. We consider this as a reasonably approximation to the factual PPP policy whose forgiveness requirements were quite loose from the beginning (see Section 2). As a robustness experiment, we also analyze the case of a conditional grant which imposes a minimum employment requirement in the period the grant is received. Results are presented in Appendix C. In comparison to the baseline grant, the conditional grant mitigates the employment loss in the impact quarter while the initial decline of the firm exit rate is a bit smaller. Intuitively, while jobs in impacted firms are saved under the conditional grant, other firms now prefer to exit. Importantly, the differences between the two policies are rather small and the main reallocation effects are also observed in the alternative scenario.

5.2 Targeted Rescue Policy

The PPP aids were dispersed in a highly timely fashion, but there is a lack of targeting such that the take-up of the loans is almost universal (Autor et al., 2022). The lack of targeting

[15] The grant also has a small effect on the real wage and on the interest rate (see Figure 15 in Appendix D.1), which in turn affects labor demand of small firms. However, this is of secondary importance relative to the direct labor demand response to the pandemic shock.

has been criticized for incurring an unprecedented fiscal cost and the potential of tying up resources in unproductive firms that could find more productive use elsewhere in the economy. In this subsection, we quantify the economic tradeoff of targeting rescue grants.[16] To this end, we consider a counterfactual rescue grant that targets only impacted firms. Other aspects of the policy, including grant amount and timing, remain the same as the baseline rescue grant. Since only a fraction $\eta_i = 11\%$ of small firms is shut down in the first period, the fiscal cost of the grant is now substantially reduced. The first row of Table 6 shows that the baseline grant is roughly 7 times more expensive than the targeted grant.

	Baseline grant	Targeted grant	Targeted/Baseline
Fiscal cost (frac. GDP)	6.28%	0.91%	14.5%
Small-firm emp. saved over 10 years	0.53%	0.12%	22.6%
Cost per perc. emp. saved (frac. GDP)	11.8%	7.4%	62.7%

Notes: The fiscal cost is computed as a fraction of GDP in the steady state. Employment saved is computed as the per-quarter difference in small-firm employment relative to the laissez-faire economy over a 10-year period from the onset of the pandemic. The last column shows the ratio between the first two columns.

Table 6: Cost per Job Saved in Small Firms

The targeted grant leads to a more limited improvement in small firm employment over the laissez-faire economy: over a 10-year horizon, the average employment improvement under the targeted grant is only 0.12%, less than a quarter of that under the baseline grant. To compare the cost-effectiveness of the two rescue grants, we compute the cost of saving one percent of job-quarters under the two rescue policies by taking the ratio of the first two rows of Table 6. We find that the cost of saving 1% of jobs (over a 10-year period) under the targeted grant is 63% of the cost under the baseline grant.

As another robustness experiment, we vary the size of the targeted grant in order to examine how the number of jobs saved and the cost effectiveness change with the size of the grant. While a much larger targeted grant saves more jobs, it becomes much less cost effective. This is because a larger grant provides windfall gains to those impacted firms which are already saved with a smaller grant while the number of saved firms diminishes at the margin. Conversely, a targeted grant half as large as considered in the benchmark is a bit more cost effective, but it saves about 40% fewer jobs (see Appendix B for details).

[16] Our analysis does not account for the potential administrative costs associated with granting targeted aids in a timely fashion. In addition, we only focus on employment in small firms here.

5.3 Zombie Firms

The rescue policy prevents the liquidation of otherwise viable firms that do not have access to external funding at the time of the shock. On the other hand, it might inadvertently prolong the life of unproductive firms whose capital may be more valuable in other (corporate or non-corporate) firms. Such adverse consequences might be expected, especially because the rescue grant is untargeted. In this section we use our model to quantify the risk of generating such zombie firms.

Specifically, a firm is *saved* by a grant in period t if the firm chooses to exit in the laissez-faire environment but stays active in the grant environment. We call a saved firm a *zombie* if the firm's liquidation value is larger than its hypothetical value if financial constraints were not present. In other words, if the rescue policy not merely alleviates financial constraints but instead prevents a more efficient reallocation of capital, the firm survives although it is socially inefficient that it does.

Formally, a firm with productivity x, capital κ, and impact status $\iota \in \{0,1\}$ is a zombie firm in t if $V_t(x,\kappa,\iota) < \theta\kappa$, where V_t is the value of an unconstrained firm (net of financial assets) in period t. As further outlined in Section 3.2.3 and in Appendix E.2, this value satisfies the recursion

$$V_t(x,\kappa,\iota) = \max_{\ell \geq 0} \left\{(1+\iota\nu_t^n)xf(x,\ell) - w_t\ell - \delta\kappa - c^f(\kappa)\right\} + q_t\mathbb{E}_{x'|x}V_{t+1}^0(x',\kappa,\iota)\ .$$

The zombie condition $V_t(x,\kappa,\iota) < \theta\kappa$ says that the value of liquidated capital exceeds the expected discounted profit value of the firm, so that the household would be better off if capital were installed somewhere else.

Table 7 shows the fraction of saved firms and the fraction of zombies among saved firms on impact under the baseline grant policy and under the counterfactual targeted grant policy. Upon impact, 16.6% of firms saved by the baseline grant are zombies. In contrast, the targeted grant saves as many firms as the baseline grant, while it only creates a negligible number of zombies.

The reason why the targeted grant policy creates substantially less zombie firms is that the targeted policy saves in proportion many more high-productivity firms, namely all those impacted firms which would exit without the grant. While these firms are also saved with the untargeted grant, the latter policy also saves many low-productivity firms that are not impacted by the pandemic, a non-negligible fraction of which turn out to be zombie firms.

	Baseline grant	Targeted grant
Fraction of saved firms	42.5%	43.0%
Fraction of zombie firms	16.6%	1.3%

Notes: Fraction of saved firms is the measure of saved firms in period $t = 0$ divided by the measure of firms that exit under the laissez-faire environment. Fraction of zombie firms is the measure of zombie firms divided by the measure of saved firms. See the text for definitions of saved firms and zombie firms.

Table 7: Zombie Firms

5.4 Short- and Long-Run Effects

The impulse response functions in Section 5.1 highlight the persistent effects of the rescue grant on aggregate capital and on the sectoral allocation of labor and capital. To better understand the impact of the rescue grant over time, we decompose the cumulative effect of the pandemic into short-run, medium-run and long-run effects and compare the baseline policy environment with the two counterfactual environments: laissez-faire and targeted grant.

