ALEXANDER MEYER-GOHDE, EKATERINA SHABALINA

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Institute for Monetary and Financial Stability
Goethe University Frankfurt
House of Finance
Theodor-W.-Adorno-Platz 3
D-60629 Frankfurt am Main
www.imfs-frankfurt.de | info@imfs-frankfurt.de

Estimation and Forecasting Using Mixed-Frequency DSGE Models*

Alexander Meyer-Gohde†
Goethe University Frankfurt and IMFS

Ekaterina Shabalina‡
Goethe University Frankfurt

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Abstract

In this paper, we propose a new method to forecast macroeconomic variables that combines two existing approaches to mixed-frequency data in DSGE models. The first existing approach estimates the DSGE model in a quarterly frequency and uses higher frequency auxiliary data only for forecasting (see Giannone, Monti and Reichlin (2016)). The second method transforms a quarterly state space into a monthly frequency and applies, e.g., the Kalman filter when faced missing observations (see Foroni and Marcellino (2014)). Our algorithm combines the advantages of these two existing approaches, using the information from monthly auxiliary variables to inform in-between quarter DSGE estimates and forecasts. We compare our new method with the existing methods using simulated data from the textbook 3-equation New Keynesian model (see, e.g., Galí (2008)) and real-world data with the Smets and Wouters (2007) model. With the simulated data, our new method outperforms all other methods, including forecasts from the standard quarterly model. With real world data, incorporating auxiliary variables as in our method substantially decreases forecasting errors for recessions, but casting the model in a monthly frequency delivers better forecasts in normal times.

JEL: E12, E17, E37, E44, C61, C68

Keywords: Mixed-frequency data, DSGE models, Forecasting, Estimation, Temporal aggregation

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† meyer-gohde@econ.uni-frankfurt.de

‡ ekaterina@shabalins.com

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1. Introduction

In this paper, we blend two mixed frequency approaches, higher frequency auxiliary variables and estimating missing intermediate observations, to estimate and forecast from higher frequency DSGE models with lower frequency primary and higher frequency auxiliary observables. This allows us to consistently model, estimate, and forecast GDP within a DSGE model at a monthly frequency. We show analytically that our approach improves the estimation and forecasting of the DSGE models and outperforms alternative mixed-frequency methods for monthly and quarterly forecasts in simulated data. In a medium scale application to real world data, the picture is less clear cut, however, we show that information from auxiliary variables substantially improves forecasts during the Great Recession.

Many new data sources with different observation frequencies have emerged over the past few decades. Giannone et al. (2016) show that an introduction of auxiliary variables[1] into DSGE models to capture data at different frequencies can significantly improve the nowcasting of DSGE models. This result is supported by other studies such as Boneva, Fawcett, Masolo and Waldron (2019) who find improvements in forecasting when augmenting DSGE models with survey expectations, Červená and Schneider (2014), who find improvements in both nowcasts and short-term forecasts for DSGE models when auxiliary variables are included, and by VAR studies, e.g., Kohns and Bhattacharjee (2019), who supplement traditional macro variables with internet search data, thereby improving GDP nowcasts. The idea of bringing more data/information to bear on a model goes back to Geweke (1977) and the influential work of Sargent and Sims (1977) who showed that additional information could be incredibly useful in explaining the dynamics of macro variables. [2]

From Christiano and Eichenbaum (1987) onwards, we also know that if the data is sampled at a lower frequency than the frequency of the decisions of economic agents, a temporal aggregation bias is present in the parameter estimates. Foroni and Marcellino (2014) show analytically and in simulations that using mixed-frequency data can improve identification,

1 Auxiliary variables would be higher frequency variables that are not directly modelled but likely contain information regarding lower frequency variables included in the model, such as monthly industrial production, which contains information about quarterly GDP, and also many different financial variables, survey variables and so on.

2 Although in this paper we are agnostic about whether this information should be introduced necessarily via factors.

reduce this temporal aggregation bias, and provide estimates closer to the data generating process. Improved estimates using a mixed-frequency approach were also obtained in Kim (2010) and Christensen, Posch and van der Wel (2016). Yau and Hueng (2019) found that a mixed-frequency method delivers more accurate nowcasts.[3]

Building on the work of Giannone et al. (2016), we introduce an additional equation into the traditional state space representation of DSGE models to enable our blending of the mixed-frequency approach and auxiliary variables to estimate and forecast from the DSGE models. This additional equation allows us to use information from auxiliary variables (that are not part of model equations) observed at a monthly frequency. However, and differently from Giannone et al.'s (2016) quarterly estimation, we use this information not only in forecasting but also in the estimation of the DSGE model itself. Hence, we estimate the quarterly DSGE model at a monthly frequency and employ approaches developed in Kim (2010) and Foroni and Marcellino (2014) to utilize the monthly auxiliary observables for the monthly DSGE model with quarterly primary observables.

This mixture of the two ideas (mixed-frequency and auxiliary variables) is necessary if one wants to fully exploit the information from both model variables observed at a monthly frequency and auxiliary variables observed at a monthly frequency. Note that omitting either of these two ingredients would make it either not possible to use information from variables that are not part of the model equations or not possible to exploit monthly information from the propagation of the model variables in the estimation of DSGE models. We show analytically and in the simulations that our approach improves the estimation and forecasting of the DSGE models. To evaluate the contribution of the introduction of mixed-frequency and the introduction of auxiliary variables, we compare results from our approach with the results from Giannone et al. (2016) and Foroni and Marcellino (2014) / Kim (2010) using both a simple 3-equation textbook DSGE model from Ch.3 Galí (2008) and the medium scale, policy relevant Smets and Wouters (2007) model.

Comparing the estimation results for the 3-equation DSGE model with simulated data shows that mixed-frequency estimation delivers better estimates, especially for monetary

3 For the results of mixed-frequency VAR models see, for example, Cimadomo, Giannone, Lenza, Monti and Sokol (2020) or Schorfheide and Song (2015) among the others who found that mixed-frequency approaches improve forecasts relative to lower frequency counterparts.

policy parameters and the standard deviations of the shocks. Kim's (2010) method outperforms Giannone et al.'s (2016) method in estimation and in forecasting. Our new method outperforms both the alternative mixed-frequency methods and quarterly forecasts.

As noted in Chauvet and Potter (2013) and Siliverstov (2020), monthly information improves forecasting power differently in expansions and recessions. While in expansions, the more frequently observed information contributes little to forecasts, in fast moving recessions, however, there is a significant difference between methods that use more timely information and those that do not, with the latter producing much worse forecasts than the former. Therefore, we compare our method's performance to the alternatives with Smets and Wouters's (2007) model and evaluate forecasts separately for the Great Recession and surrounding expansion periods. Our estimation and forecasts show that information from auxiliary variables substantially improves forecasts for the Great Recession. In contrast to conventional wisdom, estimating the model at a monthly frequency is important and delivers marginally better forecasts in expansions. Our new method produces forecasts that compromise between the forecasts of the two existing methods for both the expansion and Great Recession periods. In all estimations and forecasts for the Smets and Wouters (2007) model, we use vintages of data to make the forecasting exercise as close to the real-time situation as possible.

The rest of the paper proceeds as follows. First, in Section 2 we describe our new method for estimation and forecasting procedures and the two existing methods that it combines. In Section 3 we apply the methods to a 3-equation New Keynesian model in a Monte Carlo experiment. Then we turn to estimating and forecasting the Smets and Wouters (2007) model in Section 4. Before we conclude, Section 5 applies the methods and their estimates of the Smets and Wouters (2007) model to the Great Recession. [4]

4 Our focus is mainly on GDP nowcasts and forecasts - inflation and the interest rate could be found in the Appendix C.

2. Methods for Mixed-Frequency DSGE Models

In this section we present our new method that combines the two main existing approaches to the mixed-frequency estimation and forecasting of DSGE models. In presenting existing approaches, we focus on those introduced in Giannone et al. (2016), Kim (2010) and Foroni and Marcellino (2014) as other methods that use mixed-frequency data in DSGE estimation involve at most small modifications of the latter two and the former is a parameter frequency transformation of quarterly estimation that offers a technically different but related approach. We show analytically that our method is an improvement in terms of information extracted from the data. This improvement stems from the equivalence between Foroni and Marcellino's (2014) and Kim's (2010) methods in the absence of measurement errors, which our auxiliary variables minimizes in a mean square sense when the state is imperfectly, i.e. infrequently, observed.

2.1. Forecasting with Auxiliary Variables Using Parameter Frequency Transformation

In Giannone et al. (2016), the authors concentrate on the state space representation of the DSGE models, estimating at a quarterly frequency, then transforming the parameters to a monthly frequency to enable the use of monthly auxilliary observables in forecasting. After estimating the state space model at a quarterly frequency, they transform the state and observer equations into a monthly frequency. They complete their representation by adding equations that connect higher frequency auxiliary variables to the lower frequency observables. The Kalman filter provides optimal linear estimates of the unobserved variables when producing forecasts from the model. The additional equations are used in the Kalman procedure only for variables that are not observed at a monthly frequency. Specifically, when a variable is not observed, the filter operates as if it were to observe its estimate from the additional equations (estimated by OLS regression). The method also takes into account the noise with which estimates from these additional equations were obtained.

Precisely, the authors firstly transform the estimated standard state space at a quarterly frequency

$$\begin{cases} s_{t_q} &= T_q s_{t_{q-1}} + B_q \epsilon_{t_q} \\ Y_{t_q} &= M_q s_{t_q} \end{cases} \tag{1}$$

into the monthly state-space

$$\begin{cases} s_{t_m} &= T_m s_{t_{m-1}} + B_m \epsilon_{m,t_m} \\ Y_{t_m} &= M_m s_{t_m} \end{cases} \tag{2}$$

with $T_m = T_q^{\frac{1}{3}} = VD^{\frac{1}{3}}V^{-1}$ and $M_m = M_q$,

$vec(B_m B_m') = (I + T_m \otimes T_m + T_m^2 \otimes T_m^2)^{-1} vec(B_q B_q')$

where s_{t_m} and s_{t_q} are monthly and quarterly state vectors accordingly, Y_{t_m} and Y_{t_q} are monthly and quarterly observables. Note that Y_{t_m} is constructed in such a way that it corresponds directly to its quarterly counterpart, i.e., for example, monthly unemployment in $Y_{t,m}$ is $u^*_{t_m} = \frac{1}{3}(u_{t_m} + u_{t_m-1} + u_{t_m-2})$, or one-third (one month's contribution) of the quarterly value and not $u_{t,m}$ the month's unemployment directly. To derive[5] this correspondence between state spaces they assume invertability of V, equality of shocks $\epsilon_{m,t_m} = \epsilon_{m,t_m-1} = \epsilon_{m,t_m-2} = \epsilon_{t_q}$ and that

$$s_{t_m} = T_m s_{t_m-1} + B_m \epsilon_{m,t_m} \tag{3}$$

i.e., s_{t_m} follows an AR(1).

Secondly, they estimate the relation $X_{t_q} = \mu + \Lambda Y_{t_q} + \epsilon_{t_q}$ at a quarterly frequency, where X_{t_q} are auxiliary variables that are observable at a monthly frequency and contain information about the variables that are observed only at the quarterly frequency. They assume that this

5 For the detailed derivation see Giannone et al. (2016)

quarterly relation between observables and auxiliary variables also holds at the monthly frequency and, therefore, add it to the monthly frequency state-space.

$$\begin{cases} s_{t_m} &= T_m s_{t_{m-1}} + B_m \epsilon_{m,t_m} \\ Y_{t_m} &= M_m s_{t_m} + V_{t_m} \\ X_{t_m} &= \mu + \Lambda Y_{t_m} + \epsilon_{t_m} \end{cases} \tag{4}$$

where $V_{t_m} = (v_{1,t_m}, ..., v_{k,t_m})$ is such that $var(v_{i,t_m}) = 0$ if y_{i,t_m} is observable at a monthly frequency and $var(v_{i,t_m}) = \infty$ otherwise. The auxiliary variables are transformed to correspond to their quarterly counterparts. The variance of the shocks in the auxiliary monthly equation is assumed to be the same as their variance in the estimated quarterly counterpart. Lastly, they run the Kalman filter with these three equations using Y_{t_m} as observables if available at a monthly frequency and taking their predicted values from the auxiliary equation if not (taking the associated observation noise into account).

Note that this method uses monthly observations of observables and auxiliary variables only while forecasting, neglecting the additional information available for estimation. We now turn to methods that make use of this information and estimate models at a monthly frequency.

2.2. Mixed Frequencies: Optimal Filtering for Missing Observations

Kim (2010) assumes that the true data generating process is at a monthly frequency and can be represented by a state space model

$$\begin{cases} s_{t_m} &= T_m s_{t_{m-1}} + B_m \epsilon_{m,t_m} \\ Y_{t_m} &= M_m s_{t_m} \end{cases} \tag{5}$$

The problem is that not all variables in Y_{t_m} are observed at the monthly frequency and variables only observed only at a quarterly frequency are some function of their monthly counterparts. For example, let $Y_{t_m} = \begin{pmatrix} w_{t_m}, & z_{t_m} \end{pmatrix}'$ where w_{t_m} is observed at a monthly frequency and z_{t_m} is observed at a quarterly frequency. What we actually observe is $\tilde{Y}_{t_m} = \begin{pmatrix} w_{t_m}, & \frac{1}{3}(z_{t_m} + z_{t_m-1} + z_{t_m-2}) \end{pmatrix}'$ where for $t = 2,3,5,6,8,9...$ observations of $(z_{t_m} + z_{t_m-1} + z_{t_m-2})$ are missed. Therefore, we need to transform the state-space model to include z_{t_m-1} and z_{t_m-2} as observables. With the example of the 3-month averages as observables at a quarterly frequency, the monthly state-space form transforms into

$$\begin{cases} \begin{pmatrix} s_{t_m} \\ s_{t_m-1} \\ s_{t_m-2} \end{pmatrix} = \begin{bmatrix} T_m & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{pmatrix} s_{t_m-1} \\ s_{t_m-2} \\ s_{t_m-3} \end{pmatrix} + \begin{bmatrix} B_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \epsilon_{m,t_m} \\ 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} B & A & A \end{bmatrix} \begin{pmatrix} Y_{t_m} \\ Y_{t_m-1} \\ Y_{t_m-2} \end{pmatrix} = \begin{bmatrix} BM_m & AM_m & AM_m \end{bmatrix} \begin{pmatrix} s_{t_m} \\ s_{t_m-1} \\ s_{t_m-2} \end{pmatrix} \end{cases} \tag{6}$$

where $B = \begin{bmatrix} I & 0 \\ 0 & \frac{1}{3}I \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{3}I \end{bmatrix}$

By transforming the state-space, we have solved the issue of including z_{t_m-1} and z_{t_m-2} as observables, but for $t = 2,3,5,6,8,9...$ their observations are still missed. This means that for these periods, we do not know the state variable and are missing data for some of the observables. This problem could be overcome with the help of data augmentation and optimal filtering. Specifically, for each period t, as in the standard Kalman filter, the value of the state variable is predicted from the transition equation. If all variables are observed in this period, the error from the observation equation is calculated and the prediction is updated. If not all of the variables are observed, first, missing observables are predicted using the predicted state variable and available observations (see eq. 7), then, the prediction of the state is updated given the available observations and the predictions of the missing observations as in standard Kalman filtering.

$$\begin{pmatrix} s_{t_m} \\ z_{t_m} \end{pmatrix} | \begin{pmatrix} z^{t_m-1} \\ w^{t_m-1} \\ w_{t_m} \end{pmatrix} \sim N(\mu_{sz} + \Sigma_{sz,w}\Sigma_w^{-1}(w_{t_m} - \mu_w), \Sigma_{sz} - \Sigma_{sz,w}\Sigma_w^{-1}\Sigma_{sz,w}^{'}) \quad (7)$$

where $Y_{t_m} = \begin{pmatrix} w_{t_m}, & \tilde{z}_{t_m} \end{pmatrix}^{'} = \tilde{M}_m \tilde{s}_{t_m} = \begin{pmatrix} \tilde{M}_{m,w} \\ \tilde{M}_{m,z} \end{pmatrix} \tilde{s}_{t_m}$,

$\mu_{sz} = \begin{pmatrix} \tilde{s}_{t_m|t_m-1}, & \tilde{M}_{m,z}\tilde{s}_{t_m|t_m-1} \end{pmatrix}^{'}$, and $\mu_w = \tilde{M}_{m,w}\tilde{s}_{t_m|t_m-1}$

$\Sigma_{sz,w} = \begin{bmatrix} \tilde{P}_{t_m|t_m-1}\tilde{M}_{m,w}^{'} \\ \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1}\tilde{M}_{m,w}^{'} \end{bmatrix}$ and $\Sigma_w = \tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}_{m,w}^{'}$,

$$\Sigma_{sz} = \begin{bmatrix} \tilde{P}_{t_m|t_m-1} & \tilde{P}_{t_m|t_m-1}\tilde{M}_{m,z}^{'} \\ \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1} & \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1}\tilde{M}_{m,z}^{'} \end{bmatrix}$$

The time subscripts follow standard Kalman filter notation and denote, first, the timing of the variable and, second, the conditioning set. $P_{t_m|t_m-1}$ stands for the predicted variance of the shocks in the transition equation and tildes emphasize that we use matrices and state vectors from the transformed steady state Equation (6), i.e., $\tilde{s}_{t_m} = \begin{pmatrix} s_{t_m}, s_{t_m-1}, s_{t_m-2} \end{pmatrix}^{'}$. $M_{m,w}$ is $n_w \times n_s$ and $M_{m,z}$ is $n_z \times n_s$, where n_w, n_z, n_s are the number of observed at a monthly frequency variables, the number of variables with missing observations at a monthly frequency and the length of the state vector accordingly.

