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Abstract

This paper shows that investors financing a portfolio of projects may use the depth of their financial pockets to overcome entrepreneurial incentive problems. Competition for scarce informed capital at the refinancing stage strengthens investors' bargaining positions. And yet, entrepreneurs' incentives may be improved, because projects funded by investors with "shallow pockets" must have not only a positive net present value at the refinancing stage, but one that is higher than that of competing portfolio projects. Our paper may help to understand provisions used in venture capital finance that limit a fund's initial capital and make it difficult to add more capital once the initial venture capital fund is raised.

Venture capital finance takes place in an environment of severe informational asymmetries and incentive problems. Venture capitalists not only must assess the quality of investment proposals submitted to them for funding, but once the initial funding has taken place, entrepreneurs must be given the right incentives, and the performance of portfolio companies must be monitored on an ongoing basis. This paper departs from most of the existing literature by recognizing that venture capitalists manage a portfolio of projects. The need for portfolio management arises if the amount of capital—both financial and human—available to a venture capital fund is limited, implying that venture capitalists must carefully choose which projects to allocate their scarce financial and human resources to.[1] By staging their investments, venture capitalists retain the right to deny capital infusions to particular projects in favor of other, more promising ones:

> "The most important mechanism for controlling the venture is staging the infusion
> of capital. ... Capital is a scarce and expensive resource for individual ventures. ...
> The credible threat to abandon a venture, even when the firm might be economically
> viable, is the key to the relationship between the entrepreneur and the venture capi-
> talist. ... The seemingly irrational act of shutting down an economically viable entity
> is rational when viewed from the perspective of the venture capitalist confronted with
> allocating time and capital among various projects" (Sahlman (1990)).

Allocating scarce resources to the most potent portfolio projects implies that projects effectively compete with one another for limited "informed" capital at the refinancing stage.[2] As this naturally increases venture capitalists' ex post bargaining power, one would expect that entrepreneurs' ex ante incentives are reduced. As we will show, however, the opposite may be true. While entrepreneurs' expected payoff from a *given* effort level is reduced ("bargaining power effect"), the difference in expected payoffs across effort levels may be increased ("competition effect"). Competition for scarce informed capital introduces an additional incentive to have not only a positive net present value (NPV) at the refinancing stage, but one that is higher than that of competing portfolio projects. If the competition effect outweighs the bargaining power

effect, limiting the amount of informed capital can improve entrepreneurial incentives.

This paper compares "constrained finance" (or "shallow pockets")—i.e., committing to scarce informed capital to induce competition among entrepreneurs—with "unconstrained finance" (or "deep pockets"). Constrained finance may improve entrepreneurial incentives, but it also entails allocational inefficiencies, as successful projects may not obtain capital at the refinancing stage. Accordingly, constrained finance should not be used for projects with a high likelihood of success. Indeed, venture capitalists acknowledge that they "go for the home run" to offset the large number of failures in their portfolios (Sahlman (1990), Bygrave and Timmons (1992)).[3]

While our model focuses mainly on moral hazard, we show that constrained finance may also have advantages in dealing with adverse selection problems. In particular, separation between good and bad entrepreneurs may be impossible if investors have deep pockets, but possible if investors can choose between deep and shallow pockets. For certain parameter values, the unique equilibrium in our model is a separating equilibrium in which good entrepreneurs choose constrained finance and bad ones choose unconstrained finance.

Evidence from venture capital funds and the partnership agreements governing them support the notion of competition for scarce financial and human capital among portfolio companies. As is well known, "venture organizations will limit both how often they raise funds and the size of the funds that they raise" (Gompers and Lerner (1996)). Moreover, while venture capitalists raise a new fund every few years, partnership agreements often include covenants preventing venture capitalists from co-investing in companies managed by other funds of the same venture capitalist, implying that once a fund is raised, it cannot be easily augmented by adding more capital (Sahlman (1990), Fenn, Liang, and Prowse (1995), Gompers and Lerner (1996)).[4] A fund's human capital is also often limited fom the outset: Partnership agreements often include covenants that restrict the ability to add more general partners—i.e., experienced venture capitalists—to an existing fund (Gompers and Lerner (1996)).[5] As a consequence, venture capitalists must carefully choose to which portfolio companies they allocate their scarce financial

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and human capital, leading to precisely the sort of competition envisioned here.

Most of the theoretical literature on venture capital finance considers the financing of a single project. Exceptions are Kanniainen and Keuschnigg (2003), Bernile, Cumming, and Lyandres (2005), and Fulghieri and Sevilir (2005), who all consider the optimal span of a venture capitalist's portfolio. In contrast, holding the span of the venture capitalist's portfolio fixed, we consider the benefits and costs of venture capitalists being capital constrained.

In a broader context, this paper shows that prominent arguments made in other strands of economics are also relevant for venture capital portfolio financing. Without trying to be exhaustive, let us point out three important parallels.

First, in our model, a potential disadvantage of constrained finance is that it weakens entrepreneurs' bargaining position, thus reducing their incentives to exert effort. However, *if* entrepreneurs can be motivated to exert high effort, because the competition effect outweighs the bargaining power effect, then this disadvantage can become an advantage: Due to the investor's stronger bargaining position, projects that would otherwise not be financially viable may now become viable. The idea of strengthening the bargaining position of the party whose contribution is relatively more important is analyzed in several papers, notably Grossman and Hart (1986), Hart and Moore (1990), Aghion and Tirole (1997), and—in a corporate financing context—Aghion and Bolton (1992) and Gertner, Scharfstein, and Stein (1994). In particular, Aghion and Bolton argue that strengthening the position of investors may render projects financially viable that might not be viable otherwise.

Second, the idea that competition for scarce capital may increase incentives to effort ("competition effect") borrows from the labor tournament literature (Lazear and Rosen (1981), Nalebuff and Stiglitz (1983)). There is one subtle qualification: In many real-world tournaments, prizes are exogenously given, e.g., there is only one CEO position in a firm. In contrast, our model implies that in a context of portfolio financing, investors can provide optimal incentives by carefully choosing the ratio of available capital to projects.

Third, there is an obvious parallel to the literature on soft-budget constraints, started by Kornai (1979, 1980) in the context of socialist economies and applied by Dewatripont and Maskin (1995) to financial commitment problems. There is, again, a subtle but noteworthy difference: In Dewatripont and Maskin's model, the role of hard budget constraints is to deter bad entrepreneurs from seeking financing ex ante. In our model, by contrast, the role of hard budget constraints, or shallow pockets, is to credibly commit to a tournament to elicit greater entrepreneurial effort.

The literature on internal capital markets also addresses issues similar to those in this paper. On the positive side, internal capital markets may allow for an efficient ex post reallocation of resources, sometimes known as "winner-picking" (Stein (1997), Matsusaka and Nanda (2002)).[6] On the negative side, the prospect of having resources reallocated away may weaken division managers' ex ante incentives (Brusco and Panunzi (2005)).[7] In our model, the positive and negative sides are reversed: Unlike in an internal capital market, the ex post resource allocation is *less* efficient under constrained finance, while entrepreneurs' ex ante incentives may be *improved*.[8]

Finally, our paper relates to the capital budgeting literature, notably Harris and Raviv (1996, 1998). The authors show that imposing a fixed spending limit—which can be relaxed at the cost of a subsequent audit—may be part of an optimal capital budgeting procedure. As in our model, it may thus be optimal to ration capital, even if doing so means foregoing positive NPV investments. The reasons for doing so are different, though. In Harris and Raviv's models, capital rationing induces truthful revelation of division managers' private information. In our model, capital rationing improves entrepreneurs' ex ante incentives.

The rest of this paper is organized as follows. Section 1 describes the model. Section 2 examines the benefits and costs of constrained finance with respect to effort incentives. Section 3 considers the optimal choice between constrained and unconstrained finance. Section 4 discusses the role of ex ante and interim asymmetric information. Section 5 concludes. All proofs are in the Appendix.

1   The Model

Agents and Technology

There are two types of agents: entrepreneurs, who have no wealth, and investors. Each entrepreneur has a project that requires an initial capital outlay of $I_1 > 0$ at $t = 0$. Projects can be refinanced at $t = 1$ at cost $I_2 > 0$. Refinancing is best understood as an expansion of the project. Projects that are not refinanced continue on a smaller scale in a sense made precise below.[9] At $t = 2$, each project generates a verifiable payoff of either $R > 0$ or zero.

At $t = 1$, when the refinancing decision is made, a project's "interim type" is $\psi \in \{n, l, h\}$, which is only observed by the investor and entrepreneur. Projects with interim type $\psi = n$ are failures and generate a certain zero payoff. Projects with interim type $\psi = l$ or $\psi = h$ are successful, implying that it is efficient to refinance them. If a project with interim type $\psi \in \{l, h\}$ is refinanced, the probability that it generates R is p_ψ, where $p_h > p_l$, implying an expected payoff of $R_\psi := p_\psi R$. By contrast, if a project with interim type $\psi \in \{l, h\}$ is not refinanced, the probability that it generates R is p_0, implying an expected payoff of $R^0 := p_0 R$.[10] Hence, the overall surplus from refinancing a project with interim type $\psi \in \{l, h\}$ is $r_\psi := R_\psi - R^0 - I_2$, which is positive, and where $r_h > r_l$ follows from our assumption that $p_h > p_l$.

With probability $1 - \tau$, the project's interim type is $\psi = n$, and with probability τ, its interim type is either $\psi = l$ or $\psi = h$. Conditional on success, the probability of having interim type $\psi = h$ is q_θ, and the probability of having interim type $\psi = l$ is $1 - q_\theta$, where $\theta \in \{g, b\}$ represents the project's "ex ante type." Accordingly, the total probability that the project has interim type $\psi = h$ is τq_θ, and the total probability that it has interim type $\psi = l$ is $\tau(1 - q_\theta)$. We assume that $q_g > q_b$, i.e., good projects have a higher probability of becoming interim type $\psi = h$ than do bad projects. Figure 1 summarizes the project technology.

[Figure 1 here]

We assume that entrepreneurs can choose their ex ante type at $t = 0$. This choice is only

observed by the entrepreneur ("moral hazard"). Choosing ex ante type θ yields private benefits B_θ at $t = 2$, where $B_b = B > B_g = 0$. These benefits are only obtained if the project is successful. As B constitutes the opportunity cost of choosing $\theta = g$ instead of $\theta = b$, we refer to B simply as "effort cost" and to the entrepreneur's choice of $\theta = g$ and $\theta = b$ as "high effort" and "low effort", respectively. Finally, we assume that $(q_g - q_b)(r_h - r_l) > B$, implying that high effort is socially efficient.

Financing

Investors compete at $t = 0$ to provide financing to entrepreneurs. We specify that each investor optimally provides start-up finance to two entrepreneurs.[11] In principle, investors can raise enough capital initially that at $t = 1$, they are able to refinance all projects that are worth refinancing. The central claim of this paper, however, is that investors may sometimes deliberately limit the amount of capital raised to create competition among entrepreneurs at the refinancing stage. As noted in the Introduction, evidence from venture capital funds and the partnership agreements governing them supports the notion of competition for scarce financial and human capital envisioned here.

A priori, it is not clear why the investor would not attempt to raise additional capital at $t = 1$ if both projects turn out to be successful, and we do not preclude the investor from trying to do so. However, as only the (inside) investor and entrepreneur know the project's interim type, there exists a lemons problem vis-à-vis outside investors that may render outside financing infeasible, as in Rajan's (1992) model. We relegate a formal analysis of this issue to Section 4.2. For the time being, we assume that the lemons problem at $t = 1$ is sufficiently strong to render outside financing infeasible.

The investor's choice is between what we call *unconstrained finance* (or "deep pockets") and *constrained finance* (or "shallow pockets"). This choice is observable by entrepreneurs. Under unconstrained finance, the investor raises enough capital to potentially refinance both portfolio projects at $t = 1$, i.e., she raises $2I_1 + 2I_2$. Under constrained finance, in contrast, the investor

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only raises $2I_1 + I_2$ initially. Any capital currently not used is invested in liquid securities, whose interest rate is normalized to zero.

