

What can we learn from fluctuations of particle ratios?

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We explain how fluctuations of ratios can constrain and falsify the statistical model of particle production in heavy ion collisions, using K/π fluctuations as an example. We define an observable capable of determining which statistical model, if any, governs freeze-out in ultrarelativistic heavy ion collisions. We calculate this observable for K/π fluctuations, and show that it should be the same for RHIC and LHC energies, as well as independent of centrality, if the Grand-Canonical statistical model is an appropriate description and chemical equilibrium applies. We describe variations of this scaling for deviations from this scenario, such as light quark chemical non-equilibrium, strange quark over-saturation and local conservation (canonical ensemble) for strange quarks. We also introduce a similar observable capable, together with the published K^*/K measurement, of ascertaining if an interacting hadron gas phase governs the system between thermal and chemical freeze-out, and of ascertaining its duration and impact on hadronic chemistry

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1. Introduction

One of the main objectives of heavy ion physics is to study the collective properties of strongly interacting matter. It's equation of state, transport coefficients, degree of equilibration and phase structure, and the dependence of these on energy and system size.

Thus, the natural approach to study soft particle production in heavy ion collisions is through statistical mechanics techniques. Such an approach has a long and illustrious history [1, 2, 3, 4]. However, the systematic quantitative comparison of data to the statistical model is a comparatively recent field ; A consensus has developed that the statistical hadronization model can indeed fit most or all particles for AGS,SPS and RHIC energies [5, 6, 7, 8, 9, 10, 11].

This consensus, is, however, somewhat superficial. While it is true that one can get a reasonably nice-looking fit with a statistical model ansatz, it does not follow that the system is actually thermally and chemically equilibrated at freeze-out: Considering the paucity of data points when particle abundances are modeled, such a fit is by itself not a guarantee of the physical significance of parameters such as temperature and chemical potential. When statistical significance of these fits is calculated, it is apparent that the statistical model is nowhere near “proven” according to the standards generally accepted in particle physics [12] To compound this point, it should be remembered that a roughly “thermal” distribution is also observed in systems that appear too small to be equilibrated, such as $p - p$ and even e^+e^- [13].

Furthermore, an unambiguous link between heavy ion phenomenology and statistical model parameters is still missing. It is unclear whether strangeness enhancement, or features such as the horn, are due to canonical effects of incomplete chemical equilibration. It is unclear if [9], and at what energy, does light and/or strange quark chemical equilibration occur. It is unclear what bias, if any, does in-medium mass modification of short-lived resonances introduce into statistical model parameters. A more stringent test of the statistical model could serve both as a strong confirmation that statistical physics is an appropriate description for heavy ion collisions, and as a tool for deciding *which* statistical model is more appropriate.

Particle yield fluctuations are a promising observable to falsify the statistical model and to constrain its parameters (choice of ensemble, strangeness/light quark chemical equilibrium) [14]. One can immediately see that fluctuations are a stringent statistical model test by considering the fluctuation of a ratio between two random variables.

$$\sigma_{N_1/N_2}^2 = \frac{\langle (\Delta N_1)^2 \rangle}{\langle N_1 \rangle^2} + \frac{\langle (\Delta N_2)^2 \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle \Delta N_1 \Delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}. \quad (1.1)$$

Since, for an equilibrated system, $\langle (\Delta N_1)^2 \rangle \sim \langle N \rangle \sim \langle V \rangle$, where $\langle V \rangle$ is the system volume [15], it is clear that σ_{N_1/N_2}^2 depends on the hadronization volume in a manner *opposite* to that of particle yields, inversely rather than directly linearly proportionally. Volume fluctuations (which make a comparison between of statistical model calculations to experimental data problematic), both resulting from dynamics and from fluctuations in collision geometry, should not alter this very basic result since volume cancels out event by event [16], *provided* hadronization volume is the same for all particles (a basic statistical model requirement).

Thus, observables such as $\langle N_1 \rangle \sigma_{N_1/N_2}^2$, provided $\langle N_{1,2} \rangle$ and σ_{N_1/N_2}^2 are measured using the same kinematic cuts, should be strictly independent of multiplicity and centrality, as long as the statistical

model holds and the physically appropriate ensemble is Grand Canonical. If the temperature and chemical potentials between two energy regimes are approximately the same at freezeout (this should be the case for RHIC top energies and LHC, provided chemical equilibrium holds), this observable should also be identical across energy regimes. This could be used as a stringent test of the statistical model.