Kim (2010) model and estimate at a monthly frequency and use the Kalman filter to estimate the missing observables for variables observed only at the quarterly frequency. While this allows for the joint modelling, estimation and forecasting with mixed frequency, it does not permit the use of auxiliary variables. This lack of flexibility forces all the variables to be jointly modelled at the highest frequency in the observables.

2.3. Mixed Frequencies: State Space Transformation for Missing Observations

In Foroni and Marcellino (2014), the authors construct an expanded state vector consisting of the state variables and lags of measurement errors in the observables. The inclusion of these errors allows lower-frequency observables to be correctly linked through their monthly counterparts to the state vector. This follows Kim (2010), but allows for measurement errors and generalizes the link between quarterly and monthly observations beyond the 3-month averages explained above. Specifically, we start again with the monthly state space model.

$$\begin{cases} s_{t_m} &= T_m s_{t_{m-1}} + B_m \epsilon_{m,t_m} \\ Y_{t_m} &= M_m s_{t_m} + u_{t_m} \end{cases} \tag{8}$$

where the additional term is u_{t_m}, and therefore the transformed state-space is

$$\begin{cases} f_{t_m} &= \tilde{T}_m f_{t_{m-1}} + \tilde{B}_m \xi_{m,t_m} \\ \tilde{Y}_{t_m} &= \tilde{M}_m f_{t_m} \end{cases} \tag{9}$$

$f_{t_m} = (s_{t_m}, s_{t_m-1}, s_{t_m-2}, u_{t_m}, u_{t_m-1}, u_{t_m-2})'$, and $\xi_{t_m} = (\epsilon_{m,t_m}, 0, 0, u_{t_m}, 0, 0)'$,

$\tilde{Y}_{t_m} = (H(0)Y_{t_m} + H(1)Y_{t_m-1} + H(2)Y_{t_m-2})'$ and

$\tilde{M}_m = \begin{bmatrix} H(0)M_m & H(1)M_m & H(2)M_m & H(0) & H(1) & H(2) \end{bmatrix}$

$$\tilde{T}_m = \begin{bmatrix} T_m & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \qquad \tilde{B}_m = \begin{bmatrix} B_m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $H(0), H(1), H(2), H(l)$ here replace the matrices A and B in Kim's (2010) notation and allow for arbitrary linear dependencies between quarterly and monthly variables (e.g., the sum of all three monthly values, their average, the value in the last month of the quarter, etc.). Moreover, in principle, we allow for more than three relevant lags (which could be relevant for growth rate variables) but discard them for the simplicity of the notation and connection to the Kim (2010) algorithm.[6]

After transforming the steady state into this form, we still have missing observables as does Kim (2010) and again appeal to optimal filtering to fill in the gaps. Foroni and Marcellino (2014) appeal to the Kalman filter with missing observations (in periods with missing observations, the missed data is "skipped", or, alternatively, a value with infinite measurement error is observed) and an expectations-maximization algorithm is used to obtain maximum likelihood estimates. As one might suspect, as both Kim (2010) and Foroni and Marcellino (2014) use linear transformations and optimal filtering, their approaches can be connected analytically:

Proposition 1 (Equivalence between Kim (2010) and Foroni and Marcellino (2014) under perfect observation.)
Let there be no measurement errors ($u_{t_m} = 0\ \forall t$), the methods of Kim (2010) and Foroni and Marcellino (2014) deliver the same predictions of the state vector and its variance.

Proof. See Appendix A. □

6 Following this, the simplified vector of shocks in the transition equation could be $(\epsilon_{m,t_m}, u_{t_m})'$ and then the matrix in front of them would consist only of the first and fourth columns, but we keep this notation for parsimony with Kim's (2010) method.

Proposition 1 shows that filtered states, as in Kim (2010), can be replaced with missing observations without any alteration of the results. The intuition behind this result is the following. The filtering in Kim (2010) uses the joint normal distribution of the state vector and the observables to augment the data. However, the distribution of the state vector conditional on the augmented data (with the variance properly accounting for) is the same as when taken directly from the joint distribution. In other words, using filtered states does not bring new information to the Kalman filter but instead uses the information that the filter already has. Therefore, in our simulations we refer to Kim (2010) and Foroni and Marcellino (2014) methods as the same approach.

Note that if there are measurement errors, then the Foroni and Marcellino (2014) transformation which expands the state vector to include the errors can be used to eliminate measurement errors in the observation equation which casts the model as an expanded state without measurement errors.[7]

2.4. Mixed Frequencies: Auxiliary Variables and State Space Transformation Method

In this subsection, we present our novel method that combines Kim's (2010) and Giannone et al.'s (2016) methods. By fully exploiting the mixed frequency of the data in both estimation and forecasting, our method produces minimum variance estimates and forecasts.

As in Kim (2010), we assume that the data generating process is at a monthly frequency. We transform monthly state space as in eq. 9 and then use observed and auxiliary variables as information from two "sources" in the update step of the Kalman filter.

We start from the system of equations

7 For example, in the case of one state variable s_t and one measurement error u_t the new state vector is two-dimensional and equal to $f_t = (s_t, u_t)'$. After the transition and measurement matrix are adjusted the problem has no measurement errors any more and we can apply solution methods that do not allow for measurement errors. The disadvantage is that the state vector is larger and the computation time is thus longer. Kim (2010) does not allow for measurement errors.

$$
\begin{cases}
s_{t_m} & = T_m s_{t_{m-1}} + B_m \epsilon_{m,t_m} \\
Y_{t_m} & = M_m s_{t_m} + u_{t_m} \\
\tilde{X}_{t_m} & = \mu + \Lambda(L)\tilde{Y}_{t_m} + R_m \zeta_{t_m}
\end{cases}
\tag{10}
$$

where $\Lambda(L)$ is a set of VAR(p) coefficients, $\epsilon_{m,t_m} \sim N(0,I)$, $\zeta_{t_m} \sim N(0,I)$, $u_{t_m} \sim N(0,\Omega)$ and, as in the previous methods, s_{t_m} stands for the state vector, Y_{t_m} stands for the for the vector of the observables, X_{t_m} is the vector of auxiliary variables, u_{t_m} are measurement errors, ϵ_{m,t_m} and ζ_{t_m} represent noise, T_m is the transition matrix, M_m is the measurement matrix, μ is a vector of constants in the OLS equation. The first two equations correspond to the monthly state space model (e.g., a standard DSGE model at a monthly frequency) and the last equation is a VAR(p) model which links observables (such as quarterly DSGE observables) to the auxiliary variables. Tildes in this last equation for $X_{t,m}$ and $Y_{t,m}$ explicitly show that the auxiliary variables are transformed to correspond to quarterly aggregation (like in Foroni and Marcellino (2014) method, i.e. $\tilde{Y}_{t_m} = H_1(L)Y_{t_m}$ and $\tilde{X}_{t_m} = H_2(L)X_{t_m}$).[8]

We transform the system according to the transformation from Equation (8) to Equation (9) in Foroni and Marcellino's (2014) method by stacking lags of the state vector and measurement errors into the state vector

$$
\begin{cases}
f_{t_m} = \tilde{T}_m f_{t_{m-1}} + \tilde{B}_m \xi_{m,t_m} \\
\tilde{Y}_{t_m} = \tilde{M}_m f_{t_m} \\
\tilde{X}_{t_m} = \mu + \Lambda\tilde{Y}_{t_m} + R_m \zeta_{t_m} = \mu + \Lambda \tilde{M}_m f_{t_m} + R_m \zeta_{t_m}
\end{cases}
\tag{11}
$$

where the matrices and variables with tildes are the same as in eq. 9. The equality in the third equation follows from substituting $\tilde{Y}_{t_m}$ from the second equation in Equation (11) into the third equation in Equation (11). We now have a system that we can estimate with a

8 $H_1(L)$ and $H_2(L)$ are determined according to the nature of quarterly aggregation, for example, it could be three month averages as is the case for the interest rates or the sum of three monthly observations which is the case for GDP.

standard Kalman filter, keeping two considerations in mind. Firstly, we have two "signals", the two last equations in the obtained system. In a standard Kalman filter at the core of the update step, two Gaussian curves are combined which, in a simple example with one-dimensional variables, gives the following result,[9]

$$
\begin{aligned}
N(x,\mu_0,\sigma_0)\cdot N(x,\mu_1,\sigma_1) &= N(x,\mu',\sigma') \\
\text{where } \mu' &= \mu_0 + \frac{\sigma_0^2(\mu_1-\mu_0)}{\sigma_0^2+\sigma_1^2} = \mu_0 + k(\mu_1-\mu_0) \\
\text{and } \sigma'^2 &= \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2+\sigma_1^2} = \sigma_0^2 - k\sigma_0^2 \\
\text{and } k &= \frac{\sigma_0^2}{\sigma_0^2+\sigma_1^2} \text{ is the Kalman gain}
\end{aligned}
$$

With two signals, we have to combine three Gaussian curves, and due to the associative property we can apply the mixing in any order

$$
\begin{aligned}
N(x,\mu_0,\sigma_0)\cdot N(x,\mu_1,\sigma_1)\cdot N(x,\mu_2,\sigma_2) &= N(x,\mu',\sigma') \\
\text{where } \mu' &= \mu_0 + \frac{\sigma_0^2(\mu_1+k_1(\mu_2-\mu_1)-\mu_0)}{\sigma_0^2+\sigma_1^2(1-k_1)} = \mu_0 + k_0(\mu_1+k_1(\mu_2-\mu_1)-\mu_0) \\
\text{and } \sigma'^2 &= \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2+\sigma_1^2(1-k_1)} = \sigma_0^2 - k_0\sigma_0^2 \\
\text{and } k_1 &= \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} \text{ and } k_0 = \frac{\sigma_0^2}{\sigma_0^2+\sigma_1^2(1-k_1)}
\end{aligned}
$$

Giving the obvious hierarchical extensions to the two curve case.

The second consideration is that we still have missing observations. Therefore, we will "skip" the entries where we have missing observations and, in these periods, use only one "signal" from the auxiliary variables in the update step of the Kalman filter.

9 For a detailed derivation of combining the three Gaussian curves in a vector case, see chapter 13.6 of Kay (1993).

Accordingly, the implementation of the algorithm is as follows. We rewrite the state space system of the new method (Equation (11)) as

$$\begin{cases} f_{t_m} = \tilde{T}_m f_{t_{m-1}} + \tilde{B}_m \xi_{m,t_m} \\ Z_{t_m} = \tilde{\mu} + \Gamma_m f_{t_m} + \Omega_m \zeta_{t_m} \end{cases} \tag{12}$$

where $Z_{t_m} = \begin{pmatrix} \tilde{Y}_{t_m} \\ \tilde{X}_{t_m} \end{pmatrix}$, $\tilde{\mu} = \begin{pmatrix} 0 \\ \mu \end{pmatrix}$, $\Gamma_m = \begin{pmatrix} \tilde{M}_m \\ \Lambda \tilde{M}_m \end{pmatrix}$, and $\Omega_m = \begin{pmatrix} 0 \\ R_m \end{pmatrix}$, which is then estimated using the Kalman filter with missing observations.

Table 1: Differences and commonalities of the methods.

Difference/commonality	Kim (2010)	Giannone et al. (2016)	New method
Frequency used in the model estimation	Monthly	Quarterly	Monthly
Usage of monthly information of model variables in estimation	Yes	No	Yes
Usage of auxiliary variables in forecasting	No	Yes	Yes
Usage of the auxiliary variables in the estimation	No	No	Yes

The three differences to the Giannone et al. (2016) method are the estimation of the model at a monthly frequency, the usage of the information from auxiliary variables in the estimation, and the usage of information from the auxiliary variables in the forecasting even when the quarterly variables (variables with missing observations at a monthly frequency) are observed. The difference to the Kim (2010) method is that auxiliary variables are used to estimate the model and to perform the forecasts (see the summary in Table 1). We demonstrate the efficiency of our new method formally in the following proposition.

Proposition 2 (The efficiency of the methods.)
Our combined method produces minimum variance linear unbiased estimates whereas the methods of Giannone et al. (2016) and Kim (2010) / Foroni and Marcellino (2014) result in an excess variance of the estimated state vector.

Proof. See Appendix A. □

Proposition 2 highlights the intuitive result that minimum variance linear unbiased estimates require the use of all available information. Ignoring any information available leads to a larger variance of the state vector, and the more information is ignored, the higher the variance. Therefore, Giannone et al.'s (2016) and Kim's (2010) / Foroni and Marcellino's (2014) estimates are inefficient as long as the additional variables used in our new method improve the estimation and forecasts of the model variables (i.e., if they contain additional information that is useful for estimation or forecasting).

This proposition does not reveal the quantitative difference between estimation and forecasting results when employing the different methods. Therefore, the rest of the paper is devoted to assessing this quantitative difference via estimation and forecasting results using the different methods in three experiments. We compare Giannone et al. (2016) method, Foroni and Marcellino (2014)/Kim (2010) method and the new method first using simulated data on a 3-equation New Keynesian model and secondly using US vintage data on the medium scale Smets and Wouters (2007) model. Finally, we apply the methods to the Great Recession to examine how well they perform in phases of the business cycle that are notoriously challenging.

3. Mixed Frequencies and a 3-Equation DSGE Model

We begin by comparing the methods in Monte Carlo experiments using the textbook 3-equation New Keynesian model (see, e.g., Galí (2008)). The model features a New Keynesian Phillips curve, a dynamic IS equation obtained from the Euler condition of the representative household, and a monetary policy rule which we take as a Taylor rule.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \varepsilon_t^{\pi} \tag{13}$$

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^*) + E_t \tilde{y}_{t+1} + \varepsilon_t^{y} \tag{14}$$

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \varepsilon_t^{i} \tag{15}$$

where $\pi, \tilde{y}, i, r^*$ are inflation, the output gap, the central bank's interest rate, and the natural rate in the economy respectively. The production function has a form $Y = AN^{1-\alpha}$, where N denotes labor hours, A is a technological process that follows an AR(1) process, and Y is GDP.[10] We parameterize the model for the Monte Carlo experiment following Galí (2008). These values, alongside the estimation results, which we explain in detail in the next section, are summarized in Table 2.

3.1. Mixed-Frequency Estimation of the 3-Equation Model

The estimation proceeds as follows. First, we simulate data from the model using the parameter values denoted in the column "DGP" in Table 2. We simulate 333 periods of monthly data, which corresponds to roughly 28 years of data.[11] Then we chose Y, i, π (GDP, annual interest rate, and annual inflation rate) as observables. For the Giannone et al. (2016) method, the model needs to be estimated at a quarterly frequency. Therefore, we also construct aggregated data using 3-month averages.[12] For the mixed-frequency estimation, we assume that GDP is observed quarterly (i.e., has missing observations) and that inflation and the interest rate are observed monthly (i.e., *no* missing observations). We also note that the parameters $\theta, \beta, \rho_a, \rho_i$ (Calvo parameter, households' discount factor, persistence parameter for the technological process and persistence parameter for the monetary policy shock respectively) differ with the data frequency, i.e., $\theta_{quarterly} = 1 - 3(1 - \theta_{monthly})$, $\beta_{quarterly} = \frac{1}{(1+3(\frac{1}{\beta_{monthly}}-1))}$, $\rho_{a,quarterly} = \rho^3_{a,monthly}, \rho_{i,quarterly} = \rho^3_{i,monthly}$.

As an auxiliary variable, we mimic Industrial Production (IP) via an estimated OLS regression for the growth rates of GDP and Industrial Production (IP as the dependent variable, no constant was included) with data from the St. Louis Fed database from 1947Q1 to 2020Q1.[13] From the estimated regression coefficient, variance of the errors, and simulated data of GDP, we constructed our IP variable as a sum of the Monte Carlo simulated GDP series multiplied by the regression coefficient from OLS regression and draws from the estimated normally distributed error term distributions.[14]

10 For additional details of the model see Galí (2008). The code for the model was taken from Pfeifer (2022).

11 To check for robustness to longer data series, we also rerun the procedure for 1000 months (roughly 83 years), the results are shown in Appendix D.

12 $Y_q = \frac{1}{3}(Y_{m,t} + Y_{m,t-1} + Y_{m,t-2})$

13 When a constant was included, the estimated value of the constant was negligible and, therefore, we chose not to include it - also that the constant is hard to interpret in this regression.

14 $\hat{IP} = \hat{\beta} GDP + \hat{\sigma}\varepsilon$, $\varepsilon \sim N(0,1)$, $\hat{\beta} = 1.352, \hat{\sigma} = 0.06$

For the priors, we follow standard DSGE estimation procedure and choose the normal distribution for the policy parameters in the Taylor rule, the beta distribution for parameters distributed between 0 and 1, the gamma distribution for preference parameters, and inverse gamma distribution for the standard deviations.[15] Prior means were set at the true values of the parameters. The complete table of the priors can be found in the Appendix B.