Contracts and Renegotiations

Investors compete ex ante by offering contracts specifying for each entrepreneur E_i a share s_i of the project's final payoff. By restricting ourselves to sharing rules, we rule out transfer payments to entrepreneurs that are independent of the project's payoff. The usual motivation for this assumption is that guaranteed transfer payments independent of payoffs would attract fraudulent entrepreneurs, or "fly-by-night operators" (Rajan (1992)), who would only apply to cash in the guaranteed transfer payment.[12]

Because the project's interim type is non-verifiable, the refinancing decision cannot be part of an initial contract. Hence, whether the project will be refinanced must be determined by negotiations between the investor and entrepreneur at $t = 1$. As part of these negotiations, the two parties may renegotiate the initial sharing rule s_i, which is why we shall use the term *renegotiations*. But even though the initial sharing rule is renegotiated, it is not meaningless: It defines the entrepreneur's and investor's payoffs if the project is not refinanced, and thus their outside options if the renegotiations break down. Where do the bargaining powers in the renegotiations stem from? The entrepreneur's bargaining power stems from his ability to withdraw his inalienable and essential human capital, while the investor's bargaining power stems from her right to decide whether to refinance.[13]

The assumption that the project's interim type is non-verifiable is important. It implies that the refinancing decision cannot be part of an initial contract, which in turn forces the investor and entrepreneur into a bargaining situation at the refinancing stage. Evidence from the venture capital literature supports this assumption. Gompers (1995) writes: "Each time capital is infused, contracts are written and negotiated ... Major review of progress, due diligence, and the decision to continue funding are generally done at the time of the refinancing." That contracts are renegotiated at the refinancing stage suggests that it might be difficult to specify ex ante

what precisely "progress" means. Indeed, Gompers (1995) rejects the alternative hypothesis of contingent follow-up financing based on observable "technology-driven milestones".[14] Similarly, Kaplan and Strömberg (2003) write, "we consider a financing round as a set of contracts agreed to on a particular date that determines the disbursements of funds from the VC to a company. A new financing round *differs from the contingent release of funds* in that the price and terms of the financing are not set in advance" (italics added).

2 Refinancing and Renegotiations

Solving the model backwards, we first consider the renegotiations at $t = 1$. Subsequently, we derive the entrepreneur's expected payoff at $t = 0$, accounting for the outcome of the renegotiations. We then compute the sensitivity of the entrepreneur's expected payoff with respect to his ex ante type. Comparing the sensitivities under unconstrained and constrained finance, we finally obtain what we call the "responsiveness condition".

2.1 Renegotiations under Unconstrained Finance

Under unconstrained finance, the investor has sufficient capital to refinance all projects that are worth refinancing. As a result, she cannot credibly threaten not to refinance a project with interim type $\psi \in \{l, h\}$, regardless of the interim type of the other portfolio project. Consequently, the refinancing decision for a particular project is independent of the other project, implying that we can analyze the renegotiations with each entrepreneur separately.

Consider the renegotiations with entrepreneur E_i. Given that the investor knows E_i's interim type, renegotiations take place under symmetric information. We adopt the standard alternating offers bargaining procedure with an open time horizon analyzed in Rubinstein (1982). While the bargaining procedure is open ended, bargaining frictions ensure that an agreement is reached immediately. For the specific type of bargaining friction employed here, we follow Binmore, Rubinstein, and Wolinsky (1986) and assume that after each round, there is a probability δ that

the renegotiations break down, in which case the project is not refinanced.[15]

Without loss of generality, we assume that the investor makes the first offer, which E_i can either accept or reject.[16] The offer is to provide refinancing in return for a share of the project's payoff. If E_i rejects the investor's offer, provided that negotiations have not yet broken down, he can make a counteroffer, and so on. It is crucial that the entrepreneur can make counteroffers. If all E_i could do is accept or reject the investor's offers, the investor could extract the entire surplus. E_i's continuation payoff at $t = 1$ would then always be $s_i R^0$ regardless of his interim type, which in turn implies that there would be no difference between constrained and unconstrained finance in terms of providing incentives. However, a bargaining procedure in which only the investor can make offers would require that she can credibly commit to not listening to any offers the entrepreneur makes, which seems to be difficult to implement in practice.[17]

The analysis of the bargaining game is straightforward. If a project with interim type $\psi_i \in \{l, h\}$ is not refinanced, it generates an expected payoff of R^0. Hence, if $\psi_i \in \{l, h\}$ the outside options in the renegotiations are $(1 - s_i)R^0$ and $s_i R^0$, respectively, while the surplus to be bargained over is r_{ψ_i}. Lemma 1 characterizes the equilibrium outcome of the bargaining game as $\delta \to 0$. The proof follows Binmore, Rubinstein, and Wolinsky (1986).

Lemma 1. *Under unconstrained finance, the investor's and entrepreneur E_i's continuation payoffs at $t = 1$ are as follows*:

i) If E_i has interim type $\psi_i = n$, both continuation payoffs are zero.

ii) If E_i has interim type $\psi_i \in \{l, h\}$, E_i's continuation payoff is $s_i R^0 + \frac{1}{2} r_{\psi_i}$ and the investor's continuation payoff is $(1 - s_i)R^0 + \frac{1}{2} r_{\psi_i}$.

Proof. See Appendix. ∎

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2.2 Renegotiations under Constrained Finance

Under constrained finance, the investor cannot refinance all projects that are worth refinancing, implying that she can credibly threaten to use her scarce capital for the other portfolio project. The renegotiations with E_i thus depend on the interim type of the other entrepreneur, E_j, for two reasons. First, who the investor picks to bargain with first depends on who has a higher interim type. Second, the investor's outside option in the renegotiations with E_i depends on E_j's interim type, and vice versa.[18]

The extensive form of the bargaining game is as follows. The investor picks one of the two entrepreneurs, say E_i, and makes him an offer. If E_i accepts, the game ends. If E_i rejects, the negotiations with E_i break down with probability δ. If there is no breakdown, E_i can make a counteroffer. If the investor accepts E_i's counteroffer, the game ends. If the investor rejects, the negotiations with E_i break down with probability δ. If there is no breakdown, the investor again picks one of the two entrepreneurs, and so on. In contrast, if the negotiations with E_i have broken down, the investor must necessarily turn to E_j. Hence, the bargaining procedure is the same alternating offer procedure, with the same open time horizon and risk of breakdown, as in the case of unconstrained finance, except that after each round, the investor can choose with whom to bargain next.

If at least one entrepreneur has interim type $\psi = n$, the outcome is trivially the same as under unconstrained finance. The interesting case is where neither entrepreneur has interim type $\psi = n$. As the following lemma shows, the investor can then extract a higher continuation payoff from her first pick, say E_i, relative to unconstrained finance. The downside is that she cannot realize any surplus with her second pick E_j, as her scarce capital has already been used up.

Lemma 2. *Under constrained finance, the investor's and the two entrepreneurs' continuation payoffs at $t = 1$ are as follows:*

i) If at least one entrepreneur has interim type $\psi = n$, all payoffs are as in Lemma 1.

ii) If neither entrepreneur has interim type $\psi = n$, and if the investor picks E_i to bargain with first, then

    a) E_i's continuation payoff is $s_i R^0 + \frac{1}{2}(r_{\psi_i} - \frac{1}{2} r_{\psi_j})$,

    b) E_j's continuation payoff is $s_j R^0$, and

    c) the investor's continuation payoff is $(1 - s_i) R^0 + (1 - s_j) R^0 + \frac{1}{2}(r_{\psi_i} + \frac{1}{2} r_{\psi_j})$.

Proof. See Appendix. ∎

If both entrepreneurs have the same interim type $\psi \in \{l, h\}$, the investor cannot extract the entire surplus from her first pick E_i even though the other entrepreneur is a perfect substitute. This may seem surprising. Why does the investor not deviate and go to the other entrepreneur E_j, who should be eager to obtain refinancing, even under less favorable conditions, given that he would otherwise only obtain $s_j R^0$? The reason is that E_j would not accept an offer that leaves him just a little more than his outside option payoff. Instead, he would reject the investor's offer, and make a counteroffer that makes the investor indifferent between accepting and going back to her first pick E_i.

Finally, we consider the issue of who the investor picks to bargain with first. Note that the initial sharing rule s_i does not affect the investor's choice; it depends exclusively on the entrepreneurs' interim types. When the two interim types are not identical, the investor bargains first with the higher interim type. When the two interim types are identical, the investor is indifferent. In this case, we specify that she picks either of the two entrepreneurs with equal probability (see proof of Lemma 2).

2.3 The Responsiveness Condition

Given Lemmas 1 and 2, we can compute the entrepreneur's expected payoff at $t = 0$. The derivation is in the Appendix. The entrepreneur's expected payoff under unconstrained finance is

$$\tau \left\{ s_i R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} \left(r_h - r_l \right) \right] \right\}. \tag{1}$$

Below, we consider the entrepreneur's effort choice problem. The more responsive the entrepreneur's expected payoff is to his ex ante type, the easier it is to motivate him to choose $\theta = g$ rather than $\theta = b$. We obtain the responsiveness under unconstrained finance by subtracting the entrepreneur's expected payoff for $\theta_i = b$ from that for $\theta_i = g$:

$$\frac{1}{2}\tau \left(q_g - q_b\right)\left(r_h - r_l\right). \tag{2}$$

Importantly, the responsiveness does not correspond to the full difference in expected project values as the investor can extract part of this value in the renegotiations.

Likewise, the entrepreneur's expected payoff under constrained finance is

$$\tau \left\{s_i R^0 + \frac{1}{2}\left[r_l + q_{\theta_i}\left(r_h - r_l\right)\right]\right\} - \frac{\tau^2}{8}\left\{r_l\left(3 - q_{\theta_i} + q_{\theta_j}\right) + 3q_{\theta_i}q_{\theta_j}\left(r_h - r_l\right)\right\}. \tag{3}$$

Under constrained finance, the two entrepreneurs compete for scarce informed capital. Consequently, if the other entrepreneur also has a profitable refinancing opportunity, the investor can extract more from a given entrepreneur than she can under unconstrained finance. Our key insight, however, is that offering constrained finance may nevertheless make an entrepreneur's expected payoff more responsive to his ex ante type: While the investor's stronger ex post bargaining position reduces the entrepreneur's expected payoff for a *given* ex ante type, the difference in expected payoffs across ex ante types can be increased. As will become clear shortly, we are interested in the case in which both entrepreneurs choose $\theta = g$. Consequently, we obtain the responsiveness under constrained finance by setting $\theta_j = g$ and subtracting the entrepreneur's expected payoff for $\theta_i = b$ from that for $\theta_i = g$:

$$\frac{1}{2}\left(q_g - q_b\right)\tau \left\{\left(r_h - r_l\right) + \frac{\tau}{4}\left[r_l - 3q_g\left(r_h - r_l\right)\right]\right\}. \tag{4}$$

Comparing the responsiveness under unconstrained finance, (2), with that under constrained finance, (4), establishes the following proposition.

Proposition 1. *The responsiveness of the entrepreneur's expected payoff to his ex ante type is*

higher under constrained finance than under unconstrained finance if and only if

$$r_h - r_l < \frac{r_l}{3q_g}. \tag{5}$$

We will henceforth refer to (5) as the "responsiveness condition." It captures the tradeoff between two effects of competition for scarce informed capital under constrained finance:

Competition Effect: Under constrained finance, not being picked first to be bargained with implies that the entrepreneur will not receive refinancing in equilibrium. Thus, competition for scarce informed capital introduces an additional incremental return to being picked first, making the entrepreneur's expected payoff more sensitive to his ex ante type.