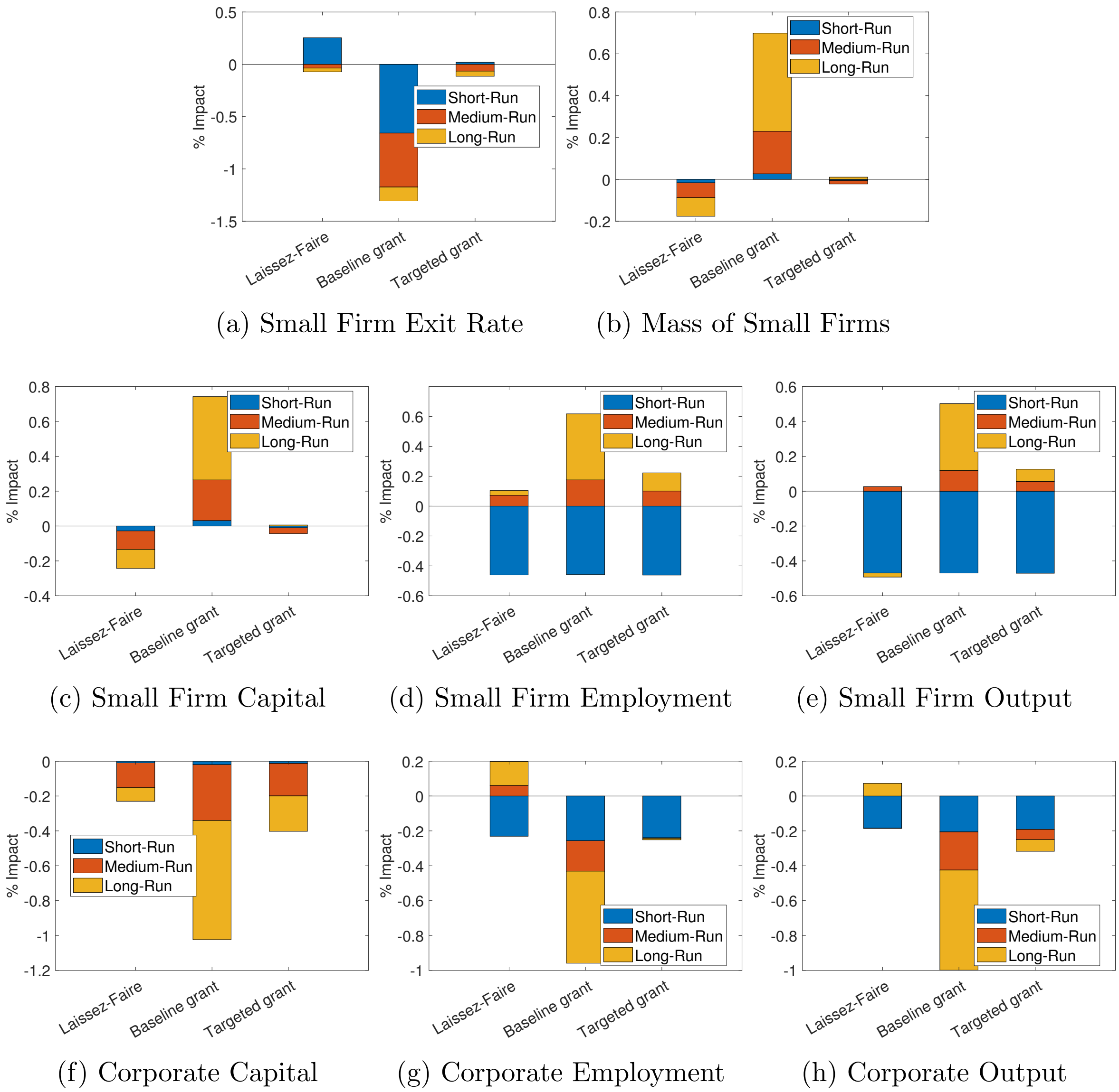
Figure 9 shows the result of the decomposition. For each policy, the net-height of the bar represents the overall average quarterly effect of the pandemic over a ten-year period. The three components representing short-run, medium-run and long-run display the decomposition of the overall average.[17]

Figures 9a and 9b show that, in the absence of government intervention, the pandemic leads to a short-run spike in firm exits and a persistent negative impact on the mass of small firms. While the targeted grant largely eliminates these impacts, the baseline grant reduces firm exits not only on impact, but also in the medium and long run, resulting in a highly persistent increase in the mass of small firms. The long-run effect on firm survival stems from the fact that the rescue grant improves the balance sheet of its recipients, including those that do not face the immediate risk of exiting.

Focusing on output in the two sectors, Figure 9e shows that the short-run output loss in small firms is similar across the three policy environments. This is because the output loss is mainly due to the productivity loss and labor demand adjustments, while firm exits play a less important role (see Section 5.1). Figure 9h shows that the short-run output loss in the corporate sector is also common across the three policy environments. Here the reason is that the short-run response is mostly due to falling TFP and employment, while capital adjusts only little in the first two quarters.

In contrast, the medium- and long-run output effects of the pandemic exhibit quite different patterns under the three policy environments. In the laissez-faire environment, the forced

[17] A detailed explanation of how this decomposition is performed is given in Appendix D.2.



(a) Small Firm Exit Rate (b) Mass of Small Firms

(c) Small Firm Capital (d) Small Firm Employment (e) Small Firm Output

(f) Corporate Capital (g) Corporate Employment (h) Corporate Output

Notes: The net height of the bars represents the average quarterly effect of the pandemic over 10 years. Short-run: first two quarters; medium-run = quarters 3 to 12; long-run = quarters 13 to 40.

Figure 9: Short- and Long-Run Effects

liquidation of small firms and the slow recovery of the non-corporate sector allow the corporate sector to absorb more labor to expand its production (see Figure 9g), which explains the long-run positive effect of the pandemic on corporate output. Furthermore, the exit of small firms in the laissez-faire scenario has long-lasting consequences since it takes time for new firms to enter, which slows down the build-up of small firm capital.

The baseline grant results in markedly larger long-run impacts compared to the targeted grant environment. Specifically, the baseline grant leads to a persistent positive effect on small firm output and an opposite impact on corporate output. This is because the grant, due to its improvement on balance sheets of financially constrained small firms, preserves capital in the non-corporate sector at the expense of capital in the corporate sector. This reallocation of capital plays out in the medium- and long-run, and hence has long-lasting consequences which are also seen in the employment and output changes.

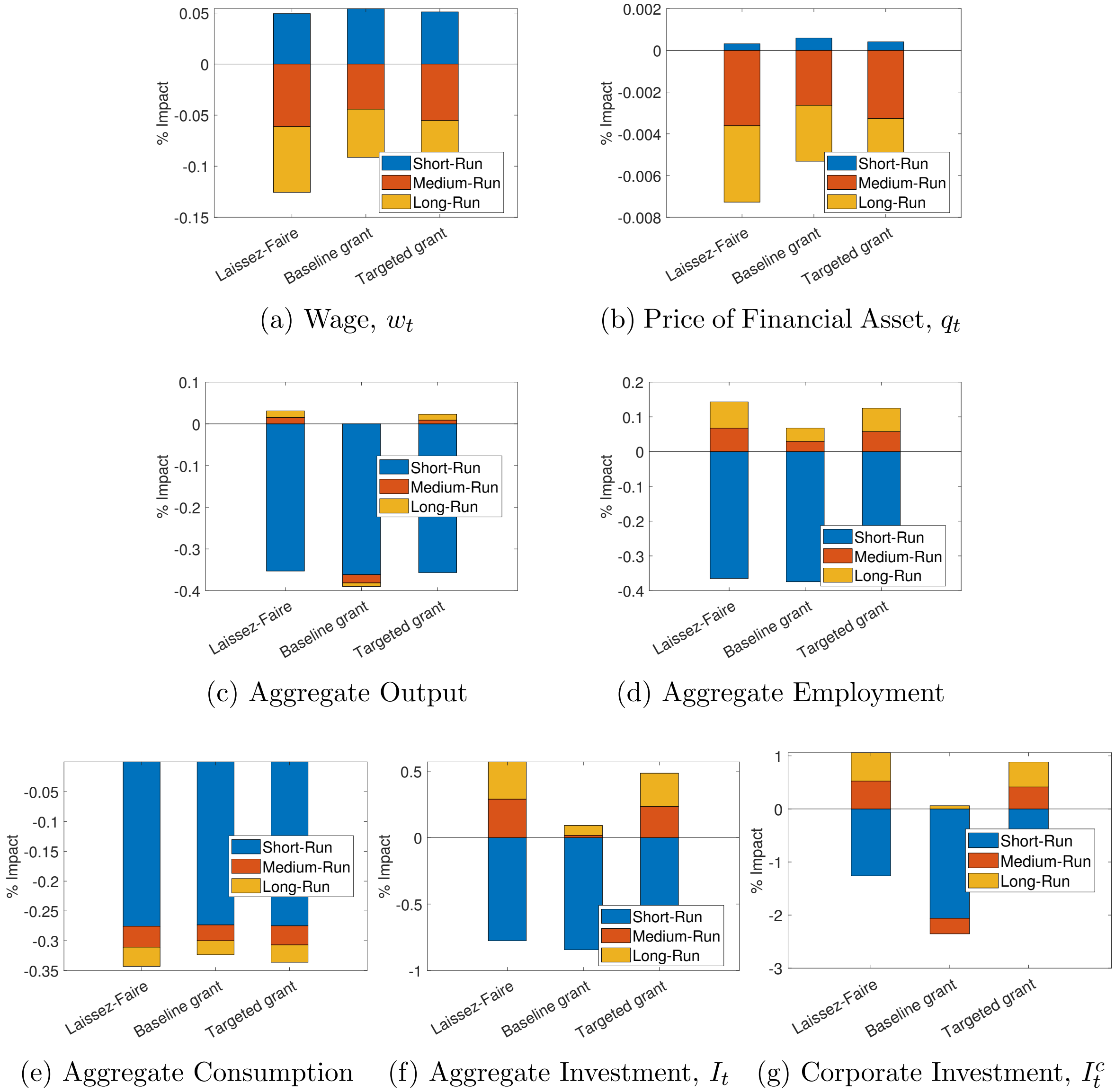
Turning our attention to aggregate variables, Figure 10 shows that the pandemic shock has mostly a short-run impact on output, employment and consumption under all scenarios considered. Most of the drop in consumption happens in the short-run in all three cases, largely due to the sizable but short-lived demand shock. The baseline grant has a positive, albeit small, effect on consumption in the medium- and long-run. The reason is the following: the baseline grant prevents firm liquidation and therefore the household suffers less from losses of capital liquidation, leaving more resources for consumption and savings. At the same time, the discount factor falls by less (i.e., the increase of the interest rate is dampened), thus inducing the household to save less and consume more in comparison to the laissez-faire (and targeted grant) scenarios.