Fluctuations are more sensitive to acceptance cuts than yields. A partial “fix” for acceptance cuts that does not require detector-specific analysis is mixed event subtraction, based on the idea that fluctuation effects resulting from acceptance cuts are present both in real and mixed events (this is the case for fluctuations, but not for correlations). Thus, an appropriate observable to model would be [17]

$$\sigma_{dyn}^2 = \sigma^2 - \sigma_{mix}^2 \quad (1.2)$$

where σ_{mix}^2 is the mixed event width. In the absence of any correlations, it reduces itself to the Poisson expectation,

$$(\sigma_{mix}^2)_{N_1/N_2} = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle}$$

putting everything together, the right observable to model becomes equivalent to v_{N_1/N_2}^{dyn}

$$\begin{aligned} v_{N_1/N_2}^{dyn} &= (\sigma_{N_1/N_2}^{dyn})^2 = \sigma_{N_1/N_2}^2 - (\sigma_{N_1/N_2}^{Poisson})^2 = \\ &= \frac{\langle N_1(N_1 - 1) \rangle}{\langle N_1 \rangle^2} + \frac{\langle N_2(N_2 - 1) \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \end{aligned} \quad (1.3)$$

(v_{dyn} is theoretically equivalent to σ_{dyn}^2 , but experimentally it is measured by histogramming [17]). This observable is currently subject of intense experimental investigation [18, 19].

We therefore propose to use the scaling of

$$\Psi_{N_1/N_2}^{N_1} = \langle N_1 \rangle v_{dyn}^{N_1/N_2} \quad (1.4)$$

to test the statistical model validity among different energy, system size and centrality regimes.

“Primordial” fluctuations of each observable, $\langle (\Delta N_{1,2})^2 \rangle$, are calculable from Textbook methods [15] from Fermi-Dirac or Bose Einstein statistics. Observed fluctuations, however, must also include corrections from resonance decays. This correction, $\langle (\Delta N_{j \rightarrow i})^2 \rangle$, is given by

$$\langle (\Delta N_{j \rightarrow i})^2 \rangle = B_{j \rightarrow i} (1 - B_{j \rightarrow i}) \langle N_j \rangle + B_{j \rightarrow i}^2 \langle (\Delta N_j)^2 \rangle. \quad (1.5)$$

where the first term represents the “anti-correlation” due to exclusive decay channels (if the ρ decays into $\pi^+ \pi^-$, it will not decay to $\pi^0 \pi^0$) while the second term is the fluctuation in the number of resonances itself. Similarly, the correlation term in Eq. 1.1 is given by

$$\langle \Delta N_1 \Delta N_2 \rangle = \sum_j B_{j \rightarrow 12} \langle N_j \rangle \quad (1.6)$$

For the analysis described in this section, it is safer to ignore this quantity by choosing particles (such as K^- and π^-) least correlated by resonances. In the next section we will detail with the correlation term as an observable, and show that it is also very useful for distinguishing between different freeze-out scenarios.

SHAREv2.X [20, 21] provides the possibility of calculating all ingredients of $\Psi_{N_1/N_2}^{N_1}$ for any hadrons, incorporating the effect of all resonance decays, as well as chemical (non)equilibrium.

It is important to underline that the value of $\Psi_{N_1/N_2}^{N_1}$ (calculated from statistical model parameters and, as we will see, sensitive to the degree of chemical equilibration of the system) should be *constant* across any system where the intensive parameters are the same, for example different centrality regimes or system sizes at the same energy. For instance, since the chemical potential of Cu-Cu 200 GeV collisions should be comparable the chemical potential at Au-Au,

$$\Psi_{K^-/\pi^-}^{\pi^-} \Big|_{Cu-Cu} \simeq \Psi_{K^-/\pi^-}^{\pi^-} \Big|_{Au-Au} \quad (1.7)$$

or in other words

$$v_{K^-/\pi^-}^{dyn} \Big|_{Cu-Cu} \simeq \frac{\langle N_{\pi} \rangle_{Cu-Cu}}{\langle N_{\pi} \rangle_{Au-Au}} v_{K^-/\pi^-}^{dyn} \Big|_{Au-Au} \simeq 3.2 v_{K^-/\pi^-}^{dyn} \Big|_{Au-Au} \quad (1.8)$$