Table 2: The 3-equation Model from Ch. 3 Galí (2008): Estimation of the Parameters Using Quarterly and Mixed-frequency Methods. Each row presents parameter estimates using the Quarterly estimation, the Kim (2010) / Foroni and Marcellino (2014) estimation method, or the new estimation method (see Section 2 for methods' details). Standard deviations of the estimated parameters are presented in parentheses. The estimates of the quarterly estimation are transformed into their monthly counterparts.[16] The column DGP shows parameter values used to generate the simulated data (the true parameter values).

Parameter	DGP	Quarterly	Mixed Kim (2010)	Mixed new algorithm
θ	0.8889	0.9032 (0.0166)	0.8902 (0.0215)	0.8894 (0.0224)
ρ_i	0.7937	0.7606 (0.0020)	0.7931 (0.0007)	0.7935 (0.0004)
ρ_a	0.9655	0.9655 (0.0179)	0.9649 (0.0295)	0.9654 (0.0297)
β	0.9966	0.9966 (0.0017)	0.9967 (0.0028)	0.9965 (0.0029)
α	0.3333	0.3376 (0.1186)	0.3328 (0.1189)	0.3350 (0.1233)
η	4	3.9930 (2.0036)	3.9648 (1.9984)	4.1501 (2.0336)
ε	6	6.2237 (2.8789)	6.0392 (3.0019)	6.0683 (2.9546)
ϕ_π	1.5	3.2882 (0.2188)	1.5446 (0.0868)	1.5464 (0.0782)
ϕ_y	0.1250	0.0721 (0.0588)	0.1249 (0.0566)	0.1241 (0.0560)
σ_π	0.0208	0.0189 (0.0013)	0.0206 (0.0008)	0.0209 (0.0005)
σ_y	0.0208	0.0101 (0.0008)	0.0218 (0.0030)	0.0206 (0.0026)
σ_i	0.0208	0.3072 (0.0323)	0.1475 (0.0246)	0.1434 (0.0234)

The estimates are presented in Table 2. The results show that monetary policy parameters (Taylor rule coefficients, persistence, and variance of the monetary policy shocks) together with demand elasticity and all standard deviations of the shocks are estimated closer to the data generating process in the mixed-frequency estimation than in the quarterly estimation. For the standard deviations of the estimates, there is no clear-cut winner among the quarterly and mixed-frequency approaches. Between Kim (2010) / Foroni and Marcellino (2014) approach and our combined method, we see that more parameters (all except capital

15 The standard deviations were also adjusted with the data frequency, with quarterly standard deviations being $\sqrt{3}$ times larger than the monthly standard deviations.

16 θ was estimated in a quarterly model at 0.7096, ρ_i at 0.44, ρ_a at 0.8999, β at 0.99, σ_π at 0.0327, σ_y at 0.0175, σ_i at 0.5320. Standard deviations were transformed using the approximation $\text{Var}(g(x)) = (g'(x))^2\text{Var}(x)$ with evaluation of the derivative at the estimated value of the parameters.

Table 3: The 3-equation Model from Ch. 3 Galí (2008): RMSE of the Filtered GDP series, obtained Using Mixed-frequency Methods. The Kalman filter generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The obtained series is compared to the simulated series. Each row presents an out-of-sample (sample different from the one on which the model was estimated) RMSE for the filtered monthly GDP series, values of which are based on the information at time t, or the information available at the end of the sample. The columns present RMSE for the filtered values, obtained using the Kalman filter of the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), or the new method, suggested in the Section 2. The errors are averaged over 1000 simulated sample points.

Filtered statistic	RMSE Giannone et al. (2016)	RMSE Kim (2010)	RMSE new method
$E(y_t\|t)$	0.18195	0.02622	0.02616
$E(y_t\|T)$	0.18195	0.02312	0.02307

share, semi elasticity of money demand, demand elasticity, and Taylor rule coefficients) are estimated closer to the DGP with our method. However, the overall estimates from these two methods are close to each other. Regarding the standard deviations, the estimates do not appear to favor one method over the other.

Thus, the estimation results clearly show that mixed-frequency approaches deliver better estimates than the quarterly estimation. The reason for the minimal difference between the two mixed-frequency approaches, Kim's (2010) / Foroni and Marcellino's (2014) method and our combined method, is likely that one auxiliary variable for only one variable unobserved at a monthly frequency variable is used to improve estimates - using more variables will likely increase the estimation improvements.

Another way to compare the model estimates for the different methods is to compare their filtered and smoothed unobserved (at a monthly frequency) GDP series. Table 3 gives the RMSE statistics of the filtered and smoothed series and Figure 10 shows these GDP estimates on out-of-sample data. Both in-sample and out-of-sample estimates of unobserved GDP show that Kim (2010) / Foroni and Marcellino (2014) and our new approach are closer to the true DGP than Giannone et al.'s (2016) method with roughly one order of magnitude smaller RSMEs. As expected, the model estimated at a quarterly frequency with Giannone et al.'s (2016) method misses higher frequency spikes. Our new (combined) approach delivers closer estimates in terms of RMSFE, but the improvements are modest relative to Kim's (2010) / Foroni and Marcellino's (2014) method.[17]

In sum, the mixed-frequency approach improves the estimation of the model parameters

17 The same graph but with a difference of the filtered variables to the data is shown in Figure 8 of Appendix C.

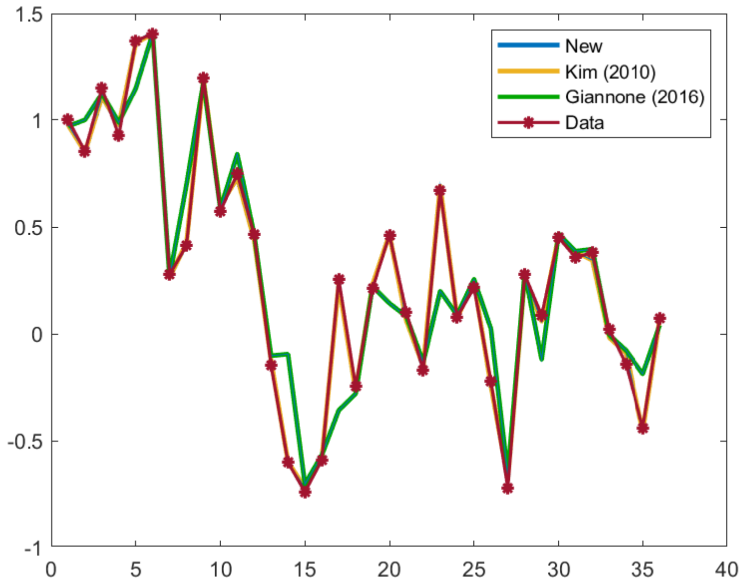

Figure 1: The 3-equation Model from Ch. 3 Galí (2008): Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$) for the out-of-sample data. The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green). The simulated data is shown in red. X-axis shows time in months.

and can better assess the current state of the economy relative to the standard quarterly procedure. Furthermore, our introduction of auxiliary variables into mixed-frequency estimation further improves the estimation and filtering of unobserved variables.

3.2. Mixed-Frequency Now/Forecasting with the 3-Equation Model

We now assess the now and forecasting performance of the different methods for GDP with 1000 out-of-sample simulated data points. In terms of nowcasting, our method that combines Kim's (2010) state space transformation and auxiliary variables improves forecasts and delivers RMSFE substantially smaller than the quarterly 1-step ahead RMSFE, which is 0.7846 (the entries are expressed for all methods relative to the RMSE of our method). All forecasts are statistically different from each other at a 1% significance level. The results also imply that using mixed-frequency methods improves nowcasting more than using additional variables (as the relative RMSE in the first row are substantially higher than in the second), however, we used only one auxiliary variable and, for more complicated models with many

Table 4: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.5012, 0.2710, 0.0108. The columns present months in which nowcasts are made. The nowcast errors are averaged over 1000 simulated sample points. *** indicates the forecasts that are statistically significantly different from the other forecasts with a 1% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation. Quarterly 1-step ahead RMSFE is 0.7846.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.3540***	1.3833***	9.7469***
Kim (2010)	1.0006***	1.0018***	1.0084***
new method	1***	1***	1***

auxiliary variables, this ranking can reverse (as indeed is the case for the Smets and Wouters (2007) model when forecasting in the Great Recession).

Figure 2 and Table 5 show the forecasting results. As with estimation and nowcasting, mixed-frequency methods produce better forecasts than the standard quarterly approach. Auxiliary variables also improve forecasts, but for some quarters, we obtained even worse results for the Giannone et al. (2016) method than the standard quarterly forecasts. The largest difference between the Kim (2010) / Foroni and Marcellino (2014) method and our combined method is for one quarter ahead forecasts, with our combined method delivering lower RMSFE. The forecasts are statistically significantly different between these two methods for the two and three quarters ahead forecasts, with the Kim (2010) RMSFE being smaller. However, the difference of the RMSFE for two and three quarters ahead is smaller than the difference for one quarter ahead forecast by order of magnitude. The forecasts from the three methods are statistically significantly different from each other for one, two, three, and seven quarters ahead forecasts. The statistical difference with the quarterly forecasts is shown in the Table 5.

The reason for the rather poor performance of Giannone et al.'s (2016) method in forecasting is the transformation of the parameters from quarterly to monthly frequency values. When

Table 5: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7045, 0.6119, 0.5899, 0.5857, 0.5841, 0.5852, 0.5857, 0.5862, 0.5869, 0.5878, 0.5887, 0.5892. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over 1000 simulated sample points. *** (**) indicates the forecasts that are statistically significantly different from quarterly forecasts with a 1% (5%) significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	1.1137	1.0215	1.0098	1.0094	1.0108	1.0086	1.0072	1.0058	1.0045	1.0029	1.0015	1.0006
Giannone et al. (2016)	1.3100***	1.0754***	1.0132***	1.0026	1.0007	1.0003	1.0002**	1.0000	1.0000	1.0000	1.0000	1.0000
Kim (2010)	1.0002***	1.0000***	1.0000	1.0000	1.0000***	1.0000***	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
New Method	1***	1***	1	1	1***	1***	1	1	1	1	1	1

the transformation of the parameters estimated at a quarterly frequency is done according to their true link[18] to the monthly counterparts (and not by the powering of the transition matrix to the power $\frac{1}{3}$) the forecasts produced by Giannone et al. (2016) method are better than the quarterly forecasts. These forecasts even outperform Kim (2010) and our new method for the forecasts two and three quarters ahead. The nowcasting RMSFE with this parameter transformation of the parameters also decreases but remain larger than those from Kim (2010) and our combined method.

To disentangle the contribution of potentially better estimation from potentially better forecasting for the mixed-frequency algorithms, we also assess the performance of all methods when the parameters are set to their pseudo-true values and not to the posteriors from their respective estimations. The results remain roughly unchanged except that the difference between our new method and Kim's (2010) method becomes statistically insignificant for all quarters in the forecasting exercise.

The results for monthly forecasts for GDP, inflation, and the interest rate are presented in the Appendix C. The forecasts for GDP and inflation are improved when using mixed-frequency approaches (Kim (2010) / Foroni and Marcellino (2014) or our new approach), but for the interest rate is forecasted best by Giannone et al.'s (2016) method. This supports the findings that both the introduction of the mixed-frequency approach and the introduction of

18 For example, the capital share should be the same for quarterly and monthly frequency, the Calvo parameter should be transformed according to $\theta_{monthly} = 1 - \frac{1}{3}(1 - \theta_{quarterly})$

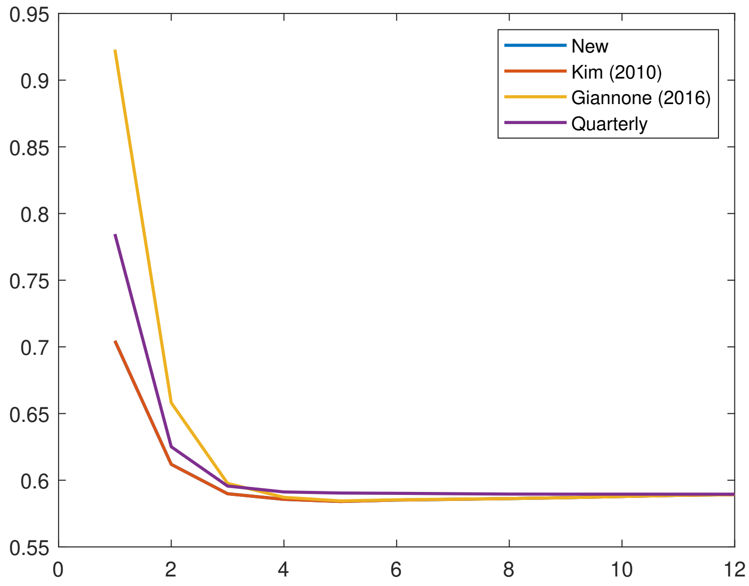

Figure 2: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

auxiliary variables can improve forecasts and the decision, perhaps, ought to be to combine the two as in our method and not choose one over the other.

Before we conclude the Monte Carlo exercise and turn to real-world data, it is informative to note the robustness of the results. Appendix D provides one such check estimating with more data points. The results from this section remain essentially unchanged, except for the forecasting exercise where the difference between Kim (2010) and our combined method becomes insignificant for all quarters, consistent with known effectiveness of the Kalman filter with large samples specifically here relative to the additional information provided by the auxiliary data in smaller samples.

4. Mixed Frequencies and the Smets and Wouters (2007) Model

In this section we turn to real-world data and a medium scale, policy relevant model, namely the Smets and Wouters (2007) model. This exercise aims to take the insights from the Monte Carlo experiment in the textbook model from the previous section and apply them to assess quantitative differences between the methods in a setting with actual data. It also serves as an example of the robustness of the results across models, which exploiting information from many auxiliary variables in contrast to the previous section a priori should benefit Giannone et al.'s (2016) method.

4.1. Description of the Data

We follow Smets and Wouters (2007) in choosing data series for the observed model variables; thus, we use seven observables (GDP, consumption, investments, labor hours, wages, inflation, and the interest rate). To make the exercise as close to real-time forecasting as possible, we use data vintages of the model variables from the ALFRED database. For GDP, consumption, investments, inflation, and the interest rate, the same series were taken as in Smets and Wouters (2007). For the population series, the seasonally unadjusted series instead of the seasonally adjusted one is used due to the availability of vintages - the two series are almost indistinguishable.[19] For wages, the FRED counterpart of the BLS wage series is used. For hours, "AWHNONAG" (FRED code name) is chosen over the original hours series "PRS85006023" (FRED code name) as vintages are available for the latter only beginning in 2011.[20] Also, in Appendix B all FRED codes for the series used are listed.

In transforming variables, we also follow Smets and Wouters (2007). In a monthly model, the Federal Funds Rate and labor hours are observed every period while GDP, investments, wages, inflation, and consumption are observed only every third month. Data from FRED-MD provided by Michael W. McCracken from St. Louis Fed was used for the auxiliary variables.[21]

19 For the sample that was used in Smets and Wouters (2007) they differ in eight observations, three of which are larger than 1, but they all are smaller than 107. The average observation for the sample is 155673.

20 See Appendix C for a plot of both series.

21 https://research.stlouisfed.org/econ/mccracken/fred-databases/

It is a compiled dataset from the FRED database, which is updated in real-time, and contains vintages of data for more than one hundred variables related to the state of the economy. We use nearly all the variables from this dataset.[22] Thus, we are left with 114 variables that correspond broadly to the categories output and income; labor market; housing; consumption, orders, and inventories; money and credit; prices, interest and exchange rates; and the stock market.[23] The variables are transformed according to the suggestions of the FRED-MD. Note that while the FRED-MD also contains vintages of the main macro variables used as model observables, these vintages are available only from 2015 and, thus, we use the corresponding ALFRED series as explained above.

To obtain quarterly counterparts of the auxiliary variables that were used in the regression for the Giannone et al. (2016) method and the combined algorithm, we use the state-space correspondence between monthly and quarterly series from Section 2.2 for the GDP series (the same correspondence is valid for the investments, consumption, and wages as well - only labor hours are transformed differently). [24]

For this section of the paper we used the 1966Q1:2010Q1 time span as a sample for estimation, which comprises 80% of the dataset. This cut of the sample into training and test subsamples is widely used in the data science literature (see Hyndman and Athanasopoulos (2018), Ch. 5.8). In forecasting using DSGE models there is no standard approach as to how to split the sample (see, for example Elliott and Timmermann (2013) Ch.2 Table A-2 for a summary). Moreover, 2010Q1 is a very convenient point to stop as it includes the Great Recession and the three quarters subsequent, thus forecasting on a span from 2010Q2:2020Q1 is an evaluation of the forecasting performance of the methods in expansions. [25] In Section 5, we apply all methods to the Great Recession to assess the performance of the

22 The exceptions were: the Federal Funds Rate as it is contained as an observable in our dataset of model variables and thirteen variables with many missing observations: Moody's Aaa Corporate Bond Minus FEDFUNDS, New Orders for Consumer Goods, New Orders for Nondefense Capital Goods, Moody's Baa Corporate Bond Minus FEDFUNDS, Consumer Motor Vehicle Loans Outstanding, Total Consumer Loans and Leases Outstanding, Help-Wanted Index for the United States, Ratio of Help Wanted/No. Unemployed, 10-Year Treasury C Minus FEDFUNDS, 5-Year Treasury C Minus FEDFUNDS, Trade Weighted US Dollar Index, Consumer Sentiment Index, VXO.