For these reasons, aggregate investment is considerably lower in the medium- and long-run, compared to the laissez-faire environment. The improvement in firms' balance sheets has long-lasting dampening effects on firm exit. As the household continues to hold and replace depreciated capital in these firms, investment in the corporate sector or for the build up of new small firms is reduced. In turn, the baseline grant reduces aggregate employment and output in the medium- and long-term because lower investment in the corporate sector eventually outweighs the capital preservation in smaller firms.

6 Conclusions

The COVID-19 pandemic caused a deep but short-lived economic recession, which prompted many governments to enact massive rescue policies targeted at small firms. In order to evaluate the macroeconomic impact of such policies, we build a general equilibrium model where firms are subject to financial constraints and can only liquidate their capital at a loss.

Based on our calibrated model, we find that an unconditional grant policy, such as the PPP enacted in the U.S. in the year 2020, is highly effective in preventing the exit of small businesses, but has only a modest impact on aggregate outcomes such as consumption, employment and output. This happens for two reasons. First, the grant mostly prevents less productive small firms from exiting. Second, it reallocates resources towards firms that have lower productivity



(a) Wage, w_t (b) Price of Financial Asset, q_t

(c) Aggregate Output (d) Aggregate Employment

(e) Aggregate Consumption (f) Aggregate Investment, I_t (g) Corporate Investment, I_t^c

Figure 10: Short- and Long-Run Effects (Other Variables)

at the expense of more productive and less impacted corporate and non-corporate firms. This result echoes previous findings in the literature (see Crouzet and Mehrotra, 2020) showing that while small firms are typically hit harder during recessions, their volatility has only a small effect on aggregate fluctuations and that policies relaxing financing constraints are unlikely to generate a sizable macroeconomic impact.

We also find that much of the reallocation of capital and labor across firms occur after the pandemic episode is over, although the grant policy is only applied when the shock hits the economy. This is because the grant has a lasting impact on the financial position of small firms, thus preventing firm exits for many years. As a consequence, the reallocation of capital and employment across firms is very persistent due to the policy.

This is much different when the universal grant is replaced by a targeted grant policy that specifically targets impacted firms. With a targeted grant, there is no persistent decline of firm exit, and the reallocation of capital across firms is much more muted. Moreover, the targeted grant policy largely prevents the creation of zombie firms which are those businesses whose liquidation would be socially beneficial but is prevented due to the grant.

By focusing on the impact of business grants on firm entry, exit and factor reallocation, our paper does not analyze how government-subsidized loans, possibly in combinations with different conditions for partial loan forgiveness, would affect business dynamics and macroeconomic aggregates. We also abstract from public debt and the timing of taxation by assuming that the government grants are financed by lump-sum taxes. Both are highly-relevant issues for future research.

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Appendix

A PPP Loan Terms

Eligibility The following requirements apply to first draw PPP loans:

- The business was operational before February 15, 2020, and is still open and operational.
- There are no more than 500 employees. If a business has multiple locations, there must be no more than 500 employees per location.

Loan Amount The maximum amount of a PPP loan is determined based on the average monthly payroll up to a threshold. The maximum amount of first draw PPP loans is 2.5 times the average monthly payroll up to $10 million. In the Accommodation and Food services sector, it is 3.5 times the average monthly payroll up to $10 million. For non-employers, the maximum amount is based on the business net profit.

Interest Rate PPP loans have an interest rate of 1%.

Maturity PPP loans issued after June 5, 2020, have a maturity of five years. PPP loans issued prior to June 5, 2020, have a maturity of two years. However, by mutual agreement, the maturity can be extended to 5 years.

Covered Period The covered period is the 8- or 24-week after loan disbursement. The 24-week period applies to all borrowers that received forgiveness prior to December 27, 2020, but borrowers that received an SBA loan number before June 5, 2020, have the option to use an eight-week period.

Conditions for Loan Forgiveness Full loan forgiveness is possible if, during the 8- or 24-week covered period following loan disbursement, all of the following are achieved:

- Employee and compensation levels are maintained (temporary layoff is allowed if the workers are rehired),
- all loan proceeds are spent on payroll costs and other eligible expenses, and
- at least 60% of the proceeds are spent on payroll costs.

Partial Forgiveness "If a borrower uses less than 60% of the loan amount for payroll costs during the forgiveness covered period, the borrower will continue to be eligible for partial loan forgiveness, subject to at least 60 percent of the loan forgiveness amount having been used for payroll costs."[18]

B Targeted Grant

In addition to the baseline grant and the targeted grant described in Section 5.2, we consider two additional targeted grant policies with different grant amounts. In "targeted grant small", we set $X_p^{small} = 0.5X_p$ and in "targeted grant large", we set $X_p^{large} = 5X_p$, where X_p is amount of the grant as a fraction of the quarterly payroll in the baseline grant and the targeted grant scenarios in Section 5.2.

Table 8 compares the cost-effectiveness of the baseline grant to three targeted grants. As the grant amount increases, the targeted grant becomes less cost-effective. In particular, a targeted grant which is almost as costly as the untargeted baseline grant turns out to be much less cost effective. Intuitively, while a larger targeted grant saves more impacted firms (and jobs in these firms), it does so with a diminishing impact at the margin. At the same time, larger grants are merely windfall gains for those firms that are already saved with much smaller government transfers.

	Baseline grant	Targeted grant small	Targeted grant	Targeted grant large
Cost (Frac. GDP)	6.28%	0.45%	0.91%	4.55%
Emp. save (Frac. Emp)	0.53%	0.07%	0.12%	0.30%
Cost per perc. jobs saved	11.8%	6.1%	7.4%	15.3%

Notes: The fiscal cost is computed as a fraction of GDP in the steady state. Employment saved is computed as the per-quarter difference in small-firm employment relative to the laissez-faire economy over a 10 year period from the onset of the pandemic.