Bargaining Power Effect: Under constrained finance, the investor can threaten to refinance the other entrepreneur when bargaining with her first pick. This provides the investor with additional bargaining power, which reduces the entrepreneur's expected return from being refinanced, thus reducing the responsiveness.

If the responsiveness condition (5) holds, the entrepreneur's expected payoff under constrained finance is more sensitive to his ex ante type than it is under unconstrained finance. Put simply, constrained finance then provides stronger effort incentives than does unconstrained finance. Intuitively, unconstrained finance provides effort incentives through the difference in final payoffs $r_h - r_l = R_h - R_l$ (see (2)). If this difference is large, the incentives provided under unconstrained finance are already quite substantial. Accordingly, the additional incentives under constrained finance created through competition for scarce informed capital have relatively little value, and the competition effect is dominated by the bargaining power effect. Conversely, if $r_h - r_l$ is small, the incentives provided under unconstrained finance are relatively small, and the additional incentives under constrained finance through competition for scarce informed capital offset the negative bargaining power effect. As we will show in the following section, (5) is a necessary but not sufficient condition for constrained finance to be chosen.

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3 Constrained versus Unconstrained Finance

3.1 Analysis

We now analyze the investor's choice between constrained and unconstrained finance. There are exactly two cases in which the investor will choose constrained finance: when constrained finance is the only viable alternative, i.e., the investor can only break even under constrained finance, and when both alternatives are viable, but constrained finance gives entrepreneurs a higher expected payoff. As there is ex ante competition for entrepreneurs, investors choose constrained finance in this case.

It is easy to show that neither of the cases is possible if constrained and unconstrained finance both implement the same level of effort. Hence, constrained finance is chosen only if it implements higher effort. That is, constrained finance must implement $\theta = g$ while unconstrained finance must implement $\theta = b$. By (2) and (4), this in turn implies, first, that the responsiveness condition (5) must hold, and second, that the effort cost B must lie in the intermediate range

$$\frac{1}{2}\left(q_g - q_b\right)\left(r_h - r_l\right) \leq B < \frac{1}{2}\left(q_g - q_b\right)\left\{\left(r_h - r_l\right) + \frac{\tau}{4}\left[r_l - 3q_g\left(r_h - r_l\right)\right]\right\}. \tag{6}$$

The condition (6) has an intuitive interpretation.[19] If effort is not particularly costly, so that even unconstrained finance can induce high effort, constrained finance cannot play out its advantage of providing relatively stronger effort incentives. Conversely, if effort is extremely costly so that even constrained finance cannot induce high effort, then, again, it does not matter that constrained finance provides relatively stronger effort incentives.

If the necessary conditions hold, the choice between constrained and unconstrained finance becomes straightforward. If only constrained finance is viable—i.e., the investor can only break even under constrained finance—then clearly, constrained finance is chosen. Likewise, if only unconstrained finance is viable, then unconstrained finance is chosen. Finally, if constrained and unconstrained finance are both viable, competition for entrepreneurs implies that the investor chooses the financing mode that gives entrepreneurs a higher expected payoff.

To see whether a project is financially viable, we must derive the investor's expected payoff at $t = 0$. The derivation is analogous to that of (1) and (3), with the addition that $\theta_i = b$ for unconstrained and $\theta_i = \theta_j = g$ for constrained finance (see proof of Proposition 2). As the investor's expected payoff decreases in the entrepreneur's payoff share, the project is viable if and only if the investor's expected payoff is non-negative at $s_i = 0$. Accordingly, the project is viable under unconstrained finance if and only if

$$\pi_U^I := \tau \left\{ R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\} \geq I_1, \tag{7}$$

and it is viable under constrained finance if and only if

$$\pi_C^I := \tau \left\{ R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} \geq I_1. \tag{8}$$

If constrained and unconstrained finance are both viable, ex ante competition among investors implies that they will choose the financing mode that is better for entrepreneurs. The entrepreneur's expected payoff in this case can be easily derived from (1) and (3), and the investors' zero-profit condition (see proof of Proposition 2). The following proposition summarizes the investors' optimal choice between unconstrained and constrained finance.

Proposition 2. *Suppose that the responsiveness condition (5) holds and B satisfies (6). For any given investment cost I_1, projects whose success probability τ is sufficiently low are not financially viable. For projects that are financially viable, the following holds:*

i) For projects with a sufficiently high investment cost—provided the project is financed at all—only unconstrained finance is chosen.

ii) For projects with low investment costs, other things equal, constrained finance is chosen if the project's success probability is low, and unconstrained finance is chosen if the project's success probability is high.

Proof. See Appendix. ∎

By Proposition 2, if either i) the responsiveness condition (5) is violated, implying that unconstrained finance provides relatively stronger effort incentives than does constrained finance, or if ii) the effort cost B is either too low or too high, so that (6) is violated, implying that constrained and unconstrained finance both implement the same effort, or if iii) the investment cost is too high, then constrained finance will not be chosen.[20] Conversely, if i)-iii) hold, then constrained finance will be chosen for relatively low success probabilities, and unconstrained finance will be chosen for relatively high success probabilities.

Proposition 2 is illustrated in Figure 2. The success probability τ is depicted on the x-axis, and the investment cost I_1 is depicted on the y-axis. The vertically and horizontally shaded areas depict all (τ, I_1) combinations for which constrained and unconstrained finance are chosen, respectively. The unshaded area depicts all (τ, I_1) combinations for which the project is not financially viable.

Perhaps the simplest way to illustrate Proposition 2 is by fixing I_1 and drawing an imaginary horizontal line originating at I_1 that runs parallel to the x-axis. In Proposition 2, "fixing I_1" is implied by "other things equal," which implies that projects are only compared with respect to their success probabilities. Holding I_1 fixed, the intersection of the horizontal line with the unshaded area shows all of the success probabilities for which the project is not financially viable, the intersection with the vertically shaded area shows all success probabilities for which constrained finance is chosen, and the intersection with the horizontally shaded area shows all success probabilities for which unconstrained finance is chosen.[21]

[Figure 2 here]

Part i) of Proposition 2 refers to values of I_1 that lie above the point where π_U^I and π_C^I intersect. For such high investment costs, the project is only viable if the probability of success is high, in which case unconstrained finance is chosen. Intuitively, for high success probabilities, the allocational inefficiency induced by constrained finance—namely, that if both projects are successful, one of them will not be refinanced—weighs heavily in expected terms.

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Part ii) of Proposition 2 refers to values of I_1 that lie below the intersection of π_U^I and π_C^I. Holding I_1 fixed, the horizontal line originating at I_1 intersects first with the unshaded area, then with the vertically shaded area, and finally with the horizontally shaded area. Projects with relatively low success probabilities are thus financed under constrained finance, while projects with high success probabilities are financed under unconstrained finance.

In Figure 2, $\tau = \widehat{\tau}$ marks the critical success probability at which the entrepreneur's expected payoffs under constrained and unconstrained finance intersect.[22] If both financing modes are financially viable, constrained finance is chosen for success probabilities $\tau \leq \widehat{\tau}$, and unconstrained finance is chosen for success probabilities $\tau > \widehat{\tau}$. In the (vertically shaded) "lens-shaped" area, unconstrained finance is not financially viable, implying that constrained finance is chosen also for success probabilities $\tau > \widehat{\tau}$.

Proposition 2 lends itself to two intuitive empirical implications. The first is that projects with very high investment costs should not be financed under constrained finance. This statement is independent of whether the two necessary conditions (5) and (6) hold. Unfortunately, a similarly strong statement cannot be made about when projects *should* be financed under constrained finance, for two reasons: The necessary conditions (5) and (6) may not hold, and the investment cost may be too high, so that part i) of Proposition 2 applies. However, one can argue the converse and in some sense weaker statement that if projects are financed under constrained finance, then, other things equal, they must have lower success probabilities than comparable projects financed under unconstrained finance.

We conclude with a comparative statics exercise. The benefit of constrained finance in our model is that it may induce high effort when unconstrained finance can only induce low effort. But if the efficiency loss from exerting low effort is relatively small, the benefit is also small. Intuitively, we might thus expect that constrained finance is more likely if the efficiency loss from exerting low effort is large, which is the case when q_b—the likelihood that exerting low effort generates a high interim type $\psi = h$—is small. The following corollary formalizes this

intuition.

Corollary 1. *Other things equal, an increase in the efficiency loss from having low entrepreneurial effort makes it more likely that constrained finance is chosen.*

Given the analysis in the proof of Proposition 2, the proof of Corollary 1 is immediate. In Figure 2, a decrease in q_b shifts both $\widehat{\tau}$ and π_U^I to the right, thus strictly expanding the range of success probabilities for which constrained finance is chosen.[23]

3.2 Empirical Implications

The first implication summarizes a key insight of our model:

Implication 1. *Other things equal, projects financed under constrained finance should have lower success probabilities than comparable projects financed under unconstrained finance.*

The intuition, which is at the heart of our model, is that for high success probabilities the allocational inefficiency induced by constrained finance—namely, that successful projects may not be refinanced—weighs heavily in expected terms, implying that such projects are optimally financed under unconstrained finance.

Like Implication 1, the following implication has been discussed in the previous section:

Implication 2. *Other things equal, projects with very high investment costs should not be financed under constrained finance.*

The intuition is closely related to that of Implication 1. Projects with very high investment costs require a high success probability to break even. But for high success probabilities, the benefits of constrained finance are outweighed by the costs.

The next empirical implication is a restatement of Corollary 1.

Implication 3. *Other things equal, projects are more likely to be financed under constrained finance if the efficiency loss from having low entrepreneurial effort is large.*

There are two aspects to the entrepreneurs' effort problem in our model. The first, addressed in Implication 3, regards the importance of entrepreneurial effort—that is, what is the efficiency loss from having low (instead of high) entrepreneurial effort? Intuitively, if the efficiency loss from having low effort is small, the benefits of constrained finance, namely, that it provides relatively stronger effort incentives, are also small and likely to be outweighed by the allocational inefficiency associated with constrained finance.

The second aspect concerns the severity of the effort problem: How costly is entrepreneurial effort? In this regard, a necessary condition for constrained finance to be chosen is that effort is sufficiently costly. If effort is not particularly costly, so that even unconstrained finance can induce high effort, constrained finance cannot play out its advantage of providing relatively stronger effort incentives. By the same token, entrepreneurial effort must not be too costly. If effort is extremely costly, so that even constrained finance cannot induce high effort, constrained finance again loses its advantage. We thus have:

Implication 4. *Projects for which inducing entrepreneurial effort is either not particularly costly or extremely costly should be financed under unconstrained finance.*

An immediate corollary to Implication 4 is that, other things equal, we should see that projects financed under constrained finance exhibit higher entrepreneurial effort. Importantly, our model does not predict that projects financed under constrained finance should have a higher ex post likelihood of success. While in our model constrained finance is chosen only if it induces higher effort, Implication 1 states that projects financed under constrained finance should have a lower ex ante success probability. As the two effects move in opposite directions, the overall effect on the project's ex post success likelihood remains ambiguous.

Under unconstrained finance, there is no allocational inefficiency: Projects rejected at the refinancing stage are always negative NPV projects. By contrast, under constrained finance, rejected projects may have either a negative or positive NPV.

Implication 5. *Projects rejected under constrained finance should on average have a higher NPV than do projects rejected under unconstrained finance.*

It would seem that a natural corollary to Implication 5 is that projects rejected under constrained finance should find it easier to obtain outside finance. As Section 4.2 shows, however, this may or may not be true. In particular, if the lemons problem that outside investors face is sufficiently strong, then projects rejected under constrained and unconstrained finance may both find it impossible to attract outside finance.