A large deviation from this value, or a systematic variation of $\Psi_{N_1/N_2}^{N_1}$ with centrality, should be taken as indication of an admixture of non-statistical behavior (for example, a significant effect of a non-thermalized “corona” [22, 23]). In the presence of a description for the corona, the formulae for Ψ can be easily extended with

$$\langle N \rangle \rightarrow \langle N \rangle_{core} + \langle N \rangle_{corona} \quad (1.9)$$

$$\langle (\Delta N)^2 \rangle \rightarrow \langle (\Delta [N_{core} + N_{corona}])^2 \rangle \simeq \langle (\Delta N_{core})^2 \rangle + \langle (\Delta N_{corona})^2 \rangle \quad (1.10)$$

A break of the flat scaling of $\Psi_{N_1/N_2}^{N_1}$ would strongly motivate such a description, and modeling it could help in constraining the dynamics of the corona.

The calculation for $\Psi_{K^-/\pi^-}^{\pi^-}$, as well as $\Psi_{K^+/\pi^-}^{\pi^-}$ is shown in Fig. 1. These species were chosen because their correlations (from resonance decays, $N^* \rightarrow N_1 N_2$), which would need corrections for limited experimental acceptance, are small.

The methods described in this section can be used to ascertain the origin of the “horn”, either at NA49 or in the future low energy RHIC runs. If the “horn” is due to a smooth transition between baryon-dominated and meson-dominated freeze-out, than $\Psi_{K^-/\pi^-}^{\pi^-}$ should not significantly change between energies to the left and the right of the horn’s tip. (μ_B will change, smoothly, but K and π are only relatively sensitive to this).

On the other hand, if, as hypothesized in [9], the “horn” is due to a sudden jump in the light quark phase space occupancy, than π fluctuations should be correspondingly enhanced, resulting in a jump of $\Psi_{K^-/\pi^-}^{\pi^-}$. When performing this scan, care should be taken that the yield and the fluctuation are measured within the same kinematic cuts (this is the reason why such an analysis is not possible with currently available data, but could become possible in future SPS and low energy RHIC runs).

A more quantitative, and striking, signature for deviation from equilibrium can be made between RHIC and LHC energies. Equilibrium thermal and chemical parameters are very similar at RHIC and the LHC (the baryo-chemical potential will be lower at the LHC, but it is low enough at RHIC that the difference should not significantly affect π and K abundance). Thus, $\Psi_{K^-/\pi^-}^{\pi^-}$ should be identical, to within experimental error, for both the LHC and RHIC, over all multiplicities where the statistical model is thought to apply.

According to [24], chemical conditions at freeze-out (at SPS, RHIC and LHC) deviate from equilibrium, and reflect the higher entropy content and strangeness per entropy of the early de-confined phase through an over-saturated phase space occupancy for the light and strange quarks ($\gamma_s > \gamma_q > 1$). If this is true, then $\Psi_{N_1/N_2}^{N_1}$ should still be independent of centrality for a given energy range, but should go markedly up for the LHC from RHIC, because of the increase in γ_q and γ_s .

We have calculated $\Psi_{N_1/N_2}^{N_1}$ for RHIC and LHC energies, for the sets of parameters used in [24]. The left and right panel in Fig. 1 shows what effect three different sets of $\gamma_{q,s}$ inferred in [24] would have on $\Psi_{K^-/\pi^-}^{\pi^-}$ and $\Psi_{K^-/K^+}^{\pi^-}$. In the left panel we have also included the value of $\Psi_{K^-/\pi^-}^{\pi^-}$ for top energy RHIC. As shown in [25], this value for top centrality matches expectations for non-equilibrium freeze-out (and is significantly above equilibrium freeze-out). A centrality scan of $\Psi_{K^-/\pi^-}^{\pi^-}$, necessary to confirm the consistency of this result has not, however, as yet been performed.

If non-statistical processes (mini-jets, string breaking etc.) dominate event-by-event physics, the flat $\Psi_{N_1/N_2}^{N_1}$ scaling on centrality/multiplicity should be broken, and $\Psi_{N_1/N_2}^{N_1}$ would exhibit a non-trivial dependence on N_{part} or dN/dy .