Table 8: Cost per Job Saved in Small Firms

C Conditional Grant

As a robustness check, we assume that the grant comes with a minimum employment requirement in period $t = 0$ that resembles the condition for forgiveness in the PPP program (see

[18]Source: https://home.treasury.gov/news/press-releases/sm1026

Appendix A).[19] That is, firms that choose to take up the grant must maintain employment at 60% of the pre-pandemic level. The grant is only given in period $t = 0$, as in our benchmark model, and the minimum employment condition only applies in period $t = 0$. This is consistent with the length of the covered period in the PPP program that ranges from 8 to 24 weeks. Other than the minimum employment requirement, all policy assumptions are the same as in Section 3.3.

Specifically, an η fraction of small firms have the opportunity to take up the grant. They choose to do so if $v_0(x, b, \kappa, \iota, s = 1) \geq v_0(x, b, \kappa, \iota, s = 0)$, where $s \in \{0, 1\}$ indicates grant take-up and $\iota \in \{0, 1\}$ is the impact status. The profit of a firm that takes up the grant in $t = 0$ changes from equation (23) to

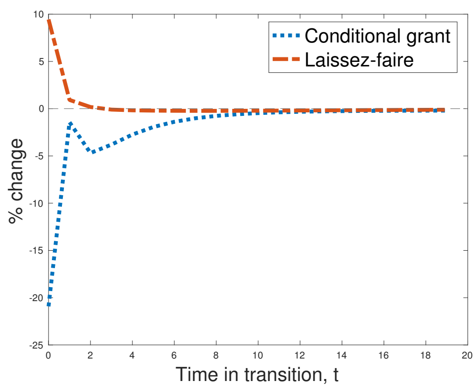
$$\pi_0(x, b, \kappa, \iota, s = 1) = \max_{\ell}(1 + \nu_0^n \iota)xf(\kappa, \ell) + b_p(x, \kappa) - w_0\ell - \delta\kappa - c^f(\kappa)$$
$$\text{such that} \quad \ell \geq 0.6 \times \ell(x, \kappa)$$

where $b_p(x, \kappa)$ is the grant amount defined in equation (22), $\ell(x, \kappa)$ is the labor demand in the steady state, and $0.6 \times \ell(x, \kappa)$ is the minimum employment requirement.

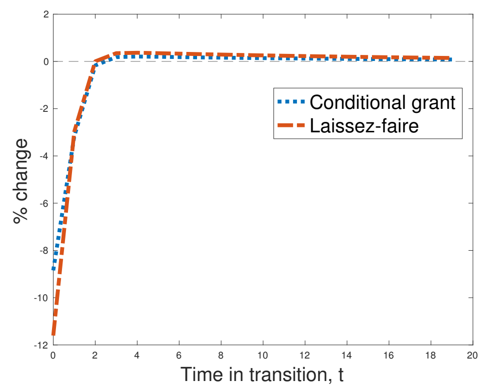
We simulate the economy with the conditional grant using the same calibrated parameter values reported in Section 4.2. The results indicate that, although the minimum employment condition stimulates short-run employment in small firms, the conditional grant delivers similar medium- and long-run impacts on aggregate output, employment, and capital as the baseline, unconditional grant. Thus, the main conclusions of the paper carry over. The conditional grant modeled here can be viewed as an extreme case because, in practice, the conditionality of the PPP loan forgiveness is not strictly enforced and partial forgiveness is possible.

Figures 11 and 12 show the impulse responses to a number of variables of interest, comparing the laissez-faire to the conditional grant. Figures 13 and 14 show the short- and long-run impact of the conditional grant policy. In comparison to the impulse responses under the unconditional grant, the conditional grant dampens the initial employment decline while mitigating the initial decline in firm exits. This is because employment in staying impacted firms remains higher while other impacted firms prefer to exit rather than accepting the conditional grant. From period $t = 1$ onward, however, the observed reallocation effects are similar to those under the unconditional grant scenario.

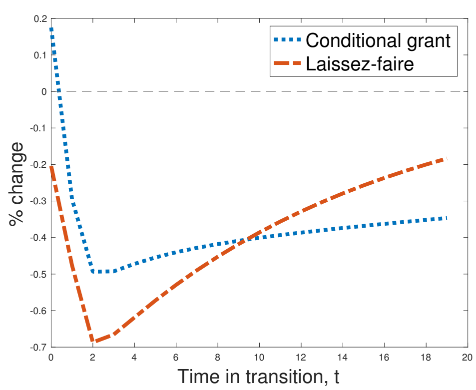
[19] For simplicity, we do not model the option of not fulfilling the condition, thus abandoning forgiveness.



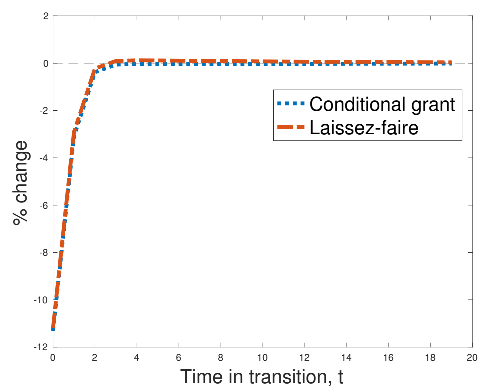
(a) Small Firm Exit Rate



(b) Aggregate Employment

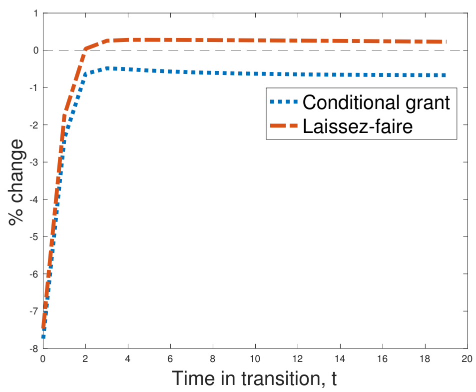


(c) Aggregate Capital

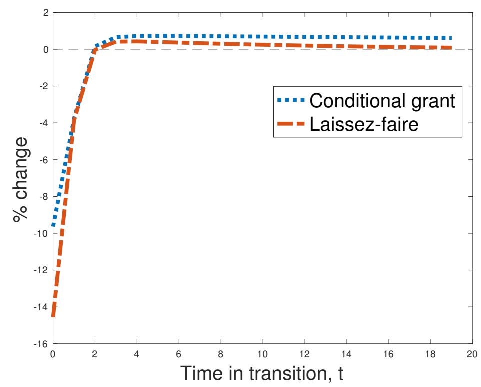


(d) Aggregate Output

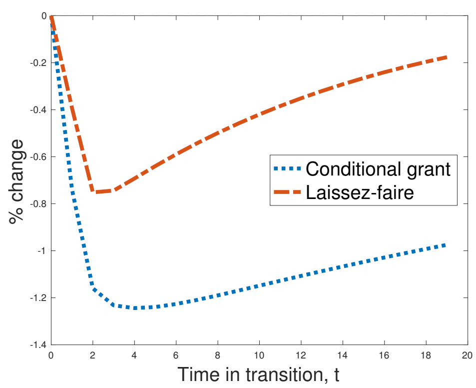
Figure 11: Impulse Response to the Pandemic Shock (Conditional Grant)