23 Due to the large number of variables and the associated prohibitive computation time, we use eight factors constructed from these variables.

24 $dy_t^q = \frac{1}{3}(dy_t^m + 2dy_{t-1}^m + 3dy_{t-2}^m + 2dy_{t-3}^m + dy_{t-4}^m)$ for $y \in$ {GDP, Investments, Wages, Consumption, Inflation}.

25 In 2010Q1, the GDP growth rate was near its sample average for the first time since the onset of the financial crisis.

methods during busts. For the quarterly estimation, our priors match those of Smets and Wouters (2007); for the monthly estimation, parameters were transformed according to their relation to monthly counterparts (see Appendix B for details).

4.2. Mixed-Frequency Estimation of the Smets and Wouters (2007) Model

Table 6 and Table 7 present the results of the estimation of the Smets and Wouters (2007) model using different datasets. The first column refers to the original dataset, and the second column shows results for the estimation using the "AWHNONAG" instead of the "PRS85006023" series for the labor hours (see Section 4.1 for explanation). The next column presents results for the estimation using the 2005/02 vintage instead of the 2005/04 vintage used in Smets and Wouters (2007) to see how real-time data changes the results compared to the later vintages. The columns "Till 2010" and "Till 2010, all data" show estimations using data until 2010 from the 2010/05 vintage and the 2021/08 vintage (the last available vintage when the estimation was performed), to evaluate how the posteriors change with more data and, again, to evaluate differences between real-time / not real-time estimation. Finally, the last two columns present estimations using Kim's (2010) and our new method, both using the 2010/05 vintage of the data.

Table 6: The Smets and Wouters (2007) Model: Estimation of the Parameters Using Quarterly and Mixed-frequency Methods. Each row presents parameter estimates from the original estimation, using the Quarterly estimation and a sample 1966Q1:2010Q1, using the estimation method from Kim (2010) / Foroni and Marcellino (2014) or the new method suggested in the Section 2. Standard deviations of the estimated parameters are presented in parentheses. The estimates of the mixed-frequency estimation are transformed into their quarterly counterparts.[26]

Parameter	Original	Other Hours	Vintage data	Till 2010	Till 2010, all data	Mixed Kim (2010)	Mixed new algorithm
φ	5.77 (1.04)	5.69 (1.03)	5.69 (1.04)	4.82 (1.03)	4.47 (0.99)	6.17 (0.71)	6.34 (0.68)
σ_c	1.38 (0.13)	1.35 (0.14)	1.35 (0.13)	1.26 (0.16)	1.34 (0.17)	0.86 (0.07)	0.81 (0.12)
h	0.71 (0.04)	0.71 (0.04)	0.71 (0.05)	0.64 (0.06)	0.60 (0.06)	1.35 (0.10)	1.12 (0.04)
ξ_w	0.70 (0.07)	0.72 (0.06)	0.72 (0.06)	0.81 (0.05)	0.78 (0.06)	0.71 (0.08)	0.69 (0.06)
σ_l	1.84 (0.57)	2.04 (0.58)	2.05 (0.58)	2.03 (0.58)	1.97 (0.57)	0.43 (0.15)	0.37 (0.19)
ξ_p	0.65 (0.05)	0.70 (0.05)	0.70 (0.05)	0.79 (0.04)	0.77 (0.04)	0.85 (0.03)	0.85 (0.02)
i_w	0.58 (0.12)	0.55 (0.13)	0.54 (0.13)	0.58 (0.12)	0.56 (0.13)	0.64 (0.06)	0.54 (0.05)
i_p	0.24 (0.09)	0.24 (0.09)	0.24 (0.09)	0.25 (0.09)	0.32 (0.10)	0.42 (0.07)	0.59 (0.11)
ψ	0.55 (0.11)	0.56 (0.11)	0.56 (0.11)	0.68 (0.10)	0.72 (0.09)	0.57 (0.07)	0.76 (0.16)
Φ	1.61 (0.08)	1.62 (0.08)	1.62 (0.08)	1.53 (0.08)	1.56 (0.08)	1.86 (0.04)	1.63 (0.08)
r_π	2.04 (0.18)	2.01 (0.18)	2.01 (0.17)	1.79 (0.17)	1.76 (0.17)	1.54 (0.08)	1.46 (0.20)
ρ	0.81 (0.02)	0.82 (0.02)	0.82 (0.02)	0.82 (0.03)	0.80 (0.03)	0.97 (0.003)	0.91 (0.01)
r_y	0.09 (0.02)	0.09 (0.03)	0.09 (0.03)	0.07 (0.03)	0.06 (0.03)	0.19 (0.01)	0.22 (0.02)
$r_{\Delta y}$	0.22 (0.03)	0.24 (0.03)	0.24 (0.03)	0.25 (0.03)	0.25 (0.03)	0.07 (0.01)	0.10 (0.02)
$\bar{\pi}$	0.79 (0.10)	0.80 (0.10)	0.80 (0.10)	0.78 (0.10)	0.80 (0.10)	0.62 (0.13)	0.88 (0.11)
$100(\frac{1}{\beta}-1)$	0.17 (0.06)	0.17 (0.06)	0.17 (0.06)	0.16 (0.06)	0.16 (0.06)	0.19 (0.09)	0.01 (0.04)
$\bar{l}$	0.51 (1.08)	0.98 (1.05)	1.01 (1.06)	0.25 (1.09)	0.96 (1.14)	3.05 (0.52)	2.64 (0.49)
$\bar{\gamma}$	0.43 (0.01)	0.43 (0.01)	0.43 (0.01)	0.42 (0.02)	0.41 (0.02)	1.08 (0.05)	1.02 (0.06)
α	0.19 (0.02)	0.19 (0.02)	0.19 (0.02)	0.17 (0.02)	0.19 (0.02)	0.09 (0.01)	0.09 (0.02)

Table 7: The Smets and Wouters (2007) Model: Estimation of the Shock Processes Using Quarterly and Mixed-frequency Methods. Each row presents parameter estimates from the original estimation, using the Quarterly estimation and a sample 1966Q1:2010Q1, using the estimation method from Kim (2010) / Foroni and Marcellino (2014) or the new method suggested in the Section 2. Standard deviations of the estimated parameters are presented in parentheses. The estimates of the mixed-frequency estimation are transformed into their quarterly counterparts.

Parameter	Original	Other Hours	Vintage data	Till 2010	Till 2010, all data	Mixed Kim (2010)	Mixed new algorithm
ρ_a	0.96 (0.01)	0.95 (0.02)	0.95 (0.02)	0.96 (0.01)	0.96 (0.01)	0.89 (0.01)	0.94 (0.01)
ρ_b	0.22 (0.09)	0.25 (0.10)	0.25 (0.10)	0.58 (0.12)	0.57 (0.13)	0.97 (0.01)	0.97 (0.01)
ρ_g	0.98 (0.01)	0.98 (0.01)	0.98 (0.01)	0.98 (0.01)	0.97 (0.01)	0.73 (0.12)	0.62 (0.09)
ρ_I	0.71 (0.06)	0.73 (0.06)	0.73 (0.06)	0.79 (0.05)	0.82 (0.05)	0.13 (0.08)	0.76 (0.07)
ρ_r	0.15 (0.06)	0.15 (0.06)	0.15 (0.06)	0.18 (0.06)	0.21 (0.07)	0.05 (0.01)	0.05 (0.05)
ρ_p	0.89 (0.05)	0.86 (0.06)	0.86 (0.06)	0.84 (0.06)	0.85 (0.06)	0.73 (0.06)	0.98 (0.004)
ρ_w	0.97 (0.02)	0.96 (0.02)	0.96 (0.02)	0.96 (0.02)	0.95 (0.03)	0.98 (0.01)	0.53 (0.05)
μ_p	0.70 (0.10)	0.69 (0.11)	0.68 (0.11)	0.72 (0.09)	0.69 (0.10)	0.94 (0.01)	1.00 (0.003)
μ_w	0.84 (0.06)	0.85 (0.06)	0.85 (0.06)	0.91 (0.04)	0.90 (0.05)	0.99 (0.003)	0.85 (0.09)
σ_a	0.46 (0.03)	0.46 (0.03)	0.46 (0.03)	0.49 (0.03)	0.47 (0.03)	0.87 (0.05)	0.94 (0.05)
σ_b	0.24 (0.02)	0.24 (0.03)	0.24 (0.03)	0.17 (0.03)	0.17 (0.03)	0.06 (0.01)	0.07 (0.01)
σ_g	0.53 (0.03)	0.52 (0.03)	0.52 (0.03)	0.50 (0.03)	0.49 (0.03)	0.84 (0.05)	0.80 (0.05)
σ_I	0.45 (0.05)	0.44 (0.05)	0.44 (0.05)	0.42 (0.04)	0.36 (0.04)	0.63 (0.15)	0.22 (0.03)
σ_r	0.24 (0.02)	0.24 (0.02)	0.24 (0.02)	0.24 (0.01)	0.24 (0.01)	0.10 (0.01)	0.11 (0.01)
σ_p	0.14 (0.02)	0.14 (0.02)	0.14 (0.02)	0.15 (0.02)	0.12 (0.01)	0.29 (0.03)	0.24 (0.02)
σ_w	0.24 (0.02)	0.24 (0.02)	0.24 (0.02)	0.27 (0.02)	0.31 (0.02)	0.51 (0.03)	0.51 (0.06)
ρ_{ga}	0.52 (0.09)	0.54 (0.09)	0.54 (0.09)	0.53 (0.08)	0.55 (0.08)	0.59 (0.05)	0.52 (0.13)

The only substantial difference between the first two columns in the Table 6 and Table 7 is the steady state value of labor hours, all other parameters were unaffected by the usage of a different labor hours series. Moving to the real-time data does not significantly change the posteriors,[27] showing that the final data points do not contain a significant amount of different information.

Adding more data (until 2010) delivers different estimates of many parameters, however - although the standard deviations do not change. The most significant differences are in the persistence of the risk premium shock and the steady state level of hours. The risk premium becomes more persistent (with innovations a bit less volatile), and steady state labor hours are even smaller than the original estimation. Apart from that, prices are estimated to be stickier, and it is costlier to change capital utilization.

With more months between vintages, the posteriors change more significantly (compare "Other hours" and "Vintage data" vs. "Till 2010" and "Till 2010, all data"). Using the most recent data vintage delivers posterior estimates of the parameters different from 2010/05 vintage whereas the posterior standard deviations of the parameters are quite robust to this modification. Capacity utilization costs increase even more, indexation to past prices increases, and monetary policy is estimated to put less weight on the output gap.

Mixed-frequency estimation delivers substantially different results. The persistence of the risk premium is much larger and the steady state level of labor hours is greater. Trend growth is twice as large as in the quarterly estimation, and capital share is minor. Habits are estimated to be more intense, and the Frisch elasticity of labor is smaller. The price indexation parameter increases even more, and price and wage markups, together with the productivity shocks, are more volatile. Monetary policy puts more weight on the output gap rather than its growth. Kim's (2010) and our new combined method provide similar estimations with our method estimating a larger volatility of investment shocks and a larger time discount factor. These differences may stem from observation errors or differences in the priors when changing frequency, but in either case, we will use the quarterly estimates in the forecasting exercises to maintain comparability between the different methods.[28]

In sum, all estimations find a highly persistent risk premium. The most volatile estimates

27 Only steady state labor hours, Frisch elasticity of labor hours, indexation parameter of wages, and the coefficient on MA term in price markup insignificantly differ from the second column, which indicates some uncertainty of steady state labor hours parameter

28 Forecasting results using the estimates from the monthly mixed methods can be found in the appendix.

are obtained for the steady state level of the labor hours. Prices are estimated to be stickier and more volatile than the original estimation, and the price indexation parameter is also larger. Monetary policy's reaction to inflation is weaker and closer to the original estimates from Taylor (1993).

4.3. Mixed-Frequency Now/Forecasting with the Smets and Wouters (2007) Model

We begin with the performance of the different methods during the expansion following the Financial Crisis. This allows us to assess the different methods during a period of more standard macroeconomic developments before we then turn in the next section to the performance of the different methods during the Financial Crisis itself.

The nowcasting performace for 2010Q2:2019Q4 can be found in Table 8, where we calculated rolling 1-3 month nowcasts and measure them against the one quarter ahead forecasts. In line with the controlled Monte Carlo results with simulated data, our new combined method produces the smallest root mean squared errors among all mixed-frequency methods. Kim's (2010) method delivers smaller forecasting errors than Giannone et al.'s (2016) method, showing the importance again of using a monthly model, which the auxiliary variables alone are unable to fully compensate for. Additionally, when using mixed-frequency estimates instead of estimates from quarterly estimation Kim's (2010) method outperforms quarterly one-step ahead forecast in the third month (see Appendix D.5). Our combined method takes the monthly frequency and further adds the information from the auxiliary variables and, accordingly, improves on both.

Table 8: The Smets and Wouters (2007) Model, Expansion: Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Nowcasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 1.24, 0.76, 0.64. The columns present months in which nowcasts are made. The forecast errors are averaged over a sample 2010Q2:2019Q4. ***/**/* indicates the forecasts that are statistically significantly different from the one-step ahead quarterly forecast with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation. Quarterly 1-step ahead RMSFE is 0.59.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.5933***	2.2234***	2.3991***
Kim (2010)	1.0929***	1.4739***	1.6264***
New Method	1***	1	1

The forecasting results are summarized in Table 9 and Figure 3. Here we give the RSMFE for GDP forecasts 1 to 12 quarters ahead. The quarterly model performs best in predicting quarterly GDP, however our new combined method produces forecasts that differ statistically from those of the quarterly model only at intermediate horizons. While at higher horizons, all forecasts are statistically indistinguishable, our new method is clearly superior to the other mixed-frequency methods for one-quarter ahead forecasts, successfully leveraging the two other mixed-frequency approaches' different introduction of higher frequency (monthly) noise.

Table 9: The Smets and Wouters (2007) Model, Expansion: Relative RMSFE of the GDP Forecasts Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7388, 0.9106, 0.7282, 0.6535, 0.5669, 0.5010, 0.4515, 0.4377, 0.4748, 0.4577, 0.4305, 0.4320. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over a sample 2010Q2:2019Q4. ***/**/* indicates the forecasts that are statistically significantly different from the quarterly forecasts with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	0.7938	0.4747	0.5878	0.6186	0.7306	0.7392	0.7911	0.7464	0.6988	0.7278	0.7300	0.7456
Giannone et al. (2016)	3.4846***	3.3361***	3.5186***	3.2665**	3.1285**	3.0140*	2.8508	2.6514	2.1911	2.1297	2.1272	2.0116
Kim (2010)	1.8739***	1.6796***	1.8951***	1.9303**	2.0486*	2.1708*	2.2415	2.1452	1.8978	1.8829	1.9407	1.8692
New Method	1	1***	1**	1**	1**	1	1	1	1	1	1	1

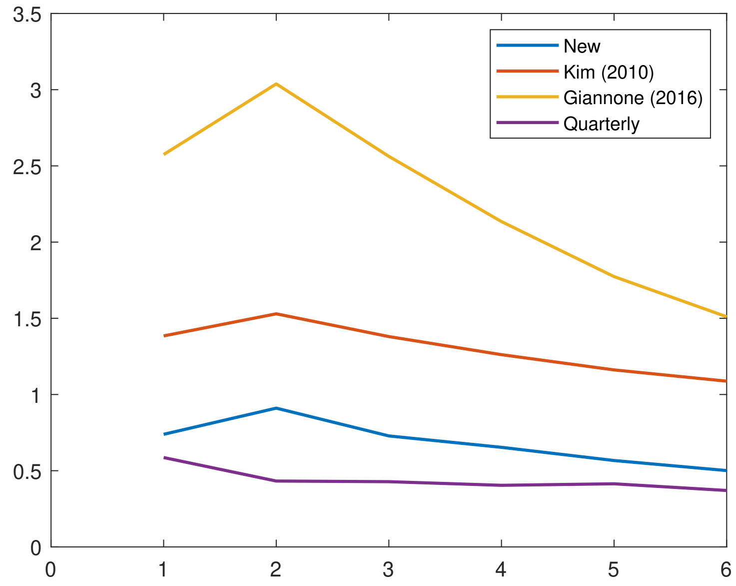

Figure 3: The Smets and Wouters (2007) Model, Expansion: RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are presented for the Smets and Wouters (2007) model and are averaged over a sample 2010Q2:2019Q4. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

Figure 4 depicts an out-of-sample exercise, where the different methods estimate monthly GDP. These estimates are plotted against the observed quarterly GDP in the figure. Giannone et al.'s (2016) method misses the first moment, predicting a far higher growth of GDP that was then actually observed, though these estimates are so imprecise as they roughly contain quarterly observed GDP within one standard deviation bounds. Comparatively, both our method and the method of Kim (2010) do much better, yet Kim's (2010) method produces much larger and persistent deviations from the quarterly data series, particularly in, e.g., 2016 onward.