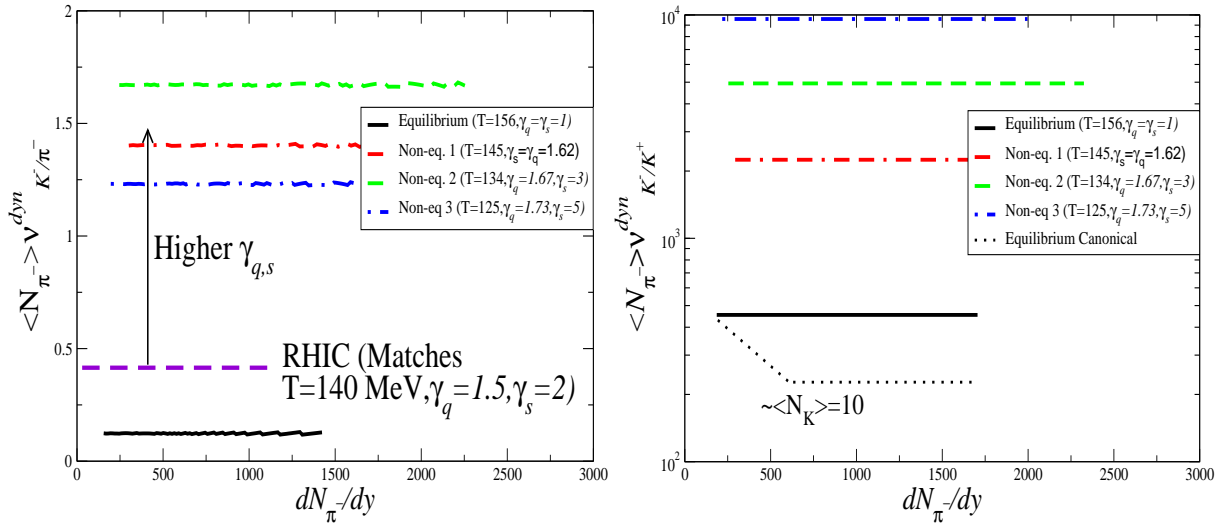


Figure 1: (color online) $\Psi_{K^-/\pi^-}^{\pi^-}$ (Left panel) and $\Psi_{K^-/K^+}^{\pi^-}$ (right panel) calculated for various statistical hadronization parameters [24] at the LHC. The left panel also shows the RHIC calculation [25]

This is also true if global correlations persist, such as is the case if the Canonical and micro-canonical ensembles [26, 27] are physically more appropriate to describe the system than the Grand-Canonical ensemble. Whether this is in fact the case is not immediately apparent: Higher energy RHIC and LHC experiments are only capable of observing the mid-rapidity region, a small fraction of the system where the energy density and chemical potential appear to be constant. The 4 π acceptance limit (explored at the SPS), where conservation laws have to be obeyed exactly, could also correspond to a highly inhomogeneous system (E.g. Baryo-chemical potential is thought to vary markedly with rapidity), where *no* ensemble is appropriate.

Thus, it would appear that the Grand Canonical ensemble is more appropriate. On the other hand, micro-canonical calculations of some SPS observables are remarkably successful [28], sug-

gesting the possibility that correlations from conservation laws should be strictly taken into account even within systems where their applicability is not intuitively clear. Furthermore, models have appeared in the literature where strangeness is produced and strictly conserved *locally*, necessitating a Canonical approach [29]. This theoretical ambiguity makes further experimental study desirable, and, due to their ensemble-specificity even in the thermodynamic limit [26, 27], fluctuations are a very convenient probe.

If global correlations persist for particle N_2 and/or N_1 , then $\Psi_{N_1/N_2}^{N_1}$ becomes reduced, and starts strongly varying with centrality in lower multiplicity events. Thus, if strangeness at RHIC/the LHC is created and maintained locally, $\Psi_{N_1/N_2}^{N_1}$ should develop a “wobble” at low centrality, and be considerably lower than Grand Canonical expectation. For $\Psi_{K^+/K^-}^{\pi^-}$ it should be lower by a factor of two.

In conclusion, measuring $\Psi_{K^-/\pi^-}^{\pi^-}$ and $\Psi_{K^+/K^-}^{\pi^-}$, at comparing the results between the LHC and RHIC can provide an invaluable falsification of the statistical model, as well as constraints as to *which* statistical model applies in these regimes.