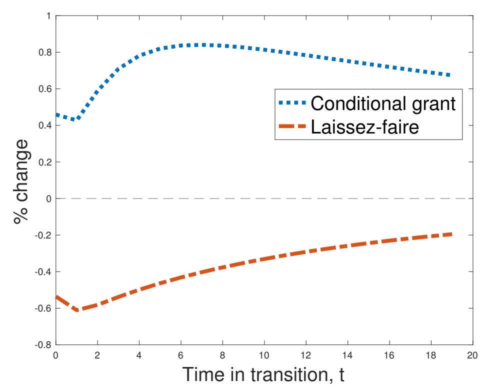
(a) Employment in the Corporate Sector



(b) Employment in Small Firms

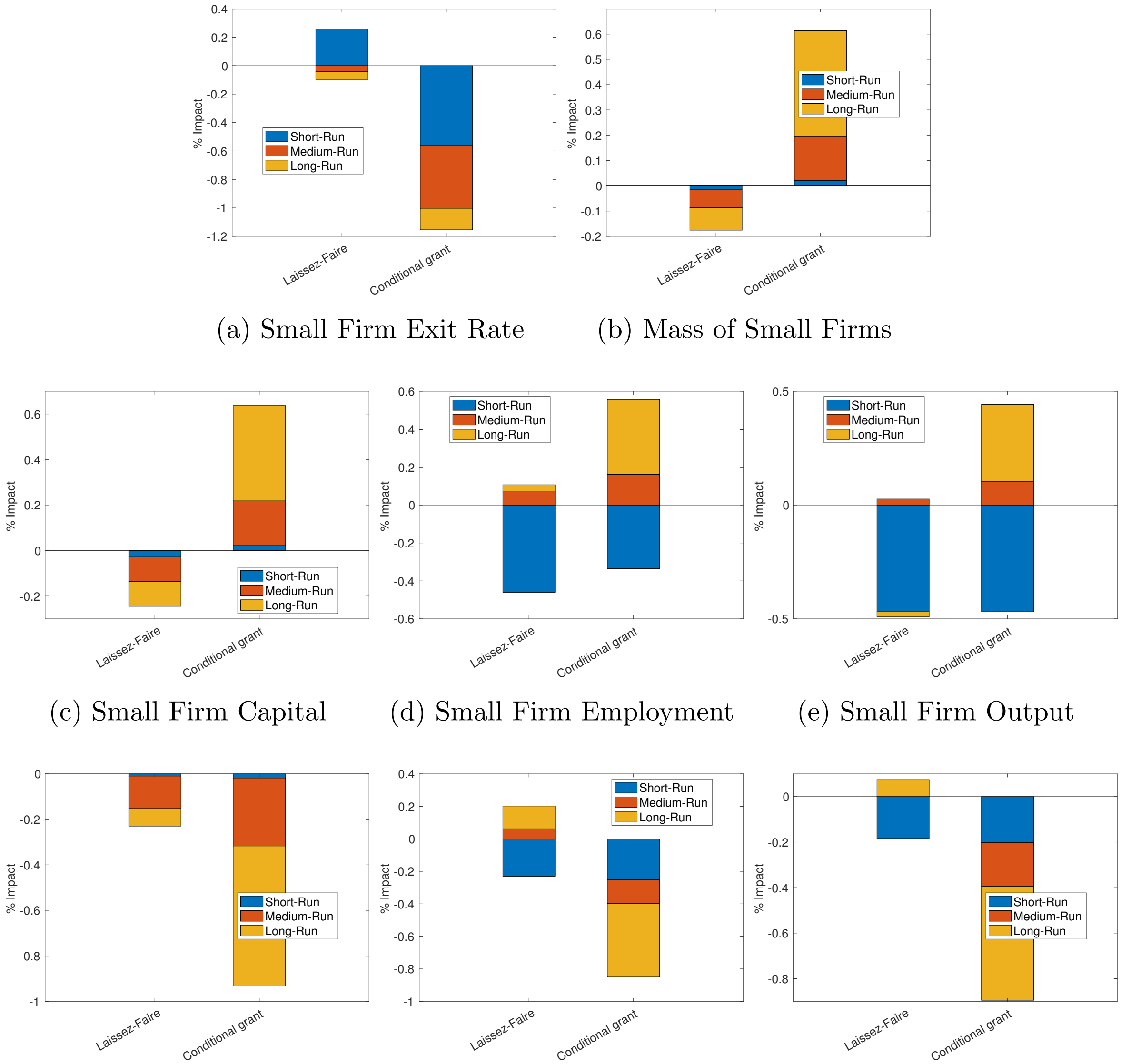


(c) Capital in the Corporate Sector



(d) Capital in Small Firms

Figure 12: Impulse Response to the Pandemic Shock by Sector (Conditional Grant)



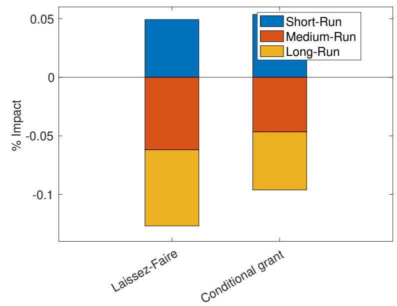
(a) Small Firm Exit Rate (b) Mass of Small Firms

(c) Small Firm Capital (d) Small Firm Employment (e) Small Firm Output

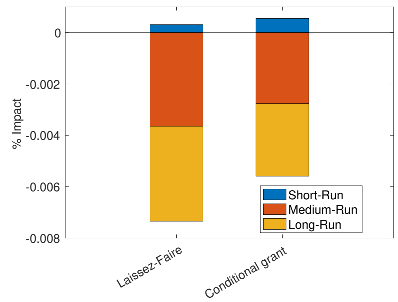
(f) Corporate Capital (g) Corporate Employment (h) Corporate Output

Notes: The net height of the bars represents the average quarterly effect of the pandemic over 10 years. Short-run: first two quarters; medium-run = quarter 3 to 12; long-run = quarter 13 to 40.

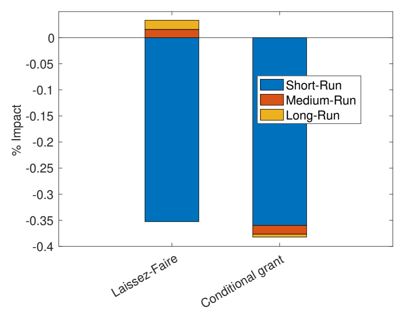
Figure 13: Short- and Long-Run Effects (Conditional Grant)