Using mixed frequency methods brings potential gains, as additional information in the form of higher frequency data is included in forecasting, but also potential losses, as this data introduces additional noise. Both effects also play out with the backdrop of temporal aggregation bias when the frequency of data sampling and agents' decision making differ. We find that our new method that combines higher frequency decision making and

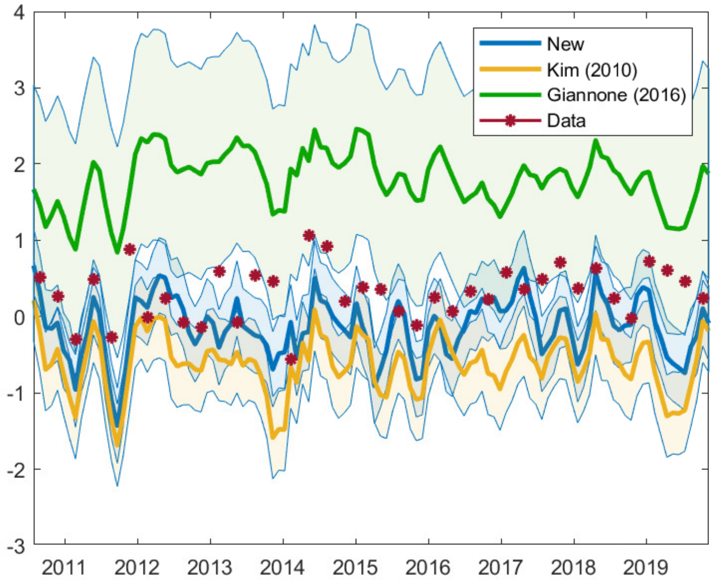

Figure 4: The Smets and Wouters (2007) Model, Expansion: Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in burgundy dots. Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). Shaded areas correspond to the one standard deviation around the mean. X-axis shows time in months.

auxiliary variables improves on existing methods that do only one of the two, both in terms of nowcasting within a quarter and forecasting on a quarterly basis.

5. Mixed-Frequency DSGE Forecasting in the Great Recession

Of particular interest is the performance of forecasting methods during particularly turbulent times, such as the Great Recession (the period 2008Q1:2009Q2),[29] as these times are marked by sequences of less probable shocks that strain the abilities of linear forecasting methods. Yet precisely in such rapidly changing environments does the inclusion of higher frequency data - intermediate observations or auxiliary variables - hold great promise. By drawing on a larger body of data at higher frequencies, the forecasting models might be better informed and caught less off guard by the rapidly changing economic environment. To this end, we

29 Recessions and expansions were identified using the NBER recession index.

Table 10: The Smets and Wouters (2007) Model, the Great Recession: Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Nowcasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 1.47, 0.85, 0.85. The columns present months in which nowcasts are made. The forecast errors are averaged over a sample 2008Q1:2009Q2. ***/**/* indicates the forecasts that are statistically significantly different from the one-step ahead quarterly forecast with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation. Quarterly 1-step ahead RMSFE is 1.30.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	0.8148	1.1997	1.0100
Kim (2010)	1.1777	1.9622	1.6709
New Method	1	1	1*

keep our focus on forecasting real economic activity as measured by GDP using the same set of methods within the context of the Smets and Wouters (2007) model as in the previous section.

The RMSFE are contained in Table 10 and the first observation is that during the Great Recession all RMSFE are larger in comparison with the expansionary period. Contrary to normal times, however using auxiliary variables substantially decreases forecasting errors, leading to the Giannone et al. (2016) method to demonstrate much smaller errors relative to the other methods in comparison to the previous section, even providing superior forecasting power compared with our combined method one month out. Interestingly, Kim's (2010) method now delivers worse nowcasts than the quarterly one-quarter ahead forecast, whereas Giannone et al. (2016) and our combined method deliver better nowcasts starting from the second and third month accordingly. Our combined method, as in the expansionary period that followed (the period 2010Q2:2019Q4, see the previous section), delivers arguably better results than the two methods alone by combining their approaches.

We now turn to the forecasted path of GDP during the Great Recession. In Figure 5, the estimates of quarterly GDP from the different mixed frequency and the standard quarterly method are depicted. For the standard quarterly method, the one-quarter ahead forecasts are plotted and it is clear that the strong mean-reversion and low frequency of the data causes the method to miss the recession in its forecasts almost entirely. The different mixed-frequency methods are given different conditioning sets, data from the first month of the quarter (m1), then, additionally the second month (m2), and so on. Both Kim's (2010) and Giannone et al.'s (2016) methods do forecast the recession, but miss the trough and identify it one quarter too late. Giannone et al.'s (2016) method, but importantly not Kim's (2010) method, profits from having data from additional months within the quarter. This is precisely the expected advantage of auxiliary variables. Our new, combined method succeeds in forecasting both the magnitude and timing of the trough more accurately than the other methods.

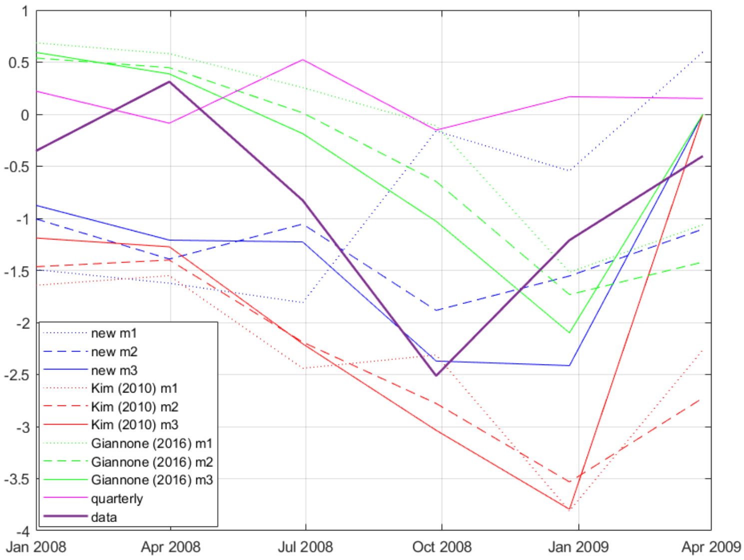

Figure 5: The Smets and Wouters (2007) Model, the Great Recession: GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. The figure shows GDP forecasts, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in green), and the standard quarterly forecasting (in magenta). The data is shown in purple. Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The forecasts are presented for the Smets and Wouters (2007) model. "m1", "m2", and "m3" correspond to the first, the second and the third months from which the nowcasts are made. X-axis shows time in quarters (January stands for the first quarter).

Comparing the smoothed GDP estimates from the different methods in Figure 6, where the first and second months of GDP in the quarter are not observed, again our new, combined method provides the best estimate. Our method can be seen as a weighted average between the Giannone et al. (2016) and Kim (2010) methods and tracks the trajectory of GDP more closely than either of the other methods over the whole period.

The RMSFE of forecasts during the Great Recession are contained in Table 11 and Fig-

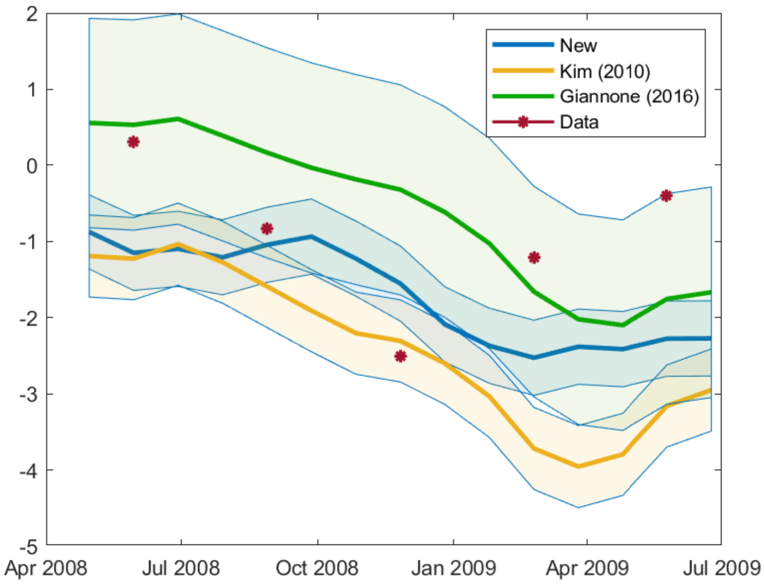

Figure 6: The Smets and Wouters (2007) Model, the Great Recession: Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in burgundy dots. Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). Shaded areas correspond to the one standard deviation around the mean. X-axis shows time in months.

ure 7 for forecasting horizons of one through six quarters. As in expansion that followed - see Table 9, the standard quarterly linear times series model generally provides the best quarterly forecasts of GDP during the Great Recession. At higher forecasting horizons, Kim's (2010) method shows smaller average forecasting errors, but the differences in the forecasts are statistically insignificantly different from the standard quarterly method at all horizons for all methods. The exception being at the one quarter horizon where our new methods provides better average forecasts than Giannone et al.'s (2016) and Kim's (2010) methods, but statistically significantly worse forecasts than the standard linear quarterly model.

Table 11: The Smets and Wouters (2007) Model, the Great Recession: Relative RMSFE of the GDP Forecasts Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 2.0824, 1.3559, 1.3390, 1.2513, 0.6773, 0.1139. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over a sample 2008Q1:2009Q2. ***/**/* indicates the forecasts that are statistically significantly different from the quarterly forecasts with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	**1**	**2**	**3**	**4**	**5**	**6**
Quarterly	0.6225	1.0743	1.1897	1.3454	1.3576	3.1904
Giannone et al. (2016)	0.8188	1.9380	1.9826	2.1317	2.8385	12.6333
Kim (2010)	0.9805	1.3623	0.8889	0.6480	0.6576	5.8290
New Method	1*	1	1	1	1	1

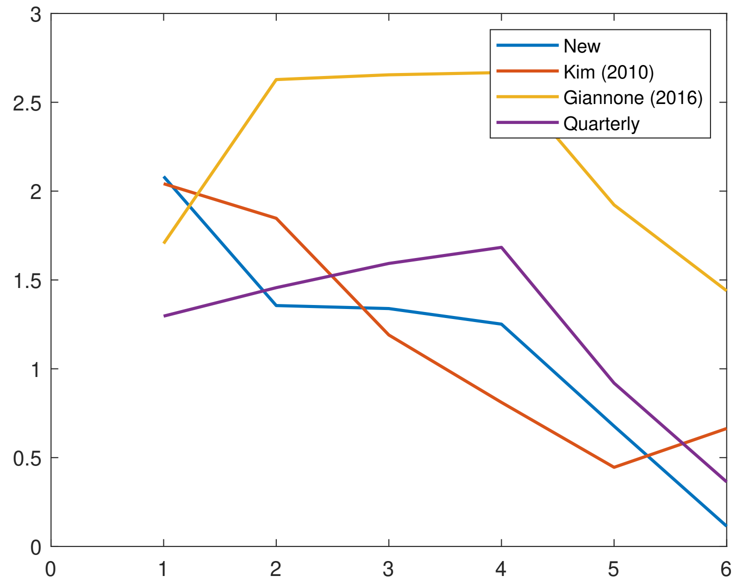

Figure 7: The Smets and Wouters (2007) Model, the Great Recession: RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). Forecasts for the mixed-frequency methods are obtained using quarterly estimates of the parameters (which are transformed into the monthly frequency using Table 14). The RMSFE are presented for the Smets and Wouters (2007) model and are averaged over a sample 2008Q1:2009Q2. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

The more mixed nature of our results can be clearly seen in Figure 7. For one quarter predictions, a standard quarterly state-observer (linear hidden Markov) model is clearly superior. Our new method along with the method of Kim (2010) then overtake the standard quarterly model, though the difference is not significant.

6. Conclusion

We have shown that including higher frequency information improves the forecasting performance of DSGE models, specifically with our new method that combines existing methods to consistently model, estimate and forecast within a DSGE model with mixed frequencies. We find that during expansions in our real world experiment and with the simulated data from our Monte Carlo exercise that using monthly data of model variables and casting the model in a monthly frequency generally decreases forecasting errors. On the other hand, incorporating information from auxiliary variables is crucial in obtaining better forecasts during recessions. Our combined method, which blends the approaches of incorporating higher frequency auxiliary variables and using monthly data of the model variables, improves forecasts in all Monte Carlo exercises with the 3-equation New Keynesian DSGE model. With real data and the medium scale policy relevant model of Smets and Wouters (2007) model, our combined method produces more mixed results but definitively tracks the timing and the depth of the Great Recession better than existing mixed-frequency DSGE methods.

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A. Derivations of the Analytical Results

Proposition 1

Proof. First of all, notice that the state space transformations done in the two algorithms are the same if there are no measurement errors ($u_{t_m} = 0\ \forall t_m$). To see this in Foroni and Marcellino (2014) algorithm we substitute f_{t_m} with $f_{t_m} = (s_{t_m}, s_{t_m-1}, s_{t_m-2})'$ and ignore the corresponding equations for the $u_{t_m}, u_{t_m-1}, u_{t_m-2}$ terms in the transition equation (as they all deliver equations like $0 = 0$ in the transition equation; in the observation equation these terms add only zeros). The system becomes identical to the one in Kim (2010).

Next, we need to prove that the Kalman filter with missing observations and Kim (2010) algorithm with data augmentation deliver the same results. The prediction step in both methods is the same as it is based on the transition equation that is the same for both methods (due to the same state space formulation). Thus the difference in methods could only come from the update step of the Kalman filter.

In Kim (2010) the prediction value of missing observations is

$$\hat{z}_{t_m} = \tilde{M}_{m,z}\tilde{s}_{t_m|t_m-1} + \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}(w_{t_m} - \tilde{M}_{m,w}\tilde{s}_{t_m|t_m-1}) \quad \text{(A.1)}$$

and the variance of the prediction is

$$V_{\hat{z}} = \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1}(I - \tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1})\tilde{M}'_{m,z} \quad \text{(A.2)}$$

where I is the identity matrix (see Equation (7)).

Therefore, the error in the update state of the Kalman filter with data augmentation reads as following

$$v_{t_m} = (\tilde{Y}_{t_m} - \tilde{M}_m \tilde{s}_{t_m|t_m-1}) = \begin{pmatrix} w_{t_m} \\ \hat{z}_{t_m} \end{pmatrix} - \begin{pmatrix} \tilde{M}_{m,w} \\ \tilde{M}_{m,z} \end{pmatrix} \tilde{s}_{t_m|t_m-1}$$

$$= \begin{pmatrix} I \\ \tilde{M}_{m,z}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1} \end{pmatrix} (w_{t_m} - \tilde{M}_{m,w}\tilde{s}_{t_m|t_m-1})$$

$$= \tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}(w_{t_m} - \tilde{M}_{m,w}\tilde{s}_{t_m|t_m-1}) \tag{A.3}$$

where in the second equation we used Equation (A.1) for $\hat{z}_{t_m}$ and in the last equation we represented $I = \tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}$. The predicted value of the state variable is[30]

$$\tilde{s}_{t_m|t_m} = \tilde{T}_m\tilde{s}_{t_m|t_m-1} + \tilde{T}_m[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})][\text{Var}(v_{t_m})]^{-1}v_{t_m} \tag{A.4}$$

The covariance and the variance are equal to

$$\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m}) = \tilde{P}_{t_m|t_m-1}(\tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}\tilde{M}_{m,w})' \tag{A.5}$$

$$\text{Var}(v_{t_m}) = \tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\cdot$$
$$\cdot(\tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}\tilde{M}_{m,w})'$$
$$= \tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_{m,w})^{-1}\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}'_m \tag{A.6}$$

and the error term is given in Equation (A.3). When we plug in the covariance, the variance and the error term into the predicted value of the state vector (Equation (A.4)) we obtain

30 Formula (4.14) in Durbin and Koopman (2012).

$$\tilde{s}_{t_m|t_m} = \tilde{T}_m \tilde{s}_{t_m|t_m-1} + \tilde{T}_m \tilde{P}_{t_m|t_m-1} (\tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w})^{'} \cdot$$
$$(\tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m}^{'})^{-1} \cdot$$
$$\tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} (w_{t_m} - \tilde{M}_{m,w} \tilde{s}_{t_m|t_m-1}) \quad \text{(A.7)}$$

Notice that

$$(\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m}^{'} \cdot$$
$$\cdot (\tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m}^{'})^{-1} \tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'}$$
$$= B(AB)^{-1}A = B(AB)^{-1}ABB'(BB')^{-1} = I \quad \text{(A.8)}$$

where $B = (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m}^{'}$, $A = \tilde{M}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'}$ and I is an identity matrix. Thus the predicted vector could be simplified to

$$\tilde{s}_{t_m|t_m} = \tilde{T}_m \tilde{s}_{t_m|t_m-1} + \tilde{T}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} (w_{t_m} - \tilde{M}_{m,w} \tilde{s}_{t_m|t_m-1}) \quad \text{(A.9)}$$

and the predicted variance is[31]

$$\tilde{P}_{t_m|t_m} = \tilde{T}_m (\tilde{P}_{t_m|t_m-1} - [\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})][\text{Var}(v_{t_m})]^{-1}[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})]^{'}) \tilde{T}_m^{'} + \tilde{B}_m \tilde{Q}_m \tilde{B}_m'$$
$$= \tilde{T}_m \tilde{P}_{t_m|t_m-1} \tilde{T}_m^{'} - \tilde{T}_m \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'} (\tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{M}_{m,w}^{'})^{-1} \tilde{M}_{m,w} \tilde{P}_{t_m|t_m-1} \tilde{T}_m^{'} + \tilde{B}_m \tilde{Q}_m \tilde{B}_m' \quad \text{(A.10)}$$

where $\tilde{Q}_m$ is the variance of the errors in the transition equation and for $[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})][\text{Var}(v_{t_m})]^{-1}[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})]^{'}$ we plugged in the formulas from Equation (A.6) and again used the Equation (A.8) to simplify the terms (see Equation (A.11) below).