2. Fluctuations and resonances

A still unresolved ambiguity of statistical models, with profound repercussions within other branches of heavy ion physics is the duration, and impact on hadronic observables, of the phase between hadronization (the moment at which particles become the effective degrees of freedom) and freeze-out (the moment at which particles stop interacting).

If chemical freezeout temperature is $T_{chem} \sim 170$ MeV, as deduced from equilibrium statistical model fits, and thermal freezeout temperature is $T_{therm} \sim 100$ MeV, as deduced from fits to particle spectra, it follows that there is a significant interacting hadron gas phase that has the potential of altering all soft hadronic signatures.

The failure to solve the HBT problem [30], combined with acceptable fits obtained by simultaneous freeze-out models [31, 32], suggests however that we are missing something fundamental, and more direct probes of freeze-out dynamics should be needed.

The measurement of Resonance yields offers such a probe [33, 34], since short-lived hadronic resonances decay before the interacting hadron gas phase (if it exists) is over. Thus, rescattering of decay products can deplete the amount of observable resonances, while regeneration could create additional resonances not present at hadronization.

The observation $\Lambda(1520)$ and $K^*(892)$ [35, 36, 37], at abundances below equilibrium statistical model expectations, could be interpreted as an indication of such reinteraction, with rescattering predictably dominating over regeneration. This interpretation, however, is not unique: Chemical non-equilibrium fits recover the resonance abundance exactly, with no need for an interacting hadron gas phase [33, 34].

Thus detection of short lived resonances can not tell us freezeout dynamics unless a different signature, more sensitive to chemical freeze-out, is obtained. As is apparent [38, 16] from Eqs 1.1 and 1.6 the correlation term is precisely the required observable, since correlations between multiplicities are fixed at *chemical* freeze-out. This correlation term can be measured by comparing observables such as $\Psi_{K^+/\pi^-}^{\pi^-}$ (correlated by $K^*(892)$) with $\Psi_{K^-/\pi^-}^{\pi^-}$ (not correlated by resonances).

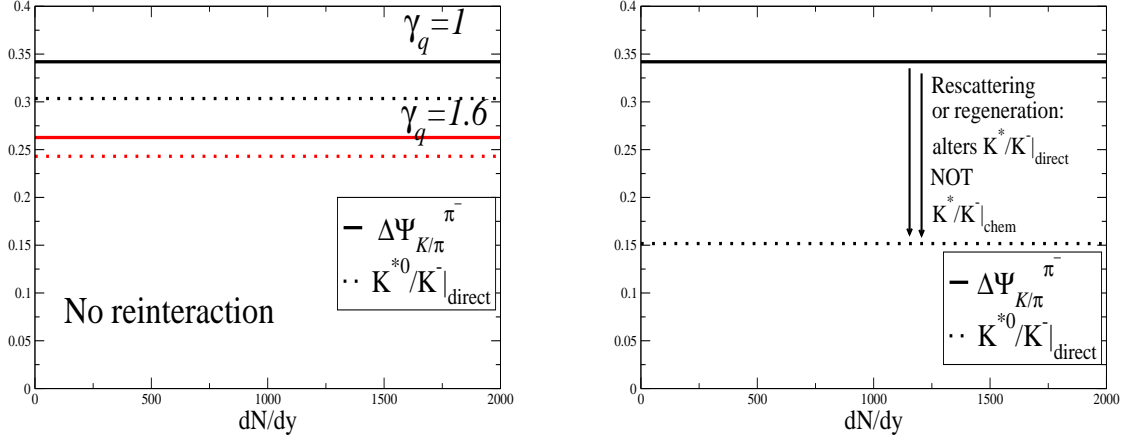


Figure 2: (color online) $\Delta\Psi_{K/\pi}^{\pi^-}$ and K^*/K^- calculated within the equilibrium and non-equilibrium statistical models

In particular

$$\langle \pi^- \rangle \left(v_{dyn}^{K^+/\pi^-} - v_{dyn}^{K^-/\pi^-} \right) \simeq 2 \frac{\langle \Delta K^+ \Delta \pi^- \rangle}{\langle K^- \rangle} \sim \frac{4}{3} \frac{\langle K^*(892) \rightarrow K^+ \pi^- \rangle}{\langle K^- \rangle} \Big|_{chemical \text{ freeze-out}} \quad (2.1)$$