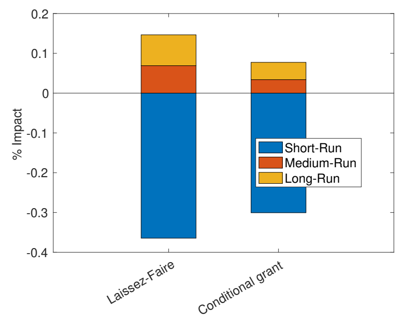
(a) Wage, w_t



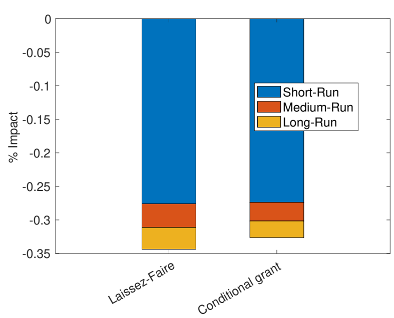
(b) Price of financial asset, q_t



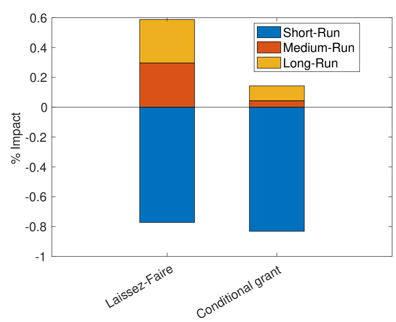
(c) Aggregate Output



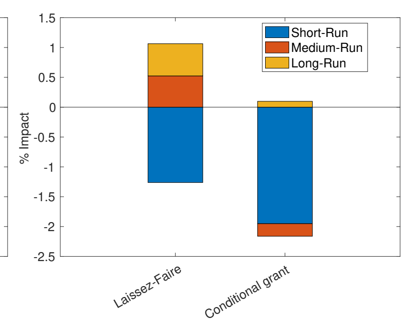
(d) Aggregate Employment



(e) Aggregate Consumption



(f) Aggregate Investment, I_t

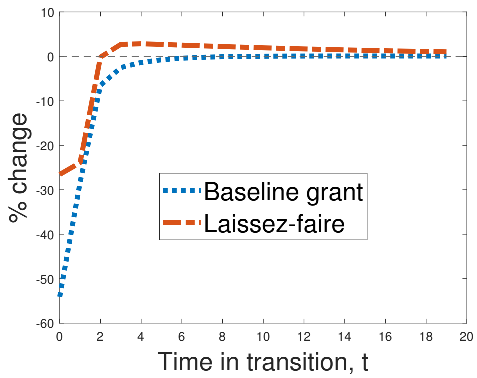


(g) Corporate Investment, I_t^c

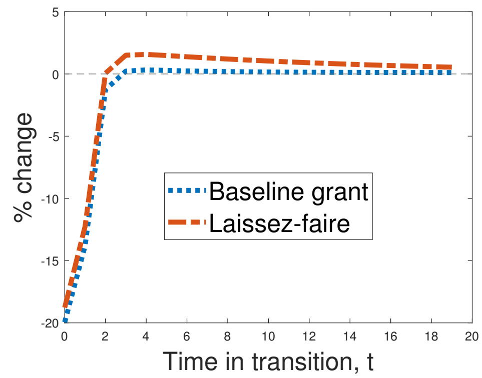
Figure 14: Short- and Long-Run Effects (Conditional Grant)

D Further Details

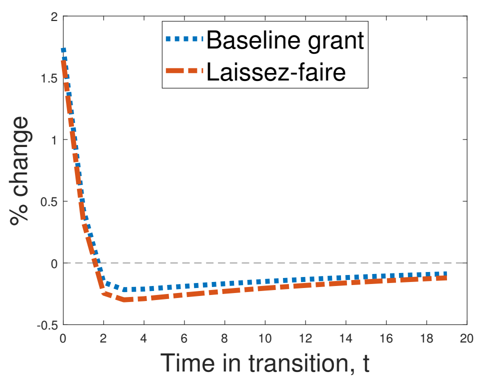
D.1 Additional Figures



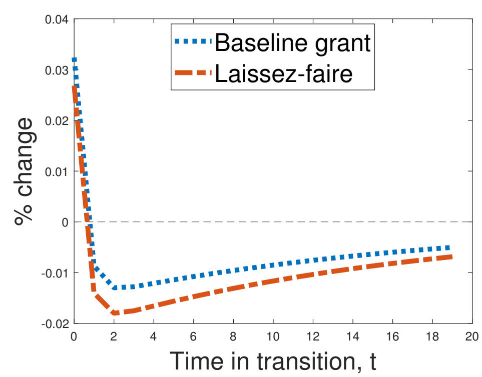
(a) Corporate Sector Investment, I_t^c



(b) Aggregate Investment, I_t



(c) Wage, w_t



(d) Price of Financial Asset, q_t

Figure 15: IRFs: Investment and Prices

D.2 Short- and Long-Run Decomposition

We decompose the total cumulative effect of the pandemic shock in a 10-year period into short-run, medium-run, and long-run effects. Specifically, let $irf_t = \frac{y_t - y_{ss}}{y_{ss}}$ be the percentage effect of the pandemic on variable y in period t, and let $\overline{irf}$ be the average effect of the pandemic on a variable over a period of T_{LR} periods, i.e.

$$\overline{irf} = \frac{1}{T_{LR}} \sum_{t=0}^{T_{LR}-1} irf_t \ .$$

We compute $\overline{irf}_{SR}, \overline{irf}_{MR}$ and $\overline{irf}_{LR}$ such that

$$\overline{irf} = \overline{irf}_{SR} + \overline{irf}_{MR} + \overline{irf}_{LR},$$

where

$$\overline{irf}_{SR} = \frac{1}{T_{LR}} \sum_{t=0}^{T_{SR}-1} irf_t \ ,$$

$$\overline{irf}_{MR} = \frac{1}{T_{LR}} \sum_{t=T_{SR}}^{T_{MR}-1} irf_t \ ,$$

$$\overline{irf}_{LR} = \frac{1}{T_{LR}} \sum_{t=T_{MR}}^{T_{LR}-1} irf_t \ .$$

We consider the short-run to be the first two quarters post-pandemic, the medium-run to be quarter 3 to year 3, and the long-run to be year 4 to year 10.

E Computational Appendix

E.1 Computation of the Stationary Equilibrium

The financial discount factor is $q = \beta$. The capital-labor ratio in the corporate sector is pinned down from $1 = \beta[1 - \delta + F_K]$. Using the functional form $F(K^c, L^c) = A(K^c)^\alpha (L^c)^{1-\alpha}$, we obtain

$$1 = q \left[1 - \delta + A\alpha \left(\frac{K^c}{L^c} \right)^{\alpha - 1} \right] \ ,$$

from which we obtain the capital-labor ratio in corporate firms

$$k^* = \frac{K^c}{L^c} = \left(\frac{A\alpha}{\frac{1}{q} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} .$$

The real wage follows from $w = F_L$, i.e.

$$w = A\left(1 - \alpha\right)\left(k^*\right)^\alpha .$$

Consumption follows from the first order condition $u_C\left(C, 1 - L\right) w = u_{1-L}\left(C, 1 - L\right)$, i.e.

$$C^\sigma = \frac{w}{\zeta} \ .$$

This permits the calculation of $\pi(x,\kappa) = \max_{\ell \geq 0} x f(\kappa,\ell) - w\ell - \delta\kappa - c^f(\kappa)$. The optimal labor demand can be derived as

$$\ell(x,\kappa) = \left(\frac{w}{xA(1-\gamma_1)\gamma_2}\right)^{\frac{1}{(1-\gamma_1)\gamma_2-1}} \kappa^{-\frac{\gamma_1\gamma_2}{(1-\gamma_1)\gamma_2-1}} .$$

Given $\pi(x,\kappa)$, $V(x,\kappa)$ can be obtained from (18), which allows us to compute $\tilde{x}(\kappa)$, and $\tilde{b}(\kappa) = \pi(\tilde{x}(\kappa),\kappa)/(1-q)$. Discretize the interval of relevant debt levels for each κ as $[\tilde{b}(\kappa),\theta\kappa]$ such that $b = \tilde{b}(\kappa)$ (unconstrained firms) is one point, while $(\tilde{b}(\kappa),\theta\kappa]$ is cut into intervals of equal lengths. Value and policy functions are then defined on the finite set $\mathbb{X} \times B \times \mathbb{K}$. Value functions for unconstrained firms are $v(x,b,\kappa) = V(x,\kappa) - b$; for constrained firms they follow from (19)–(21). Liquidation and entry policy functions follow from (12) and (13).