A related empirical implication concerns the likelihood that projects are rejected at the refinancing stage. Under unconstrained finance, this likelihood is simply $1 - \tau$. By contrast, under constrained finance, the likelihood of rejection is strictly higher.[24] Moreover, we know from Implication 1 that projects for which constrained finance is chosen should have lower ex ante success probabilities to begin with. As both effects move in the same direction, we have:

Implication 6. *Projects financed under constrained finance should have a higher likelihood of being rejected at the refinancing stage than projects financed under unconstrained finance.*

4 Adverse Selection

This section considers the role of asymmetric information both at the ex ante and the refinancing stages. Our base model assumed that entrepreneurs can choose their ex ante type. In Section 4.1, we assume instead that ex ante types are chosen by nature, and that only the respective entrepreneur can observe his ex ante type. Hence, we consider an adverse selection problem instead of a moral hazard problem.

In Section 4.2, we consider the role of asymmetric information at the refinancing stage. The (inside) investor and entrepreneur know the project's interim type, but outside investors do not. Our base model assumes that the resulting lemons problem is sufficiently strong to render outside financing at the refinancing stage infeasible. We now formally show under what conditions this

is the case. Moreover, we show that our results hold qualitatively even in cases in which outside financing at the refinancing stage is feasible.

4.1 Ex Ante Asymmetric Information

Contrary to our base model, we now assume that the entrepreneur's ex ante type is chosen by nature prior to $t = 0$. With probability α, nature chooses $\theta = g$, and with probability $1 - \alpha$, nature chooses $\theta = b$. Entrepreneurs know their ex ante types, but investors do not. Hence, at $t = 0$, when investors compete for entrepreneurs, the former face an adverse selection problem. To simplify the exposition, we assume that projects are financially viable. From our previous analysis, we know that this is the case if the initial investment I_1 is not too large.

Suppose for the moment that unconstrained finance is the only financing mode available to investors. We consider competitive equilibria à la Rothschild and Stiglitz (1976). As explained previously, the initial sharing rule s_i does not affect the investor's choice as to which project she refinances. Consequently, separation between ex ante types $\theta = g$ and $\theta = b$ cannot be achieved by offering a menu of initial sharing rules, as both types of entrepreneurs would strictly prefer the highest sharing rule offered. The following result is then immediate.

Lemma 3. *Suppose unconstrained finance is the only financing mode available to investors. Then the unique competitive equilibrium is a pooling equilibrium in which all entrepreneurs receive the same sharing rule regardless of their ex ante type.*

We now argue that allowing investors to choose between constrained and unconstrained finance may enable them to separate type $\theta = g$ from type $\theta = b$ entrepreneurs. Recall from Proposition 1 that if the responsiveness condition (5) holds, the payoff differential across ex ante types is larger under constrained finance. This implies that (5) is necessary but not sufficient to achieve separation across types. To achieve separation, the difference in the responsiveness between constrained and unconstrained finance must additionally be sufficiently large so that separation can be achieved at sufficiently favorable terms for type $\theta = g$ entrepreneurs. Moreover,

the allocational inefficiency induced by constrained finance must not be too large. Otherwise, investors offering constrained finance will be unable to offer mutually profitable contracts that can achieve separation.

In addition to these conditions, we obtain the usual condition arising in competitive screening models that the probability α of type $\theta = g$ entrepreneurs must not be too large. The following proposition establishes conditions under which all of the above requirements are met. As in Rothschild and Stiglitz (1976), we restrict consideration to pure-strategy equilibria.

Proposition 3. *Consider the following separating equilibrium: Entrepreneurs with ex ante type $\theta = b$ receive unconstrained finance, and entrepreneurs with ex ante type $\theta = g$ receive constrained finance. Suppose the responsiveness condition (5) holds. Then this separating equilibrium exists and is the unique competitive equilibrium if*

$$\tau \leq \frac{(q_g - q_b)(r_h - r_l)}{r_l + q_g^2 (r_h - r_l)}$$

and

$$\alpha \leq \min \left\{ \frac{\tau}{8} \frac{r_l - 3q_g (r_h - r_l)}{r_h - r_l}, \frac{1}{2} \left[1 - \tau \frac{r_l + q_g^2 (r_h - r_l)}{(q_g - q_b)(r_h - r_l)} \right] \right\}.$$

Proof. See Appendix. ∎

4.2 Interim Asymmetric Information and Outside Finance

While there is perfect competition for entrepreneurs at $t = 0$, we have assumed that the (inside) investor is the only source of funding at the refinancing stage—that is, projects that are not refinanced by the inside investor cannot obtain refinancing from outside investors. Intuitively, the market for outside finance may shut down at the refinancing stage due to a "lemons problem." The insiders, namely the entrepreneur and inside investor, know the project's interim type, but outside investors do not. If successful projects are pooled with "lemons"—i.e., projects with

interim type $\psi = n$—then outside investors may be unable to make an offer that can both attract successful projects and allow the investors to break even.

We proceed as follows. First, we show that there is always an equilibrium in which the market for outside finance shuts down at the refinancing stage, validating the assumption in our base model. Second, to the extent that there is also an equilibrium in which outside finance is feasible, we show that our results hold qualitatively. The inside investor is then no longer the only potential provider of capital at the refinancing stage, but she is still the only provider of *informed* capital, as only she, but no outside investor, knows the project's interim type. Accordingly, outside finance commands a lemons premium, providing the inside investor (again) with a strong bargaining position: While projects do not compete for *scarce* capital at the refinancing stage, they now compete for *cheaper* (informed) capital.

For a lemons problem to exist at the refinancing stage, type $\psi = n$ projects must have an incentive to seek outside finance. Otherwise, the pool of projects seeking outside finance would consist only of positive NPV projects. In our model thus far, insiders do not strictly benefit from luring outside investors into refinancing a type $\psi = n$ project. But they do if we change our model as follows: Suppose type $\psi = n$ projects, instead of having a zero success probability, have a small but positive probability p_n of generating $R > 0$. If p_n is small, refinancing a type $\psi = n$ project remains a negative NPV investment.[25] Most importantly, this modification has no effect on our previous results. In particular, the renegotiatons between the entrepreneur and the inside investor remain exactly the same: There is still no refinancing of type $\psi = n$ projects by the inside investor, and type $\psi = n$ projects still generate a zero payoff if they are not refinanced. However, the insiders now strictly benefit from luring outside investors into refinancing a type $\psi = n$ project: They have nothing to lose, but they may gain $R - D$ with probability p_n.

The market for outside finance at $t = 1$ operates as follows. Projects, represented by the insiders, express their willingness to seek outside finance. Outside investors then compete to provide funds I_2 in return for a share $D \leq R$ of the project's payoff.[26] Given the modification

introduced above, the insiders now strictly prefer to seek outside finance for unsuccessful projects. In contrast, the insiders may have something to lose from seeking costly outside finance for *successful* projects. As successful projects are pooled with lemons, outside finance may only be available at unfavorable terms. If these terms are sufficiently unfavorable, the insiders may prefer not to refinance a successful project—thus realizing R^0—instead of seeking costly outside finance. Formally, the insiders will seek outside finance for a type $\psi \in \{l, h\}$ project if and only if

$$\lambda_\psi := p_\psi(R - D) - R^0 \geq 0. \tag{9}$$

The difference

$$r_\psi - \lambda_\psi = p_\psi R - I_2 - p_\psi(R - D) = p_\psi D - I_2$$

represents the lemons premium associated with costly outside finance. If there was no asymmetric information vis-à-vis outsiders, the insiders could always obtain funds I_2 in return for a repayment $F = I_2/p_\psi$, realizing an expected payoff of $p_\psi(R - F) = p_\psi R - I_2$. If there is asymmetric information, however, outside investors will demand a higher repayment $D > F$ due to the possibility of financing a lemon.

Our equilibrium concept is that of perfect Bayesian Nash equilibrium in which outside investors rationally anticipate which projects seek outside finance. Given these rational beliefs, outside investors compete themselves down to zero profits. The following result characterizes all (pure-strategy) equilibria under constrained and unconstrained finance.

Proposition 4. *Under unconstrained finance, the market for outside finance at the refinancing stage shuts down completely. Likewise, under constrained finance, there is always an equilibrium in which the market for outside finance shuts down. Depending on τ, there may exist two additional equilibria under constrained finance: If τ is sufficiently large, there exists an equilibrium in which all three interim types have access to costly outside finance at the refinancing stage, while if τ lies in some intermediate range, there exists an equilibrium in which only interim types $\psi \in \{n, h\}$ have access to costly outside finance.*

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Proof. See Appendix. ∎

The intuition underlying Proposition 4 is straightforward. Given that any offer that outside investors make also attracts all lemons, outside investors must set D relatively high to break even. Outside finance thus involves a lemons premium, which makes it costly. Under unconstrained finance, the inside investor has sufficient funds to refinance all successful projects. There is thus no need to draw on costly outside finance. This implies that the only projects seeking outside finance are lemons, which in turn implies that the market for outside finance shuts down. Likewise, under constrained finance, there is always an equilibrium in which the market for outside finance shuts down. Irrespective of τ or other parameter values, if outside investors believe that only lemons seek outside finance, then outside finance becomes infeasible. This validates the assumption in our base model that the only source of funding at the interim stage is the inside investor.

But Proposition 4 also shows that, at least for certain parameter values, there may be additional equilibria under constrained finance in which outside finance is feasible at the refinancing stage.[27] Arguably, since outside finance commands a lemons premium, the inside investor will always find it optimal to use up her capital of I_2 to refinance *one* of the two projects (unless both are failures, of course). But if outside finance is feasible, then the other project may *also* be refinanced—depending on the project's interim type, of course—implying that inside and outside finance may coexist at the refinancing stage.

Given that there may be an equilibrium in which projects that are not refinanced by the inside investor have access to outside finance, it is important to check whether our previous results hold qualitatively if outside finance is costly but feasible. For the sake of brevity, we only consider the equilibrium in Proposition 4 in which all three interim types have access to costly outside finance. It is easy to verify that qualitatively similar results obtain regarding the other equilibrium in which only type $\psi = n$ and type $\psi = h$ projects have access to costly outside finance. The following proposition establishes the analogue of the responsiveness condition (5)

for the case in which outside finance is costly but feasible.

Proposition 5. *Consider the equilibrium in Proposition 4 in which all three interim types have access to costly outside finance at the refinancing stage. Given this equilibrium, the responsiveness of the entrepreneur's expected payoff to his ex ante type is higher under constrained finance than under unconstrained finance if and only if*

$$(r_h - \lambda_h) - (r_l - \lambda_l) < \frac{r_l - \lambda_l}{3q_g}. \tag{10}$$

Proof. See Appendix. ■

The responsiveness condition is now expressed in terms of the lemon premium $r_\psi - \lambda_\psi$, as the insiders now bargain over the cost savings from using cheaper informed capital at the refinancing stage. Most importantly, the responsiveness condition retains its basic qualitative structure from Proposition 1. This points to the crucial driver behind the responsiveness condition: There must be a benefit to being refinanced by the *inside* investor. This implies that there will be a benefit to being a high interim type, which in turn implies a benefit to exerting high effort. Whether this benefit arises because not being refinanced by the inside investor means not being refinanced at all, as in our base model, or whether it arises because not being refinanced by the inside investor means a lower surplus due to the use of costly outside finance, as above, is irrelevant for our model's central argument.

5 Conclusion

This paper shows that investors financing a portfolio of investment projects may use the depth of their financial pockets to overcome entrepreneurial agency problems. Limiting the amount of capital allows investors to credibly commit to a tournament among portfolio projects for (cheaper) informed capital at the refinancing stage. While this improves the investor's ex post bargaining position, thus reducing the entrepreneur's expected payoff, it may nevertheless also

improve the entrepreneur's incentives. This is because projects funded by investors with scarce capital must have not only a positive NPV at the refinancing stage, but one that is higher than that of competing portfolio projects. As a consequence, committing to "shallow" pockets may be optimal despite the allocational inefficiency when positive NPV projects are not refinanced.

Committing to shallow pockets (or "constrained finance") may have also benefits in dealing with adverse selection problems. If all investors have deep pockets ("unconstrained finance"), it may be impossible to separate good from bad entrepreneurs. If investors can choose between constrained and unconstrained finance, however, such separation may be possible. In the separating equilibrium in question, bad entrepreneurs are financed under unconstrained finance, and good ones are financed under constrained finance.