31 Formula (4.18) and (4.23) in Durbin and Koopman (2012).

$$[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})][\text{Var}(v_{t_m})]^{-1}[\text{Cov}(\tilde{s}_{t_m|t_m-1}, v_{t_m})]^{'} = \tilde{P}_{t_m|t_m-1}(\tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}\cdot$$
$$\cdot(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}\tilde{M}_{m,w})^{'}(\tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m})^{-1}\cdot$$
$$\cdot(\tilde{P}_{t_m|t_m-1}(\tilde{M}_m\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}\tilde{M}_{m,w})^{'})^{'}$$
$$= \tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1} \tag{A.11}$$

The missing observations Kalman filter can be represented as a standard Kalman filter with the observation equation like[32]

$$W\tilde{Y}_{t_m} = W\tilde{M}_m\tilde{s}_{t_m} \tag{A.12}$$

in the time periods with missing observations where W selects only observed variables, i.e. $W = \begin{pmatrix} I & 0 \end{pmatrix}$, W is $n_w \times (n_w + n_z)$, I is $n_w \times n_w$.

The predicted value in the periods with missing observations is thus

$$\tilde{s}_{t_m|t_m} = \tilde{T}_m\tilde{s}_{t_m|t_m-1} + \tilde{T}_m\tilde{P}_{t_m|t_m-1}(W\tilde{M}_m)'(W\tilde{M}_m\tilde{P}_{t_m|t_m-1}(W\tilde{M}_m)')^{-1}W(\tilde{Y}_{t_m} - M_m\tilde{s}_{t_m|t_m-1}) =$$
$$\tilde{T}_m\tilde{s}_{t_m|t_m-1} + \tilde{T}_m\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}(w_{t_m} - \tilde{M}_{m,w}\tilde{s}_{t_m|t_m-1}) \tag{A.13}$$

where in the last equation we used the fact that $W\tilde{M}_m = \tilde{M}_{m,w}$. The predicted variance is

$$\tilde{P}_{t_m|t_m} = \tilde{T}_m\tilde{P}_{t_m|t_m-1}(\tilde{T}_m - \tilde{T}_m\tilde{P}_{t_m|t_m-1}(W\tilde{M}_m)^{'}(W\tilde{M}_m\tilde{P}_{t_m|t_m-1}(W\tilde{M}_m)^{'})^{-1}W\tilde{M}_m)' + \tilde{B}_m\tilde{Q}_m\tilde{B}'_m$$
$$= \tilde{T}_m\tilde{P}_{t_m|t_m-1}(\tilde{T}_m - \tilde{T}_m\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w}(\tilde{M}_{m,w}\tilde{P}_{t_m|t_m-1}\tilde{M}^{'}_{m,w})^{-1}\tilde{M}_{m,w})' + \tilde{B}_m\tilde{Q}_m\tilde{B}'_m \tag{A.14}$$

The expressions of the predicted state vector (Equation (A.9) and Equation (A.13)) and

32 See Section 4.10 in Durbin and Koopman (2012) for details.

the predicted variance of the state vector (Equation (A.10) and Equation (A.14)) in the two methods are the same. □

Proposition 2

Proof. Firstly, the state space system of the new method (Equation (11)) could be rewritten as

$$\begin{cases} f_{t_m} = \tilde{T}_m f_{t_{m-1}} + \tilde{B}_m \xi_{m,t_m} \\ \begin{pmatrix} \tilde{Y}_{t_m} \\ \tilde{X}_{t_m} \end{pmatrix} = \begin{pmatrix} 0 \\ \mu \end{pmatrix} + \begin{pmatrix} \tilde{M}_m \\ \Lambda \tilde{M}_m \end{pmatrix} f_{t_m} + \begin{pmatrix} 0 \\ R_m \end{pmatrix} \zeta_{t_m} \end{cases} \tag{A.15}$$

or simply

$$\begin{cases} f_{t_m} = \tilde{T}_m f_{t_{m-1}} + \tilde{B}_m \xi_{m,t_m} \\ Z_{t_m} = \tilde{\mu} + \Gamma_m f_{t_m} + \Omega_m \zeta_{t_m} \end{cases} \tag{A.16}$$

From Lemma 2 in Durbin and Koopman (2012) we know that the estimates of this system obtained using the Kalman filter are minimum variance linear unbiased estimates (MVLUE). In the new method exactly this system is estimated using Kalman filter with missing observations. Therefore, its estimates are MVLUE.

The Giannone et al. (2016) method does not use the information from the auxiliary variables when estimating the model and the model is estimated at a quarterly frequency. Abstracting from the difference that could arise due to the estimation at quarterly frequency instead of monthly frequency with missing observations, in their method for the estimation they use only part of the observations available (use only $\tilde{Y}_{t_m}$ and not $\tilde{X}_{t_m}$ when updating the guesses). That is equivalent to using Kalman filter with missing observations where all $\tilde{X}_{t_m}$ are missing.

The Kim (2010) / Foroni and Marcellino (2014) method also uses only $\tilde{Y}_{t_m}$ and not $\tilde{X}_{t_m}$ in their estimation (because they do not use auxiliary variables in their method).

The forecasting is done only in Giannone et al. (2016) and even for the forecasting the method uses only part of the observations available. It disregards $\tilde{X}_{t_m}$ when $\tilde{Y}_{t_m}$ is available.

Therefore, all other methods (apart from the new method) estimate the system Equation (A.16) and produce forecasts from it but as if some observations are missing. This leads to larger variance of the state vector

$$\tilde{P}_{t_m|t_m} = \tilde{T}_m \tilde{P}_{t_m|t_m-1}(\tilde{T}_m - \tilde{T}_m \tilde{P}_{t_m|t_m-1}(W\tilde{\Gamma}_m)^{'}(W\tilde{\Gamma}_m \tilde{P}_{t_m|t_m-1}(W\tilde{\Gamma}_m)^{'})^{-1} W\tilde{\Gamma}_m)' + \tilde{B}_m \tilde{Q}_m \tilde{B}'_m \tag{A.17}$$

where for the new method W is an identity matrix and for all other methods W has rows equal to zero for the variables that are treated as missing. In case all rows in W are zero the term in the round brackets is equal to $\tilde{T}_m$ and the more rows in W are non-zero, the smaller is the term in the round brackets and thus the smaller is the variance of the state vector.

□

B. Appendix Tables

Table 12: Priors of the Parameters in the 3-equation Model from Ch. 3 Galí (2008). Each row presents a parameter, its mean, standard deviation and the functional form of the prior. All parameters are presented in quarterly and monthly frequencies. For the parameters that change with a change in a frequency the values in the "Monthly" and "Quarterly" columns differ. For the parameters that are independent of frequencies (i.e. preference parameters) the values in the "Monthly" and "Quarterly" columns are the same.

Parameter	Description	Distribution	Monthly Prior mean	Monthly Prior std	Quarterly Prior mean	Quarterly Prior std
θ	Calvo parameter	Beta	8 / 9	$\frac{0.15}{\sqrt(3)}$	2 / 3	0.15
ρ_v	Persistence of monetary policy shock	Beta	$0.5^{\frac{1}{3}}$	$\frac{0.12}{\sqrt(3)}$	0.5	0.12
ρ_a	Persistence of TFP	Beta	$0.9^{\frac{1}{3}}$	$\frac{0.05}{\sqrt(3)}$	0.9	0.05
β	Households' discount factor	Beta	$\frac{1}{(1+\frac{1}{3}(\frac{1}{0.99}-1))}$	$\frac{0.005}{\sqrt(3)}$	0.99	0.005
α	Capital share	Beta	$\frac{1}{3}$	0.12	$\frac{1}{3}$	0.12
η	Semi-elasticity of money demand	Gamma	4	2	4	2
ε	Demand elasticity	Gamma	6	3	6	3
ϕ_π	Taylor coefficient in front of inflation	Normal	1.5	0.75	1.5	0.75
ϕ_y	Taylor coefficient in front of output gap	Normal	0.5 / 4	0.06	0.5 / 4	0.06
σ_π	Std cost-push shock	Inverse Gamma	$\frac{0.25^2}{3}$	$\frac{0.12}{\sqrt(3)}$	0.25^2	0.12
σ_y	Std demand shock	Inverse Gamma	$\frac{0.25^2}{3}$	$\frac{0.12}{\sqrt(3)}$	0.25^2	0.12
σ_i	Std monetary policy shock	Inverse Gamma	$\frac{0.25^2}{3}$	$\frac{0.12}{\sqrt(3)}$	0.25^2	0.12

Table 13: Code names from FRED of the variables used in the Smets and Wouters (2007) model.

Variable	Code	Description
GDP	GDPC1	Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate
Population	CNP16OV	Population Level, Persons, Quarterly, Not Seasonally Adjusted
Investments	FPI	Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
GDP deflator	GDPDEF	Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted
Hours	AWHNONAG	Average Weekly Hours of Production and Nonsupervisory Employees, Total Private, Monthly, Seasonally Adjusted
Employment	CE16OV	Employment Level, Thousands of Persons, Quarterly, Seasonally Adjusted
Wages	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, Index 2012=100, Quarterly, Seasonally Adjusted
Fed Funds Rate	FEDFUNDS	Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted
Consumption	PCE	Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate

Table 14: Transformation of the parameters from quarterly to monthly frequency for the Smets and Wouters (2007) model. All standard deviations were divided by $\sqrt{3}$, all persistence parameters were powered to the power of $\frac{1}{3}$. Transformations for other parameters (that needed transformation) are presented in the table below.

Variable	Transformation formula	Description
c_τ	$c_\tau^m = \frac{c_\tau^q}{3}$	Depreciation rate
c_β	$c_\beta^m = \frac{1}{\frac{1}{3}(\frac{1}{c_\beta^q}-1)+1}$	Discount factor
c_γ	$c_\gamma^m = \frac{c^m}{100}+1 = \frac{c^q}{300}+1$	c^q is a quarterly trend growth rate to GDP
c_{π_e}	$c_{\pi_e}^m = \frac{c_\pi^m}{100}+1 = \frac{c_\pi^q}{300}+1$	c_π^q is a quarterly steady state inflation rate
c_{habbit}	$c_{habbit}^m = \frac{c_{habbit}^q}{3}$	Habit formation parameter
c_{prob_p}	$c_{prob_p}^m = 1-\frac{1-c_{prob_p}^q}{3}$	Calvo parameter for prices
c_{prob_w}	$c_{prob_w}^m = 1-\frac{1-c_{prob_w}^q}{3}$	Calvo parameter for wages

C. Appendix Graphs

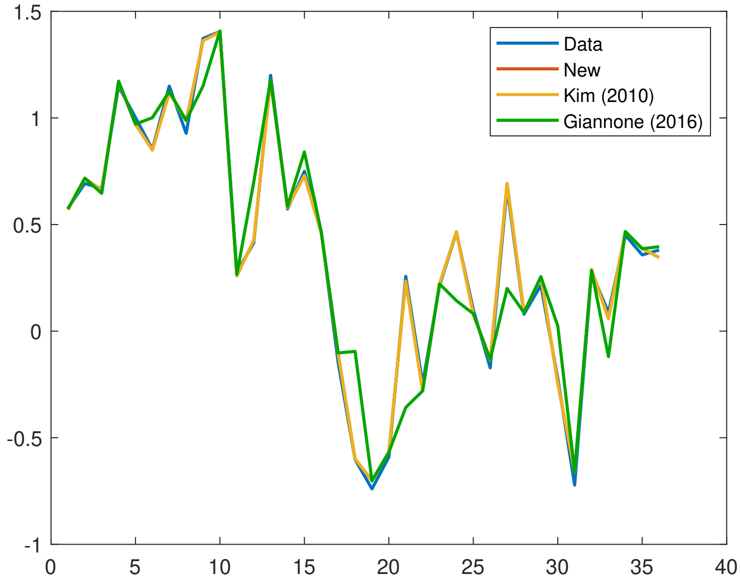

Figure 8: The 3-equation Model from Ch. 3 Galí (2008): Updated Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|t})$) for the out-of-sample data. The smoothed series are obtained using the Kalman smoother of the new method (in red), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green). X-axis shows time in months and Y-axis shows filtered GDP.

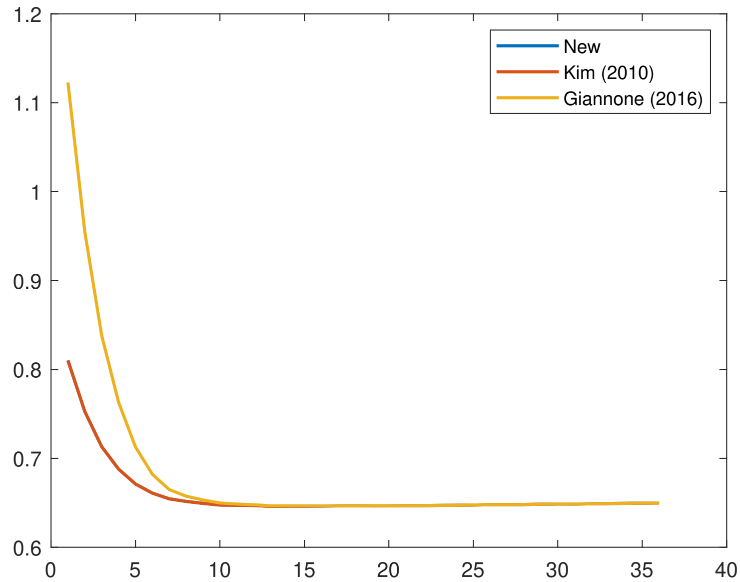

Figure 9: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the Monthly GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows monthly GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow). The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in months and Y-axis shows RMSFE for each method.

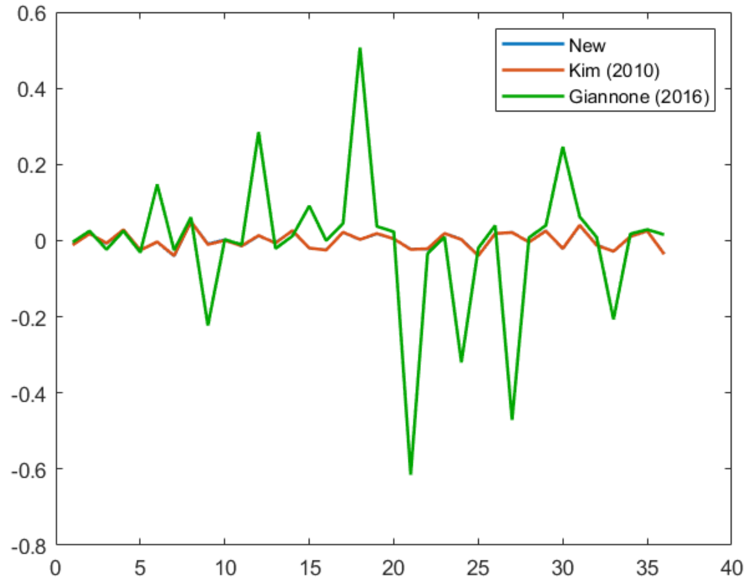

Figure 10: The 3-equation Model from Ch. 3 Galí (2008): Smoothed Out-of-Sample GDP, Difference to the Data. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$) for the out-of-sample data. The smoothed series are obtained using the Kalman smoother of the new method (in red), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green). All series are presented as deviations from the data. X-axis shows time in months.

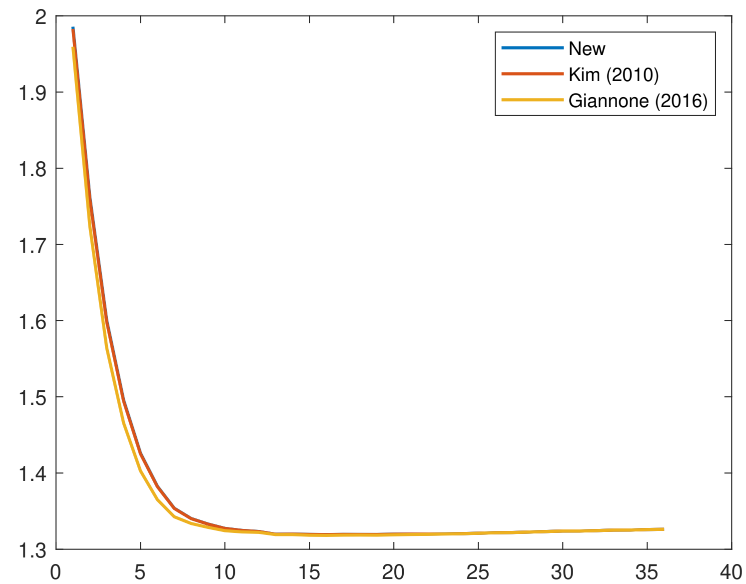

Figure 11: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the Interest Rate Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows Interest Rate RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow). The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in months and Y-axis shows RMSFE for each method.