This permits calculation of the stationary measure of small firms. Combining (5) and (8) yields one equation for the stationary measure μ^0:

$$\mu^0(A) = \int \mathbb{I}_{(x',b'(x,b),\kappa')\in A}\, g(x'|x)\left(1-d^l(x,b,\kappa)\right) d\mu^0(x,b,\kappa) + M\int \mathbb{I}_{(x',b'(x,b),\kappa')\in A}\, g(x'|x)\, d^e(x,b,\kappa)\, d\Phi(x,b,\kappa) ,$$

for all Borel subsets $A \subset \mathbb{X} \times [\tilde{b}(\kappa),\theta\kappa] \times \mathbb{K}$. Again, note that small firm capital is fixed, i.e. $\kappa' = \kappa$. After discretization, μ^0 is a vector with dimension equal to the cardinality of $\mathbb{X}\times B\times\mathbb{K}$, and the above equation can be solved by matrix inversion or by fixed point iteration. This also defines the measure of active firms $\mu(x,b,\kappa) = (1-d^l(x,b,\kappa))\mu^0(x,b,\kappa) + Md^e(x,b,\kappa)\Phi(x,b,\kappa)$ for $(x,b,\kappa) \in \mathbb{X}\times B\times\mathbb{K}$. Note that both μ^0 and μ are linear in M; hence parameter M linearly scales the size of the non-corporate sector.

The capital stock in the corporate sector can be backed out from the goods-market equilibrium condition

$$C+\delta\int \kappa\, d\mu(x,b,\kappa) + M\int \kappa d^e(x,b,\kappa) d\Phi(x,b,\kappa) - \int [xf(\ell(x,\kappa)) - c^f(\kappa)]\, d\mu(x,b,\kappa) - \theta\int \kappa d^l(x,b,\kappa)\, d\mu^0(x,b,\kappa)$$
$$= K^c[F(1,1/k^*) - \delta] ,$$

with capital-labor ratio k^* obtained before. Finally, employment in the corporate sector is $L^c = K^c/k^*$ and labor supply is $L = L^c + \int \ell(x,\kappa)\, d\mu(x,b,\kappa)$.

E.2 Computation of the Transition Path

We describe how the transition path after the unexpected pandemic shock at $t=0$ back to the original steady state can be calculated numerically. Choose large enough T and suppose that the economy has approximately reached the steady state from period $T+1$ onward. That is, all endogenous variables attain their steady state values for $t \geq T+1$.

1. Start with a guess for initial consumption C_0. Then use the following first-order conditions of the household and corporate sector decision problems:

$$q_t = \beta \frac{D_{t+1}}{D_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} , \tag{24}$$

$$w_t = \zeta_t C_t^{\sigma} , \tag{25}$$

$$w_t = (1-\alpha) A_t k_t^{\alpha} , \tag{26}$$

$$1 = q_t [1 - \delta + \alpha A_{t+1} k_{t+1}^{\alpha-1}] , \tag{27}$$

   which hold for all $t = 0, \ldots, T$, where $A_t = A\,(1+\nu_t^c)$, $D_t = \left(1+\nu_t^d\right)$, and $\zeta_t = \left(1+\nu_t^{\ell}\right)$ are time-varying multipliers of corporate productivity, utility of consumption, and utility of leisure, respectively. k_t is the capital-labor ratio in the corporate sector. The guess for C_0 directly yields w_0 and k_0. Then use all four equations to obtain a dynamic equation for k_t:

$$\beta k_t^{\alpha} = \frac{A_{t+1} D_t \zeta_t}{A_t D_{t+1} \zeta_{t+1}} \cdot \frac{k_{t+1}^{\alpha}}{1 - \delta + \alpha A_{t+1} k_{t+1}^{\alpha-1}} .$$

   This equation can be inverted numerically to obtain iteratively $k_1, \ldots, k_T$.

   Given this solution, the above equations can be used to back out C_t, w_t and q_t for all $t = 0, 1, \ldots, T$. This obtains operating profits of small firms $\pi_t(x, \kappa, \iota)$ for all periods $t = 1, \ldots, T$ where dummy variable ι indicates whether the firm is impacted by the shock. In period 0, write $\pi_0(x, \kappa, \iota, s)$, where index $s = 1$ indicates that these firms receive the grant in $t = 0$.

2. Iterate (10) and (11) (adjusted for the pandemic shock) backwards from steady state value functions yields value functions $v_t(x, b, \kappa, \iota)$ and $v_t^0(x, b, \kappa, \iota)$ for $t = 0, \ldots, T$ and $\iota \in \{0, 1\}$. See below for a description how this can be done without optimization over assets. Regarding period $t = 0$, calculate $v_0(x, b, \kappa, \iota, s)$ and $v_0^0(x, b, \kappa, \iota, s)$ separately for firms with and without grant as these have different profits in $t = 0$.

   Calculate entry and exit decision rules $d_0^e(x, b, \kappa, \iota)$ and $d_0^l(x, b, \kappa, \iota, s)$ and $d_t^e(x, b, \kappa, \iota)$, $d_t^l(x, b, \kappa, \iota)$ for $t = 1, \ldots, T$. Note that the grant is only paid in period $t = 0$ to incumbent firms which is why s enters only the exit policy function d_0^l.

   Calculate the distribution of active firms in $t = 0$, $\mu_0(x, b, \kappa, \iota, s)$ from (5) and the given steady-state distribution of incumbent firms at the beginning of the period $\mu^0(x, b, \kappa)$. Here we impose that fraction η_i of incumbent firms and potential entrants are hit by the pandemic and that fraction η of continuing firms in period $t=0$ receive the grant, while entrants in period $t=0$ are not eligible for the grant. Hence,

$$\mu_{t=0}(x,b,\kappa,\iota,s) = \begin{cases} \eta_i \eta \mu^0(x,b,\kappa)(1-d_0^l(x,b,\kappa,1,1)) & ,\ \iota=1,\ s=1, \\ \eta_i \Big[(1-\eta)\mu^0(x,b,\kappa)(1-d_0^l(x,b,\kappa,1,0)) & \\ +M d_0^e(x,b,\kappa,1)\Phi(x,b,\kappa)\Big] & ,\ \iota=1,\ s=0, \\ (1-\eta_i)\eta\mu^0(x,b,\kappa)(1-d_0^l(x,b,\kappa,0,1)) & ,\ \iota=0,\ s=1, \\ (1-\eta_i)\Big[(1-\eta)\mu^0(x,b,\kappa)(1-d_0^l(x,b,\kappa,0,0)) & \\ +M d_0^e(x,b,\kappa,0)\Phi(x,b,\kappa)\Big] & ,\ \iota=0,\ s=0. \end{cases}$$

Continue to iterate over (5) and (8), adjusted for the additional state variable ι, to obtain firm distributions before and after entry/exit in all subsequent periods $t=1,\dots,T$, $\mu_t^0(x,b,\kappa,\iota)$ and $\mu_t(x,b,\kappa,\iota)$ (note that state variable s only matters in $t=0$). Calculate the net output of small firms:

$$\begin{aligned} Y_0^n =& \int [(1-\nu_0^n\iota)xf(\kappa,\ell((1-\nu^n)x,\kappa)) - \delta\kappa\ - c^f(\kappa)]\ d\mu_0(x,b,\kappa,\iota,s) \\ &+ \theta\kappa \int \Big[\eta_i\big(\eta d_0^l(x,b,\kappa,1,1) + (1-\eta)d_0^l(x,b,\kappa,1,0)\big) \\ &\qquad + (1-\eta_i)\big(\eta d_0^l(x,b,\kappa,0,1) + (1-\eta)d_0^l(x,b,\kappa,0,0)\big)\Big]\ d\mu^0(x,b,\kappa) \\ &- M \int \big[\eta_i d_0^e(x,b,\kappa,1) + (1-\eta_i)d_0^e(x,b,\kappa,0)\big]\kappa d\Phi(x,b,\kappa)\ , \\ Y_t^n =& \int (1-\nu_t^n\iota)xf(\kappa,\ell(x,\kappa)) - \delta\kappa\ d\mu_t(x,b,\kappa,\iota) + \theta\kappa\int d_t^l(x,b,\kappa,\iota)\ d\mu_t^0(x,b,\kappa,\iota) \\ &- M\int [\eta_i d_t^e(x,b,\kappa,1) + (1-\eta_i)d_t^e(x,b,\kappa,0)]\kappa d\Phi(x,b,\kappa)\ ,\ t=1,\dots,T. \end{aligned}$$