Our model lends itself to several testable implications. A key implication of our model is that, other things equal, projects financed under constrained finance should have lower ex ante success probabilities than comparable projects financed under unconstrained finance. The intuition, which lies at the heart of our model, is that for high success probabilities, the allocational inefficiency induced by constrained finance weighs heavily in expected terms, implying that such projects are better financed under unconstrained finance. The same intuition holds for projects with high investment costs, as such projects require a high probability of success to be financially viable. On the other hand, the main benefit of constrained finance in our model is that it may provide stronger effort incentives to entrepreneurs. Hence, another empirical implication is that constrained finance should be more likely if the efficiency loss from having low entrepreneurial effort is large.

Figure 1: Summary of Project Technology. In the figure, τ denotes the probability that the project is successful, meaning it has interim type $\varphi \in \{l, h\}$, and $1-\tau$ denotes the probability that the project fails, meaning it has interim type $\varphi = n$. Conditional on being successful, the probability that the project has interim type $\varphi = h$ ($\varphi = l$) is q_θ ($1-q_\theta$), where $\theta \in \{g, b\}$ denotes the project's ex ante type. A successful project that is refinanced (not refinanced) generates an expected payoff of $R\varphi$ (R^0), while a project that fails generates a certain zero payoff.

Figure 2: Illustration of Proposition 2. In the figure, π_U^I represents the investor's expected gross payoff under unconstrained financed as defined in (7), π_C^I represents the investor's expected gross payoff under constrained financed as defined in (8), I_1 represents the project's ex ante investment cost, and τ represents the project's probability of success. The entrepreneur's expected payoff is larger (smaller) under constrained finance if $\tau < \widehat{\tau}$ (if $\tau > \widehat{\tau}$). The vertically (horizontally) shaded area depicts all combinations of I_1 and τ for which constrained (unconstrained) finance is chosen. The non-shaded area depicts all combinations of I_1 and τ for which the project is not financially viable.

6 Appendix

Proof of Lemma 1. Claim i) is obvious. As for claim ii), denote by $y_i := (1 - s_i)R^0$ and $z_i := s_i R^0$ the investor's and E_i's continuation payoffs, respectively, if the project is not refinanced, and by $v_i := R_{\psi_i} - I_2$ and $w_i := v_i - (y_i + z_i) = r_{\psi_i}$ their combined continuation payoffs and the net surplus, respectively, from refinancing a project with interim type $\psi_i \in \{l, h\}$.

Given that the proof is standard, we shall be brief. We characterize offers by the continuation payoff X which the offer leaves to E_i. The investor always offers X^I, while E_i always offers X^E. If the investor must respond to E_i's offer, she accepts any X^E satisfying

$$v_i - X^E \geq \delta y_i + (1 - \delta)(v_i - X^I). \tag{11}$$

The right-hand side in (11) represents the investor's payoff from rejecting E_i's offer: With probability δ, the negotiations with E_i break down, and the investor receives y_i. If negotiations do not break down, the investor makes her counteroffer X^I. Similarly, if E_i must respond to the investor's offer, he accepts any X^I satisfying

$$X^I \geq \delta z_i + (1 - \delta)X^E. \tag{12}$$

As usual, offers along the equilibrium path must make the counterparty indifferent to accepting and rejecting, implying that (11)-(12) must hold with equality. Solving (11) for X^E and inserting the result in (12), we have

$$X^I = \frac{\delta z_i + (1 - \delta)\delta(v_i - y_i)}{\delta(2 - \delta)}, \tag{13}$$

which E_i accepts immediately.

By L'Hôpital's rule, E_i's equilibrium continuation payoff as $\delta \to 0$ is

$$\lim_{\delta \to 0} X^I = \frac{v_i - y_i + z_i}{2} = z_i + \frac{w_i}{2} = s_i R^0 + \frac{r_{\psi_i}}{2}, \tag{14}$$

implying that the investor's equilibrium continuation payoff as $\delta \to 0$ is

$$\lim_{\delta \to 0} v_i - X^I = v_i - z_i - \frac{w_i}{2} = y_i + \frac{w_i}{2} = (1 - s_i)R^0 + \frac{r_{\psi_i}}{2}.$$

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Note that the same equilibrium continuation payoffs would obtain if, instead of solving for X^I, we solved for X^E and took the limit as $\delta \to 0$, i.e., $\lim_{\delta \to 0} X^I = \lim_{\delta \to 0} X^E$. Consequently, instead of letting the investor make the first offer, we could have assumed that E_i makes the first offer; the equilibrium continuation payoffs are identical. ■

Proof of Lemma 2. Claim i) is obvious. As for claim ii), we use the same notation as in the proof of Lemma 1, except that we use subscripts i and j to distinguish between E_i and E_j. If $\psi_i \in \{l, h\}$, $\psi_j \in \{l, h\}$, and $\psi_i \neq \psi_j$, we specify that the investor picks the entrepreneur with the higher interim type. Without loss of generality, we assume that this is E_i. We confirm below that this strategy on the part of the investor is optimal. If $\psi_i = \psi_j$, the investor is indifferent. In this case, we specify that the investor randomly picks an entrepreneur (with equal probability), with whom she then bargains until there is either a breakdown or an agreement.[28] Again without loss of generality, we assume that this is E_i.

Analogous to the proof of Lemma 1, the investor always offers x_i^I and accepts any counteroffer x_i^E that satisfies

$$v_i - x_i^E + y_j \geq \delta(y_i + v_j - X_j^I) + (1 - \delta)(y_j + v_i - x_i^I). \tag{15}$$

In (15), X_j^I denotes the investor's offer to E_j if he is the only entrepreneur present, i.e., if the negotiations with E_i have broken down. We already know from Lemma 1 what this offer is going to be. In contrast, x_i^E and x_i^I denote E_i's and the investor's offers, respectively, if both entrepreneurs are still present. Note the difference to (11): If the investor accepts E_i's offer, she realizes, in addition to $(v_i - x_i^E)$, also her outside option payoff y_j with E_j, whose project is not refinanced. By contrast, if the investor rejects E_i's offer, the negotiations with E_i break down with probability δ, in which case she continues with E_j. Finally, if the negotiations with E_i do not break down, the investor makes her counteroffer x_i^I. As for E_i, he always offers x_i^E and accepts any counteroffer x_i^I satisfying

$$x_i^I \geq \delta z_i + (1 - \delta)x_i^E. \tag{16}$$

Analogous to the proof of Lemma 1, (15)-(16) must hold with equality. Solving (15) for x_i^E and inserting the result in (16), we obtain

$$x_i^I = \frac{\delta z_i + (1-\delta)\delta(v_i - y_i + y_j - v_j + X_j^I)}{\delta(2-\delta)},$$ (17)

which E_i accepts immediately.

Analogous to (14), we obtain $\lim_{\delta \to 0} X_j^I = z_j + w_j/2$. Using L'Hôpital's rule, we thus have that E_i's equilibrium continuation payoff as $\delta \to 0$ is

$$
\begin{aligned}
\lim_{\delta \to 0} x_i^I &= \frac{v_i - y_i - \frac{w_j}{2} + z_i}{2} = z_i + \frac{1}{2}(w_i - \frac{w_j}{2}) \\
&= s_i R^0 + \frac{1}{2}(r_{\psi_i} - \frac{r_{\psi_j}}{2}),
\end{aligned}
$$

which implies that the investor's total equilibrium continuation payoff (i.e., including her outside option payoff y_j realized with E_j) as $\delta \to 0$ is

$$
\begin{aligned}
v_i - z_i - \frac{1}{2}(w_i - \frac{w_j}{2}) + y_j &= y_i + \frac{1}{2}(w_i + \frac{w_j}{2}) + y_j \\
&= (1-s_i)R^0 + (1-s_j)R^0 + \frac{1}{2}(r_{\psi_i} + \frac{r_{\psi_j}}{2}).
\end{aligned}
$$

As in the proof of Lemma 1, we could have equally solved for x_i^E and taken the limit as $\delta \to 0$; the equilibrium continuation payoffs are identical.

It remains to show that if both entrepreneurs are still present and $\psi_i \neq \psi_j$, the investor does not find it profitable to deviate and make an offer to the entrepreneur with the lower interim type, E_j. Suppose the investor deviates and offers x_j^I to E_j while accepting any x_j^E that satisfies

$$v_j - x_j^E + y_i \geq \delta(y_j + v_i - X_i^I) + (1-\delta)(y_j + v_i - x_i^I).$$ (18)

In (18), if the investor rejects E_j's offer and the negotiations with E_j break down, the investor must necessarily switch back to E_i. However, the investor also switches back to E_i if the negotiations with E_j did not break down.[29] As for E_j, he offers x_j^E and accepts any x_j^I satisfying

$$x_j^I \geq \delta z_j + (1-\delta)x_j^E.$$ (19)

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As previously, (18)-(19) must hold with equality. Solving (18) for x_j^E and inserting the result in (19) yields

$$x_j^I = \delta z_j + (1 - \delta)(v_j + y_i - y_j - v_i + \delta X_i^I + (1 - \delta)x_i^I). \tag{20}$$

To confirm that the investor does not find it profitable to deviate, we must show that

$$v_i - x_i^I + y_j \geq v_j - x_j^I + y_i. \tag{21}$$

Inserting x_j^I from (20) into (21) and rearranging, (21) becomes

$$\delta z_j - \delta(v_j + y_i - y_j - v_i) + (1 - \delta)\delta X_i^I \geq x_i^I \delta(2 - \delta). \tag{22}$$

Next, inserting (17) into (22), dividing through by δ, and rearranging, (22) becomes

$$(1 - \delta)(X_i^I - X_j^I) \geq (z_i - z_j) - \delta(v_i - y_i + y_j - v_j). \tag{23}$$

Note that from (13) we have that

$$X_i^I = \frac{\delta z_i + (1 - \delta)\delta(v_i - y_i)}{\delta(2 - \delta)}$$

and

$$X_j^I = \frac{\delta z_j + (1 - \delta)\delta(v_j - y_j)}{\delta(2 - \delta)}.$$

Finally, inserting X_i^I and X_j^I into (23), multiplying through by $\delta(2 - \delta)$, and rearranging, (23) becomes

$$\delta[(v_i - y_i - z_i) - (v_j - y_j - z_j)] = \delta(w_i - w_j) = \delta(r_{\psi_i} - r_{\psi_j}) \geq 0,$$

which holds by assumption. ∎

Proof of Proposition 1. It remains to derive (1) and (3). Consider first the derivation of (1). Under unconstrained finance, the probabilities of having interim type $\psi = n$, $\psi = l$, and $\psi = h$ are $1 - \tau$, $\tau(1 - q_{\theta_i})$, and τq_{θ_i}, respectively. Multiplying these probabilities with the respective continuation payoffs from Lemma 1 and rearranging yields (1).