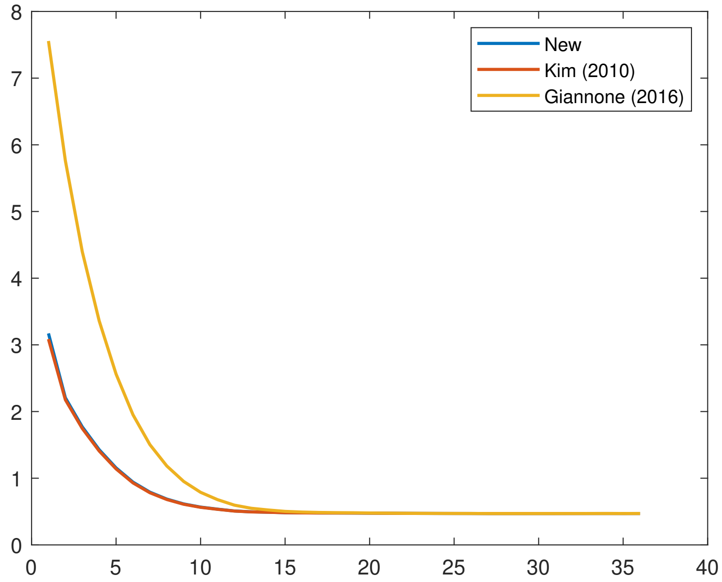

Figure 12: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the Inflation Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows Inflation RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow). The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in months and Y-axis shows RMSFE for each method.

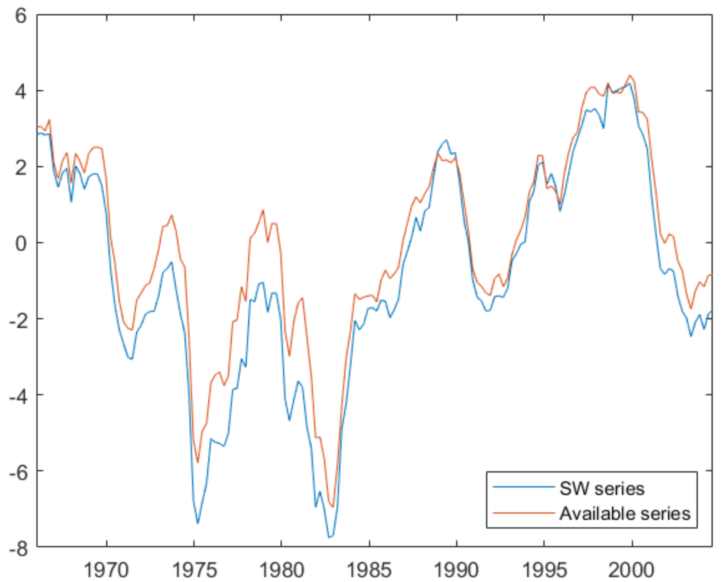

Figure 13: Labor hours data. The figure shows demeaned series of labor hours. Available series refers to AWHNONAG series, SW hours refer to the series used in the Smets and Wouters (2007) paper. X-axis shows time in years.

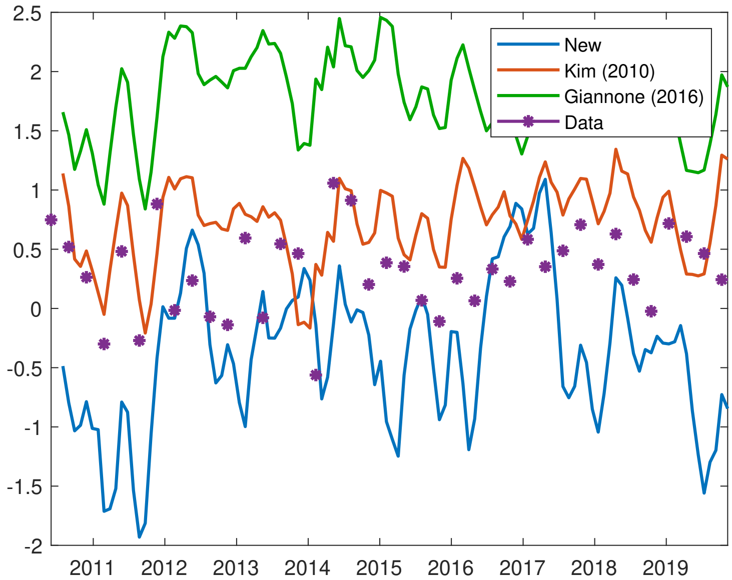

Figure 14: The Smets and Wouters (2007) Model, Expansion: Updated Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|t})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in purple dots. X-axis shows time in months and Y-axis shows smoothed GDP.

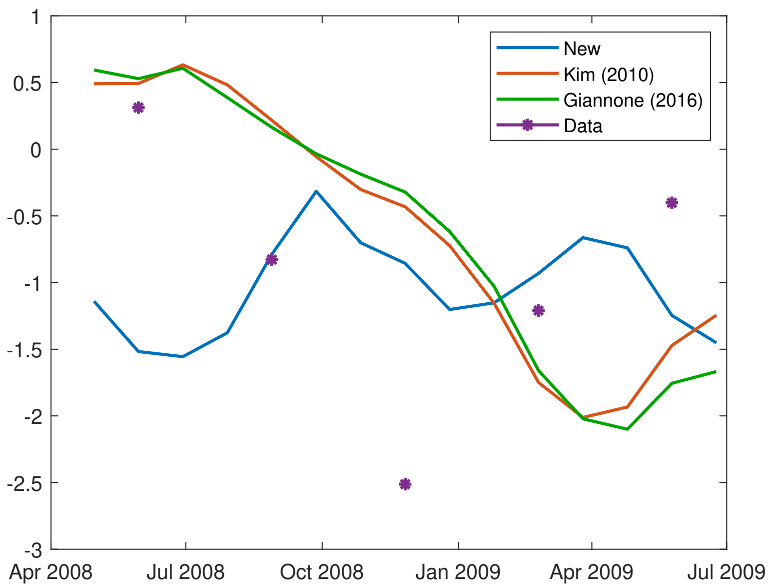

Figure 15: The Smets and Wouters (2007) Model, the Great Recession: Updated Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|t})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in purple dots. X-axis shows time in months and Y-axis shows smoothed GDP.

D. Robustness Checks

D.1. Larger sample size for a 3-equation DSGE model

Table 15: The 3-equation Model from Ch. 3 Galí (2008): Estimation of the Parameters Using Quarterly and Mixed-frequency Methods. Each row presents parameter estimates using the Quarterly estimation, the Kim (2010) / Foroni and Marcellino (2014) estimation method, or the new estimation method (see Section 2 for methods' details). Standard deviations of the estimated parameters are presented in parentheses. The estimates of the quarterly estimation are transformed into their monthly counterparts.[33] The column DGP shows parameter values used to generate the simulated data (the true parameter values).

Parameter	DGP	Quarterly	Mixed Kim (2010)	Mixed new algorithm
θ	0.8889	0.9060 (0.0152)	0.8903 (0.0216)	0.8903 (0.0218)
ρ_i	0.7937	0.7611 (0.0013)	0.7935 (0.0004)	0.7935 (0.0005)
ρ_a	0.9655	0.9655 (0.0177)	0.9659 (0.0282)	0.9688 (0.0252)
β	0.9966	0.9966 (0.0017)	0.9967 (0.0029)	0.9968 (0.0027)
α	0.3333	0.3475 (0.1197)	0.3352 (0.1160)	0.3508 (0.1235)
η	4	3.9593 (1.9678)	3.9600 (1.9871)	4.2016 (2.1196)
ε	6	5.3966 (2.2140)	5.9440 (3.0260)	5.8319 (3.1064)
ϕ_π	1.5	3.3430 (0.1932)	1.5354 (0.0799)	1.5408 (0.0818)
ϕ_y	0.1250	0.0665 (0.0585)	0.1164 (0.0571)	0.1191 (0.0584)
σ_π	0.0208	0.0191 (0.0008)	0.0209 (0.0005)	0.0209 (0.0005)
σ_y	0.0208	0.0095 (0.0004)	0.0215 (0.0029)	0.0213 (0.0029)
σ_i	0.0208	0.2936 (0.0262)	0.1401 (0.0237)	0.1416 (0.0243)

D.2. Giannone with other transformation to monthly frequency DSGE for a 3-equation DSGE model

Below are presented results when the parameters from quarterly estimation are transformed into their monthly counterparts not by powering the transition matrix into the power $\frac{1}{3}$, but by doing different transformations for each parameter according to its specific nature (for example, the capital share parameter is not transformed, but the Calvo parameter is transformed according to $\theta_{monthly} = 1 - \frac{1}{3}(1 - \theta_{quarterly})$).

For the nowcasts the new method outperforms the other methods. For the forecasts the

33 θ was estimated in a quarterly model at 0.7096, ρ_i at 0.44, ρ_a at 0.8999, β at 0.99, σ_π at 0.0327, σ_y at 0.0175, σ_i at 0.5320. Standard deviations were transformed using the approximation $\text{Var}(g(x)) = (g'(x))^2 \text{Var}(x)$ with evaluation of the derivative at the estimated value of the parameters.

Table 16: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.5010, 0.2706, 0.0108. The columns present months in which nowcasts are made. The nowcast errors are averaged over 10000 simulated sample points. *** indicates the forecasts that are statistically significantly different from the other forecasts with a 1% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.3549***	1.3868***	9.7494***
Kim (2010)	1.0005***	1.0014***	1.0056***
New Method	1***	1***	1***

Table 17: The 3-equation Model from Ch. 3 Galí (2008): RMSE of the Filtered GDP series, obtained Using Mixed-frequency Methods. The Kalman filter generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The obtained series is compared to the simulated series. Each row presents an out-of-sample (sample different from the one on which the model was estimated) RMSE for the filtered monthly GDP series, values of which are based on the information at time t, or the information available at the end of the sample. The columns present RMSE for the filtered values, obtained using the Kalman filter of the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), or the new method, suggested in the Section 2. The errors are averaged over 10000 simulated sample points.

Filtered statistic	RMSE Giannone et al. (2016)	RMSE Kim (2010)	RMSE new method
$E(y_t \mid t)$	0.18196	0.02622	0.02618
$E(y_t \mid T)$	0.18196	0.02316	0.02310

Table 18: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7045, 0.6119, 0.5899, 0.5857, 0.5841, 0.5852, 0.5857, 0.5862, 0.5869, 0.5878, 0.5887, 0.5892. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over 10000 simulated sample points. *** (**) indicates the forecasts that are statistically significantly different from quarterly forecasts with a 1% (5%) significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	1.1137	1.0215	1.0098	1.0094	1.0108	1.0086	1.0072	1.0058	1.0045	1.0029	1.0015	1.0006
Giannone et al. (2016)	1.3100***	1.0754***	1.0132***	1.0026	1.0007	1.0003	1.0002**	1.0000	1.0000	1.0000	1.0000	1.0000
Kim (2010)	1.0002***	1.0000***	1.0000	1.0000	1.0000***	1.0000***	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
New Method	1***	1***	1	1	1***	1***	1	1	1	1	1	1

Table 19: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.5012, 0.2710, 0.0108. The columns present months in which nowcasts are made. The nowcast errors are averaged over 1000 simulated sample points. *** indicates the forecasts that are statistically significantly different from the other forecasts with a 1% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.0152***	1.0490***	10.2028***
Kim (2010)	1.0006***	1.0018***	1.0084***
New Method	1***	1***	1***

Table 20: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7045, 0.6119, 0.5899, 0.5857, 0.5841, 0.5852, 0.5857, 0.5862, 0.5869, 0.5878, 0.5887, 0.5892. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over 1000 simulated sample points. *** (**) indicates the forecasts that are statistically significantly different from quarterly forecasts with a 1% (5%) significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	1.1137	1.0215	1.0098	1.0094	1.0108	1.0086	1.0072	1.0058	1.0045	1.0029	1.0015	1.0006
Giannone et al. (2016)	0.9968***	0.9938***	0.9976	0.9996	1.0000	0.9996**	0.9996**	0.9999	1.0000	1.0000	1.0000	1.0000
Kim (2010)	1.0002***	1.0000***	1.0000	1.0000	1.0000**	1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
New Method	1***	1***	1	1	1**	1**	1	1	1	1	1	1

new method has statistically significantly different forecasts from Kim (2010) in the first three quarters (one, two and three quarters ahead forecasts). In the first quarter the forecast is better using the new method and in the other two quarters forecasts are worse for the new method. Both methods are statistically significantly different from Giannone et al. (2016) forecasts in the second and the third quarter. They are worse than Giannone et al. (2016) in these quarters.

D.3. Forecasts with all methods and with parameters set to the true values for a 3-equation DSGE model

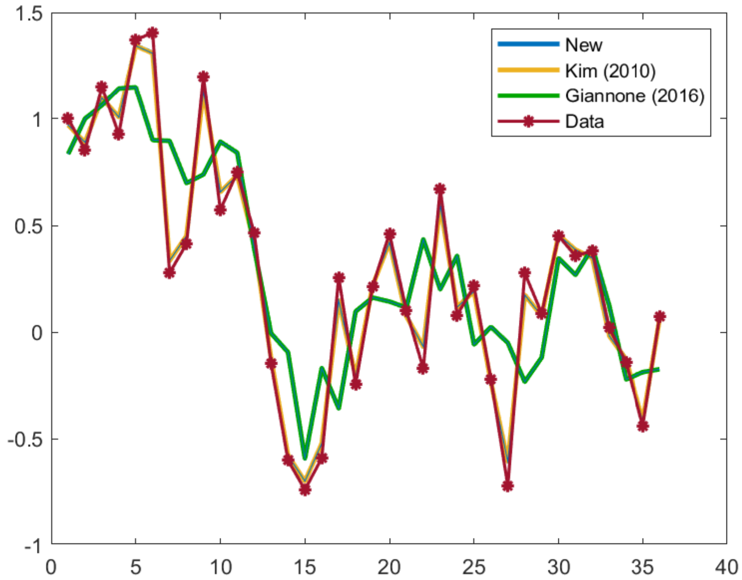

Figure 16: The 3-equation Model from Ch. 3 Galí (2008): Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$) for the out-of-sample data. The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green). The simulated data is shown in red. Forecasts are obtained using DGP estimates. X-axis shows time in months.

The new method outperforms other methods in nowcasting for all months. For the forecasting the new method and Kim (2010) are not statistically significantly different from each other. But they are both statistically significantly different from Giannone et al. (2016) in the first three quarters. They are better in these quarters.

Table 21: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Nowcasts are obtained using DGP estimates. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.5058, 0.2815, 0.0206. The columns present months in which nowcasts are made. The nowcast errors are averaged over 1000 simulated sample points. *** indicates the forecasts that are statistically significantly different from the other forecasts with a 1% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.3101***	1.4699***	7.7376***
Kim (2010)	1.0000***	1.0002***	1.0032***
New Method	1***	1***	1***

Table 22: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Forecasts are obtained using DGP estimates. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7031, 0.6116, 0.5898, 0.5857, 0.5841, 0.5852, 0.5857, 0.5862, 0.5869, 0.5878, 0.5887, 0.5892. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over 1000 simulated sample points. *** (**) indicates the forecasts that are statistically significantly different from quarterly forecasts with a 1% (5%) significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	1.0834	1.0242	1.0119	1.0102	1.0116	1.0094	1.0077	1.0060	1.0045	1.0030	1.0015	1.0006
Giannone et al. (2016)	1.2284***	1.0680***	1.0156***	1.0036	1.0012	1.0012	1.0010	1.0003	1.0001	1.0000	1.0000	1.0000
Kim (2010)	1.0000***	1.0000***	1.0000	1.0000	1.0000	1.0000**	1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000
New Method	1***	1***	1	1	1	1**	1**	1	1	1	1	1

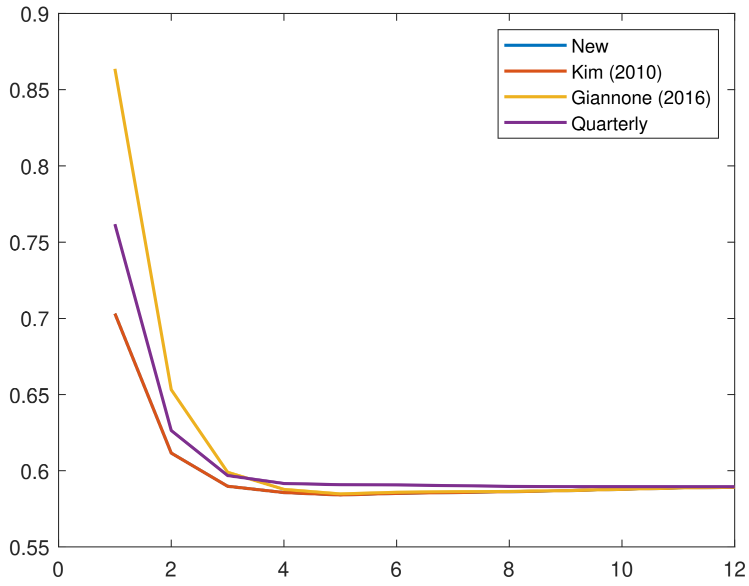

Figure 17: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). Forecasts are obtained using DGP estimates. The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

D.4. Forecasts with all methods and with parameters set to the estimates from quarterly estimation of a 3-equation DSGE model

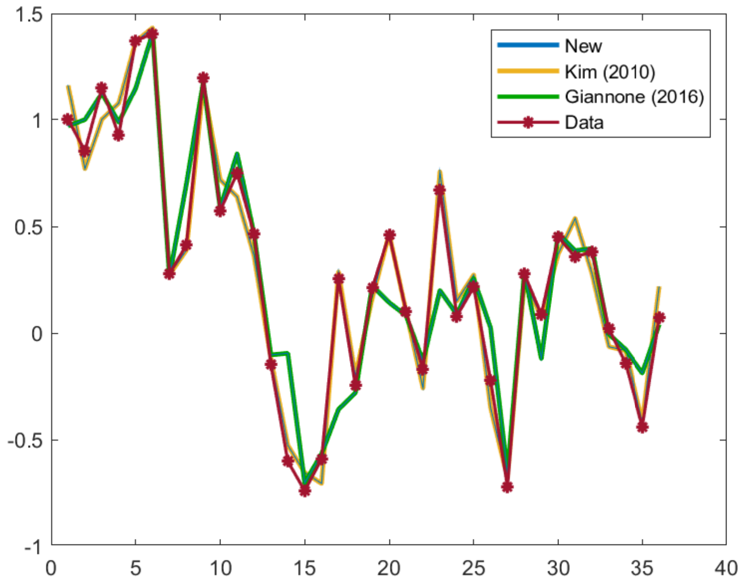

Figure 18: The 3-equation Model from Ch. 3 Galí (2008): Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$) for the out-of-sample data. The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green). Forecasts of the mixed-frequency methods are obtained using quarterly estimates for the parameters (which are transformed into the monthly frequency using transformation in Section 3.1). X-axis shows time in months and Y-axis shows smoothed GDP.