Starting at $t=0$, given the initial steady-state capital stock in the corporate sector $K_0^c = K^{c*}$ and the previously obtained path for k_t, calculate L_t^c, output Y_t^c, investment I_t^c, and K_{t+1}^c iteratively for all $t=0,\dots,T$:

$$L_t^c = K_t^c/k_t\ \Rightarrow\ Y_t^c = A_t F(K_t^c, L_t^c)$$

$$\Rightarrow I_t^c = Y_t^n + Y_t^c - C_t\ \Rightarrow K_{t+1}^c = (1-\delta)K_t^c + I_t^c\ .$$

If $K_{T+1}^c > K^{c*}$, increase C_0; if $K_{T+1}^c < K^{c*}$, decrease C_0. Repeat until K_{T+1}^c is reasonably close to K^{c*} (saddle path).

Regarding value function iteration (step 2), borrowing/savings policies can be directly obtained using a similar logic as in the stationary equilibrium. Firms become unconstrained when their financial savings exceed a certain threshold level after which they are able to pay positive dividends. The value function of these unconstrained firms takes the form $v_t(x, b, \kappa, \iota) = V_t(x, \kappa, \iota) - b$ where

$$V_t(x, \kappa, \iota) = \pi_t(x, \kappa, \iota) + q_t \mathbb{E}_{x'|x} \max[\theta\kappa, V_{t+1}(x', \kappa, \iota)] \ . \tag{28}$$

This equation can be solved backwards starting from $V_{T+1} = V$ (steady state). Then define cutoff productivity levels $\tilde{x}_t(\kappa, \iota) = \min\{x \in \mathbb{X} | V_t(x, \kappa, \iota) \geq \theta\kappa\}$ such that unconstrained firms with productivity below $\tilde{x}_t(\kappa, \iota)$ liquidate the firm in period t, while all others stay.

To determine the level of debt below which a firm becomes unconstrained, calculate backward the debt levels $\tilde{b}_t(\kappa, \iota) = \pi_t(\tilde{x}_t(\kappa, \iota), \kappa, \iota) + q_t \tilde{b}_{t+1}(\kappa, \iota)$ for $t = 0, \dots, T$ with $\tilde{b}_{T+1}(\kappa, \iota) = \tilde{b}(\kappa)$ for $\iota \in \{0, 1\}$ (steady state). Verify that $\tilde{b}_t(\kappa, \iota) < \theta\kappa$ (otherwise there cannot be unconstrained firms in $t - 1$). Then, in any period t, a firm is unconstrained if $b \leq \tilde{b}_t(\kappa, \iota)$ in which case it sets $b'_t(x, b, \kappa, \iota) = \tilde{b}_{t+1}(\kappa, \iota)$ if $x \geq \tilde{x}_t(\kappa, \iota)$. Otherwise, if $x < \tilde{x}_t(\kappa, \iota)$, the firm is liquidated, $d^l_t(x, b, \kappa, \iota) = 1$. Note that in the pandemic period $t = 0$, $\tilde{b}_0(\kappa, \iota)$ differs between firms with grant and firms without grant so that the cutoff that differentiates constrained from unconstrained firms depends on the grand receipt indicator s.

Any firm with $b > \tilde{b}_t(\kappa, \iota)$ is constrained. This firm is also liquidated if $x < \tilde{x}_t(\kappa, \iota)$. Otherwise it pays zero dividends until savings exceed $-\tilde{b}_{t+1}(\kappa, \iota)$. This implies that the value and policy functions of constrained firms are

$$v_t(x, b, \kappa, \iota) = \max[0, \pi_t(x, \kappa, \iota) - b - q_t \tilde{b}_{t+1}(\kappa, \iota)] + q_t \mathbb{E}_x v^0_t(x', b'_t(x, b, \kappa, \iota), \kappa, \iota) \ , \tag{29}$$

$$v^0_t(x, b, \kappa, \iota) = \max[\theta\kappa - b, v_t(x, b, \kappa, \iota)] \ , \tag{30}$$

$$b'_t(x, b, \kappa, \iota) = \max\left[\tilde{b}_{t+1}(\kappa, \iota), \frac{1}{q_t}(b - \pi_t(x, \kappa, \iota))\right] \ , \tag{31}$$

if $\pi_t(x, \kappa, \iota) - b + q_t \theta\kappa \geq 0$, and $v_t(x, b, \kappa, \iota) = v^0_t(x, b, \kappa, \iota) = \theta\kappa - b$ otherwise. Again, in period $t = 0$, the value function of constrained firms differs between firms with grant and firms without grant via the profit function $\pi_0(x, \kappa, \iota, s)$ in equation (23).

F Moment Computation

Firm exit rate. The firm exit rate is the quarterly rate at which small firms permanently exit. In the model, we calculate the firm exit rate as the measure of firms that exit in a period divided by the measure of firms at the beginning of the period. In the steady state, the exit rate is

$$r_{exit} \;=\; \frac{\int_{x,b,\kappa} d^l\left(x,b,\kappa\right)\; d\mu^0\left(x,b,\kappa\right)}{\int_{x,b,\kappa} d\mu^0\left(x,b,\kappa\right)}.$$

The data counterpart is computed based on data from the BDS. Since the BDS data is annual, we first compute the annual exit rate by dividing the number of establishment death in year t, and then convert them into the quarterly rate. The relationship between annual exit rate (r^y_{exit}) and quarterly exit rate (r^q_{exit}) is as follows.

$$r^y_{exit} \;=\; 1 - \prod_{q=1}^{4}(1 - r^q_{exit}).$$

We use the establishment exit rate to proxy for the firm exit rate since we do not have high-quality micro data on firm exits.

Fixed expense to revenue ratio. Fixed expenses are firms' non-payroll overhead expense. We compute the data target based on a table prepared by Sageworks published in the Washington Post, which is constructed based on the 2007 U.S. Economic Census.[20] We weight the expense-to-revenue ratios of the nine types of small businesses by their yearly revenues.

The model counterpart of fixed expenses of small firms include the maintenance cost of capital and the additional fixed cost, and the model counterpart of small firm revenue is their output. We compute fixed expense to revenue ratio in the steady state as

$$\frac{\int_{x,b,\kappa}\left[\delta\kappa + c^f(\kappa)\right]\; d\mu(x,b,\kappa)}{\int_{x,b,\kappa} x f(\kappa,\ell(x,\kappa))\; d\mu(x,b,\kappa)}.$$

Debt to Asset Ratio. The data target for the debt-to-asset ratio is computed based on the KFS. It is computed as the ratio between the sum of positive net debt of small firms divided by the sum of their real assets. The net debt of a firm is its total debt minus financial assets; we treat firms with a negative net debt (positive financial asset) as having zero debt. The model counterpart of the debt to asset ratio is

$$\frac{\int_{x,b,\kappa} \max\{b,0\}\; d\mu(x,b,\kappa)}{\int_{x,b,\kappa} \kappa\; d\mu(x,b,\kappa)}.$$

[20]Link: https://www.washingtonpost.com/wp-srv/special/business/costofrunningabusiness.html, accessed on May 28, 2021.

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