Consider next the derivation of (3). Given that the investor picks the entrepreneur with the higher interim type first, and if she indifferent, she picks each of the two entrepreneurs with equal probability (see Proof of Lemma 2), Lemma 2 implies the following expected continuation payoffs at $t = 1$ for E_k, an arbitrary entrepreneur: zero if $\psi_k = n$, $s_k R^0$ if $\psi_k = l$ and $\psi_{j \neq k} = h$, $s_k R^0 + \frac{1}{2}(r_h - \frac{1}{2}r_l)$ if $\psi_k = h$ and $\psi_{j \neq k} = l$, $s_k R^0 + \frac{1}{2}r_{\psi_k}$ if $\psi_k \in \{l, h\}$ and $\psi_{j \neq k} = n$, and $s_k R^0 + \frac{1}{8}r_\psi$ if $\psi_k = \psi_{j \neq k} = \psi \in \{l, h\}$. Multiplying these expected continuation payoffs with the respective joint probabilities for interim types (ψ_i, ψ_j) and rearranging yields (3). The respective joint probabilities are $\tau^2 q_{\theta_i} q_{\theta_j}$ for (h, h), $\tau^2 (1 - q_{\theta_i})(1 - q_{\theta_j})$ for (l, l), $(1 - \tau)^2$ for (n, n), $\tau(1 - q_{\theta_i})(1 - \tau)$ for (l, n), $\tau(1 - q_{\theta_j})(1 - \tau)$ for (n, l), $\tau q_{\theta_i}(1 - \tau)$ for (h, n), $\tau q_{\theta_j}(1 - \tau)$ for (n, h), $\tau^2 q_{\theta_i}(1 - q_{\theta_j})$ for (h, l), and $\tau^2 q_{\theta_j}(1 - q_{\theta_i})$ for (l, h). ∎

Proof of Proposition 2. Analogous to the derivation of (1) and (3) in the proof of Proposition 1, we can derive the investor's expected payoff at $t = 0$. Under unconstrained finance, the investor's expected payoff at $t = 0$ is

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} (r_h - r_l) \right] \right\} - I_1, \tag{24}$$

and under constrained finance, it is

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} (r_h - r_l) \right] \right\} - \frac{\tau^2}{8} \left\{ \left(1 + 3q_{\theta_j} - 3q_{\theta_i} \right) r_l + q_{\theta_i} q_{\theta_j} (r_h - r_l) \right\} - I_1. \tag{25}$$

If (5) and (6) hold, we have $\theta_i = b$ in the case of unconstrained finance and $\theta_i = \theta_j = g$ in the case of constrained finance. Accordingly, (24) and (25) become

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_b (r_h - r_l) \right] \right\} - I_1 \tag{26}$$

and

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_g (r_h - r_l) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 (r_h - r_l) \right\} - I_1, \tag{27}$$

respectively. Setting $s_i = 0$ in (26) and (27), respectively, we obtain $\pi_U^I - I_1$ and $\pi_C^I - I_1$ as defined in (7) and (8) in the main text.

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We next derive the entrepreneur's expected payoff at $t = 0$ if the project is financially viable and investors compete themselves down to zero profits. Setting (26) and (27) equal to zero, solving for s_i, and inserting the result in (1) (with $\theta_i = b$) and (3) (with $\theta_i = \theta_j = g$), respectively, we have that E_i's equilibrium expected payoff under unconstrained finance is

$$\pi_U^E - I_1 := \tau \left\{ R^0 + r_l + q_b \left(r_h - r_l \right) + B \right\} - I_1, \tag{28}$$

and his equilibrium expected payoff under constrained finance is

$$\pi_C^E - I_1 := \tau \left\{ R^0 + r_l + q_g \left(r_h - r_l \right) \right\} - \frac{\tau^2}{2} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} - I_1. \tag{29}$$

We finally establish the functional properties of π_U^I, π_C^I, π_U^E, and π_C^E. Once these properties have been established, the rest of the proof is trivial. By inspection, π_U^I and π_U^E are both linear and strictly increasing in τ. Moreover, both are zero at $\tau = 0$, and π_U^E lies strictly above π_U^I for all $\tau > 0$.[30] Likewise, it is easily shown that π_C^I and π_C^E are both strictly concave, increasing in τ, and zero at $\tau = 0$. Note that

$$\lim_{\tau \to 0} \frac{d\pi_C^E}{d\tau} - \lim_{\tau \to 0} \frac{d\pi_U^E}{d\tau} = \left(q_g - q_b \right) \left(r_h - r_l \right) - B > 0,$$

where the inequality follows from our assumption that $\theta = g$ is socially optimal. Hence, π_C^E lies strictly above π_U^E for small τ, implying that it crosses π_U^E exactly once from the left. In Figure 2, this intersection point is denoted by $\widehat{\tau}$. Straightforward calculations show that

$$\widehat{\tau} := 2 \frac{\left(q_g - q_b \right) \left(r_h - r_l \right) - B}{r_l + q_g^2 \left(r_h - r_l \right)} < 1,$$

where the inequality follows from $2(q_g - q_b) \left(r_h - r_l \right) < r_l + q_g^2 \left(r_h - r_l \right)$.[31] Likewise, note that

$$\lim_{\tau \to 0} \frac{d\pi_C^I}{d\tau} - \lim_{\tau \to 0} \frac{d\pi_U^I}{d\tau} = \frac{1}{2}(q_g - q_b) \left(r_h - r_l \right) > 0,$$

which establishes that π_C^I lies strictly above π_U^I for small τ, implying that it crosses π_U^I exactly once from the left as depicted in Figure 2. Denote the intersection of π_C^I and π_U^I by $\widetilde{\tau}$. Straightforward calculations show that

$$\widetilde{\tau} := 4 \frac{\left(r_h - r_l \right) \left(q_g - q_b \right)}{r_l + q_g^2 \left(r_h - r_l \right)} > \widehat{\tau}.$$

Associated with $\widetilde{\tau}$ is a critical value of I_1, which is equal to the value of π_U^I (or, equivalently, the value of π_C^I) at $\tau = \widetilde{\tau}$. Denote this critical value by $\widetilde{I}_1$. From (7), we have that

$$\widetilde{I}_1 := \widetilde{\tau} \left\{ R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\}. \tag{30}$$

Case i) of Proposition 2 then holds for $I_1 > \widetilde{I}_1$ while case ii) holds for $I_1 \leq \widetilde{I}_1$. ∎

Proof of Proposition 3. Denote by s_C and s_U the equilibrium sharing rules offered by constrained and unconstrained investors, respectively. A separating equilibrium in which type $\theta = g$ entrepreneurs prefer constrained finance and type $\theta = b$ entrepreneurs prefer unconstrained finance exists if i) s_C and s_U are incentive compatible, ii) the investors' and entrepreneurs' participation constraints hold, and iii) there exists no other offer that can break the proposed separating equilibrium. We now address each of these three conditions in turn.

Consider incentive compatibility first. In the proposed equilibrium, unconstrained investors attract only type $\theta = b$ entrepreneurs and make zero profits. Setting (24) with $\theta_i = b$ and $s_i = s_U$ equal to zero and solving for s_U, we obtain

$$s_U = 1 + \frac{1}{2} \frac{r_l + q_b \left(r_h - r_l \right)}{R^0} - \frac{I_1}{\tau R^0}. \tag{31}$$

Consider next s_C. Incentive compatibility for type $\theta = b$ entrepreneurs requires that constrained investors offer s_C such that type $\theta = b$ entrepreneurs weakly prefer unconstrained finance. Consequently, s_C must satisfy[32]

$$\tau \left\{ s_U R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\}$$
$$\geq \tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l \left(3 - q_b + q_g \right) + 3 q_b q_g \left(r_h - r_l \right) \right\},$$

which becomes

$$s_C \leq s_U + \frac{\tau}{8} \frac{r_l \left(3 - q_b + q_g \right) + 3 q_b q_g \left(r_h - r_l \right)}{R^0}, \tag{32}$$

where s_U is defined in (31).

Incentive compatibility for type $\theta = g$ entrepreneurs requires that they weakly prefer constrained finance:[33]

$$\tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{3\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} \geq \tau \left\{ s_U R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\},$$

which becomes

$$s_C \geq s_U + \frac{3\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R^0}. \tag{33}$$

By inspection, (32) and (33) can be jointly satisfied if and only if $\frac{r_l}{3q_g} > r_h - r_l$, i.e., if and only if the responsiveness condition (5) holds.

Consider next the participation constraints. The entrepreneurs' expected payoff is always non-negative, while s_U was constructed such that unconstrained investors break even. From (25) with $\theta_i = \theta_j = g$ and $s_i = s_C$, we have that the expected payoff of constrained investors is non-negative if

$$\tau \left\{ \left(1 - s_C \right) R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} - I_1 \geq 0,$$

which becomes

$$s_C \leq s_U + \frac{1}{2} \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R_0}. \tag{34}$$

This condition is compatible with (33) if

$$s_U + \frac{1}{2} \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R_0} \geq s_U + \frac{3\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R^0},$$

which becomes

$$\tau \leq \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{r_l + q_g^2 \left(r_h - r_l \right)}.$$

Finally, existence of the proposed separating equilibrium requires that there exists no other—in this case: pooling—offer that can break the separating equilibrium and allows investors to break even. Analogous to (31), the zero-profit pooling offer is given by

$$s_P = 1 + \frac{1}{2} \frac{r_l + \left(\alpha q_g + \left(1 - \alpha \right) q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{I_1}{\tau R^0}.$$

For type $\theta = g$ entrepreneurs to prefer s_C to s_P, it must hold that

$$\tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{3\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} \geq \tau \left\{ s_P R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\},$$

which becomes

$$s_C \geq s_P + \frac{3\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R^0}. \tag{35}$$

Condition (35) is compatible with (32) if

$$s_U + \frac{\tau}{8} \frac{r_l \left(3 - q_b + q_g \right) + 3 q_g q_b \left(r_h - r_l \right)}{R^0} \geq s_P + \frac{3\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R^0},$$

which becomes

$$\alpha \leq \frac{\tau}{8} \frac{r_l - 3 q_g \left(r_h - r_l \right)}{r_h - r_l}.$$

Likewise, (35) is compatible with the zero-profit constraint (34) if

$$s_U + \frac{1}{2} \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R_0} \geq s_P + \frac{3\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R^0},$$

which becomes

$$\alpha \leq \frac{1}{2} \left[1 - \tau \frac{r_l + q_g^2 \left(r_h - r_l \right)}{\left(q_g - q_b \right) \left(r_h - r_l \right)} \right].$$

Finally, if the above conditions hold, any candidate pooling equilibrium can be broken by the separating offers s_U and s_C, which establishes uniqueness. ∎

Proof of Proposition 4. The argument for why under unconstrained finance the market for outside finance shuts down at $t = 1$ has been given in the main text. Consider next constrained finance. By (9), if interim type $\psi = l$ weakly prefers to seek outside finance, then interim type $\psi = h$ strictly prefers to seek outside finance. This immediately implies that we have three equilibrium candidates under constrained finance: (i) no project has access to outside finance, (ii) all three interim types have access to outside finance, and (iii) only interim types $\psi = n$ and $\psi = h$ have access to outside finance. Importantly, there cannot exist an equilibrium in which only interim types $\psi = n$ and $\psi = l$ have access to outside finance at $t = 1$, and there

obviously cannot exist an equilibrium in which only successful projects have access to outside finance: Any offer that attracts successful projects also attracts all lemons. We now consider all three candidate equilibria in turn.

Equilibrium in which no project has access to outside finance at the refinancing stage

This is trivially always an equilibrium. If outside investors believe that only lemons seek outside finance, the market for outside finance shuts down completely.