Table 23: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Nowcasts of the mixed-frequency methods are obtained using quarterly estimates for the parameters (which are transformed into the monthly frequency using transformation in Section 3.1). The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.5087, 0.2840, 0.0373. The columns present months in which nowcasts are made. The nowcast errors are averaged over 1000 simulated sample points. *** indicates the forecasts that are statistically significantly different from the other forecasts with a 1% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	1.3340***	1.3199***	2.8341***
Kim (2010)	1.0000***	1.0002***	1.0007***
New Method	1***	1***	1***

Table 24: The 3-equation Model from Ch. 3 Galí (2008): Relative RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. Forecasts of the mixed-frequency methods are obtained using quarterly estimates for the parameters (which are transformed into the monthly frequency using transformation in Section 3.1). The RMSFE are shown as a ratio to the RMSFE of the New Method. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7073, 0.6093, 0.5887, 0.5854, 0.5840, 0.5849, 0.5855, 0.5862, 0.5869, 0.5878, 0.5887, 0.5892. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over 1000 simulated sample points. *** (**) indicates the forecasts that are statistically significantly different from quarterly forecasts with a 1% (5%) significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	1.1093	1.0260	1.0118	1.0099	1.0110	1.0090	1.0076	1.0059	1.0045	1.0030	1.0015	1.0006
Giannone et al. (2016)	1.3049***	1.0801***	1.0152	1.0030	1.0009	1.0007**	1.0006**	1.0002	1.0000	1.0000	1.0000	1.0000
Kim (2010)	1.0000***	1.0000***	1.0000	1.0000	1.0000	1.0000**	1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000
New Method	1***	1***	1	1	1	1**	1**	1	1	1	1	1

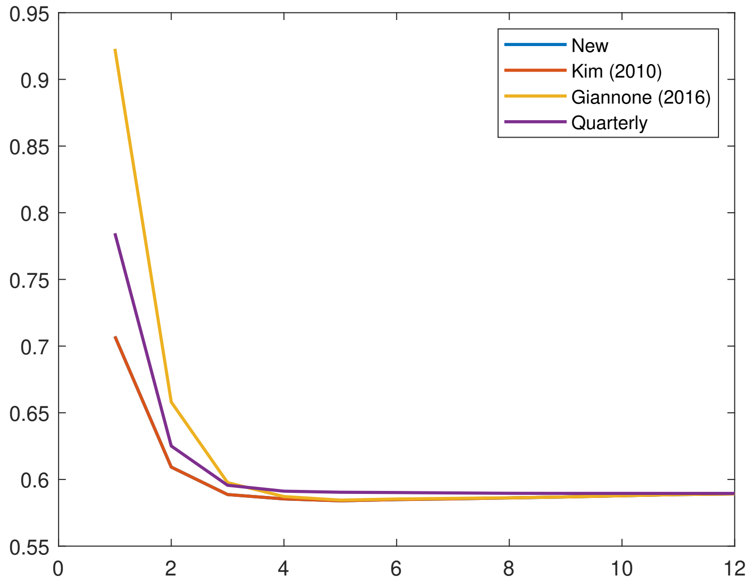

Figure 19: The 3-equation Model from Ch. 3 Galí (2008): RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). Forecasts for the mixed-frequency methods are obtained using quarterly estimates for the parameters (which are transformed into a monthly frequency using transformation in Section 3.1). The RMSFE are presented for the 3-equation DSGE model from Ch.3 Galí (2008) and are averaged over 1000 simulated sample points. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

D.5. Forecasting results with parameters set to the mixed-frequency estimates of the Smets and Wouters model: expansion

Table 25: The Smets and Wouters (2007) Model, Expansion: Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.92, 0.86, 0.94. The columns present months in which nowcasts are made. The forecast errors are averaged over a sample 2010Q2:2019Q4. ***/**/* indicates the forecasts that are statistically significantly different from the one-step ahead quarterly forecast with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation. Quarterly 1-step ahead RMSFE is 0.59.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	2.1443***	1.9493***	1.6293***
Kim (2010)	0.9620*	0.7667	0.6303
New Method	1**	1*	1**

Table 26: The Smets and Wouters (2007) Model, Expansion: Relative RMSFE of the GDP Forecasts Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 0.7443, 2.5841, 2.8309, 2.5389, 2.1733, 1.8701, 1.5974, 1.4002, 1.2054, 1.0596, 0.9287, 0.8153. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over a sample 2010Q2:2019Q4. ***/**/* indicates the forecasts that are statistically significantly different from the quarterly forecasts with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6	7	8	9	10	11	12
Quarterly	0.7879	0.1673	0.1512	0.1592	0.1906	0.1980	0.2236	0.2333	0.2753	0.3144	0.3384	0.3951
Giannone et al. (2016)	3.4586***	1.1756***	0.9052***	0.8407**	0.8161**	0.8075*	0.8057	0.8288	0.8631	0.9199	0.9861	1.0659
Kim (2010)	1.4042***	0.5521***	0.4493***	0.4361**	0.4380**	0.4557*	0.4821*	0.5232*	0.5720	0.6391	0.7109	0.7874
New Method	1	1***	1***	1**	1**	1*	1*	1	1	1	1	1

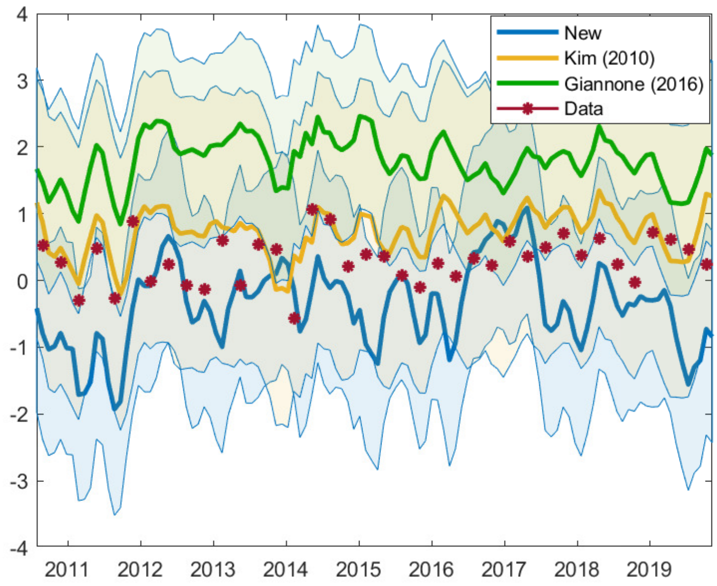

Figure 21: The Smets and Wouters (2007) Model, Expansion: Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in burgundy dots. Shaded areas correspond to the one standard deviation around the mean. X-axis shows time in months.

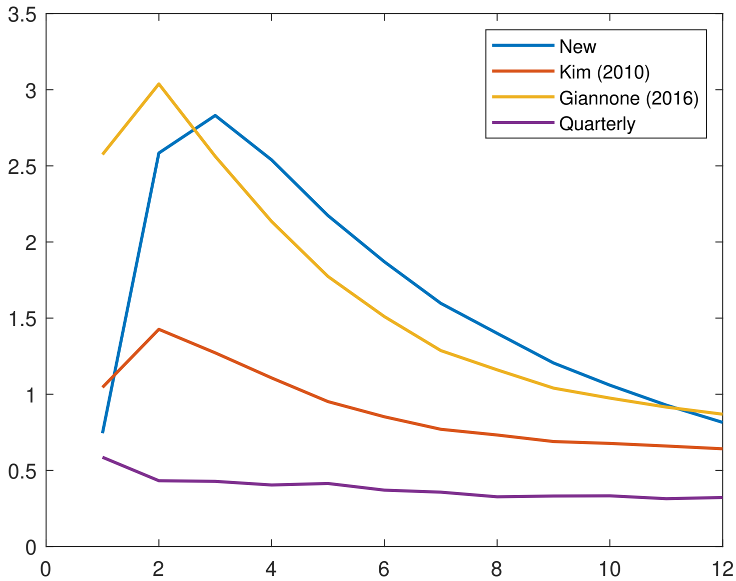

Figure 20: The Smets and Wouters (2007) Model, Expansion: RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). The RMSFE are presented for the Smets and Wouters (2007) model and are averaged over a sample 2010Q2:2019Q4. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

D.6. Forecasting results with parameters set to the mixed-frequency estimates of the Smets and Wouters model: the Great Recession

Table 27: The Smets and Wouters (2007) Model, the Great Recession: Relative RMSFE of the GDP Nowcasts, Obtained Using Mixed-frequency Methods. Each row presents RMSFE nowcasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 3.64, 1.45, 0.98. The columns present months in which nowcasts are made. The forecast errors are averaged over a sample 2008Q1:2009Q2. ***/**/* indicates the forecasts that are statistically significantly different from the one-step ahead quarterly forecast with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation. Quarterly 1-step ahead RMSFE is 1.30.

Method	Month 1	Month 2	Month 3
Giannone et al. (2016)	0.3296	0.7075	0.8838*
Kim (2010)	0.3565	0.6164*	0.7870*
New Method	1**	1	1

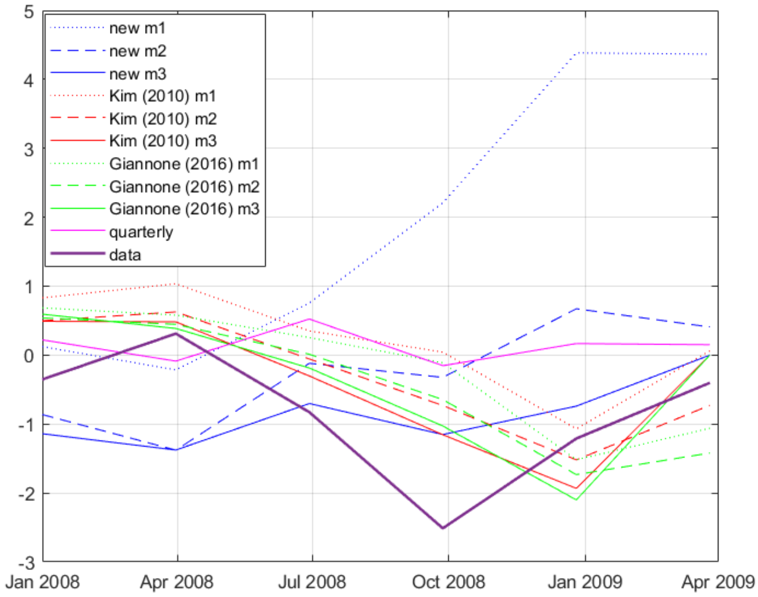

Figure 22: The Smets and Wouters (2007) Model, the Great Recession: GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in green), and the standard quarterly forecasting (in magenta). The data is shown in purple. The forecasts are presented for the Smets and Wouters (2007) model. "m1", "m2", and "m3" correspond to the first, the second and the third months from which the nowcasts are made. X-axis shows time in quarters (January stands for the first quarter).

Table 28: The Smets and Wouters (2007) Model, the Great Recession: Relative RMSFE of the GDP Forecasts Obtained Using Quarterly and Mixed-frequency Methods. Each row presents RMSFE forecasts obtained using the Giannone et al. (2016), the Kim (2010) / Foroni and Marcellino (2014), and the new method, suggested in the Section 2. The RMSFE are shown as a ratio to the RMSFE of the new method. The RMSFE of the new method are 1.5566, 3.5307, 3.9524, 3.8064, 2.9431, 2.2300. The columns present quarters ahead for which forecasts are made. The forecast errors are averaged over a sample 2008Q1:2009Q2. ***/**/* indicates the forecasts that are statistically significantly different from the quarterly forecasts with a 1%/5%/10% significance level based on the Diebold and Mariano (1991) test, where Newey–West standard errors are used to deal with the autocorrelation.

Method	1	2	3	4	5	6
Quarterly	0.8328	0.4126	0.4031	0.4423	0.3124	0.1630
Giannone et al. (2016)	1.0954	0.7443	0.6717	0.7008	0.6532	0.6453
Kim (2010)	1.0243	0.7292	0.6412	0.6383	0.5382	0.4605
New Method	1*	1	1	1	1	1

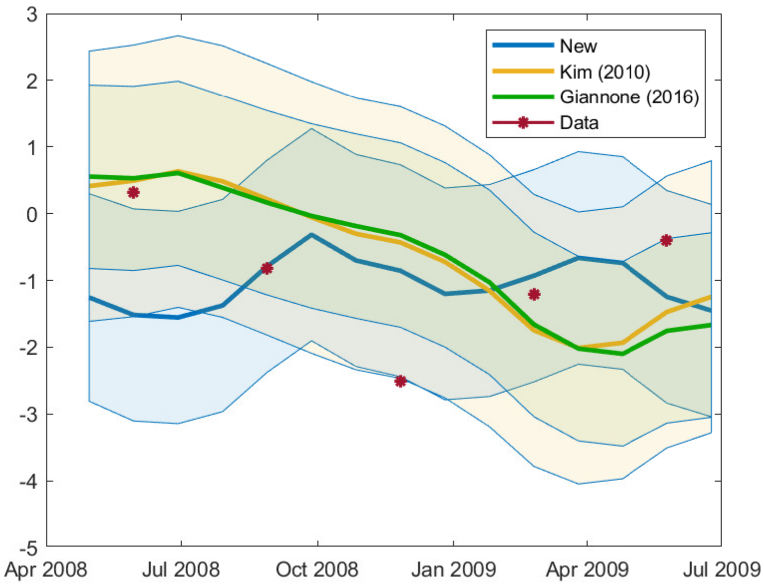

Figure 23: The Smets and Wouters (2007) Model, the Great Recession: Smoothed Out-of-Sample GDP. The Kalman smoother generates monthly GDP series, values of which are missed in the first and the second months of each quarter. The figure shows these smoothed GDP series ($E(y_{t|T})$). The smoothed series are obtained using the Kalman smoother of the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in yellow), and the Giannone et al. (2016) method (in green), and the actual data is shown in burgundy dots. Shaded areas correspond to the one standard deviation around the mean. X-axis shows time in months.

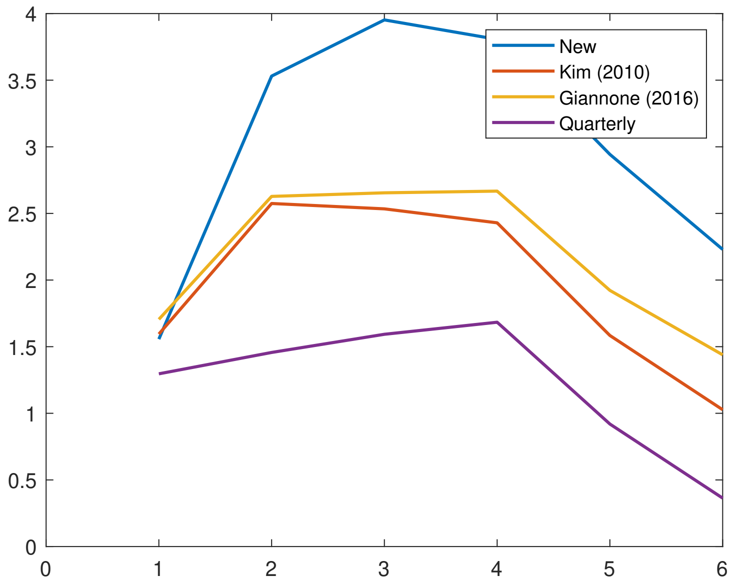

Figure 24: The Smets and Wouters (2007) Model, the Great Recession: RMSFE of the GDP Forecasts, Obtained Using Quarterly and Mixed-frequency Methods.. The figure shows GDP RMSFE, obtained using the new method (in blue), the Kim (2010) / Foroni and Marcellino (2014) method (in red), the Giannone et al. (2016) method (in yellow), and the standard quarterly forecasting (in purple). The RMSFE are presented for the Smets and Wouters (2007) model and are averaged over a sample 2008Q1:2009Q2. X-axis shows forecasting horizon in quarters and Y-axis shows RMSFE for each method.

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