We therefore define

$$\Delta\Psi_{K/\pi}^{\pi^-} = \frac{3}{4} \langle \pi^- \rangle \left(v_{dyn}^{K^+/\pi^-} - v_{dyn}^{K^-/\pi^-} \right) \simeq \frac{\langle K^*(892) \rightarrow K^+ \pi^- \rangle}{\langle K^- \rangle} \Big|_{chemical \text{ freeze-out}} \quad (2.2)$$

this result is somewhat spoiled by finite baryochemical potential, as well as higher lying resonances (including the anti-correlation term in Eq. 1.5). To ascertain the size of these corrections, we have used SHARE to calculate both the K^*/K^- and $\Delta\Psi_{K/\pi}^{\pi^-}$. As shown in Fig. 2 (left panel), these corrections make up a 10 % effect, less than the expected experimental error and not enough to alter the difference between a single freeze-out and two simultaneous ones.

A long reinteracting hadron gas phase would in general bring the observed (final) abundance of K^*/K away from the chemical freezeout value (either up, by regeneration, or down, by rescattering). Thus, $\Delta\Psi_{K/\pi}^{\pi^-}$ would become different from K^*/K^- (Fig. 2 right panel). In the weak interaction limit, regeneration would presumably be rarer than rescattering so the observable K^*/K^- abundance would be suppressed by a factor that combines the interaction width $\Delta\Gamma = \Gamma_{rescattering} - \Gamma_{regeneration}$ with the lifetime of the interacting phase τ

$$\frac{K^*}{K^-} \Big|_{observed} \sim \Delta\Psi_{K/\pi}^{\pi^-} \exp[-\Delta\Gamma\tau] \quad (2.3)$$

In the strong reinteraction limit, rescattering and regeneration would reach detailed balance until a lower freeze-out T_{therm} , so the observed K^*/K^- would be sensitive to the difference between the

two temperatures as well as the mass difference (Δm) between K^* and K

$$\left. \frac{K^*}{K^-} \right|_{observed} \sim \Delta \Psi_{K/\pi}^{\pi^-} \exp \left[\frac{\Delta m}{T_{therm}} - \frac{\Delta m}{T_{chem}} \right] \quad (2.4)$$

One untested effect that could spoil this result is strong rescattering capable of bringing the K and π out of the detector's acceptance region in phase space [14, 16]. This effect can not be taken into account by mixed event techniques described in the previous section (since mixed events retain no two-particle correlations), and calculating it in a model-independent way is problematic.

Inferring the presence of such a correction is however relatively straight-forward: The probability of such rescattering strongly depends on the width of the acceptance region. Thus, if $\Delta \Psi_{K/\pi}^{\pi^-}$ as a function of the rapidity window should go from zero (at small rapidity no multiplicity correlation survives) and saturate at a constant value (where the full resonance derived correlation is recovered). This constant value, as long the rapidity window is much smaller than the total extent of rapidity of the system, is the quantity that can be related to $\left. \frac{K^*}{K^-} \right|_{chem}$.

The dependence on centrality of $\Delta \Psi_{K/\pi}^{\pi^-}$, on the other hand, has to remain flat, since in the Grand Canonical limit the ratio of two particles should be independent of centrality, and the total system size should not alter the probability of a local process (such as scattering in/out of the acceptance region) to occur.

If $\Delta \Psi_{K/\pi}^{\pi^-}$ obeys the consistency checks elucidated here (saturating value when rapidity window is varied, flat when centrality is varied), it should be taken as a reliable indication that the $\Delta \Psi_{K/\pi}^{\pi^-}$ measurement in fact reflects the value of $\left. \frac{K^*}{K^-} \right|_{chem}$ at chemical freezeout.

If $\Delta \Psi_{K/\pi}^{\pi^-}$ depends on rapidity up to the acceptance region of the detector, a more thorough effort to account for acceptance corrections to the correlation is needed. This can be done by using the same techniques utilised for direct resonance reconstruction [35, 36, 37]. However, such an endeavour is detector specific, and goes well beyond this write-up.

In conclusion, we have shown that observables incorporating both yields and fluctuations give a stringent test of statistical models. We have also argued that such observables can be used to infer the duration of the interacting hadron gas phase, and its effect on hadronic observables. We expect that forthcoming experimental data, together with the methods elucidated here, will allow us to clarify some of the outstanding puzzles apparent in the study of heavy ion collisions.

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