Equilibrium in which all three interim types have access to outside finance

We first characterize outside investors' rational beliefs, which we denote by $\pi(\psi)$. In the proposed equilibrium, there is exactly one project seeking outside finance in every state of nature.[34] With probability $\tau^2 q_{\theta_i} q_{\theta_j}$, both projects have interim type $\psi = h$. Hence, the conditional probability that the project seeking outside finance has interim type $\psi = h$ is $\pi(h) = \tau^2 q_{\theta_i} q_{\theta_j}$. Likewise, with probability $1 - \tau^2$, at least one project has interim type $\psi = n$. The conditional probability that the project seeking outside finance has interim type $\psi = n$ is thus $\pi(n) = 1 - \tau^2$. Finally, with probability $\tau^2(1 - q_{\theta_i} q_{\theta_j})$, at least one project has interim type $\psi = l$ and no project has interim type $\psi = n$. The conditional probability that the project seeking outside finance has interim type $\psi = l$ is thus $\pi(l) = \tau^2(1 - q_{\theta_i} q_{\theta_j})$.[35]

Given these beliefs, the zero-profit repayment D required by outside investors is

$$D = \frac{I_2}{\tau^2 q_{\theta_i} q_{\theta_j} p_h + \tau^2(1 - q_{\theta_i} q_{\theta_j}) p_l + (1 - \tau^2) p_n}. \tag{36}$$

The proposed equilibrium exists if i) interim types $\psi = l$ and $\psi = h$ weakly prefer outside finance, and ii) there exists a repayment $D \leq R$ satisfying (36). By our previous arguments, if interim type $\psi = l$ weakly prefers outside finance, then interim type $\psi = h$ strictly prefers outside finance. Hence, the proposed equilibrium exists if and only if (36) and

$$p_l(R - D) \geq R^0 \tag{37}$$

hold. Note that (37) implies that $D < R$. Inserting (36) into (37) and rearranging, we obtain

the requirement that

$$\tau^2 \geq \left(\frac{p_l I_2}{p_l R - R^0} - p_n \right) \left(\frac{1}{q_{\theta_i} q_{\theta_j} (p_h - p_l) + p_l - p_n} \right), \tag{38}$$

which implies that for an equilibrium to exist in which all three interim types have access to costly outside finance, τ must be sufficiently large.[36]

Equilibrium in which only interim types $\psi = n$ and $\psi = h$ have access to outside finance

In this equilibrium, there is exactly one project seeking outside finance if either both projects have interim type $\psi = h$, or if at least one project has interim type $\psi = n$. Hence, the conditional probability that the project seeking outside finance has interim type $\psi = l$ is $\pi(l) = 0$, the conditional probability that it has interim type $\psi = h$ is $\pi(h) = \tau^2 q_{\theta_i} q_{\theta_j} / [(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}]$, and the conditional probability that it has interim type $\psi = n$ is $\pi(n) = (1 - \tau^2) / [(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}]$.

Given these beliefs, the zero-profit repayment D required by outside investors is

$$D = \frac{I_2}{\xi(\tau)}, \tag{39}$$

where

$$\xi(\tau) := \frac{\tau^2 q_{\theta_i} q_{\theta_j} p_h + (1 - \tau^2) p_n}{\tau^2 q_{\theta_i} q_{\theta_j} + 1 - \tau^2}$$

is strictly increasing in τ with $\lim_{\tau \to 0} \xi(\tau) = p_n$ and $\lim_{\tau \to 1} \xi(\tau) = p_h$.

The proposed equilibrium exists if i) interim type $\psi = h$ weakly prefers outside finance, ii) interim type $\psi = l$ prefers no refinancing to outside finance, and iii) there exists a repayment $D \leq R$ satisfying (39). Hence, the proposed equilibrium exists if and only if (39) and

$$p_h (R - D) \geq R^0 > p_l (R - D) \tag{40}$$

hold. Note that the first inequality implies that $D < R$. Inserting (39) with equality into (40), we obtain

$$p_h \left(R - \frac{I_2}{\xi(\tau)} \right) \geq R^0 > p_l \left(R - \frac{I_2}{\xi(\tau)} \right).$$

Because $r_\psi := p_\psi R - R^0 - I_2 > 0$ for $\psi \in \{l, h\}$, the second inequality is violated if $\xi(\tau) \geq p_l$. Given that $\xi(\tau)$ is increasing in τ, this implies that τ must not be too large. On the other hand,

given that $\lim_{\tau \to 0} \xi(\tau) = p_n$ and our assumption that p_n is small, the first inequality is violated if τ is sufficiently small. ∎

Proof of Proposition 5. As in Section 2.2, we first derive the entrepreneurs' continuation payoffs at $t = 1$ under constrained finance. The basic structure of the bargaining game is the same as in Section 2.2, so we confine ourselves to reporting the equilibrium continuation payoffs as $\delta \to 0$. The main difference to our base model concerns the insiders' total payoff if a project is not refinanced by the inside investor. In our base model, this payoff was zero for projects with interim type $\psi = n$ and R^0 for projects with interim type $\psi \in \{l, h\}$. Now, given that projects have access to costly outside finance, the insiders' total payoff if the project is not refinanced by the inside investor is $\lambda_\psi := p_\psi(R - D)$ for all three interim types, where, unlike our base model, it now holds that $p_n > 0$. The only exception is when both projects have interim type $\psi = n$: As only one project can be presented to outside investors, the insiders' total payoff in this case is λ_n from the project presented to outside investors and zero from the other project.

Consider first the case in which $\psi_i = \psi_j = n$. If E_j is the last entrepreneur to be bargained with, E_j and the investor each realize $\frac{1}{2}\lambda_n$. Consider next the negotiations with E_i, who the investor picks first. As the investor can credibly threaten to present E_j's project to the outside investors instead, equilibrium continuation payoffs are, analogous to Lemma 2, $\frac{1}{2}(\lambda_n - \frac{1}{2}\lambda_n) = \frac{1}{4}\lambda_n$ for E_i and zero for E_j.

Consider next the case in which $\psi_i \in \{l, h\}$ and $\psi_j = n$. By optimality (see proof of Lemma 2), the investor bargains first with E_i. Moreover, if the negotiations with E_i break down, it is optimal to present E_i's project to the outside investors, not E_j's.[37] Hence, the investor and E_i bargain over the cost savings from using inside funds, $r_{\psi_i} - \lambda_{\psi_i}$, implying that E_i's equilibrium continuation payoff is the sum of $s_i R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $\frac{1}{2}(r_{\psi i} - \lambda_{\psi_i})$, which equals $s_i R^0 + \frac{1}{2}r_{\psi_i}$. Naturally, E_j's equilibrium continuation payoff is then $\frac{1}{2}\lambda_{\psi_j}$.

Consider finally the case in which $\psi_i \in \{l, h\}$ and $\psi_j \in \{l, h\}$. Suppose E_j is the last entrepreneur to be bargained with. The payoffs now depend on whether the investor has already

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used up her funds for E_i. If the investor's funds have already been used up, E_j realizes $s_j R^0 + \frac{1}{2}\lambda_{\psi_j}$, and the investor realizes $(1-s_j) R^0 + \frac{1}{2}\lambda_{\psi_j}$ from bargaining with E_j. If the investor's funds are still available, E_j and the investor bargain over the cost savings from using inside funds, $r_{\psi_j} - \lambda_{\psi_j}$. Consequently, E_j realizes the sum of $s_j R^0 + \frac{1}{2}\lambda_{\psi_j}$ and $\frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$, which equals $s_j R^0 + \frac{1}{2}r_{\psi_j}$, and the investor realizes the sum of $(1-s_j) R^0 + \frac{1}{2}\lambda_{\psi_j}$ and $\frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$ from bargaining with E_j, which equals $(1-s_j) R^0 + \frac{1}{2}r_{\psi_j}$. Consider next the negotiations between the investor and her first pick, E_i. If the negotiations break down, E_i realizes $s_i R^0 + \frac{1}{2}\lambda_{\psi_i}$, and the investor realizes the sum of $(1-s_i) R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $(1-s_j) R^0 + \frac{1}{2}r_{\psi_j}$. On the other side, the surplus over which E_i and the investor bargain is $r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$. Hence, E_i's equilibrium continuation payoff is the sum of $s_i R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $\frac{1}{2}[r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$, which equals $s_i R^0 + \frac{1}{2}[r_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$. Naturally, E_j's equilibrium continuation payoff is then $s_j R^0 + \frac{1}{2}r_{\psi_j}$.

Consider next the issue who is picked to be bargained with first. If $\psi_i \neq \psi_j$, we know from the proof of Lemma 2 that the investor picks the entrepreneur with the higher interim type first. In contrast, if $\psi_i = \psi_j$, the investor picks both entrepreneurs with equal probability. We thus have the following *expected* continuation payoffs for E_k, an arbitrary entrepreneur: $\frac{1}{8}\lambda_n$ if $\psi_k = \psi_{j \neq k} = n$, $s_k R^0 + \frac{1}{2}r_{\psi_k}$ if $\psi_k \in \{l, h\}$ and $\psi_{j \neq k} = n$, $\frac{1}{2}\lambda_{\psi_n}$ if $\psi_k = n$ and $\psi_{j \neq k} \in \{l, h\}$, $s_k R^0 + \frac{1}{8}(r_\psi + 3\lambda_\psi)$ if $\psi_k = \psi_{j \neq k} = \psi \in \{l, h\}$, $s_k R^0 + \frac{1}{2}[r_h - \frac{1}{2}(r_l - \lambda_l)]$ if $\psi_k = h$ and $\psi_{j \neq k} = l$, and $s_k R^0 + \frac{1}{2}\lambda_l$ if $\psi_k = l$ and $\psi_{j \neq k} = h$.

Given these expected continuation payoffs, we can, analogous to (3), compute E_i's expected payoff at $t = 0$. We obtain[38]

$$
\tau^2 q_{\theta_i} q_{\theta_j} \left\{ s_i R^0 + \frac{1}{8}\left[r_h + 3\lambda_h\right] \right\} + \tau^2 q_{\theta_i} \left(1 - q_{\theta_j}\right) \left\{ s_i R^0 + \frac{1}{2}\left[r_h - \frac{1}{2}\left[r_l - \lambda_l\right]\right] \right\}
$$
$$
+ \tau^2 \left(1 - q_{\theta_i}\right) q_{\theta_j} \left\{ s_i R^0 + \frac{1}{2}\lambda_l \right\} + \tau^2 \left(1 - q_{\theta_i}\right)\left(1 - q_{\theta_j}\right) \left\{ s_i R^0 + \frac{1}{8}\left[r_l + 3\lambda_l\right] \right\}
$$
$$
+ \tau \left(1 - \tau\right) \left\{ s_i R^0 + \frac{1}{2}\left[r_l + q_{\theta_i}\left(r_h - r_l\right)\right] \right\} + \left(1 + 2\tau - 3\tau^2\right) \frac{1}{8}\lambda_n,
$$

which simplifies to

$$\tau \left\{ s_i R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} \left(r_h - r_l \right) \right] \right\} + \left(1 + 2\tau - 3\tau^2 \right) \frac{1}{8} \lambda_n \tag{41}$$

$$-\frac{\tau^2}{8} \left\{ \left(r_l - \lambda_l \right) \left(3 - q_{\theta_i} + q_{\theta_j} \right) + 3 q_{\theta_i} q_{\theta_j} \left[r_h - \lambda_h - \left(r_l - \lambda_l \right) \right] \right\}.$$

Having derived E_i's expected payoff at $t = 0$, we next compute the responsiveness under constrained finance when all three interim types have access to costly outside finance. Analogous to (4), we obtain the responsiveness from (41) by setting $\theta_j = g$ and subtracting E_i's expected payoff for $\theta_i = b$ from that for $\theta_i = g$. We have

$$\frac{1}{2} \tau \left(q_g - q_b \right) \left[\left(r_h - r_l \right) - \frac{\tau}{4} \left[3 q_g [r_h - \lambda_h - \left(r_l - \lambda_l \right)] - \left(r_l - \lambda_l \right) \right] \right]. \tag{42}$$

Comparing (2) with (42), we obtain the responsiveness condition (10). ∎

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Notes

[1]As Silver (1985) writes, "the need for greater amounts of venture capital, frequently not cited in the business plan, occurs sooner than expected. Because the Murphy's law affliction attacks most venture capital portfolios, there arises a serious need for portfolio management."

[2]Refinancing by uninformed outside investors is at best more costly, and at worst unavailable: "If the original partnership is unwilling to arrange for additional financing, it is unlikely that any other partnership will choose to do so; the reluctance of the original partnership is a strong signal that the company is a poor investment" (Fenn, Liang, and Prowse (1995)). Consistent with this notion, Bruno and Tyebjee (1983) find that being denied follow-up financing by a previous-round venture capitalist reduces a portfolio company's chances of obtaining financing from outside investors by 74 percent. See Section 4.2 for a formal analysis.

[3]Sahlman (1990) reports the results of one survey of venture capital investments showing that 34.5 percent of invested capital resulted in a loss, and another 30 percent resulted in returns in the low- to middle-single digits. Less than 7 percent of invested capital resulted in payoffs of more than ten times the original amount invested.

[4]Bartlett (1995) and Brooks (1999) provide discussions of venture partnership agreements.

[5]This practice may seem peculiar at first glance, but the motive stems from limited partners' concerns that "by adding less experienced general partners, venture capitalists may reduce the burden on themselves" (Gompers and Lerner (1996)). Besides, it is not easy to find skilled venture capitalists that can be added to an existing fund: "[T]he skills needed for successful venture capital investing are difficult and time-consuming to acquire. During periods when the ... demand for venture capital has shifted, adjustments in the number of venture capitalists ... take place very slowly" (Gompers (1995)).

[6]A distinct though somewhat related point is made by Gertner, Scharfstein, and Stein (1994), who argue that assets from defaulting projects can be redeployed more efficiently in an internal capital market.

[7]For related arguments, see Rotemberg and Saloner (1994), Gautier and Heider (2005), and Inderst and Laux (2005). In contrast, in Stein's (2002) model, managerial incentives to produce information may be either weaker or stronger in a hierarchy.

[8]In winner-picking models à la Stein (1997), the amount of resources that can be allocated across projects in an internal capital market is the same as under stand-alone finance. However, headquarters has the authority to redistribute assets from "losers" to "winners," while stand-alone financiers lack this authority. Hence, headquarters has advantages but no disadvantages. In contrast, in our model, constrained and unconstrained investors have the same authority to reallocate resources, but constrained investors have fewer resources available. Hence, in allocating resources, constrained investors have disadvantages but no advantages.

[9]While it is natural to think of I_2 as financial capital, it may alternatively represent human capital on the part of the investor, who must expend time and resources to coach the project.

[10]That R^0 does not depend on the project's interim type simplifies the analysis, but is not crucial.

[11]By managing more than two projects—the optimal span of the investor's portfolio in our model—the investor would spread herself too thin in the projects' critical start-up phase.

[12]Suppose there is a potentially large pool of such fly-by-night operators—ex ante indistinguishable from genuine entrepreneurs—who have projects generating a certain zero payoff. Knowing that they will receive a guaranteed payment, all of those operators would apply for financing, in which case the investor's expected profit would quickly become negative. In contrast, under a sharing rule, the fly-by-night operators have nothing to gain from applying. Indeed, if

there is an epsilon cost, they will strictly prefer not to apply.

[13]Leaving the decision rights with regard to the refinancing decision with the investor is optimal given our fly-by-night operator assumption. If the entrepreneur had decision rights, a fraudulent entrepreneur could extract a bribe at $t = 1$ by forcing the investor to invest I_2 at the refinancing stage, which is a negative NPV undertaking given that projects by fly-by-night operators generate a certain zero payoff. The two sides will thus strike a deal whereby the operator cedes his decision rights to the investor in return for a bribe. Anticipating this bribe, all operators would apply for financing.

[14]Gompers (1995) writes: "Tangible assets may be easy to monitor without formal evaluation. A venture capitalist can tell if a machine is still bolted to the floor. ... Conversations with practitioners, however, indicate that they normally make continuation decisions when a new financing round occurs. Venture capitalists evaluate a firm based on performance progress, not whether a machine is still bolted down."

[15]Modeling bargaining frictions by a risk of breakdown is standard. In contrast to the case in which bargaining frictions take the form of delay, the risk of breakdown ensures that the two parties' outside options are always relevant. That bilateral bargaining with a risk of break-down, but not bargaining with delay, can support the axiomatic Nash bargaining solution with threatpoints, is shown in Binmore, Rubinstein, and Wolinsky (1986).

[16]As is standard in the literature, we consider the limit as bargaining frictions go to zero, i.e., $\delta \to 0$. In the limit, it is irrelevant who makes the first offer. See the proof of Lemma 1 for details.

[17]Besides, the notion that the investor can extract the entire surplus at $t = 1$ does not square with our assumption that the entrepreneur is essential to continue the project.

[18]This is provided both entrepreneurs are still present, i.e., there is no breakdown.

[19]If entrepreneurs are indifferent between $\theta = b$ and $\theta = g$, we assume without loss of generality that they choose $\theta = b$. Note that if the responsiveness condition (5) holds, there exists always a nonempty set of B values that satisfy (6).

[20]To be precise, Proposition 2 does not require that (5) and (6) hold for all $\tau > 0$. The two conditions only need to hold for sufficiently large success probabilities for which constrained finance is viable.

[21]It is easy to construct a numerical example. If $q_b = 1/4$, $q_g = 1/2$, $r_l = 7$, $r_h = 11$, $R^0 = 8$, and $B = 1/2$, then (5) and (6) hold for all $\tau > 0$. Given the expressions for the investor's and entrepreneur's expected payoffs derived in the Appendix, it can be easily verified that $\widehat{\tau} = 1/8$, while π_U^I and π_C^I intersect at $\tau = 1/2$, implying that case i) of Proposition 2 holds if $I_1 \geq 6$, and case ii) holds if $I_1 < 6$. For example, when $I_1 = 1$, the project is not viable if $\tau < 0.0805$, constrained finance is chosen if $0.0805 \leq \tau \leq 1/8$, and unconstrained finance is chosen if $\tau > 1/8$.

[22]The derivation of $\widehat{\tau}$ and the entrepreneur's payoffs under constrained and unconstrained finance are found in the proof of Proposition 2, which also shows that $\hat{\tau}$ lies to the left of the intersection of π_U^I and π_C^I as depicted in Figure 2.

[23]Moreover, a decrease in q_b makes it more likely that case ii) in Proposition 2 applies, for two reasons: The set of admissible B values satisfying (6) becomes larger, and the fact that π_U^I shifts to the right implies that the critical investment cost above which case i) applies is shifted upwards.

[24]Straightforward calculations show that the likelihood that a project is rejected at the refinancing stage under constrained finance is $1 - \tau + \frac{1}{2}\tau^2$.

[25]Precisely, it must hold that $p_n < I_2/R$.

[26]In a two-payoff model, with one payoff being $R > 0$ and the other payoff being zero, any feasible financial contract must necessarily involve a positive repayment if the payoff is R.

[27]The conditions for an equilibrium in which all three interim types have access to costly outside finance, and the one in which only interim types $\psi \in \{n, h\}$ have access to costly outside finance, are not mutually exclusive. It is easy to find values of τ for which both equilibria exist (in addition to the equilibrium in which the market for outside finance shuts down, which always exists).

[28]One can show that in the limit as $\delta \to 0$, the same outcome would obtain if the investor randomizes in every round rather than staying with her first pick. The analysis involves somewhat longer equations, though.

[29]To prove that the investor's strategy is optimal, it suffices to consider one-stage deviations. See Fudenberg and Tirole (1992), Theorem 4.2.

[30]Strictly speaking, (28) and (29) are only meaningful for values of τ for which the project is viable, i.e., values for which (26) and (27) are non-negative. This rules out $\tau = 0$. However, given that all functions in question are strictly increasing and either linear or strictly concave, considering the functions' behavior at $\tau = 0$ tells us their behavior relative to each other for larger, admissible values of τ.

[31]Dividing through by $(r_h - r_l)$ and rearranging, we obtain $2(q_g - q_b) - q_g^2 < \frac{r_l}{r_h - r_l}$, which holds by (5).

[32]The left-hand side corresponds to (1) with $\theta_i = b$ and $s_i = s_U$, and the right-hand side corresponds to (3) with $\theta_i = b$, $\theta_j = g$, and $s_i = s_C$.

[33]The left-hand side corresponds to (3) with $\theta_i = \theta_j = g$ and $s_i = s_C$, and the right-hand side corresponds to (1) with $\theta_i = g$ and $s_i = s_U$.

[34]If both projects have interim type $\psi = n$, it is optimal for the insiders to present only one project to outside investors as the latter would otherwise rationally conclude that both projects are unsuccessful.

^{35}If one project has interim type $\psi = h$ and the other has interim type $\psi = l$, it is optimal for the insiders to finance the former internally and to present the latter to outside investors.

^{36}Recall that p_n is assumed to be small. If, e.g., p_n is close to p_l, (38) holds trivially for all $\tau \geq 0$.

^{37}We assume that if the negotiations with E_i over using inside funds break down, the investor and E_i can still negotiate over the surplus realized from using costly outside funds. An alternative assumption would be that the breakdown is "complete" in the sense that *any* negotiations with E_i are impossible. While the precise definition of a breakdown of negotiations affects the form of the responsiveness condition derived below, our qualitative results do not hinge on it.

^{38}Recall the joint probabilities for interim types (ψ_i, ψ_j) stated in the proof of Lemma 2.
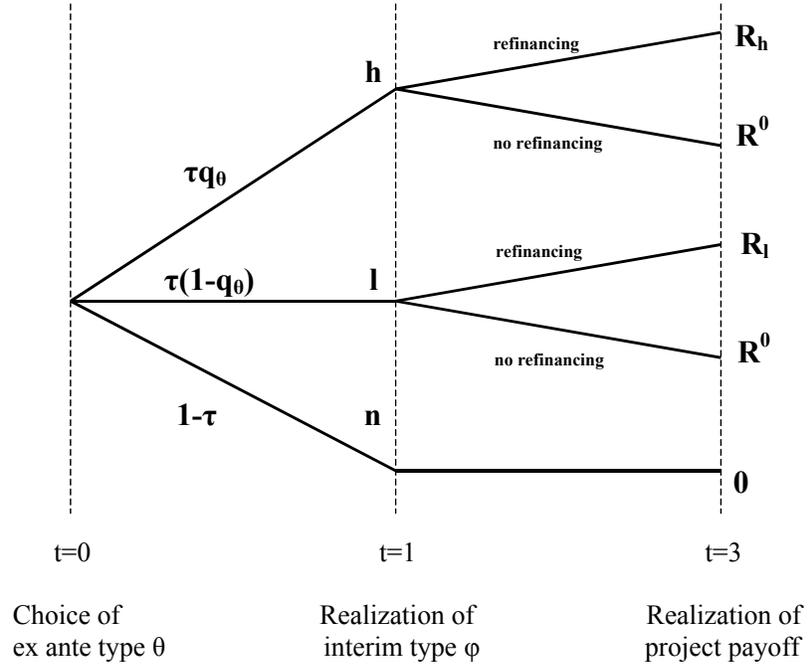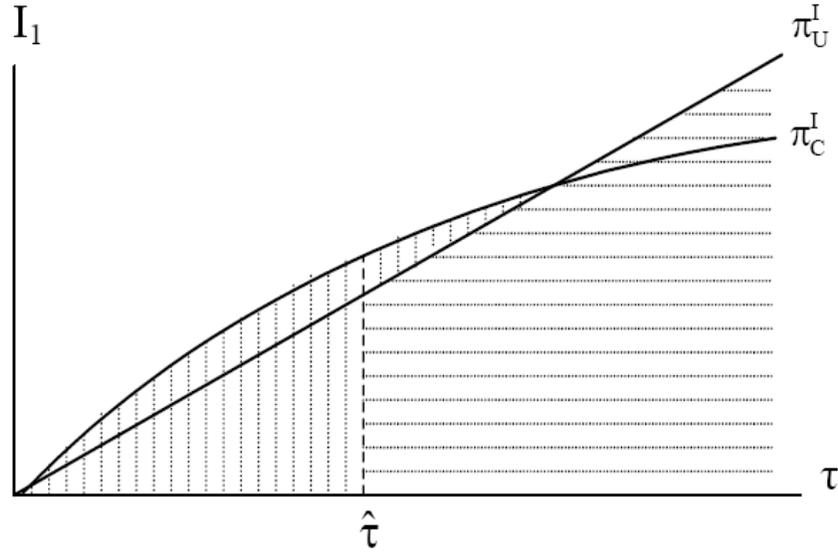
Figure 1: Summary of Project Technology.

Figure 2: Illustration of Proposition 2.